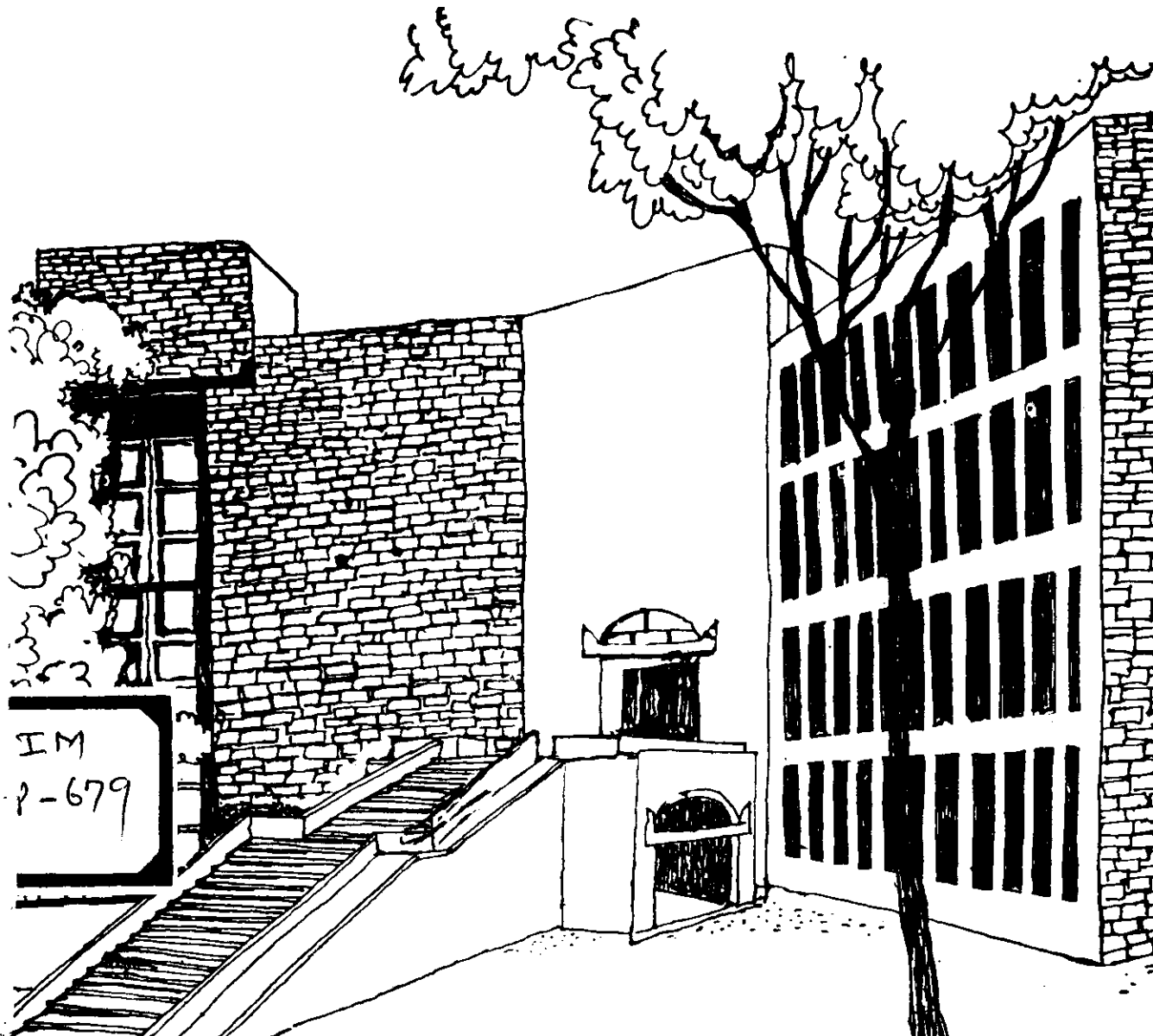




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OPTIMAL MIX OF COUPONS IN A MESS SYSTEM

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## OPTIMAL MIX OF COUPONS IN A MESS SYSTEM

### ABSTRACT

Most organisations operate staff mess or canteen facility for its employees where payments for food items are made with money coupons rather than cash. In such coupon-based transaction system, the management needs to decide the denominations in which the coupons should be printed and also the proportion in which these denominations should be combined to form a coupon book which is made available to the employees. These decisions are made with the twin objectives of minimising the total number of coupons used as well as ensuring that most transactions are feasible. This paper reports the details of the method used for studying such a problem in a specific organisation. The study arrives at an optimal mix of coupons which leads to a significant improvement over the existing practice.

