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OPTIMAL INVESTMENT PLANNING MODEL
FOR POWER SYSTEMS

by

Shishir K. Mukherjee

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ABSTRACT (within 250 words)

Availability of adequate and cheap electricty is one of the necessary requirements for industrial and agricultural development and it is also a basic requirement of urban life. Due to very rapid growth of demand for electricty and widespread shortages experienced in India, investment planning in the power sector is of prime importance. As power sector is also highly capital intensive, there is a definite need for determining least-cost investment plans and the use of operation research techniques for choosing the best expansion path from a large number of alternative system expansion plans for power generation.

The present paper describes an optimal investment planning model for determining the least-cost expansion plan for generating capacity of a power system under growing demand over a planning horizon of 15 to 25 years. Given projections of future demand for power, the expected shape of the annual load duration curve and relevant capital and operating cost information, a solution of the mixed integer linear programming model would provide the type, capacity and time schedule of commissioning the additional generating plants needed for meeting the given demand with an accepted standard of service in terms of reserve requirements or load-loss probability.

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Shishir K. Mukherjee
Signature of the Author

OPTIMAL INVESTMENT PLANNING MODEL FOR
POWER SYSTEMS

Shishir K. Mukherjee

1. INTRODUCTION

An optimal investment planning model for power systems is described in this paper. Availability of adequate electricity at low cost is one of the necessary preconditions for industrial growth and agricultural development and it is also a basic requirement of urban life. In India, the growth of demand for electricity has been very rapid with power demand doubling every 5 to 6 years and widespread shortages have been quite frequent during the last decade. Due to this high rate of demand growth and long gestation periods required for building additional generating/transmission capacity, advance planning is necessary so that /and installed capacity is available when it is required. The model described in this paper takes a reasonably longer planning horizon of 15 to 25 years, and attempts to find a least cost expansion plan for power generating capacity in a region.

Given an existing power system and its demand growth pattern over a given planning horizon, the purpose of optimal planning studies is to attempt to answer the following questions:

1) Which type of generation capacity should be added to meet the increasing load,

2) What should be the capacity of these generating plants,

3) When should these plants be commissioned,

4) Where these plants should be located, and

5) Which additional transmission lines should be built?

The model described in this paper attempts to answer the first three questions and solve what is commonly known as the long-term optimisation problem for power systems.

Leaving aside the on-line optimisation of the operation of an integrated power system, there are three other problems which have received attention in literature. The short-term optimisation problem would usually solve the economic dispatching and unit commitment decisions over a planning horizon of several weeks or a month. The medium-term optimisation problem would usually consider a planning horizon of a year and would prepare schedules for operation and maintenance of the various generating plants during the year. The long-term optimisation problem considered in this paper is not entirely independent of the short and medium-term optimisation problems and is difficult to solve without knowing the solution of the other two problems. Certain simplified assumptions are usually made to circumvent this problem. The model described also considers the requirements for planned maintenance and obtains an initial maintenance and operation schedule in relation to the load duration curve for the year.

The investment planning model could not be literally viewed as a master plan for the development of the entire system during the planning horizon due to the approximate nature of data on demand over the rather long planning horizon. The model should be used to provide a framework within which detailed project proposal and site selection studies for the next few generating units could be carried out. Necessarily, an investment planning model of this type contains fewer engineering details and the cost information used in the model would be of approximate nature. However, such a modelling framework is necessary so that individual capacity additions to the system follow an optimal pattern over the planning horizon and the characteristics of the complete system is taken into account for each decision on expansion of the system.

In the following two sections the unique characteristics of electricity demand is discussed along with the device of load duration curve which describes the variable demand pattern. The reserve requirements of a power system to attain a prescribed system reliability is discussed in section 4. The investment planning model is formulated as a mixed integer linear programming problem in section 6, the solution of which would provide a time schedule by year of investment decisions in nuclear or fossil-fuel generating plants (or hydel plants if the model is modified to consider them), and their capacities along with an initial schedule of maintenance and operation of individual plants to meet given system demand. Concluding comments are included in section 6.

2 CHARACTERISTICS OF SYSTEM DEMANDS

It is well known that electrical power demands on utility systems are not constant. They go through diurnal, weekly, and seasonal cycles of variation. Variations are caused by such factors as the living habits and work schedules of people served, the nature of the industries included, and weather extremes. As electric energy cannot be effectively stored, such variations create problems for the system planner and dispatcher.

