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ABSTRACT

When a price change is announced in an inventory system, a one time decision has to be made to purchase a large quantity Q' before the price change becomes effective to take advantage of current lower price. In this note, we consider a system having a limited storage capacity $W << Q'$, so that additional units are required to be stored in rented warehouse. Optimum value of Q' and corresponding gain are determined. The model is illustrated with an example.

INTRODUCTIN

In the classical EOQ system it is implcitly assumed that the inventory system under considerations has sufficient storage capacity to store the optimum order quantity, and that the unit cost remains unchanged during the period under considerations Naddor [4] has considered a model to determine optimum purchase quantity when the supplier announces a fixed price increase with effect from some given time instant T_*. In such a situation it is obvious that a large quantity Q' (say) should be purchased before T_*, in order to take advantage of present lower price. The inventory system under consideration has its own warehouse (OW) with a storage capacity of W units only. The present optimum order quantity does not exceed W. But, the special order quantity Q'>> W, in such a situation, the excess (Q'-W) units are required to be kept in a rented warehouse (RW), which is assumed to have sufficiently large storage capacity.

Inventory models with two levels of storage i.e. in which the optimum order quantity is required to be stored at OW and RW has been developed by Hartley [2], Sarma [5,6], Dave [1], and Murdeshwar and Sathe [3]. They have considered deterministic system with fixed price.

In this paper we consider the problem of determining a special order quantity in the face of the known price increase, when the system does not have sufficient storage capacity to store the one time special order quantity in OW.

ASSUMPTION AND NOTATIONS

i) The demand rate of R quantity units per time unit is known and constant.

ii) Lead time is zero, and shortages are not allowed.

iii) Replenishment rate is infinite. Replenishment size is the decision variable.

iv) The storage capacity of OW is W, and that of RW is infinite. If order quantity exceeds W, then excess units are kept in RW. Demands are satisfied from OW only.

v) The cost of transportation for K units from RW to OW in one transhipment is U(K).

vi) If I_1 is the carrying charge fraction for OW, and I_2 is the carrying charge fraction for RW, then $I_1 < I_2$.

Following notations are used:

R = Demand rate per time unit

d = Cost per unit at present

λ = Known price increase as of time instant T_0.

Q_0 = EOQ before price increase

Q_0^* = EOQ after price increase

Q' = Special order quantity before price increase becomes effective.

W = Storage capacity of OW

I_1 = Carrying charge fraction for OW.

I_2 = Carrying charge fraction for RW.

A = Replenishment cost per order.

T_0 = Effective time of price increase.

T_1 = Time when the special order gets depleted.

$U(K) = a + b(K-P)$, where $a \geq b > 0$ are constants.

The cost of transporting K units from RW to OW in one shipment, where 'P' is the maximum number of units which can be transported under the fixed charge 'a' and for every additional unit after 'P' a fixed charge 'b' is to be paid.

KNOWN PRICE INCREASE

Note that at present the optimum order quantity is:

$$Q_0 = \sqrt{2AR/dI_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

After the price change becomes effective the new price will be $(d+\lambda)$ per unit and the optimum lot-size will then be

$$Q_0^* = \sqrt{2AR/(d+\lambda)I_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Obiviously $Q_0^* < Q_0$ and less then or equal to W.

The problem for the management is to determine the size of the order, before the price change becomes effective. If a quantity Q' is purchased just before T_0, the next purchase will occur at T after an elapse (Q'/R) time units. All subsequent purchases will be made at the new price $(d+\lambda)$ and then the optimum order quantity is given by (2).

To determine the one time order size Q', to purchase prior to T_0. We maximize the cost difference during the period (T_0, T_1) with and without one time special order.

Assumed that Q'>> W, so that W units are kept in OW, and (Q'-W) units are kept in RW. Earlier, $Q_0 \leqslant W$, so that, the entire lot-size were kept in OW only.

