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1. Introduction

This paper is extracted from an earlier study by the author [3, 4]. It is argued in this study that input as well as output markets should really be assumed to be simultaneously imperfect because perfect markets may exist only in rare cases. In that case decision making process at the firm level is actually constrained by the technology of production as embodied in its production function, market demand function of the product as it is observed empirically, input supply functions as they actually are during the period in question and demand for inventory if any. Symbolically, these functions can be expressed as

$$(1.1)\qquad \begin{aligned} Q &= f(K, L, \ldots) \\ D &= f(P, \ldots\ldots) \\ K &= f(R, \ldots\ldots) \\ L &= f(W, \ldots\ldots) \\ S &= f(I, \ldots\ldots) \\ Q &= D + S \end{aligned}$$

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where dots represent other relevant variables not stated explicitly, including errors in equation, f represents function whose form could differ from function to function, Q means production, K means capital employed, L means labour employed, D means market demand, P means product price, R means cost of capital, W means wages, S means stock, and I means cost of maintaining inventory of the finished product. One may incorporate inventory function of the inputs also if situation demands so.

The behavioural process of the firm may be specified as that mode of choice that leads to achievement of chosen objectives under the constraints given in (1.1). Usually it is assumed that profit maximisation is the objective preferred by the firm but several other objectives have been followed in actual practice and one may like to keep the objective to be variable. But for any chosen objective one can work out the equilibrium condition of the firm corresponding to that objective. For the sake of comparability with most of the past work in the field, we choose the objective to be profit maximisation and similar approach can be followed if the chosen objective is different one.

Profit II in the present case can be expressed as difference between revenue, R, and cost, C, so that

(1.2)
$$II = R - C$$
$$R = PQ$$
$$C = LW + KR + SI$$

It will be noted here that all the variables involved in (1.2) are really variables in a situation of imperfection prevailing in the corresponding markets. The first order condition for profit maximisation can be obtained as in Misra [3] as follows:

$$
\begin{aligned}
(1.3) \qquad QP &= aLW + bKR + cSI + dSP \\
(1+e^{-1})a &= \alpha^{-1}(1 + e_1^{-1}) \\
(1+e^{-1})b &= \beta^{-1}(1 + e_2^{-1}) \\
(1+e^{-1})c &= \gamma^{-1}(1 + e_3^{-1}) \\
(1+e^{-1})d &= e^{-1} + \gamma^{-1} \\
e &= \frac{\partial \log Q}{\partial \log P} \\
\alpha &= \frac{\partial \log Q}{\partial \log L} \\
\beta &= \frac{\partial \log Q}{\partial \log K} \\
e_1 &= \frac{\partial \log L}{\partial \log W} \\
e_2 &= \frac{\partial \log K}{\partial \log R} \\
e_3 &= \frac{\partial \log S}{\partial \log I} \\
\gamma &= \frac{\partial \log Q}{\partial \log S}
\end{aligned}
$$

It has been shown in Misra [3] as to how equilibria condition corresponding to different market situations can be obtained as special cases of the general condition in (1.3). It has also been described in Misra [3] as to how one can ascertain as to which objective prevailed during a chosen period in any actual case of empirical study. Empirically the parameters involved in (1.3) will differ from firm to firm, or from industry to industry depending upon the level of aggregation involved in respect of commodities, firms, regions and industries.

2. Direct Tax Functions

Using the equilibrium condition (1.3) one can conclude on similar lines as in Misra [3] that the interests of firms are served best if they are aware of supply and demand patterns throughout the country, where they operate, and act accordingly. It is inferred that adoption of labour intensive technology is better when economy is faced with labour abundance and adoption of capital intensive technology is better when the economy is faced with labour scarcity. But in actual situation the information base of the firms is not as adequate as implied by above statement. Particularly, in inflationary situation, several firms have tended to borrow more from financial institutions, got controlled over means of production and kept the capacity unutilised because observed price rise and interest gap was high enough to let them behave so. In that case a case arises as to whether a taxation mix on output, labour and capital can be utilised to put the firms on the desired track. There is already a variety of concession if capital intensification is increased. Should one not wonder as to how labour bias can be introduced in taxation system itself if one wants to increase employment. One may also wonder as to how taxation instrument could be utilised to promote capacity utilisation, to supply the goods where they are needed most and to reduce stocks of hoarded goods.

Let us consider a general scheme of taxation where taxes can be imposed on not only output but use of such factors of production as capital and labour and inventory level. This scheme always provides the flexibility of dropping out a component of tax mix by setting the corresponding tax variable at zero level. Suppose further that the taxation scheme is determined according to following process:

$$(2.1) \qquad \begin{aligned} T_1 &= T(Q) \\ T_2 &= T(L) \\ T_3 &= T(K) \\ T_4 &= T(S) \end{aligned}$$

where the form of tax function for each component is sought to be determined according to some desirable rule. If the functional forms could be quantified in this sense then the taxes to be imposed in a given situation could always be worked out without much effort.