A substantial part of the electrical power demand is on a continuous basis. Loads are highest during the normal working hours of weekdays and drop off during late-night hours and on weekends. Whether the peak demands of a system last for a few minutes or a few hours, generating capacity must be available to supply the demand at the moment it is needed. Thus, while about 50 percent of the system capacity must operate almost continuously, the remainder, meant to serve the peaks of the load and provide reserve capacity, is idle for portions of time.

The usual utility practice is to use new, efficient steam-electric units, either fossil-fueled or nuclear, to serve the base portion of the load. The older, less efficient steam-electric capacity is used to serve the higher portions of the load and therefore operates at a lower utilization factor. Conventional hydroelectric projects, pumped storage, and gas turbine plants are well suited for peak-load operations due to their flexibility and low turn-on time.

3. LOAD DURATION CURVE

In system planning, it is necessary to know not only the peak demands but also the shape of the load, most effectively as a load duration curve prepared from the chronological load curve. The ordinate of a load duration curve may be in terms of load demand or in terms of percent of peak demand, and the abscissa may be in terms of total hours or percent of total hours. The area under the curve represents the energy requirements in kilowatt-hours.

For the purpose of reducing the computational effort in the solution of the system planning model, some simplifying approximations will be made to the reclining-S-shaped load-duration curve. A stepwise curve will be used to approximate the smooth curve as shown in Figure 1. By increasing the number of steps, the smooth curve could be approached quite closely at the cost of increased computational effort. However, since the shape of the load duration curves, which will be forecast for the whole of planning horizon (15 to 25 years), are subject to error, this further simplification should not appreciably affect the optimal system plan.

As shown in Figure 1, a typical yearly load duration curve is *approximated by a discrete stepwise function with k equal steps (k=5 in Figure 1) that follow the shape of the curve. The modified load duration* curve will be used for system planning purposes. The system demand is based on k discrete values, with D_{1t} and D_{kt} the peak and base demands, respectively. The subscript t represents the year (t = 1, 2, ..., T), where T is the planning horizon. The system demand D_{it} also include allowances for transmission losses in the transmission network as these are not explicitly considered in this model. An investment planning model which considers both the generating plants and transmission network will be reported in a subsequent paper.

