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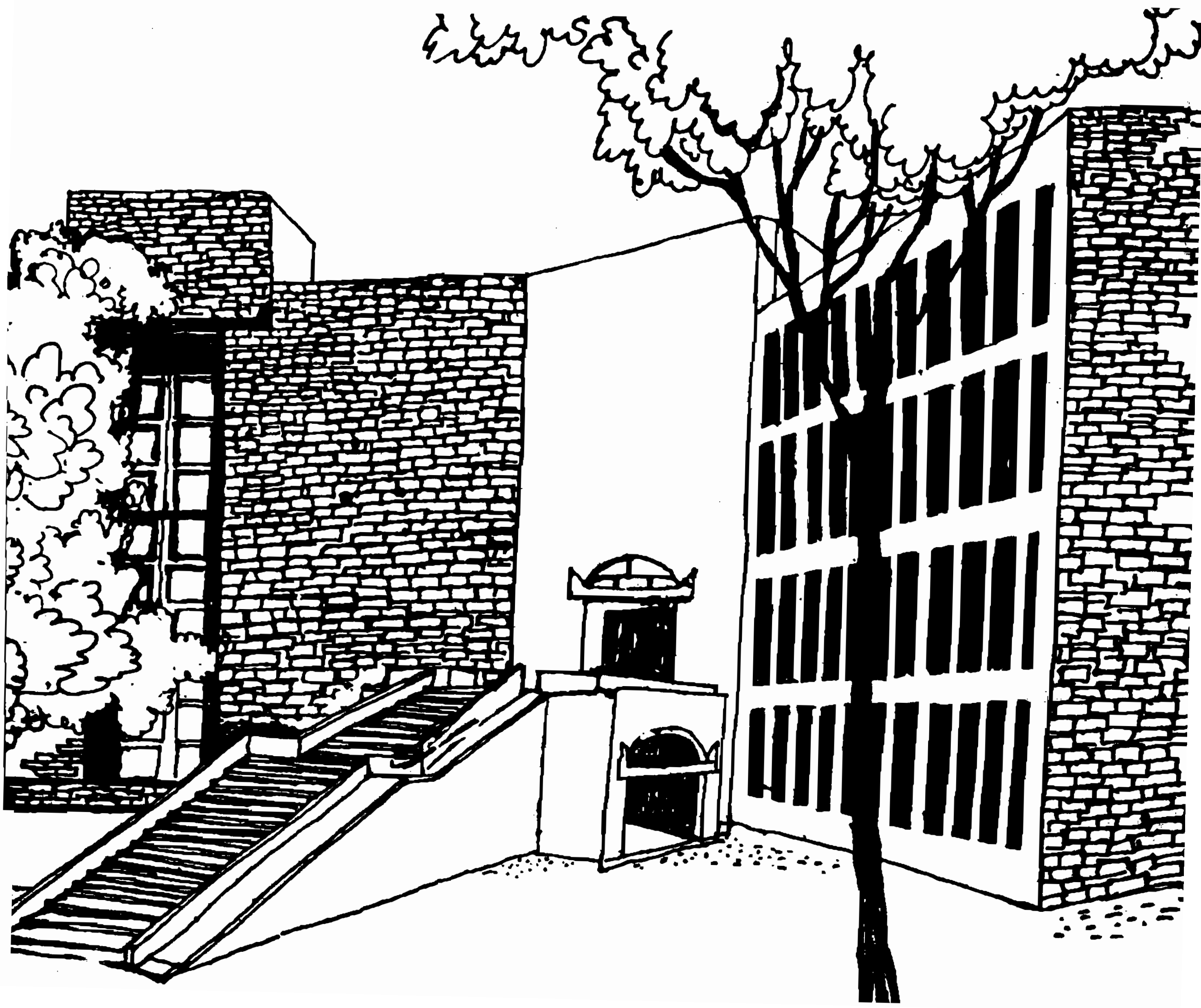

NEGOTIATION PROCEDURES CONVERGING TO BARGAINING SOLUTIONS

By

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Abstract

In this paper we study globally stable adjustment processes converging to the egalitarian and Nash solutions respectively in two person bargaining problems, that arise under bilateral monopoly. The preference structures of the individuals are allowed to exhibit consumption externalities and we impose plausible restrictions on them in order to guarantee convergence.