Following Sarma [5] and Dave [1], the total cost due to the special order quantity during (T_0, T_1) is given by

$$C'(Q',K) = dQ' + I_2 d\, Q'^2/2R + (d(I_2 - I_1)/2R)\{W(W-K) + Q'(K-2W)\}$$
$$+ (U(K)/K)(Q'-W) + A \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Let C(Q) denote the total cost of the system during (T_0, T_1), when no special purchase is made just before T_0, and several purchases of Q_0^* are made at new price of $(d+\lambda)$, then the total cost during (T_0, T_1') is

$$C(Q) = (d+\lambda)Q' + (Q_0^*/2)[(d+\lambda)(I_1 Q'/R)] + A\, Q'/Q_0^*$$
$$- Q'[(d+\lambda) + C_0^*/R] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

where

$$C_o^* = \sqrt{2A(d+\lambda)I_1 R} \quad \ldots\ldots (5)$$

To find the optimal value of K and Q', we maximize the differences between C(Q) and C'(Q',K). This difference, denoted by G(Q',K) is given by

$$G(Q',K) = Q'(\lambda + C_o^*/R - I_2 d\, Q'/2R)$$
$$- (d(I_2 - I_1)/2R)[W(W - K) + Q'(K - 2W)]$$
$$- (U(K)/K)(Q' - W) - A \quad \ldots\ldots (6)$$

For the optimum values of K_o and Q_o'

$$\frac{\partial G(Q',K)}{\partial K} = 0 \quad \text{and} \quad \frac{\partial G(Q',K)}{\partial Q'} = 0$$

give

$$K_o = \sqrt{2R(a-bP)/d(I - I)} \quad \ldots\ldots (7)$$

$$Q_o' = [R\lambda + C_o^* + Wd(I_2 - I_1) - Rb - \sqrt{2R(a-bP)d(I_2-I_1)}\,]/I_2 d \quad \ldots\ldots (8)$$

The optimum gain is

$$G_o(Q_o',K_o) = (R\lambda^2/2I_2 d) + (C_o^*\lambda/I_2 d) + (A\lambda/d) + bW$$
$$+ ((I_2-I_1)/I_2)\{(a - bP) + W/R[\lambda + C_o^* - WdI_1/2 - Rb]$$
$$+ (\sqrt{2R(a-bP)(I_2-I_1)d}\,/\,I_2 d)[WdI_1/R - \lambda - C_o^*/R + b]$$
$$- (b/I_2 d)[\lambda R + C_o^* - Rb/2] \quad \ldots\ldots (9)$$

Note that when $I_2 = I_1$, and U(K)--->0, then $A = C_3$ and the total maximum gain equation of the system becomes

$$G_o(Q_o') = R\lambda^2/I_1 d + C_o^*\lambda/I_1 d + C_3\lambda/d \quad \ldots\ldots (10)$$

and $Q_o' = (C_o^* + R\lambda)/I_1 d$ .... (11)

Equations [10] and [11] are the same those given by Naddor [4] for the single storage model.

For the feasibility of the two storage facilities model, we should have $Q_0' > W$ and $K \leq W$.

Numerical Example

We discuss the results obtained with the help of a hypothetical numerical example.

Consider an inventory systems with following parameter values:

$R = 10000$ units per annum, $d = $ Rs. 10 per unit $\lambda = 1(0.25)3$,

$I_1 = 0.2$, $I_2 = 0.4$, $A = $ Rs. 100, $a = $ Rs 100, $W = 1000$ units and $P = 1000$ units per transhipment.

In this case, $Q_0 = 1000$, $K_0 = 1000$ units $= P$ for $\lambda = 1(0.25)3$ the values of Q_0^*, C_0^*, Q_0' and $G_0 = G_0(Q_0', K_0)$ are shown in the Table 1. From Table 1, we find that the optimum values of Q_0' and $G_0(Q_0', K_0)$ increase with λ.

In the above case we have assumed $W = 1000$ units. For different warehouse capacity taking $\lambda = 1$, we have $Q_0 = 1000$ units, $Q_0^* = 953$ units, $K_0 = 1000$ units and $C_0^* = $ Rs 2098. For different values of W, the corresponding optimum values of Q_0' and $G_0(Q_0', K_0)$ are exhibited in Table 2. Where we observe that the Values of Q_0' and $G_0(Q_0', K_0)$ increase with W.

Finally, we study the effect of variations in both λ and W on the values of Q_0' and $G_0(Q_0', K_0)$. In each entry of Table 3, the upper value denotes Q_0' and the figures in paranthesis indicates the optimum gain $G_0(Q_0', K_0)$. From the Table we observe that for any given W, values of Q_0' and $G_0(Q_0', K_0)$ increase with increase in λ. However, for any given λ, the value of Q_0' increase with

Table 1: Values Q_o^*, C_o^*, Q_o' and $G_o(Q_o', K_o)$ with increasing λ

λ	Q_o	C_o^*	Q_o'	$G_o(Q_o', K_o)$
1.00	953	2098	3024	1384
1.25	943	2121	3655	2104
1.50	933	2145	4286	2982
1.75	923	2168	1917	4019
2.00	913	2191	5548	5216
2.25	904	2214	6178	6571
2.50	894	2236	6809	8085
2.75	886	2258	7440	9758
3.00	877	2280	8070	11590

Table 2: Values of Q_o, Q_o^*, K_o, C_o^*, Q_o' and $G_o(Q_o', K_o)$ with different values of W and constant λ.