If taxes are imposed as suggested in (2.1), then, the resulting profit of the firm can be expressed as

$$(2.2) \qquad II_1 = II - T_1 - T_2 - T_3 - T_4$$

First order condition of equilibrium of the firm can be obtained in this situation as

$$(2.3) \qquad QP = aLW + bKR + cSI + dSP + (1 + e^{-1})^{-1}(t_1Q + t_2\alpha^{-1}L + t_3\beta^{-1}K + t_4\gamma^{-1}S)$$

$$t_1 = \frac{\partial T_1}{\partial Q}$$

$$t_2 = \frac{\partial T_2}{\partial L}$$

$$t_3 = \frac{\partial T_3}{\partial K}$$

$$t_4 = \frac{\partial T_4}{\partial S}$$

We may use the relations (2.2) and (2.3) to obtain optimal profit level, Π_1^*, under taxation and similarly use the relations (1.2) and (1.3) to obtain optimal profit, Π^*, without taxation, and the two are related as

$$(2.4) \qquad \Pi_1^* = \Pi^* + (1 + e^{-1})^{-1} (t_1 Q + t_2 \alpha^{-1} L + t_3 \beta^{-1} K + t_4 \gamma^{-1} S) - T_1 - T_2 - T_3 - T_4$$

Relation (2.4) suggests a criterion using the taxation scheme in such a manner that maximum profit with taxation remains same as maximum profit without taxation. In this manner taxation instrument could be used to let the firms move on to new equilibrium position without feeling the pinch. This can be achieved if we set the individual tax levels as follows:

$$(2.5) \qquad \begin{aligned} T_1 &= (1 + e)^{-1} t_1 Q \\ T_2 &= (1 + e)^{-1} t_2 \alpha^{-1} L \\ T_3 &= (1 + e)^{-1} t_3 \beta^{-1} K \\ T_4 &= (1 + e)^{-1} t_4 \gamma^{-1} S \end{aligned}$$

Remembering (2.3) we observe that the relations in (2.5) are first order differential equations which can be solved to obtain the corresponding tax functions as

$$(2.6) \qquad T_1 = A_1 Q^{(1 + e^{-1})}$$

$$T_2 = A_2 L^{\alpha}(1 + e^{-1})$$

$$T_3 = A_3 K^{\beta}(1 + e^{-1})$$

$$T_4 = A_4 S^{\gamma}(1 + e^{-1})$$

These tax functions are neutral in the sense that firms are supposed to retain their profit levels even after being taxed. Alternatively, one may like to use differential weights, positive or negative, to be associated with taxes realised corresponding to Q, L, K and S to correct any imbalances in the system. In that case the tax realisations can be expressed as

(2.7) $$t_1Q = \delta_1(1 + e^{-1})\,T_1$$

$$t_2L = \delta_2\,\alpha\,(1 + e^{-1})\,T_2$$

$$t_3K = \delta_3\,\beta\,(1 + e^{-1})\,T_3$$

$$t_4S = \delta_4\,\gamma\,(1 + e^{-1})\,T_4$$

The tax functions corresponding to this situation can be derived as

(2.8) $$T_1 = B_1\,Q^{\delta_1(1 + e^{-1})}$$

$$T_2 = B_2\,L^{\delta_2(1 + e^{-1})\alpha}$$

$$T_3 = B_3\,K^{\delta_3(1 + e^{-1})\beta}$$

$$T_4 = B_4\,S^{\delta_4(1 + e^{-1})\gamma}$$

Behavioural implications of such tax functions as in (2.6) and (2.8) can be examined easily. Let us consider the tax functions as in (2.6) and remember that if taxes are imposed in this manner then the firms would like to minimise their tax liability which gets done if the level of operation is maintained at a level where

(2.7) absolute value of e is minimum

α is minimum

β is minimum

γ is minimum

provided α, β and γ are positive. In most cases price elasticity of output, e, will follow the trend of price elasticity of demand and therefore mostly negative. In practice these need be examined corresponding to situation prevailing in each case. The markets where goods are needed to be supplied will have low values of e and therefore a producer who manages to meet the needs of consumers who need the good most will also achieve the objective of minimising tax liability on output, capital, labour and inventory. Tax liability on capital can be further reduced by operating at that point of the production function where β is lowest. Same holds good in case of labour and holding of stocks of finished product. In other words this mechanism of taxation will promote best utilisation of factors of production like labour and capital and produce enough to meet the needs of the consumers in relation to urgency of their needs. Firms which have been holding control on such means of production as land, capital, raw material etc., would get penalised automatically if taxation is geared to operational efficiency. The devices of associating weights like δ_1, δ_2, δ_3

and δ_4 in appropriate cases may serve similar purposes where undesirable practices are resorted to at higher levels and they need to be curbed down rather rapidly.

