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PRODUCTION FUNCTION IN INDIAN SUGAR INDUSTRY

by

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Kirit Patel

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ABSTRACT (within 250 words)

The purpose of the study is to (a) examine the degree of substitutability between labour and capital, (b) estimate returns to scale, (c) compute the factors' marginal productivities and relative contribution to output and to (d) test the predictive ability of the estimated relationship. These objectives are pursued with the aid of annual time series data for the period 1946 to 1966. Both inter-regional and inter-temporal comparisons have been attempted. The multiple regression technique is applied to various forms of the production functions. It is found that the elasticity of factor substitution is unity in the Indian Sugar Industry. It has experienced increasing returns to scale. Labour as a factor of production is more important both in terms of marginal productivity and contribution to the output and it is more efficient in all-India than in Uttar Pradesh and Bihar and more in Bihar than in Uttar Pradesh. All these findings imply that there is a good scope for employment of more labour and the expansion of sugar industry in India. The paper suggests that the output of a manufacturing industry can reasonably be forecasted through the multiple regression technique.

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Date 5th December, 1975

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Production Function in Indian Sugar Industry

by

G.S. Gupta and Kirit Patel*

Sugar industry is one of the very important industries in India. Its importance is in terms of the investment of country's scarce resources in it, magnitude of sugar production as proportion to world sugar production, sugar export from India, government controls in this industry, and so on. A study of the economics of this industry will therefore be highly useful. The present paper makes an attempt in this direction. To be specific, the paper purports to:

(a) examine the degree of factor substitutability and thus to see if the scarce capital could be economized in this industry;

(b) estimate returns to scale and thus to conclude as to whether the industry should be expanded or not;

(c) compute factors' marginal productivities to infer which factor should be encouraged more in industry's expansion;

(d) compute factors' relative contribution to find out what has contributed and by how much to output growth, and

(e) forecast output of sugar industry to see if supply of an industrial product could be forecasted through the production function estimation.

These objectives are pursued with the help of annual time-series data for the period 1946 to 1966, the last year for which all the required data are available. Both inter-regional and inter-temporal comparisons have been attempted. The former is carried through the study of the production function and productivity trends separately for all-India (all states), and for Uttar Pradesh and Bihar regions.[1] The latter comparison is attempted through the study of the subject-matter of this paper separately for the whole sample period, 1946-66, and for its two sub-periods, i.e. 1946-1958 and 1959-1966.[2] Although this kind of work exists in the literature but it is either out-dated or/ and subject to correctable imperfections. For example, Sastry's

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work is based on data for the period 1951 to 1961, while Murti and Sastry's work uses cross-section data in nominal terms for 1951 and 1952.[3] The results of our study are expected to be useful for all those responsible for the management and control of the Indian sugar industry.

Specification of Production Function

The production function expresses the technological or engineering relationship between various inputs and output. Output of an industry is a positive function of the quantities of land, labour and capital, and of organisation and technical progress. However, in a manufacturing industry like sugar, land usually does not pose any constraint for output expansion, for its requirement in relation to other factors of production is insignificant and it is usually a fixed factor of production or that its quantity cannot be increased significantly. Similarly, in a time-series study, which this paper purports to make, organisation ceases to explain variation in output, for it does not change. Thus, the relevant production function for a time series study of sugar industry in its unspecified form is of the following type:

$$O_t = f(L_t, K_t, T) \quad t = \overline{1;n} \qquad (1)$$

where O_t = output in period t

L_t = labour input in period t

K_t = fixed capital input in period t

T = technical progress variable

t = time subscript.

Variables and Their Measurements: Theoretically, a production function relates an output in real terms to its inputs in real terms. Furthermore, it expresses output in period t as a function of inputs of labour and capital services, and of the level of technological progress available in period t. Thus, it is a relationship between flow variables, as distinguished from stock variables. Thus, the estimation of function (1) requires a measure of each of the flow of final output, labour and of capital, and of technological progress.

Output is measured as a flow of goods and services during an accounting period. It is measured either in terms of value added (net output) or gross output. The two measures differ by raw-materials and other intermediate goods. If output is measured in terms of gross output, raw-materials, fuels and similar intermediate goods become an another input variable in the production function.

Although raw-materials and other intermediate goods cancel as inputs and outputs for the economy as a whole, they do not cancel within an industry. This argues for the use of gross output instead of value added and for use of raw-materials, etc. as a factor of production in our study. However, such a use renders other factors of production, viz. labour and capital, as insignificant determinants of output in empirical studies. Furthermore, if the coefficient of raw-materials, etc. variable is not significantly different from unity in the production function, the choice between two functions, namely net output as a function of labour, capital and technology, and raw-materials, etc., is immaterial.[4] In order to decide in favour of one of those functions, we have examined the value of the regression coefficient of raw-materials, etc. under the following partial specification:

$$y = a R^{b} \qquad (2)$$

where y = gross output of sugar and its by-products at constant prices

R = raw-materials, fuels, etc. at constant prices

a and b are parameters.

The regression results of function (2) are presented in Table 1.[5]

In Table 1 and in all other tables R^2 denotes the coefficient of determination, and DW, the Durbin-Watson statistic. Numbers in parentheses underneath parameters are the corresponding t-values. The t-values for coefficient b in Table 1 and coefficient b_2 in Table 3 correspond to the hypotheses that $b = 1$, and $b_2 = 1$, respectively; all other t-values throughout this paper correspond to the hypothesis of zero coefficient. One-star (*) throughout this paper means that the corresponding coefficient is significantly different from one or zero, as the case may be, by the t-test at the 5 per cent level of significance. These various statistics are noted down for each regression equation and their brief interpretation is attempted at appropriate places.