1. Introduction :- The simple analytical framework of a bilateral monopoly problem, reduces to a problem involving division of a fixed sum of money between two agents, as discussed in Roth (1979) for instance. An extension of the same problem, incorporating externalities in the preference structure of the individuals, is as follows :

Consider a situation with two agents 1 and 2, with utility functions $u_i : \mathbb{R}_+^2 \to \mathbb{R}$, $i=1,2$ for distributions of money between the two agents. (Here each agent's preference depends not only on his own money holdings but also on the money holdings of the other agent.) Let $w_i \geq 0$ be the initial wealth of agent i and suppose they bargain over the division of Q units of money. We will assume throughout this paper the following :

Assumption 1() (a) $u_i : \mathbb{R}_+^2 \to \mathbb{R}$ is differentiable

(b) $\frac{\partial u_i(x)}{\partial x_i} > 0$, $\frac{\partial u_i(x)}{\partial x_j} < 0 \; \forall \; x \in \mathbb{R}_+^2$, $i \in \{1,2\}$, $j \neq i$.

Assumption 1(a) facilitates exposition and permits the analysis of the negotiation procedures we propose. Assumption 1(b), requires that each agent's utility increases with a corresponding increase in his personal income and decreases with an increase in the income of his opponent.

A feasible allocation is a proposed split (c_1, c_2) such that $c_1+c_2=Q$, $c_1 \geq 0$, $c_2 \geq 0$, which leaves player i with total wealth w_i+c_i.

The purpose of this paper is to suggest negotiation procedures (or allocation adjustment processes) which converge to desirable bargaining solutions.

2. The Egalitarian Solution :- The egalitarian solution to the above problem is an allocation (c_1^*, c_2^*) such that

(i) $c_1^* + c_2^* = Q$

(ii) $u_1(w_1+c_1^*, w_2+c_2^*) - u_1(w_1, w_2) = u_2(w_1+c_1^*, w_2+c_2^*) - u_2(w_1, w_2)$.

Let us (for the purpose of this section) normalize our utility functions so that

(a) $u_1(w_1, w_2) = u_2(w_1, w_2) = 0$

(b) $u_1(w_1+Q, w_2) = u_2(w_1, w_2+Q) = 1$.

Then under Assumption (1), there exists a unique egalitarian solution, $(\bar{c}_1, \bar{c}_2)$.

We propose the following adjustment mechanism :

$$\left.\begin{aligned} &\frac{dc_1}{dt} = u_2(w_1+c_1, w_2+(Q-c_1) - u_1(w_1+c_1, w_2+(Q-c_1)) \\ &c_2(t) = Q - c_1(t), \\ &t \geqq 0 \end{aligned}\right\} \quad (1)$$

It is clear that the unique critical point (or equilibrium) of (1) is $(\bar{c}_1, \bar{c}_2)$. Our purpose in this section is to show that if $\{c_1(t), c_2(t)\}_{t \geqq 0}$ be any solution of (1), then $\lim_{t \to \infty} c_1(t) = \bar{c}_1$, $\lim_{t \to \infty} c_2(t) = \bar{c}_2$.

Theorem 1:- Let $\{c_1(t), c_2(t)\}_{t \geqq 0}$ be any solution of (1). Then, $\lim_{t \to \infty} c_1(t) = \bar{c}_1$, $\lim_{t \to \infty} c_2(t) = \bar{c}_2$.