3. Estimation of Tax Functions

Quantification of tax functions in (2.6) can be done if we can estimate the corresponding constant and the parameters occurring in the exponent. Past information on taxes and Q, L, K or S may not be useful in estimating these functions because in that case these functions will lose their ability to function as corrective instruments. A better way could be to estimate the underlying economic structure and obtain the parameters from that. In any case the parameters should be those prevailing during the period of taxation. Parameter e needs to be computed for each product while parameters α, β and γ should be computed for each firm if one wants to be realistic.

An empirical exercise of this type was done by Misra [3] where industrial sector of the economy was considered owing to reasons relating to availability of relevant data. A simultaneous equations model of the industrial sector as estimated according to Two-Stage Least-Squares procedure is reproduced below:

(3.1)

$$D = -12.53 - 0.89P - 0.07Y_a + 3.73U \; ; \quad R^2 = 0.861$$

$$P = 4.66 + 1.55Q - 0.05P_a + 0.44P_r \; ; \quad R^2 = 0.996$$

$$Q = -1.16 + 0.01K + 0.03L + 0.02E \; ; \quad R^2 = 0.988$$

$$L = -5.29 - 9.79W_i + 0.23K + 8.67W - 3.22W_a ; \quad R^2 = 0.929$$

$$K = 0.13 + 4.56\,TD + 0.11\,FA - 4.51R + 2.98\,TC; \quad R^2 = 0.992$$

$$W_i = 1.59 - 1.29Q + 0.08K + 0.45W + 0.05P_c; \quad R^2 = 0.995$$

$$B = D - \dot{Q}$$

The measure R^2 reported here corresponds to the corrected model. The symbols in this model have the following meaning:

- D = Domestic demand for industrial products
- P = Industrial price index
- Y_a = Net domestic product of agriculture sector
- U = Urban population of the country
- P_a = Agricultural price index
- Q = Industrial production
- P_r = Price index of industrial raw materials
- K = Fixed capital employed in industrial sector
- L = Workers employed in industrial sector
- E = Energy consumed by industrial sector
- W_i = Wage index of workers employed in the industrial sector
- W = Total workers in the country
- W_a = Wage index of workers employed in the agricultural sector
- TD = Total deposit money with the public
- FA = Foreign aid
- R = Loan rate of long term borrowings of manufacturing units
- TC = Total currency with the public
- P_c = Consumer price index
- B = Balance of trade as measured by imports minus exports.

The parameters involved in the equilibrium condition and tax functions can be computed with the help of model (3.1) and the data on the respective variables for each one of the sample years. These values are reported below:

Table 1

Estimated Elasticity Coefficients

Year	e	e_1	e_2	α	β
1952-53	-5.11	-5.40	-8.27	0.82	0.06
1953-54	-5.25	-5.66	-7.66	0.86	0.07
1954-55	-4.37	-5.37	-7.62	0.78	0.07
1955-56	-3.95	-5.12	-6.80	0.76	0.07
1956-57	-3.47	-5.04	-5.84	0.72	0.08
1957-58	-3.04	-5.23	-4.98	0.68	0.08
1958-59	-3.48	-5.62	-4.65	0.67	0.10
1959-60	-2.58	-3.69	-2.59	0.72	0.12
1960-61	-2.63	-3.79	-2.28	0.69	0.13
1961-62	-2.15	-3.80	-1.99	0.61	0.13
1962-63	-2.02	-3.91	-1.26	0.59	0.18
1963-64	-1.15	-3.08	-1.13	0.65	0.19
1964-65	-1.82	-3.82	-0.83	0.54	0.24
1965-66	-1.96	-4.11	-0.68	0.53	0.28
1966-67	-2.02	-4.58	-0.64	0.50	0.33
1967-68	-1.48	-3.97	-0.56	0.45	0.28

These elasticity coefficients can be used to estimate the exponents involved in the tax functions as follows:

Table 2

<u>Estimates of Tax Function Exponents</u>

Year	$1+e^{-1}$	$\alpha(1+e^{-1})$	$\beta(1+e^{-1})$
1952-53	0.804	0.659	0.048
1953-54	0.810	0.697	0.057
1954-55	0.771	0.601	0.054
1955-56	0.747	0.568	0.052
1956-57	0.711	0.512	0.057
1957-58	0.672	0.457	0.054
1958-59	0.712	0.477	0.071
1959-60	0.612	0.441	0.073
1960-61	0.619	0.427	0.080
1961-62	0.534	0.326	0.069
1962-63	0.504	0.297	0.091
1963-64	0.130	0.085	0.025
1964-65	0.450	0.243	0.108
1965-66	0.489	0.259	0.137
1966-67	0.504	0.252	0.[illegible]66
1967-68	0.324	0.146	0.041