The results in Table 1 indicate that the coefficient of raw-materials, etc., i.e. b, is close to unity in all the nine regressions and that this coefficient is not significantly different from unity in eight of these equations. Furthermore, more than

95 per cent of variation in gross output is explained by variations in raw-materials etc. alone. These results lead us to select the value added concept of output and relate it with labour, capital and technology.

Labour input in real term is measured either in terms of the number of persons on the job or the number of man-hours worked during a period. If labour is measured as the number of persons on the job, it is not measured as a flow but as a stock in existence at a point of time, or averaged over several points within an accounting period. Labour measured in terms of man-hours is a flow variable, and thus it is in the same dimensions as output. Thus, we prefer the second measure to the first measure.

Capital is a stock variable but for production function we really need its flow concept. Thus, we need a measure of capital services. In some cases it may be possible to measure capital concumption or machine hours as estimates of input flows, but generally this is not the case. If the capital stock were utilised at a constant rate throughout the sample period, one could reasonably use capital stock as an input variable, for in that case capital services would be proportional to capital stock. Since capital utilisation rate has not been constant in Indian sugar industry, we cannot use capital stock as a measure of capital input in our production function. We need to correct this variable by multiplying it by capacity utilisation factor ( ). Thus, we have used K(K') as capital input in our production function. Since data on capacity utilisation in Indian sugar industry are not published, we have computed this on the basis of actual man-hours worked per worker as a fraction of the maximum man-hours worked per worker in any year during the whole sample period.

The technical progress in time-series studies is usually measured by time variable and there is no practical alternative to this. We have, therefore, used time variable (T) as a measure of technical progress.

<u>Form of the Production Function:</u> There are five different forms of production function in the literature; Leontief (fixed coefficients), linear, Cobb-Douglas (Double-log), constant elasticity of substitution (CES) and variable elasticity of substitution (VES) production functions. The Leontief function assumes were elasticity of substituti between factors of production, the linear function assumes factors to be perfect substitutes, and the Cobb-Douglas function has a unitary elasticity of factors' substitution. The CES and VES functions do

Table 1 : Production Function: $y = aR^b$

Region	Sample Period	Regression Coefficient (and t-ratio) log a	b	R^2	DW
All-India	1946-66	0.40 (4.73)	1.01 (0.53)	0.9934	1.78
	1946-58	0.62 (5.87)	0.95 (2.00)	0.9922	2.91
	1959-66	0.10 (0.27)	1.07 (0.96)	0.9726	1.46
Uttar Pradesh	1946-66	0.58 (6.88)	0.96 (1.8)	.9847	2.25
	1946-58	0.59 (3.94)	0.95 (1.2)	.9793	2.09
	1959-66	0.70 (4.18)	0.93 (1.70)	.9884	2.66
Bihar	1946-66	0.75 (8.37)	.89 (3.1) *	.9712	2.01
	1946-58	0.66 (6.43)	0.93 (1.6)	.9769	2.20
	1959-66	0.67 (3.08)	0.91 (1.1)	.9564	1.87

Note: * indicates that the corresponding coefficient is significantly different from unity by the two-tail t-test at 5% level.

not assume any specific value for the elasticity of substitution but as their names imply, the former takes this elasticity as a constant while the latter allows it to vary. Thus, the form of the productio function is determined by the elasticity of factor substitution.

The elasticity of factor substitution is a measure of the ease with which the varying factor can be substituted for others.[6] In practice, factors of production are substitutes but this relationshi is far from perfect. This rules out the Leontief and the linear forms of the production function. In order to choose one from the remaining three forms, one needs to conduct certain tests. In the first place, we need to examine as to whether the said elasticity variable or constant in the industry under study. To do this, we have estimated the following function:

$$\frac{V}{L} = a_1 \left(\frac{W}{L}\right)^{b_1} \left(\frac{K'}{L}\right)^{c_1} e^{d_1 T} \qquad (3)$$

where V = value added at constant prices

L = number of man-hours by workers

W = real wages

K' = capital stock adjusted for capacity unilisation at constant prices

T = time variable

and a, b, c, and d, are parameters.

In this function, the elasticity of factor substitution (6) i given by

$$6 = \frac{b_1}{1 - c_1 \left(1 + \frac{wL}{rK}\right)} \qquad (4)$$

where w = wage rate per man-hour

and r = capital rental.

Function (3) can be estimated by the Ordinary Least Squares (OLS method after log transformation. This function was estimated the Indian sugar industry. The time variable, in general, was found to be insignificant; all the results without time variable reported in Table 2.

Table 2: Variable Elasticity of Factor Substitution:

$$\frac{b_1}{1-c_1\left(1+\frac{wL}{rK}\right)}$$

Function: $\frac{V}{L} = a_1 \left(\frac{W}{L}\right)^{b_1} \left(\frac{K}{L}\right)^{c_1}$

Region	Sample Period	Regression Coefficient (and t-ratio) log a_1	b_1	c_1	R^2	DW
All-India	1946–66	0.61 (0.80)	0.26 (0.75)	0.34 (6.32) *	.7558	2.14
	1946–58	1.29 (0.48)	0.16 (0.36)	0.32 (1.84) *	.2734	2.33
	1959–66	1.18 (0.27)	-.01 (.013)	0.65 (2.33) *	.6900	1.23
Uttar Pradesh	1946–66	-1.24 (1.21)	0.62 (3.82) *	0.20 (2.84) *	.5732	2.06
	1946–58	-1.44 (1.04)	0.65 (3.11) *	0.20 (0.75)	.5203	2.12
	1959–66	0.92 (0.31)	0.29 (0.53)	0.17 (0.74)	.3300	1.96
Bihar	1946–66	-1.55 (0.71)	0.64 (1.88) *	0.14 (1.62)	.2483	2.02
	1946–58	-2.22 (0.68)	0.56 (1.34)	0.43 (0.88)	.2050	1.62
	1959–66	-3.52 (0.74)	1.11 (1.31)	-0.028 (0.09)	.2835	2.7

Note: * indicates that the corresponding coefficient is significantly different from zero by the one-tail t-test at 5% level.