W	Demand	λ	Q_o	Q_o^*	K_o	C_o^*	Q_o'	$G_o(Q_o', K_o)$
1000	10000	1	1000	953	1000	2098	3024	1384
1100	10000	1	1000	953	1000	2098	3074	1394
1200	10000	1	1000	953	1000	2098	3124	1403
1300	10000	1	1000	953	1000	2098	3174	1411
1400	10000	1	1000	953	1000	2098	3224	1418
1500	10000	1	1000	953	1000	2098	3274	1424
1600	10000	1	1000	953	1000	2098	3324	1429
1700	10000	1	1000	953	1000	2098	3374	1433
1800	10000	1	1000	953	1000	2098	3424	1436
1900	10000	1	1000	953	1000	2098	3474	1438
2000	10000	1	1000	953	1000	2098	3524	1439
2500	10000	1	1000	953	1000	2098	3774	1429
3000	10000	1	1000.	953	1000	2098	4024	1394

Values of Q^*_o , $G_o(Q'_o, K_o)$ and Q'_o with varying capacity
owned warehouse(W) and known price change(λ).

	Values of λ								
	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00
	953	943	933	923	913	904	894	886	877
	3024 (1384)**	3655 (2104)	4286 (2982)	4917 (4019)	5548 (5216)	6178 (6571)	6809 (8085)	7440 (9758)	8070 (11590)
	3074 (1394)	3705 (2114)	4336 (2992)	4967 (4030)	5598 (5226)	6228 (6581)	6859 (8096)	7490 (9769)	8120 (11601)
	3124 (1403)	3755 (2123)	4386 (3001)	5017 (4039)	5648 (5235)	6278 (6591)	6909 (8106)	7540 (9779)	8170 (11611)
	3174 (1411)	3805 (2131)	4436 (3010)	5067 (4047)	5698 (5244)	6328 (6600)	6959 (8114)	7590 (9788)	8220 (11620)
	3224 (1418)	3855 (2138)	4486 (3017)	5117 (4055)	5748 (5251)	6378 (6607)	7009 (8122)	7640 (9796)	8270 (11628)
	3274 (1429)	3905 (2144)	4536 (3023)	5167 (4061)	5798 (5258)	6428 (6614)	7059 (8129)	7690 (9802)	8320 (11635)
	3324 (1431)	3955 (2149)	4586 (3028)	5217 (4066)	5848 (5263)	6478 (6619)	7109 (8134)	7740 (9808)	8370 (11641)
[illegible]	3374 (1433)	4005 (2153)	4636 (3032)	5267 (4071)	5898 (5268)	6528 (6624)	7159 (8139)	7790 (9813)	8420 (11646
[illegible]	3424 (1436)	4055 (2157)	4686 (3036)	5317 (4074)	5948 (5271)	6578 (6628)	7209 (8143)	7840 (9817)	8470 (11650
[illegible]	3474 (1438)	4105 (2159)	4736 (3038)	5367 (4076)	5998 (5274)	6628 (6630)	7259 (8145)	7890 (9820)	8520 (11653
[illegible]	3524 (1439)	4155 (2160)	4786 (3039)	5417 (4078)	6048 (5275)	6678 (6632)	7309 (8147)	7940 (9821)	8570 (11655
0	3774 (1429)	4405 (2150)	5036 (3030)	5667 (4069)	6298 (5268)	6928 (6625)	7559 (8141)	8190 (9815)	8820 (11649
0	4024 (1394)	4655 (2116)	5286 (2996)	5917 (4036)	6548 (5235)	7178 (6592)	7809 (8109)	8440 (9784)	9070 (11619

s in paranthesis indicates value of $G_o(Q'_o, K_o)$

increase in W, but the optimum gain increases only marginally upto certain fixed value of W and then it decreases, e.g. in this case the value of $G_{e}(Q_{e}', K_{e})$ increase up to W is 2000, and then starts decreasing.

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