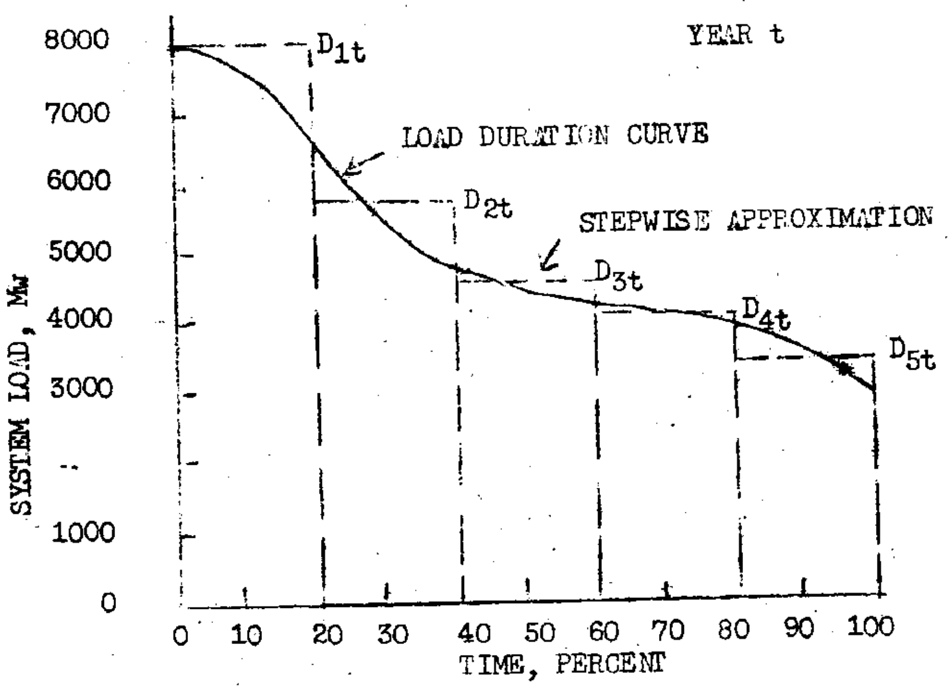

Figure 1. Stepwise Approximation of Typical Load Duration Curve

4 SYSTEM RELIABILITY AND RESERVE REQUIREMENTS

For any electric utility system, a prime requirement is its ability to fulfill customer demands for power with some prescribed measure of reliability. Since future customer demands can only be estimated, and since generators and auxiliary equipment (boilers, turbines, etc.) are subject to forced outages, a utility system is required to maintain a reserve of installed generating capacity in the form of "extra" units. Failure to have a sufficient reserve leads to customer shortages with resulting customer dissatisfaction and loss of industrial production. After the 1965 blackout in the Northeast of USA, the utility industries in that country have been very concerned about meeting system demand under almost any circumstances rather than face further public criticism. On the other hand, a surplus generating capacity means additional cost for the utility industry and in effect for the society. In India excepting a few utility industries, the generation and distribution of electricity is governed by public bodies i.e. State Electricity Boards and power shortages and load shedding has become chronic diseases resulting in heavy losses of industrial and agricultural production and inconveniences to the public. Due to long delays in the commissioning of planned generating and transmission systems there is hardly any reserve capacity in any power system in India. Considering the high cost of power shortage the need for advance planning and speedy execution in this area is obvious.

To ensure that system load demands can be met, sufficient capacity must be provided not only to meet the peaks of the load but also for adequate reserve capacity. Reserves are needed to replace generating capacity removed from service because of unscheduled or forced outages of generating or transmission equipment, to replace capacity removed from service for scheduled maintenance, to serve loads greater than anticipated, and for system control service. The total amount of reserve capacity required depends upon such factors as the magnitude and characteristics of the system load, the degree of desired reliability (expressed as loss-of-load probability), the number, sizes and nature of generating units, maintenance requirements, and the degree of interconnections with other systems.

A part of the reserve capacity must be ready for use within a relatively short time - within 10 minutes or less. Some of this kind of reserve is in the form of "spinning reserve" - synchronized with the system and ready to accept load. The amount of spinning reserve capacity required is often determined by the size of the largest unit in service and the amount of unscheduled power available from interconnections.

In the electrical utility industry, the reserve question is usually phrased as: "What is the optimum reserve generating capacity?" A rule of thumb in the industry is that the reserve capacity should generally be no less than 20-25 per cent of the system capacity and the spinning reserve capacity must be at least equal to the capacity of the largest unit in the system. However, there is no single optimum percentage, since the optimum varies with the number and size distribution of the generating units. A more meaningful question to ask is: "When should new generating capacity be added to the system?" This question, answered for a given reliability criteria, automatically determines the optimum reserve capacity for any given generating system. An associated problem is that of determining the optimum size (capacity) and nature (nuclear, fossil-fuel) of units to be added to the system. Varying the size or nature of the unit affects fuel costs, maintenance costs, labor costs, equipment cost, etc.

Optimal Investment planning model should try to answer both these questions, not only for the next unit but for all units needed during the planning horizon of 15 to 25 years. However, there is an interconnection between the investment planning model and the "reserve problem." To proceed toward the solution of the investment planning model, the reserve requirements must be known, whereas the reserve problem could not be solved unless the number, size, and nature of the generating units in the system are known. To break this chain, certain initial assumptions must be made regarding the reserve requirements in the system planning model. Once optimum solutions are obtained for this model and system configurations are known, available computer programs can be used to determine "loss-of-load probability" for each step of the load duration curve and a given generation and maintenance schedule.[1] A loss-of-load probability of 24 hours in 10 years is a well-accepted figure in USA. In India perhaps a moderate reliability standard which would be attainable within a forseeable future should be used now. The initial assumptions regarding the reserve requirements can now be checked from the load-loss probabilities and, if necessary, may be adjusted.

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[1] A generation schedule for the year is obtained from the optimal solution of the investment planning model. The solution also specifies a maintenance schedule to make sure that maintenance requirements are met. This maintenance schedule could probably be improved upon from other system considerations once the generation schedule is known.