It will be seen in Table 2 that only in one out of the nine equations, both the non-intercept coefficients are significantly different from zero. This means that only in one out of the nine equations, σ differs from zero or b_1 (a constant), for if $b_1=0, \sigma=0$ and if $c_1=0, \sigma=b_1$ (vide equation 4). This finding, thus, clearly rules out the possibility of a variable elasticity of factor substitution in the Indian cement industry. Furthermore, it suggests that this elasticity in either zero or some other constant.

A zero elasticity of substitution between labour and capital is inconceivable. In order to examine the value of the constant elasticity of facter substitution in sugar industry, we fitted the following function:

$$\frac{V}{L} = a_2 \left(\frac{W}{L}\right)^{b_2} e^{d_2 t} \tag{5}$$

In this function $\sigma = b_2$.[8] Again the inclusion of trend variable was found to be unwarranted. The estimation results of equation (5) without trend variable are provided in Table 3.

The regression results in Table 3 indicate that in none of the nine fitted equations, the elasticity of substitution coefficient (b_2) is significantly different from unity. A further examination of the b_2 coefficients would indicate that all the nine but one are significantly different from zero. These findings suggest that the elasticity of substitution between labour and capital is unity in the Indian sugar industry. This, in turn, means that the appropriate form of the production function under study is the Cobb-Douglas one.

<u>Estimation of Production Function</u>

The relationship between inputs and output variables in the Cobb-Douglas form is usually expressed in the following form:

$$V = A\, L^{\alpha}\, K'^{\beta}\, e^{\partial T} \tag{6}$$

Where constants α and β are respective input elasticities of output and ∂ is the coefficient of neutral technical progress; V is value added, L is man-hours worked, K' is fixed capital adjusted for capacity utilisation and T is trend variable; and e is natural exponent.

Table 3: Constant Elasticity of Factor Substitution: b_2

Function: $\frac{V}{L} = a_2 \left(\frac{W}{L}\right)^{b_2}$

Region	Sample Period	Regression Coefficient (and t-ratio) log a_2	b_2	R^2	DW
All-India	1946–66	-4.41 (1.81)	1.23 (0.42)	.2130	1.22
	1946–58	1.41 (0.47)	0.27 (1.51)	.027	1.81
	1959–66	-4.95 (1.06)	1.35 (0.47)	.3536	1.64
Uttar Pradesh	1946–66	-0.92 (0.77)	0.65 (1.84)	.3811	1.36
	1946–58	-1.09 (0.85)	0.66 (1.66)	.4930	1.92
	1959–66	-0.20 (0.08)	0.55 (1.17)	.2567	1.75
Bihar	1946–66	-0.79 (0.36)	0.63 (1.03)	.14	1.68
	1946–58	-0.44 (0.17)	0.56 (1.08)	.14	1.49
	1959–66	-3.46 (0.81)	1.07 (0.10)	.2823	2.19

Note: The numbers in parentheses are t-values corresponding to hypotheses that log a_2=0 and b_2=1.

Function (6) was estimated using OLS method for all-India, Uttar Pradesh and Bihar separately for the whole sample period (1946-1966), and its two sub-period, i.e. 1946-1958 and 1959-1966. Empirical results without technical progress variable (T) were preferable to those with neutral technical progress variable. The former results are presented in Table 4.

The estimation results in Table 4 are encouraging. As expected, exponents of both labour and capital inputs assume a positive value in all the nine regressions. Furthermore, ten out of a total of 18 such coefficients are significantly different from zero at the 5 per cent level of significance by the one-tail t-test. The value of the coefficient of determination (R^2) varies between a low of 0.5674 for Bihar's production function during 1946-1958 and a high of 0.9714 for production in all-India during 1959-1966. It is interesting to note that the labour and adjusted capital inputs explain a very high degree of variation in value added in all-India. The Durbin-Watson (DW) statistic generally assumes a value which rules out the presence of the first-degree autocorrelation in six of these nine equations and it is inconclusive for the remaining three equations, viz., all-India 1959-1966, Bihar 1946-1966, and Bihar 1946-1958.

Analysis and Application of Estimated Production Function

The estimated production functions as reported in Table 4 can be used to compute, if necessary, and analyse (a) factors' elasticities of output and returns to scale, (b) factors' marginal productivities, (c) each factor's relative contribution to mean output, and (d) forecasts of value added by sugar industry. All this is attempted in this section.

Elasticities of Output and Returns to Scale: In a double-log (Cobb-Douglas) function, elasticities are constants over the sample period and they are given directly by the regression coefficients. Thus, the estimated values of are the corresponding labour (man-hours) elasticities of output (value added) and those of are the corresponding capital (adjusted for capacity utilisation) elasticities of output (value added). The labour elasticity of production in Table 4 varies between a low of 0.34 in Uttar Pradesh during 1946-58 and a high of 1.41 in all-India during 1959-1966. The capital elasticity of output fluctuates between a low of 0.12 in Bihar during 1946-1966 and a high of 0.50 also in Bihar during 1959-1966. As expected, labour elasticity is generally greater than the corresponding capital elasticity; only in one out of nine cases, i.e. Bihar during 1946-1958, the opposite is true.