## OPTIMAL MIX OF COUPONS IN A MESS SYSTEM

### INTRODUCTION

Most people while going out shopping not only ensure that they have adequate money to make the purchases, but also try to ensure that they carry the amount in several denominations so that each transaction need may be met. Obviously, the surest way of ensuring the feasibility of completing all transactions would be to carry the entire amount in the lowest possible denomination of 5 paise. But that would be too cumbersome. Hence an attempt is made to project the actual need for various transactions and then carry the appropriate amount of various denominations of money. This problem of day to day faced by individuals arises in several other systems too.

For example, in the transportation system, a bus conductor would like to carry a small set of distinct denominations of tickets to be able to meet a much larger variety of fares that actually arise. The trade-off is between ease of completing a transaction and containing the variety of denominations used. The central bank of a country has to decide the number of notes of various denominations to be printed, depending on the anticipated need of the people to complete various transactions. This paper reports work done on a similar problem, connected with

the operation of a mess system.

The Indian Institute of Management, Ahmedabad employs over 600 persons and provides them dining facility at subsidised rates. This facility, known as the staff mess serves breakfast, lunch and dinner on all the seven days of a week. In addition, tea and coffee are served everyday for about an hour each in the morning and afternoon, during working hours of the institute. Since no cash transactions are allowed, individuals pay for their purchases through coupons. These coupons are printed by the institute in denomination of 5, 10, 25, 30 paise and Re.1, and are packed in two kinds of coupon books, one of Rs.20 and the other of Rs.40. The employees buy these coupon books by paying cash and subsequently use the coupons to purchase food items from the mess. The mess staff is allowed to accept payment only in the form of coupons. They are also not allowed to return coupons, implying that an exact amount has to be rendered for the purchase of a food item. The purpose of the present study was to determine:

- (a) The denominations of coupons that should be printed.
- (b) The optimal proportion of coupons of the denominations arrived at in (a) in a coupon book.

While determining the optimal combination, the following objectives were sought to be achieved:

- (a) Minimize the overall number of coupons used.
- (b) Provide sufficient number of 5 paise coupons to ensure

feasibility of payment for all transactions.

#### METHODOLOGY

The denominations to be printed and the proportions of these denominations to be used in forming a coupon book would depend upon the pattern of requirement of coupons. To arrive at this pattern, sample data on value of transactions was collected for a period of seven days for breakfast, lunch and dinner. The frequency distribution of the value of transactions is given in Appendix 1. In addition to these, the mess supervisor provided us with an estimate of tea and coffee consumption as 600 cups and 400 cups respectively on working days, and 250 cups and 150 cups respectively on weekends. The coupon book finally designed should be based on the aggregate pattern of requirement.

Since tea and coffee cost 25 and 30 paise a cup respectively, it was assumed that all transactions on tea and coffee would be done through use of 25 and 30 paise coupons. As tea and coffee account for a substantial number of transactions, it was decided that final set of denominations to be used must include 25 and 30 paise coupons. Similarly, since most lunch and dinner transactions were between Re.1 and Rs.2, it was thought imperative to include Re.1 in the final set of denominations. Finally, since every transaction becomes feasible if 5 paise coupons are made available, even this denomination had to figure in the final set. Therefore, any choice of combination of

denominations must include Re.1, and 5, 25, 30 paise coupons.

The other optional denominations considered were 10, 20 and 50 paise. This gave rise to the following eight combinations which were investigated further.

| <u>Combination</u> | <u>Denominations</u>              |
|--------------------|-----------------------------------|
| 1                  | Re.1, Paise 30, 25, 5             |
| 2                  | Re.1, Paise 50, 30, 25, 5         |
| 3                  | Re.1, Paise 30, 25, 20, 5         |
| 4                  | Re.1, Paise 30, 25, 10, 5         |
| 5                  | Re.1, Paise 50, 30, 25, 20, 5     |
| 6                  | Re.1, Paise 50, 30, 25, 10, 5     |
| 7                  | Re.1, Paise 30, 25, 20, 10, 5     |
| 8                  | Re.1, Paise 50, 30, 25, 20, 10, 5 |

In order to compute the number of coupons used for a particular transaction, it was necessary to assume the manner in which a coupon holder would tear coupons from the coupon book. It was assumed that he would tear coupons in lexicographical order rather than minimizing the number of coupons torn. This point is elaborated further through an example.