Further examination of system reliability and reserve capacity need takes place when the site selection study for the next unit to be installed is undertaken, because the system configuration is then known. After this phase of the study, the precise date (where the year is known from the system planning model) that the new unit will be required to add capacity to the system is also known, and an installation schedule can be established.

For inclusion in the system planning model, reserve-capacity coefficients r_{it} are assumed for $i = 1,2, \ldots , k$ and $t = 1,2, \ldots , T$. Then the required system capacity is given by

$$R_{it} = r_{it} \cdot D_{it} , \qquad \begin{matrix} i = 1,2, \ldots , k \\ t = 1,2, \ldots , T \end{matrix}$$

The usual range for r_{it} is of the order of 1.12 to 1.25.

5 MATHEMATICAL FORMULATION OF THE SYSTEM PLANNING MODEL

The system planning model can be formulated as a mixed-integer linear programming problem. Given the system load in the form of load duration curves for each year[2] of the planning horizon of T years (T = 15 to 25), reserve requirements, and fixed and variable production costs for the available and possible future generating units, the model would produce optimal expansion plans for the planning horizon and a generation schedule for each step or period of the load duration curve. The plan would be optimal in the sense that it would minimize the sum of yearly costs discounted to the first year of study (present worth) and would provide a feasible expansion program.

5.1 Basic Assumptions and Notations

It is assumed that two basic types of generating plants will be considered for system expansion - nuclear and fossil-fuel plants[3] - each

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[2] A period of several years could be chosen for planning with necessary modifications of the model. 2-3 years could be a good compromise between prohibitive problem size and computing effort if smaller intervals are chosen and the loss of utility as a planning device if a period consists of five or more years.

[3] Hydel plans could also be considered. But the available capacity of a hydel plant could vary from year to year and also from season to season within the same year depending on water inflow into the reservoir and this has to be accounted for. A subsequent paper will describe the modifications needed in optimal system planning model to accomoodate hydel plants.

available in n different sizes. More than two types could be considered, and n may vary over the planning horizon as larger size units become available.

Let U_{ij} and V_{ij} denote, respectively, the number of nuclear and fossil-fuel generating units of size j (j = 1,2, ... , n) commissioned during year i (i = 1,2, ... , T). U_{ij} and V_{ij} are integer variables and can take values of 0 or 1 as normally not more than one plant of of any size will be commissioned in a year.

Let h_{uj} and ℓ_{uj} be the maximum and minimum operating capacity of the nuclear plants of size j (j = 1, 2, ..., n). h_{vj} and ℓ_{vj} are similar notations for the fossil-fuel plants.

Let there be P existing generating units, and let h_{pt} and ℓ_{pt} be the maximum and minimum operating capacity of the p^{th} existing plant, available in the t^{th} year (p=1,2,..., P; t=1,2, ..., T). Some of the existing plants may become obsolete and may be put out of service before year T.

Let $W_{ij\ell m}$ and $X_{ij\ell m}$ be zero-one variables. If $W_{ij\ell m}$ takes the value 1, it indicates that the nuclear plant of size j commissioned in year i, is being scheduled to supply power in the m^{th} year during step ℓ of the load duration curve. If $W_{ij\ell m} = 0$, it indicates that either the unit was not commissioned or, if commissioned, is not scheduled for generation. If it is idle, it can undergo scheduled maintenance. If $X_{ij\ell m} = 1$, it indicates that during this period the above unit is scheduled for maintenance during step ℓ of the m^{th} year. Proper constraints are needed so that $W_{ij\ell m} = 0$ and $X_{ij\ell m} = 0$, whenever U_{ij} (denoting scheduled production) = 0. $Y_{ij\ell m}$ and $Z_{ij\ell m}$ are the corresponding zero-one variables on maintenance for fossil-fuel generating units and must be constrained to be zero if $V_{ij} = 0$.

$Q_{p\ell m}$ and $S_{p\ell m}$ are zero-one variables for existing generating units such that, when they are one, the p^{th} presently existing unit is respectively scheduled for production or maintenance in the m^{th} year during the ℓ^{th} step of the load duration curve.

$N_{ij\ell m}$ is the utilized capacity in Mw, during the ℓ^{th} step in year m, of the nuclear generating unit of size j commissioned in year i. $F_{ij\ell m}$ is the corresponding variable for the fossil-fuel generating units. Proper constraints are needed to ensure that $N_{ij\ell m} = 0$ whenever $U_{ij} = 0$ and that $F_{ij\ell m} = 0$ whenever $V_{ij} = 0$.