Table 4: Production Function $V = A L^{\alpha} K'^{\beta}$

Region	Sample Period	Regression Coefficient Log A	α	β	α+β	R^2	DW
All-India	1946-66	-1.92 (1.99)	0.91 (3.36) *	0.29 (3.32) *	1.20	.9373	1.95
	1946-58	-1.79 (0.99)	0.92 (1.88) *	0.23 (0.90)	1.15	.8402	2.27
	1959-66	-5.33 (4.53) *	1.41 (5.16) *	0.45 (4.26) *	1.86	.9714	1.32
Uttar Pradesh	1946-66	0.42 (0.51)	0.43 (1.97) *	0.28 (3.04) *	.71	.7367	2.00
	1946-58	0.72 (0.58)	0.34 (0.86)	0.32 (0.97)	.66	.5399	1.96
	1959-66	-3.15 (2.20) *	1.21 (3.60) *	0.26 (1.78)	1.47	.8389	2.53
Bihar	1946-66	-1.94 (2.45) *	0.99 (3.64) *	0.12 (1.29)	1.11	.6096	1.52
	1946-58	-1.15 (0.98)	0.41 (0.52)	0.50 (0.87)	.91	.5674	0.98
	1959-66	-3.60 (1.82)	1.27 (2.66) *	0.30 (0.94)	1.57	.6314	2.70

Note: * indicates that the corresponding coefficient is significantly different from zero by the one-tail t-test at 5% level.

Looking at the inter-temporal variations in labour-elasticity, one finds that it has increased over time in all the three regions. It increased from 0.32 to 1.41 in all-India, 0.34 to 1.21 in Uttar Pradesh, and o.41 to 1.27 in Bihar, between 1946-1958 and 1959-1966. The capital elasticity of output does not witness the similar trends. While it increased from 0.23 in 1946-1958 to 0.45 in 1959-1966 in all-India, it decreased from 0.32 to 0.26 in Uttar Pradesh and from 0.50 to 0.30 in Bihar between the same periods.

Analysing the inter-regional variations in factor elasticities, one notices that the labour elasticity of output was, in general, the highest in all-India and the least in Uttar Pradesh. A deviation to this is observed for the sample period 1946-1966, where Bihar had the highest labour elasticity. The capital elasticity fluctuates between the regions more violently than does the labour elasticity. For the sample period 1946-1966, it was the highest (0.29) in all-India and the least in Bihar (0.12); for sample period 1946-1958, it was the highest (0.50) in Bihar and the lowest in all-India (0.23); and for example period 1959-1966, it was the highest (0.45) again in all-India and the lowest in Uttar Pradesh (0.26). From this analysis, one can conclude that, in general, factor elasticities are greater in all-India than other two regions and they are greater in Bihar than in Uttar Pradesh.

In the Cobb-Dougles form of the production function, sum of factor elasticities, i.e. $(\alpha+\beta)$, gives an indication of returns to scale. These sums have been worked out for all the three regions and for all the three samples and the same are provided in Table 4, column heading $(\alpha+\beta)$. These sums vary between a low of 0.66 in Uttar Pradesh during 1946-1958 and a high of 1.86 in all-India during 1959-1966. Thus, there is evidence of both diminishing and increasing returns to scale. Of the nine values, six exceed unity and the remaining are less then unity. Thus, generally speaking, Indian sugar industry is operating under increasing returns to scale.

A careful look at the column heading ( ) in Table 4 would indicate that returns to scale have increased over time in all the three regions and that these increases are quite sizeable. An inter-regional comparison of returns to scale would indicate larger returns to scale in all-India than the other two regions and in Bihar than Uttar Pradesh. This finding suggests for a greater expansion of sugar industry in regions other than Bihar and Uttar Pradesh than in these two regions.

Marginal Productivities: In the Cobb-Douglas production function, marginal productivity of a factor of production is variable over time. It varies directly with the corresponding output-input ratio. In terms of our production function (6), the marginal productivities of labour and capital are given by

$$MP_L = \frac{\partial V}{\partial L} = \alpha\left(\frac{V}{L}\right) \qquad (7)$$

$$MP_K = \frac{\partial V}{\partial K} = \beta\left(\frac{V}{K}\right) \qquad (8)$$

where MP_L and MP_K denote marginal productivities of labour and capital, respectively.

Using these formulae, we computed MP_L and MP_K, at each observation and at the points of sample means for all the three regions and for all the three sample periods. The results of mean marginal productivities and the lower and the upper limit of these productivities are provided in Table 5.

Marginal productivity of labour is consistently larger than that of capital in all the regions and in all the sample periods. Recall that the same was true with respect to factor elasticities. Marginal productivity of labour has varied between 0.51 in Uttar Pradesh in some year during 1946-1958 and 3.91 in all-India in some year during 1946-1966. This means that at the margin the minimum contribution of one man-hour work to real value added by sugar industry was Rs.0.51 and that its maximum contribution was Rs.3.91. This finding looks very reasonable in the context of Indian sugar industry. Marginal productivity of capital fluctuated between 0.19 in all-India in some year during 1946-1966, and 1.81 in Bihar in some year during 1946-1958. This indicates that at the margin, the contribution of capital-in-use to value added was at least Rs.0.19 and almost it was Rs.1.81.

Like labour elasticity of output, marginal productivity of labour, in general, was higher in all-India than other regions and higher in Bihar than Uttar Pradesh. This suggests that a relatively larger use of labour in regions other than Bihar and Uttar Pradesh than in these two regions will be beneficiary. No region has higher marginal productivity of capital than other two regions in all the three sample periods. However, on the whole, Bihar seems to lead other regions in this respect.