Proof :- To prove this theorem we construct the function

$$V(c_1) = [u_2(w_1+c_1, w_2+(Q-c_1) - u_1(w_1+c_1, w_2+(Q-c_1))]^2 .$$

$V(\bar{c}_1) = 0$ and $V(c_1) \neq 0$ if $c_1 \neq \bar{c}_1$

Further,

$$\frac{dV(c_1(t))}{dt}=2[u_2(w_1+c_1,w_2+(Q-c_1))-u_1(w_1+c_1,w_2+(Q-c_1))]^2\left[\frac{\partial u_2}{\partial x_1}-\frac{\partial u_2}{\partial x_2}-\frac{\partial u_1}{\partial x_1}+\frac{\partial u_1}{\partial x_2}\right]<0,$$

whenever, $c_1(t)\neq\bar{c}_1$ and $c_1(t)$ solves (1).

In addition $c_1(t)\in[0,Q]$ (a compact set) for all $t\geq 0$. Hence by Liapunov's theorem on global asymptotic stability (see Luenberger (1979) or Varian (1981), $\lim_{t\to\infty} c_1(t)=\bar{c}_1$ and $\lim_{t\to\infty} c_2(t)=\bar{c}_2$.

3. <u>The</u> <u>Nash</u> <u>Solution</u> :- A <u>Nash</u> <u>Solution</u> to the problem defined in Section 1, is an allocation (c_1^*,c_2^*) such that (c_1^*,c_2^*) solves the following programming problem :

$$[u_1(w_1+c_1,w_2+c_2)-u_1(w_1,w_2)][u_2(w_1+c_1,w_2+c_2)-u_2(w_1,w_2)] \to \max$$

$$\text{s.t. } c_1+c_2 = Q$$

$$c_1\geq 0,\ c_2\geq 0.$$

For the purpose of this section we shall assume in addition to Assumption 1, the following :

<u>Assumption</u> <u>2</u> :- (a) $u_i : \mathbb{R}_+^2\to\mathbb{R}$ is concave for i=1,2.

(b) u_i is twice differentiable for i=1,2.

(c) $\frac{\partial}{\partial x_j}\left(\frac{\partial u_i}{\partial x_i}\right)\geq 0$ for i=1,2; $j\neq i$.

(d) $u_1(w_1+c_1,w_2+(Q-c_1))\neq u_1(w_1,w_2)$ for $c_1\in[0,Q]$.

By Assumption 2(a), we may conclude that there exists a unique Nash solution (c_1^*,c_2^*) to the above programming problem.

Following Roth (1979), and generalizing a concept contained there, we define player i's <u>boldness</u> with respect to the allocation (c_1,c_2) as

$$b_i((w_1,w_2);(c_1,c_2)) = \frac{\frac{\partial u_i}{\partial x_i}(w_1+c_1,w_2+c_2) - \frac{\partial u_i}{\partial x_j}(w_1+c_1,w_2+c_2)}{[u_i(w_1+c_1,w_2+c_2) - u_i(w_1,w_2)]},$$

i=1,2. Assumption 2(d), permits this definition.

As in Roth (1979), the following result is easily established.

<u>Lemma 1</u> :- (c_1^*,c_2^*) is the Nash bargaining solution if and only if $b_1((w_1,w_2);(c_1^*,c_2^*)) = b_2((w_1,w_2);(c_1^*,c_2^*))$.

<u>Proof</u> :- Parallels the proof of Theorem 7 in Roth (1979).