$G_{p\ell m}$ is a non-negative variable denoting the capacity in MW of the p^{th} existing unit being utilized in the m^{th} year during the ℓ^{th} step of the load duration curve.

5.2 Annual Costs

The present worth of all production costs incurred during the planning horizon of T years is considered the objective function which will be minimized by an optimal system expansion and generation plan. Let β be the yearly discount factor, where

$$\beta = \frac{1}{1 + I}$$

where I is the appropriate annual interest rate.

The annual costs are of two kinds:

c′ Fixed Cost. Capital costs associated with installed generating units that must be covered without consideration of the duration of the utilization are known as fixed cost. This cost includes the investment charges, depreciation, and all other fixed costs which are independent of the generated energy, such as maintenance costs, personnel cost, and overhead.

- Variable Cost. The variable costs are those costs directly related to power generation and utilization of the generating units. These basically include the fuel cost and variable O&M costs and are normally expressed as paise/kwh. It is well-known that the heat rate, and thus paise/kwh cost, will vary depending on the amount of power generated by a particular unit; for simplification, one uniform heat rate, and thus a single paise/kwh cost will be assumed for each unit. It is possible to have graduated rates, depending on the range at which the unit is operating, as is done in economic dispatching models, but such detailed formulation will increase the computational effort without improving the accuracy of the model appreciably.

Let C_{uij} be the discounted value in rupees at year i of all fixed costs for purchasing, installation, maintenance, and operation incurred during the planning horizon for the nuclear generating unit of size j commissioned during year i. C_{vij} is the corresponding cost figure for fossil-fuel plant. These costs are later discounted to year 1 to obtain the present worth of all fixed costs.

Let $f_{ij\ell m}$ be the variable cost in rupees per Mw utilized capacity during ℓ^{th} step of the load duration curve in the m^{th} year for the nuclear plant of size j commissioned in year i. Thus, $f_{ij\ell m}$, multiplied by the utilized capacity in Mw, gives the variable production cost for the unit during the period covered by the ℓ^{th} step of the load duration curve. $G_{ij\ell m}$ is the corresponding cost coefficient for the fossil-fuel plants. Let $d_{p\ell m}$ be the corresponding cost coefficient for the p^{th} existing generating unit.

It should be noted here that the cost structure assumed here is flexible enough to permit inclusion of expected changes in fuel cost and increased maintenance costs in the future.

5.3 The Mixed-Integer Linear Programming Formulation

The system planning model will now be stated in explicit mathematical terms in the form of constraint equations or inequalities and the objective function, which together comprise the mixed-integer linear programming problem. A glossary of notations already defined follows for easy reference.

Glossary of Notations

T Planning horizon in years

k Number of steps (or periods) in the modified load duration curve

D_{it} System demand in Mw during i^{th} period in the modified load duration curve including transmission losses; $i = 1,2, \ldots , k$

D_{1t} System peak demand during t^{th} year

D_{kt} System base demand during t^{th} year

r_{it} Reserve-capacity coefficient during i^{th} period in t^{th} year; $i = 1, 2 \ldots, k$; $t = 1,2, \ldots, T$

R_{it} Required system capacity including reserve during i^{th} period in the t^{th} year

P Number of existing generating units

U_{ij} Number of nuclear generating units of size j commissioned during year i; $i = 1, 2 \ldots , T$; $j = 1,2 \ldots, n$

V_{ij} Number of fossil-fuel generating units of size j commissioned during year i

h_{uj} Maximum operating capacity for nuclear plants of size j

ℓ_{uj} Minimum operating capacity for nuclear plants of size j

h_{vj} Maximum operating capacity for fossil-fuel plants of size j

ℓ_{vj} Minimum operating capacity for fossil-fuel plants of size j

h_{pt} Maximum operating capacity of p^{th} existing plant

ℓ_{pt} Minimum operating capacity of p^{th} existing plant

$W_{ij\ell m}$, $X_{ij\ell m}$, $Y_{ij\ell m}$, $Z_{ij\ell m}$, $Q_{p\ell m}$ and $S_{p\ell m}$ are zero-one variables. When they are one, they mean the following during the ℓ^{th} period in year m:

$W_{ij\ell m}$	Nuclear plant of size j commissioned in year i is scheduled to supply power
$X_{ij\ell m}$	The above nuclear plant is schedule for maintenance
$Y_{ij\ell m}$	Fossil-fuel plant of size j commissioned in year i is scheduled to supply power
$Z_{ij\ell m}$	The above fossil-fuel plant is scheduled for maintenance
$Q_{p\ell m}$	The p^{th} existing unit is scheduled to supply power
$S_{p\ell m}$	The p^{th} existing unit is scheduled for maintenance
x_{ijm}	Minimum number of days required in year m for the maintenance of nuclear unit of size j commissioned in year i
z_{ijm}	Same as x_{ijm} for fossil-fuel units
s_{pm}	Same as x_{ijm} for p^{th} existing unit
$N_{ij\ell m}$	Utilized capacity in Mw, during the ℓ^{th} period in year m, of nuclear unit of size j commissioned in year i
$F_{ij\ell m}$	Same as $N_{ij\ell m}$ but for fossil-fuel units
$G_{p\ell m}$	Utilized capacity of p^{th} existing unit during ℓ^{th} period in year m
β	Yearly discount factor, $\beta = 1/(1 + I)$, where I is the appropriate annual interest rate.
C_{uij}	Discounted value in **rupees** at year i of all fixed costs during the planning horizon for nuclear unit of size j commissioned in year i.
C_{vij}	Same as C_{uij} but for fossil-fuel units

$f_{ij\ell m}$ Variable cost in rupees per Mw of utilized capacity during ℓ^{th} period in year m for the nuclear plant of size j commissioned in year i

$G_{ij\ell m}$ Same as $f_{ij\ell m}$, but for fossil-fuel units

$d_{p\ell m}$ Same as $f_{ij\ell m}$, but for p^{th} existing unit

Reserve Requirements. The total power requirements R_{it} including reserve capacity for each year and during each step of the load duration curve must be exceeded by the combined capacity of the equipment scheduled for generation (commissioned until that time and not idle or under maintenance). This can be expressed by the equation:

$$\sum_{p=1}^{P} h_{pm} \cdot Q_{p\ell m} + \sum_{i=1}^{m} \sum_{j=1}^{n} (h_{uj} \cdot W_{ij\ell m} + h_{vj} \cdot Y_{ij\ell m}) \geqq R_{\ell n} \quad (1)$$

$$\ell = 1,2, \ldots, k$$

$$m = 1,2, \ldots, T$$

System Load Equation. During each step in the load duration curve (which may be denoted as a period) and during each year in the planning horizon, the total utilized capacity of all generating units scheduled for operation must be equal to or greater than the system load. This relationship can be expressed as follows:

$$\sum_{p=1}^{P} G_{p\ell m} + \sum_{i=1}^{m} \sum_{j=1}^{n} (N_{ij\ell m} + F_{ij\ell m}) \geqq D_{\ell m} \quad (2)$$

$$\ell = 1,2, \ldots, k$$

$$m = 1,2, \ldots, T$$

Maintenance Requirements. During each year, some time must be allocated for each of the existing generating units during which scheduled maintenance can be done.

The following three constraints express this requirement:

$$\sum_{\ell=1}^{k} \frac{365}{k} S_{p\ell m} \geq s_{pm} \qquad (3)$$

$$p = 1,2, \ldots, P$$

$$m = 1,2, \ldots, T$$

in which s_{pm} denote the minimum number of days required for such maintenance for the p^{th} existing unit in m^{th} year.

$$\sum_{\ell=1}^{k} \frac{365}{k} X_{ij\ell m} \geq x_{ijm} \qquad (4)$$

$$i = 1,2, \ldots, m$$

$$j = 1,2, \ldots, n$$

$$m = 1,2, \ldots, T$$

in which x_{ijm} is the minimum number of days required in the m^{th} year for maintenance of a nuclear plant of size j commissioned in year i.

$$\sum_{\ell=1}^{k} \frac{365}{k} Z_{ij\ell m} \geq z_{ijm} \qquad (5)$$

$$i = 1,2, \ldots, m$$

$$j = 1,2, \ldots, n$$

$$m = 1,2, \ldots, T$$

<u>Consistency Requirements</u>. These constraints are needed so that the values that the variables take are consistent with each other and represent all the relationships between them. Otherwise, the solution obtained may not be feasible.