Table 5: Factors' Marginal Productivities

(Rupees)

Region	Period	Marginal Productivity of Labour: Mean	Labour: Lower Limit	Labour: Upper Limit	Capital: Mean	Capital: Lower Limit	Capital: Upper Limit
All-India	1946-1966	2.51	1.42	3.91	0.31	0.19	0.73
	1946-1958	2.05	1.44	2.53	0.40	0.28	0.58
	1959-1966	3.07	3.17	6.05	0.38	0.30	0.43
Uttar Pradesh	1946-1966	1.03	0.69	1.33	0.42	0.24	1.11
	1946-1958	0.75	0.51	1.26	0.71	0.51	1.26
	1959-1966	3.19	2.79	3.75	0.29	0.22	0.39
Bihar	1946-1966	2.17	1.44	2.92	0.21	0.07	0.44
	1946-1958	0.86	0.60	1.08	1.32	1.02	1.81
	1959-1966	2.97	2.22	3.74	0.37	0.19	0.55

Marginal productivity of labour follows an upward trend while that of capital follows a downward trend in all the three regions. This points to greater use of additional labour input than capital input in augmenting value added by sugar industry in India.

Relative Contributions: The factor elasticities in Table 4 can be used to compute the relative contributions of each factor of production to the mean value of value added by sugar industry. To explain the computation procedure, let following be the estimated regression equation:

$$\log V = \widehat{\log A} + \hat{\alpha}\log L + \hat{\beta}\log K + \hat{u} \qquad (9)$$

where hat ($\wedge$) means estimates. Taking mean of (9) and then dividing both sides by mean of log V ($\overline{\log V}$), we obtain ($\hat{\overline{U}} = 0$):

$$1 = \widehat{\log A}\left(\frac{1}{\overline{\log V}}\right) + \hat{\alpha}\left(\frac{\overline{\log L}}{\overline{\log V}}\right) + \hat{\beta}\left(\frac{\overline{\log K}}{\overline{\log V}}\right) \qquad (10)$$

Thus, relative contributions are given by

$$R_L = \hat{\alpha}\left(\frac{\overline{\log L}}{\overline{\log V}}\right)$$

$$R_{K'} = \hat{\beta}\left(\frac{\overline{\log K'}}{\overline{\log V}}\right) \qquad (11)$$

$$R_c = \widehat{\log A}\left(\frac{1}{\overline{\log V}}\right)$$

where R_L, $R_{K'}$ and R_c denotes the relative contributions of labour, capital and constant term (i.e. omitted factors of production) to the mean level of value added, respectively.

Formulae (11) have been used to compute relative contributions of labour, capital, and other factors of production to mean value added by sugar industry. The results are presented in Table 6.

It would be seen from Table 6 that the contribution of labour to mean value of value added by Indian sugar industry has always been more than that of capital in all the three regions under study. Its

Table 6: Factors' Relative Contribution to Mean Value Added

(Percentages)

Region	Period	Relative Contribution of Labour	Capital	Others
All-India	1946-1966	68.86	26.31	4.83
	1946-1958	71.60	18.94	9.46
	1959-1966	102.20	46.35	-48.55
Uttar Pradesh	1946-1966	30.77	23.43	45.80
	1946-1958	25.03	22.82	52.15
	1959-1966	85.58	25.44	-11.02
Bihar	1946-1966	57.81	7.64	34.55
	1946-1958	24.04	21.81	54.15
	1959-1966	72.96	26.85	0.19

contribution to mean value of net output ranges between a low of 24.04 per cent in Bihar during 1946-1953 and a high of 102.20 per cent in all-India during 1959-1966. The contribution of capital to net value added varies between a low of 7.64 per cent in Bihar during 1946-1966 and a high of 46.35 per cent in all-India during 1959-1966. The contribution of other factors has fluctuated between -48.65 per cent and 54.15 per cent.

Factors' relative contributions have varied both with regions and time. Contribution of labour has been more in all-India than the other two regions and more in Uttar Pradesh than in Bihar. No such clear conclusion can be inferred about the contribution of capital. Contribution of both labour and capital have witnesses an increasing trend over time.

Forecasts: Regression results of Table 4 can be used to predict value added by sugar industry in different regions and the resulting forecasts, in turn, along with some other relations, can be used to forecast the supply of sugar and its by-products in India and in its two important regions, viz., Uttar Pradesh and Bihar. In terms of R^2, which is an important crieterion for judging the predictive efficiency of an equation, regression fits are very good only for all-India production function. Furthermore, the estimates are more reliable if they are obtained from a large sample size than if they are obtained from a large sample size then if they are obtained from a small sample size. Therefore, we shall restrict our forecasts for all-India only and we shall use the production function as estimated from annual time series data for the period 1946-1966. For convenience, we reproduce the selected production function for all-India during 1946-1966:

$$\log V = \underset{(1.69)}{-1.92} + \underset{(3.36)^*}{0.91} \log L + \underset{(3.32)^*}{0.29} \log K' \qquad (12)$$

$$R^2 = 0.9373, \; DW\text{-}1.95$$

In order to predict the future values of V with the aid of this estimated production function, we need to predict the magnitudes of independent variables L and K' in the prediction period. The latter forecaste have been obtained on the basis of the trend method. Several forms for each trend equation was tried, the best in terms of high R^2 are reported below:

$$L = 103.49 + 6.92\ T \qquad R^2 = 0.8432 \tag{13}$$

$$K' = 7.15\ e^{0.137\ T} \qquad R^2 = 0.9499 \tag{14}$$

where T = time trend, normalized to take a value of 1 in 1946, 2 in 1947, ... , and 21 in 1966.

The predicted values of the independent variables for the chosen prediction period, obtained by feeding the values of trend variable in equation (13) and (14) are presented below in Table 7.