The estimates given in Table 2 show that exponents of tax functions differ over time. These can be computed in any specific case corresponding to taxation year under consideration. The constant terms involved in the tax functions can be estimated corresponding to some standard year. In the present case the industrial sector of the economy came closest to situation of profit maximisation during 1963-64. Therefore we may consider average of three years, namely, 1962-63, 1963-64 and 1964-65 as a situation pertaining to this year and treat this as the standard year. Data on production index, fixed capital and excise duty for these years are given below:

Table 3

Data on Production, Fixed Capital and Excise Duty

(Figures in Rs 10^9)

Year	Production Index	Fixed Capital	Excise Duty
1962-63	1.32	23.20	5.99
1963-64	1.54	27.84	7.30
1964-65	1.63	37.97	8.01
Total	4.49	89.01	21.30
Average	1.497	29.67	7.10

Data on excise duty are taken from Lakdawala and Nambiar [1] and on other variables as given in Misra [3] Average of exponents for these years is 0.361. Using these figures for the average year we can write tax function for output as

$$(3.2) \qquad \log 7.10 = \log A_1 + 0.361 \log 1.497$$

This gives $A_1 = 1.65$ and therefore the tax function on output can be expressed as

$$(3.3) \qquad T_1 = 1.65\, Q^{(1+e^{-1})}$$

Determination of tax function for fixed capital is rather difficult because there are no direct taxes imposed on this though it gets subsidy in certain cases. Assuming that the observed extent of capacity underutilisation is partly for speculative reasons, we may assemble together the available facts to quantify this tax function also. A valid criterion could be to evolve tax function on capital so that it deters tendency to acquire fixed capital for nonproductive purposes. This can be achieved if investment in fixed capital for nonproductive purposes is made costlier than keeping the money in banks. In the absence of any information on returns from speculation in fixed capital we may assume that it is almost same as return from investment in stock market which is around 15 per cent. If a tax of 10 per cent is imposed upon unutilised capital the net return would be around 5 per cent which could be as attractive as putting this money in fixed deposits.

It has been estimated by Shah [2] for the period under consideration that extent of nonutilisation of capacity in Indian industries is around 50 per cent. Using these estimates tax on fixed capital equal to 29.67 comes to around 1.48. Average value of exponents of capital for the same period as obtained from Table 2 is around 0.075. Therefore with these estimates we may obtain from the tax function on capital as given in (2.6), the following relation:

$$(3.4) \qquad \log 1.48 = \log A_3 + 0.075 \log 29.67$$

This gives $A_3 = 1.15$ and the corresponding tax function on capital can be written as

$$(3.5) \qquad T_3 = 1.15\, K^{\beta(1+e^{-1})}$$

We do not endeavour here to quantify tax functions for labour and stock holdings but if one has a reasonably good criterion to promote employment and discourage holding of stocks to certain extents, then, supportive additional researches may lead to quantification of these functions on similar lines as described above.

The proposed tax functions provide sufficient scope for accounting for the product market situation, technology in vogue, and efficiency of factor utilisation. The market situation can be advantageously used by the manufacturing sector. The sector can also figure out the appropriate technology and level of utilisation of factors to minimise tax liability. Taxation can be used effectively only if coefficients β and e are known accurately. The information can be used equally well by the taxing authority and the taxed undertaking to meet their individual objectives. Since differences persist from industry to industry, and firm to firm within an industry, it is desirable to work out tax function for individual firms to ensure inter and intra industry judiciousness. Global tax functions are likely to have a streamroller effect by treating efficient and inefficient firms in the same way whereas firm level tax functions will punish the inefficient more than the better managed units.

A good consequence of the aforesaid taxation policy may be to make the firms maintain an accurate and relevant information base. For instance, data on fixed capital as used in estimating β should be only the utilised component which is not available at present but it will be in the interest of the firm itself to maintain such data. Once data is maintained, it can be used to evolve a firm level policy. Such a decision making process will make the firm conform to the needs of society and, in the process, achieve maximum possible growth in its own business.

The problem of different rates of interest on capital loaned out to various sizes and types of industrial undertakings by several financial institutions gets resolved automatically according to the above taxation procedure. Assuming that smaller units can use their fixed capital more efficiently, the incidence of tax on fixed capital will tend to be lower in their case. Thus, total cost of capital as the sum of interest payment and tax payment will tend to be lower. This advantage will be available to all firms making more efficient use of their fixed capital. Thus, if interest rate is fixed for all kinds of undertakings, the taxation procedure will help the efficient units and punish the inefficient units in terms of total cost of investing in fixed capital assets. There does not appear any merit in cheaper loan on fixed capital account except for its effective use in production. In the absence of this criterion a cheaper capital may have inbuilt tendency to cover mismanagement.

References

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