Suppose combination 7 is implemented, that is, a coupon book consists of coupons in the denominations of Re.1 and 30, 25, 20, 10 and 5 paise. Now suppose a transaction of Rs.1.40 takes place, then it was assumed that the user would complete this transaction by giving one coupon each of Rs.1, 30 paise and 10 paise. It is obvious that for this particular transaction, at least 3 coupons have to be used, and hence the above mix of



coupons would also be the optimal mix as it minimizes the number of coupons needed to complete the transaction.

Now take a case of a transaction of Rs. 1.45. As per lexicographical choice, this transaction would be completed by using one coupon each of Re.1, and 30, 10 and 5 paise, thereby requiring a total of 4 coupons. Whereas it would be easily seen that the same transaction can be completed by using only 3 coupons, namely one each of Rs.1, 25 paise and 20 paise. Therefore, lexicographical procedure does provide a feasible but not necessarily an optimal solution.

In order to arrive at the optimal solution, one can formulate the problem as follows:

Suppose a transaction of  $t$  paise is to be made, then the problem is to:

$$\begin{aligned} &\text{minimize } x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \\ &\text{s.t. } 100x_1 + 30x_2 + 25x_3 + 20x_4 + 10x_5 + 5x_6 = t \end{aligned}$$

where  $x_i$  are non-negative integers representing the number of coupons of  $i^{\text{th}}$  denomination used. This formulation is a special case of the well-known Knapsack problem where the constraint is an equality. The problem could be solved by one of the traditional methods to obtain the minimum number of coupons. It should be noted the above problem always has a feasible solution since  $t$  is an integral multiple of 5 paise.

We however did not solve all such Knapsack problems primarily because of the fact that most users would not necessarily optimize on number of coupons used while paying for their purchases. Since coupons are generally arranged in the descending order of denominations, lexicographical approach is likely to be followed in most transactions. It would be worthwhile pointing out that for most transactions the lexicographical approach also minimizes the number of coupons used.

#### RESULTS AND ANALYSIS

We worked out the total weekly requirement of coupons of each type for all the eight combinations described earlier. The results are shown in Table 1. From the Table, we can observe that combination 1 though uses the least types of coupons, will require the maximum number of coupons. Whereas, combination 8 which uses all types of coupons requires the minimum number of coupons. Combinations 3 and 6 are just as good as combination 8, but combination 3 requires only 5 types of coupons, viz. coupons of Re.1, Paise 30, 25, 20 and 5.

|         |
|---------|
| TABLE 1 |
|---------|

Since coupons have to be assembled in the form of a coupon book, we assumed that a coupon book would continue to have coupons worth Rs.40 (the existing practice) and tried to fit

coupons of various denominations in the proportion obtained in Table 1 for a comparison of the number of coupons required. Table 2 gives the number of coupons required of each type of denomination for all the eight combinations.

TABLE 2

From the Table , it is clear that combination 3 is perhaps the best as the number of coupons required is 92 which is the minimum of all and it requires only 5 types of denominations. However, it does not serve the second objective as the number of 5 paise coupons is only five. With such a small number of 5 paise coupons, it is quite likely that a user may soon find his/her booklet void of 5 paise coupons. It could then become difficult for the user to make further transactions (unless the person decides to overpay). Therefore, it was thought that it would be necessary to provide sufficient percentage of 5 paise coupons in each booklet. Hence the composition of a booklet with a given percentage of 5 Paise coupons were determined. While doing so, the proportions of coupons of other denominations were left unchanged. The resulting composition of Rs.40 booklet for a minimum of 10%, 15%, and 20% coupons of 5 paise, are given in tables 3, 4, 5 respectively.