Table 7 : Predicted Values of Independent Variables

Prediction period	All - India	
	Man-hours (L) (crores)	Capital adjusted for capacity utilization (K') (Rs.crores)
1967	25.56	144.56
1968	26.25	165.74
1969	26.95	190.01
1970	27.64	217.84
1971	28.33	249.75

The point forecasts of value added at 1960 prices by Indian sugar industry are then obtained by feeding the corresponding values of L and K' in equation (12). The so obtained point forecasts and the interval forecasts at 5 per cent significance level computed using the appropriate formula are presented below in Table 8.

Table 8: Forecasts of Value Added by Indian Sugar Industry

(Rs. crores)

Prediction period	Point Forecast	Interval Forecast (5% level)	
		Lower Limit	Upper Limit
1967	96.18	70.58	131.08
1968	102.49	74.88	140.30
1969	109.15	79.35	150.14
1970	116.17	84.00	160.65
1971	123.61	88.86	171.94

In order to get forecasts of gross sugar output, we correlated gross sugar output with value added by sugar industry. The estimated correlation equation was the following:

$$Y = \underset{(2.32)*}{12.55} + \underset{(33.02)*}{2.67}\,V \qquad (15)$$

$$R^2 = 0.9829$$

The forecasted values of value added at 1960 prices by sugar industry as reported above in Table 8 were fed to yield forecasts of sugar output at 1960 prices in India. The so obtained forecasts are presented below in Table 9.

Table 9: Forecasts of Sugar Output in India

(Rs. crores)

Prediction period	Point Forecast	Interval Forecast (5% level)	
		Lower Limit	Upper Limit
1967	269.34	248.40	290.28
1968	286.20	264.82	307.57
1969	303.97	282.10	325.84
1970	322.70	300.25	345.15
1971	342.57	319.46	365.69

Since sugar output data for our prediction period are not yet published, we are unable to check the accuracy of our forecasts. However, we do not pretend that our forecasts will be one hundred per cent correct. The predicted values could deviate from their corresponding true values for only one or more of the following reasons:

(a) predicted values of L and K' are different from their corresponding true values,

(b) estimated regression coefficients differ from their true values in the prediction period, and

(c) true disturbance term in the prediction period is different from zero.

A crude method to evaluate forecasts is to examine the accuracy of exanti (within sample period) forecasts. This is done on the basis of the percentage absolute mean error (PAME) criterion. The PAME of exanti forecasts is given by

$$PAME = \frac{100}{\bar{Y}} \frac{1}{n} \sum_{t=1}^{n} \hat{u}_t \quad \ldots \quad (13)$$

where n = sample size

$\hat{u}_t$ = estimated error in period t

$\bar{Y}$ = mean value of the variable under forecasting.

The computed PAME for value added by sugar industry in equation (12) is 2.4 per cent while that for gross output in sugar industry in equation (15) is 4.7 per cent. Both these errors are insignificant and thus our regression model is good in reproducing the history. If the structure of the economy does not undergo a significant change, our expost forecasts will also be good. If future is expected to deviate from past the forecasts given in Table 9 should be adjusted accordingly.

Conclusion

The study may be concluded by collecting together its main findings and inferring from them certain guidelines which may be recommended for further expansion of the Indian sugar industry:

(a) The industry is found to have zero neutral technical progress.

(b) The elasticity of factor substitution is found to be unity. This means that the factor ratio can be changed in proportion to change in the factor price ratio.

(c) The industry is found to be operating under increasing returns to scale. This implies that there is a scope for the further expansion of this industry.

(d) Returns to scale are the greatest in all-India and the least in Uttar Pradesh. This suggests that the industry should expand more in places other than Bihar and Uttar Pradesh, and more in Bihar than in Uttar Pradesh. This trend is, in fact, noticed in historical data.

(e) Labour is found to be more important factor than capital in terms of factor elasticity of output, marginal factor productivity and relative contribution to mean value added by sugar industry. This argues in favour of a relative increase in labour input for a larger value added by the industry.

(f) It is found that, in general, labour is marginally more efficient in all-India than in Uttar Pradesh and Bihar, and more in Bihar than in Uttar Pradesh. It may be pointed out that this is not because of heterogeneous labour in different regions but could be because of different labour quantities and/or location. This finding is supported by factor elasticities as well as relative contributions. This suggests that sugar industry in regions other than Bihar and Uttar Pradesh, on average, should become more labour intensive while that in Uttar Pradesh could become less labour intensive for improving the overall efficiency of labour.

(g) Estimated production functions can reasonably be used for forecasting industrial outputs, particularly sugar production.

Footnotes

The time series data for the whole period are published only for this regional classification, i.e. all-India, Uttar Pradesh and Bihar. The share of Uttar Pradesh and Bihar in India's total sugar production was 66.68 per cent and 18.32 per cent in 1946, 47.33 per cent and 16.40 per cent in 1958, and 36.40 per cent and 9.64 per cent in 1966, respectively.

The period classification is so chosen because the new data source, viz., Annual Survey of Industries, replaced the old data since, Census of Manufacturing Industries in the year 1959.

3. Sastry, V.S.R.K. (1966): Measurement of Productivity and Production Function in Sugar Industry in India, 1951-1961, Indian Journal of Industrial Relations, Vol.2, No.1, (July) pp.205-221.

4. This statement will be exactly correct if the two functions are linear and only approximately correct if the functions are non-linear.