The following properties are easily verified by applying Assumption 1(b), 2(a) and 2(c) :

$$\left.\begin{array}{l} \frac{\partial b_i((w_1,w_2);(c_1,c_2))}{\partial c_i} < 0, \quad i=1,2 \\ \frac{\partial b_i((w_1,w_2);(c_1,c_2))}{\partial c_j} \geq 0, \quad i=1,2 \ ; \ j \neq i. \end{array}\right\} \quad (*)$$

wherever, $c_2 = Q - c_1$ and $c_1 \in [0,Q]$

The negotiation procedure we postulate assumes that the bolder of the two players is capable of extracting a concession from his opponent. Thus we suggest the following adjustment mechanism :

$$\left.\begin{array}{l} \frac{dc_1}{dt} = b_1((w_1,w_2);(c_1,Q-c_1)) - b_2((w_1,w_2);(c_1,Q-c_1)) \\ c_2(t) = Q - c_1(t) \\ t \geq 0 \end{array}\right\} \quad (2)$$

It is clear from Lemma 1, that the only equilibrium of (2) is (c_1^*,c_2^*) i.e. the Nash solution. The main result of this section asserts that all solutions of (2) converge to (c_1^*,c_2^*) i.e. (c_1^*,c_2^*) is globally asymptotically stable.

Theorem 2 :- Let $\{c_1(t), c_2(t)\}_{t\geq 0}$ solve (2). Then

$$\lim_{t\to\infty} c_1(t)=c_1^*, \quad \lim_{t\to\infty} c_2(t)=c_2^*.$$

Proof :- The proof proceeds by constructing the function

$V(c_1) = [b_1((w_1,w_2);(c_1,Q-c_1))-b_2((w_1,w_2);(c_1,Q-c_1))]^2$.

$V(c_1^*) = 0$ and $V(c_1)>0$ for $c_1 \neq c_1^*$

$$\frac{dV(c_1(t))}{dt}=2[b_1((w_1,w_2);(c_1(t),Q-c_1(t))-b_2((w_1,w_2);(c_1(t),Q-c_1(t))]^2 \cdot \left\{\frac{\partial b_1}{\partial c_1} - \frac{\partial b_1}{\partial c_2} - \frac{\partial b_2}{\partial c_1} + \frac{\partial b_2}{\partial c_2}\right\} < 0$$

for $c_1(t)\neq c_1^*$ and $\{c_1(t)\}_{t\geq 0}$ satisfying (2). This follows by appealing to the inequalities(*).

Thus V is a Liapunov function. Furthr $c_1(t)\in[0,Q]\forall t\geq 0$. Thus by the theorem on global asymptotic stability in Luenberger (1979) or Varian (1981), $\lim_{t\to\infty} c_1(t)=c_1^*$ and $\lim_{t\to\infty} c_2(t)=c_2^*$.

4. Conclusion and Discussion :- In this paper we have established convergent negotiation procedures for the bilateral monopoly problem.

The first point we would like to make is that Assumption 2(a) is quite unnecessary. Without this assumption we do not get a unique Nash solution. However, if we strengthen Assumption 2(b) to require twice continuous differentiability we observe that the only invariant set of the adjustment mechanism (2) is the set of Nash product maximizers; further if we impose the condition that $\frac{\partial^2 u_i}{\partial x_i^2} \leq 0$ (which is weaker than concavity), then by applying the appropriate stability theorem as in Luenberger (1979), we obtain the result that all solutions to the negotiation procedure (2), converge to the Nash s[illegible] we are able to obtain results similar to those obtain[illegible] Maschler, Owen and Peleg (1988) in the context of a bilateral monopoly problem (even when environments exhibit nonconvexities).

Furth (1990) studies differential equations in the context of bargaining problems, but with an entirely different objective. He starts off with a given solution to a bargaining problem and constructs, by means of solving a set of differential equations, a new solution. He inspects properties which are "hereditary" in the above framework. At the end of the paper he poses the question, whether such an analysis could be used to model real life bargaining situations. We come close to answering this question (in terms of adjustment mechanisms) in the above analysis.

It may be asked at this point, why we chose to model negotiation as a differential equation, rather than as a non-cooperative game. The answer lies in Roth (1988), where he forcefully argues that the non-cooperative view is invalidated by abundant experimental evidence suggesting that considerations of fairness play a prominent role in controlled bargaining.

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