Only the generating units actually commissioned are available for scheduling for production or maintenance starting from that year. This is expressed by the following two constraints.

$$U_{ij} - W_{ij\ell m} - X_{ij\ell m} \geqq 0 \qquad (6)$$

$$V_{ij} - Y_{ij\ell m} - Z_{ij\ell m} \geqq 0 \qquad (7)$$

$$i = 1,2, \ldots, m$$

$$j = 1,2, \ldots, n$$

$$\ell = 1,2, \ldots, k$$

$$m = 1,2, \ldots, T$$

Only the generating units scheduled for production in a particular period (th step of the load duration curve) of year m can be utilized for power production to their maximum capacity. The generating units are also subject to the following minimum operating capacity restrictions:

$$h_{uj} \cdot W_{ij\ell m} - N_{ij\ell m} \geqq 0 \qquad (8)$$

$$h_{vj} \cdot Y_{ij\ell m} - F_{ij\ell m} \geqq 0 \qquad (9)$$

$$h_{pt} \cdot q_{p\ell m} - G_{p\ell m} \geqq 0 \qquad (10)$$

$$N_{ij\ell m} \geqq \ell_{uj} \qquad (11)$$

$$F_{ij\ell m} \geqq \ell_{vj} \qquad (12)$$

$$G_{p\ell m} \geqq \ell_{pt} \qquad (13)$$

$i = 1,2, \ldots, m$

$j = 1,2, \ldots, n$

$\ell = 1,2, \ldots, k$

$m = 1,2, \ldots, T$

$p = 1,2, \ldots, P$

Finally, the following objective function may be expressed, which equals the present worth of all fixed and variable costs incurred during the planning horizon.

$$\text{Minimize PWTC} = \sum_{i=1}^{T} \sum_{j=1}^{n} \beta^{i-1}(C_{uij}U_{ij} + C_{vij}V_{ij})$$

$$+ \sum_{i=1}^{T} \sum_{\ell=1}^{k} \sum_{j=1}^{n} \beta^{i-1}(f_{ij\ell m}N_{ij\ell m} + g_{ij\ell m}F_{ij\ell m})$$

$$+ \sum_{i=1}^{T} \sum_{\ell=1}^{k} \sum_{p=1}^{P} \beta^{i-1} \cdot d_{p\ell m}G_{p\ell m} \tag{14}$$

The expressions (1) through (14) represent a mixed-integer linear programming problem in which the variables $N_{ij\ell m}$, $F_{ij\ell m}$, and $G_{p\ell m}$ are non-negative, and the variables U_{ij}, V_{ij}, $W_{ij\ell m}$, $X_{ij\ell m}$, $Y_{ij\ell m}$ and $Z_{ij\ell m}$ are 0-1 integer variables. An optimal solution of this problem provides an optimal expansion plan and a generation and maintenance schedule for the entire planning horizon, which minimizes the present worth of all future costs to satisfy the given demand.

There are various algorithms available for solving mixed integer linear programming problems. The most efficient ones essentially partition the linear programming matrix into two parts, consisting of the continuous and integer variables, and different techniques are used to solve and relate the two parts.

It should be noted that with specific knowledge and information for any particular utility system, the above formulation could be further simplified, giving fewer integer variables. A good starting solution can also be obtained from knowledge of the system, and the number of iterations and computational time can thus be reduced appreciably. "Branch and bound" methods are also being increasingly used for the solution of mixed integer linear programs, and very efficient algorithms can often be obtained through such methods.

6 CONCLUSIONS

Various simplifying assumptions have been made in the above formulation. However, this model can handle a large number of alternative system expansion plans, which is not possible by other direct engineering cost models used for determining system operating costs. Once the few best solutions are isolated by the solution of this model, they can be further examined in detail, with all the complexities of the system, by the application of a direct engineering cost model to determine system cost once the expansion plan is known.

Parametric studies and sensitivity analyses can be performed by using the investment planning model to study the effects of fuel cost changes, price escalation, changes in reserve requirements, technological innovations, etc. It is feasible to enlarge the model by also considering investments in transmission network needed to carry enlarged system load.

The system planning model should be periodically reviewed and updated as better information is available regarding future demands, costs, and immediate expansion plans.

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