5. All the data used in this paper, their source and measurement is given in Appendix.

6. Hicks, J.R. (1932): Theory of Wages, MacMillan, London, p.117.

7. Lu, Y.C., and L.Fletcher(1968): A Generalization of the CES Production Function; Review of Economics and Statistics, Vol.L, No.4 (Nov.), pp.449-52.
Ravankar, N.S.(1971): A Class of Variable Elasticity of Substitution Production Function, Econometrica, Vol.XXXIX, No.1 (January), pp.61-71.
It should be noted that these results are on the assumption of perfect competition and the constant returns to scale.

8. Ferguson, C.B.(1965): Time Series Production Functions and Technological Progress in Manufacturing, Journal of Political Economy, Vol.LXXIII, No.2 (April), pp.135-164. It should be noted that equation (5) is derived from the CES function on the assumption of perfect competition and constant returns to scale.

9. For interval forecasts, see Johnston, J. (1972): Econometric Methods, pp.152-5.

Appendix: Data Sources and Measurements

The data as used in the paper were compiled from the data as published in Census of Manufacturing Industries (CMI) and Annual Survey of Industries (ASI). The compilation procedure used for each variable is explained below.

The data on gross output of sugar and its by-products; value added by sugar industry; various raw-materials and fuel used etc. at current prices and in physical quantities are published in CMI and ASI. First of all from their 1960 data we obtained unit value of each output and input variable at 1960 prices. The quantities of various outputs and inputs in different years were then multiplied by their corresponding unit value in 1960. Thus, we obtained time series data on various outputs and inputs at 1960 prices. By aggregating various values at 1960 prices of all products and by-products of sugar year-wise we obtained time series of gross output in sugar industry at 1960 prices for different regions. Similarly, time series of values of raw-materials, fuels, etc. at 1960 prices were obtained for all the three regions. To obtain time series for value added at 1960 prices, value of raw-material, fuels, etc. at 1960 prices was subtracted from the corresponding values of gross output in sugar industry at 1960 prices.

The data on number of man-hours worked by direct workers is available for all the years. The same data by contract workers is available for most years. Furthermore, data on number of both direct and contract workers is available for all years. The data on number of man-hours worked by contract workers in those years, for which data were not available, were obtained by multiplying the number of such workers by the number of man-hours per direct worker in the corresponding year.

The data on book value of fixed capital is published in CMI and ASI. The time series of wholesale price index of machinery (P_m) and equipment, and that of construction activity (P_e) is available in Dholakia (1974) (vide Table 1-3). We first obtained a weighted average price index for fixed capital in sugar industry from above two indices; weights used were 0.70 and 0.30 during 1946-1956, and 0.75 and 0.25 during 1957-1966 for P_m and P_e, respectively. These weights were assigned on the basis of share of plant, machinery, equipment and other fixed assets; and land and construction in total fixed capital in sugar industry. The weights are changed in the year 1957, for that year marked the change in relative shares of two types of capital. The fixed capital in 1946 was deflated by this price index for 1946 and the additional fixed capital in successive years was first deflated by the corresponding year's price index and then was added to the previous period's real fixed capital to obtain time series for fixed capital in sugar industry at 1960-61 prices.

The data on total amount disbursed by way of wages in sugar industry is available in CMI and ASI. These data were divided by the corresponding number of man-hours worked by workers to obtain money wage per man-hour. The so obtained data were then deflated by wholesale sugar price index with base 1960-61 = 100 to obtain real wage rate.

No direct data on capacity utilisation in Indian sugar industry are published. We compiled this series on the basis of actual man-hours worked per worker as a fraction of the maximum man-hours worked per worker in any year during the whole sample period.

The above procedure was followed to generate all the data for all-India and all but 1959 data for Uttar Pradesh and Bihar. Since 1959 data for Uttar Pradesh and Bihar were not available to us, the same had to be interpolated. The shares of these states in all-India with respect to variable under interpolation in 1958 and 1960 were worked out and an arithmetic average of these shares was used to interpolate 1959 data for these states from 1959 data for all-India.

The so compiled data on all the variables for regions all-India, Uttar Pradesh, and Bihar are presented in Tables A-1, A-2, and A-3, respectively.

Table A-1 : Input-Output Data of Sugar Industry (all-India)

Year	Gross output at 1960 prices (Rs.crores)	Value added at 1960 prices (Rs.crores)	Man-hours (crores)	Fixed capital at 1960 prices (Rs.crores)	Rate of capacity utilisation	Adjusted fixed capital at 1960 prices (Rs.crores)	Raw-material at 1960 prices (Rs.crores)	Wage rate per hour at 1960 prices (paise)
1946	56.01	21.32	9.62	17.63	.5257	9.27	34.02	41.52
1947	65.90	19.92	12.73	18.95	.6705	12.71	44.42	43.12
1948	67.22	22.55	13.21	21.06	.6431	13.54	43.38	47.14
1949	77.66	27.95	12.94	25.06	.5736	14.37	48.34	46.36
1950	83.57	28.19	12.01	25.07	.4447	11.15	54.13	56.49
1951	108.07	32.31	16.13	26.33	.6363	16.75	74.27	46.77
1952	95.06	33.91	14.80	26.68	.5856	15.62	59.88	48.47
1953	72.61	22.84	12.25	26.53	.5150	13.63	48.49	59.00
1954	134.73	46.58	16.96	28.99	.6709	19.45	86.18	47.38
1955	135.55	41.84	18.97	32.10	.7102	22.80	92.25	51.40
1956	152.66	49.48	20.97	42.99	.7091	30.43	101.54	53.23
1957	150.81	48.63	19.50	55.92	.6469	36.17	99.38	51.57
1958	145.14	48.31	18.72	60.63	.6454	39.13	94.83	46.69
1959	143.82	40.48	18.00	68.60	.6308	43.27	100.45	48.25
1960	135.94	66.14	21.32	77.15	.9169	70.73	116.62	45.05
1961	231.33	78.98	24.65	83.73	.9802	82.07	148.87	51.59
1962	217.98	76.91	23.87	90.56	.9662	87.50	137.59	55.18
1963	165.60	60.49	19.69	94.38	.9580	90.42	102.20	56.59
1964	200.69	73.85	21.37	104.16	.9662	100.62	123.36	52.38
1965	251.63	90.64	24.24	110.39	1.0000	110.39	156.82	54.80
1966	296.18	107.89	25.12	117.92	.9777	115.28	184.50	57.04

Sources: 1. Compiled from the data in Central Statistical Organisation: Census of Manufacturing Industries and Annual Survey of Industries (Various Issues).

2. Dholakia, B.H.(1974): The Sources of Economic Growth in India,

Table A-2 : Input-Output Data of Sugar Industry (Uttar Pradesh)

Year	Gross Output at 1960 prices (Rs.crores)	Value added at 1960 prices (Rs.crores)	Man-hours (crores)	Fixed Capital at 1960 prices (Rs.crores)	Rate of capacity utilisation	Adjusted fixed capital at 1960 prices (Rs.crores)	Raw-material at 1960 prices (Rs.crores)	Wage rate per hour at 1960 prices (Paise)
1946	37.35	13.75	5.80	10.00	.5566	5.57	23.19	44.40
1947	42.20	15.77	7.74	10.42	.7328	7.64	25.97	45.52
1948	40.67	13.00	7.50	10.81	.3048	8.29	26.94	54.69
1949	37.32	12.54	6.95	11.99	.6108	7.32	24.17	51.31
1950	44.01	14.38	9.59	11.52	.6225	7.17	29.07	41.69
1951	65.09	18.88	10.13	12.54	.6642	8.33	45.43	48.56
1952	53.04	16.94	8.83	11.96	.5849	7.00	35.24	50.94
1953	43.73	13.98	7.68	12.14	.5263	6.39	29.07	52.48
1954	80.17	32.42	10.54	12.59	.6842	8.62	47.19	52.37
1955	75.39	23.68	10.32	14.14	.6768	9.57	50.82	58.32
1956	83.00	26.49	10.66	18.21	.6765	12.32	55.63	60.94
1957	73.72	23.67	10.20	21.47	.6424	13.79	49.02	57.89
1958	68.70	21.18	9.48	21.74	.6220	13.52	46.46	49.58
1959	79.50	21.57	9.35	23.09	.6188	14.29	50.77	49.64
1960	88.86	27.23	11.34	24.42	.9346	22.82	60.37	44.99
1961	107.65	34.47	12.17	25.80	.9682	24.92	71.83	54.55
1962	93.53	29.23	11.61	26.96	.9562	25.78	63.03	58.61
1963	66.68	21.72	9.10	27.39	.9412	25.78	43.97	58.69
1964	86.19	28.45	9.97	28.69	.9727	27.91	56.75	55.37
1965	96.42	30.02	11.15	29.99	1.0000	29.99	67.24	55.87
1966	107.83	34.57	11.15	31.28	.9731	30.44	71.97	59.46

Sources: 1. Compiled from the data in Central Statistical Organisation: Census of Manufacturing Industries and Annual Survey of Industries (Various Issues)

2. Dholakia, B.H.(1974): The Sources of Economic Growth in India, Good Companions, Baroda, p.196.

Table 1-3: Input-Output Data of Sugar Industry (Bihar)

Year	Gross output at 1960 prices (Rs.crores)	Value added at 1960 prices (Rs.crores)	Man-hours (crores)	Fixed capital at 1960 prices (Rs.crores)	Rate of capacity utilisation	Adjusted fixed capital at 1960 prices (Rs.crores)	Raw-material at 1960 prices (Rs.crores)	Wage rate per hour at 1960 prices (paise)
1946	10.26	3.29	2.26	4.10	.3785	1.55	6.29	37.77
1947	12.46	4.51	2.58	4.34	.4552	1.98	7.05	41.99
1948	13.42	4.86	2.87	4.95	.4830	2.39	7.77	45.55
1949	19.05	7.39	3.19	4.77	.5168	2.47	10.60	40.76
1950	17.46	6.06	2.43	4.08	.4375	1.79	10.32	60.51
1951	16.39	5.69	2.18	3.60	.4375	1.57	9.69	48.90
1952	20.97	6.98	2.95	3.71	.5201	1.93	12.60	46.35
1953	11.21	3.48	2.05	3.74	.3765	1.41	6.76	58.75
1954	20.91	5.41	2.68	4.03	.4966	2.00	14.09	50.19
1955	26.73	7.75	3.94	4.42	.6996	3.10	15.43	46.42
1956	24.44	7.11	3.95	4.89	.6996	3.42	15.70	46.68
1957	21.57	6.86	2.80	5.11	.5133	2.62	13.24	51.55
1958	23.81	7.39	2.81	5.15	.5179	2.67	14.80	47.83
1959	21.48	5.47	2.86	5.60	.5686	3.18	15.01	44.31
1960	25.23	6.68	3.57	6.07	.9567	5.81	18.17	37.01
1961	29.66	10.33	3.51	5.90	.9504	5.60	17.80	47.10
1962	27.77	7.76	3.58	6.47	.9423	6.09	18.52	46.36
1963	12.73	4.07	2.33	6.81	.9220	6.28	7.94	47.68
1964	18.79	6.73	2.47	6.93	.9163	6.35	11.18	46.42
1965	29.99	9.09	3.27	6.99	1.0000	6.99	19.27	50.80
1966	23.55	8.25	3.30	7.29	.9776	7.13	18.70	50.75