TABLES 3 4 & 5

It is clear from the Table 3 that if we wish to provide for at least 10% coupons of 5 paise, then one can choose either of the combinations 3, 6 or 8 as they require the minimum number of coupons. Since combination 3 uses the least number of denominations, the best policy would be to print coupons in combination 3. Similarly, if we wish to provide at least 15% coupons of 5 paise, then though combination 8 uses the least number of coupons (Table 4), it may be more suitable to adopt a policy of using coupons in combination 3 to reduce the variety of denominations used. Finally, if 20% coupons of 5 paise are to be provided, the best policy as can be seen from Table 5 is to use combination 5 which uses 6 different denominations and requires 99 coupons.

The current coupon book of Rs.40 has the following composition:

| <u>Denomination</u> | <u>Number of coupons</u> |
|---------------------|--------------------------|
| Re. 1               | 20                       |
| Paise 30            | 30                       |
| Paise 25            | 30                       |
| Paise 10            | 20                       |
| Paise 5             | 30                       |
|                     | 130                      |

Since both the combinations have about 20% coupons of 5 paise, they would have roughly the same ease of completing the

transactions. A comparison of the two compositions reveals that we can reduce a booklet of 130 coupons to about 100 coupons, which is a net savings of about 25% in terms of the number of coupons used.

#### CONCLUSION

In this paper we studied the usage of coupons in a mess system. Various combinations of denominations were considered to identify the best combination which reduced the number of coupons needed to complete all the transactions and also ensured feasibility of transacting in any amount by providing sufficiently large number of coupons of the lowest denomination. The composition of the coupon book finally suggested is likely to reduce the number of coupons used by about 25%. This would not only reduce the cost of printing coupons, but would also result in significant savings in time needed for completing a transaction, thereby reducing the waiting time at the mess, especially during the peak periods.

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TABLE 1  
WEEKLY REQUIREMENT OF COUPONS FOR EACH COMBINATION

| COMBINATION | DENOMINATION |              |              |              |              |              |              | TOTAL<br>NO. OF<br>COUPONS |
|-------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|----------------------------|
|             | Re.<br>1     | Ps.<br>50    | Ps.<br>30    | Ps.<br>25    | Ps.<br>20    | Ps.<br>10    | Ps.<br>5     |                            |
| 1           | 3464<br>0.2  | ---          | 5564<br>0.32 | 3988<br>0.23 | ---          | ---          | 4516<br>0.25 | 17532<br>1.00              |
| 2           | 3464<br>0.21 | 1388<br>0.08 | 3398<br>0.21 | 3843<br>0.23 | ---          | ---          | 4357<br>0.27 | 16450<br>1.00              |
| 3           | 3464<br>0.24 | ---          | 5564<br>0.38 | 3988<br>0.27 | 927<br>0.06  | ---          | 808<br>0.05  | 14751<br>1.00              |
| 4           | 3464<br>0.22 | ---          | 5564<br>0.36 | 3988<br>0.25 | ---          | 1854<br>0.12 | 808<br>0.05  | 15678<br>1.00              |
| 5           | 3464<br>0.22 | 1388<br>0.1  | 3398<br>0.22 | 3843<br>0.24 | 293<br>0.02  | ---          | 3185<br>0.2  | 15571<br>1.00              |
| 6           | 3464<br>0.24 | 1388<br>0.09 | 3398<br>0.23 | 3843<br>0.26 | ---          | 1702<br>0.12 | 953<br>0.06  | 14748<br>1.00              |
| 7           | 3464<br>0.22 | ---          | 4786<br>0.3  | 3843<br>0.24 | 1681<br>0.11 | 1116<br>0.07 | 953<br>0.06  | 15843<br>1.00              |
| 8           | 3464<br>0.24 | 1388<br>0.09 | 3398<br>0.23 | 3843<br>0.27 | 293<br>0.02  | 1116<br>0.08 | 953<br>0.07  | 14455<br>1.00              |

NOTE : The decimal figures indicate the proportions of coupons of denomination used.

TABLE 2  
COMPOSITION OF COUPON BOOK FOR EACH COMBINATION

| COMBINATION | DENOMINATION |           |           |           |           |           |          | TOTAL<br>NO. OF<br>COUPONS |
|-------------|--------------|-----------|-----------|-----------|-----------|-----------|----------|----------------------------|
|             | Rs.<br>1     | Ps.<br>50 | Ps.<br>30 | Ps.<br>25 | Ps.<br>20 | Ps.<br>10 | Ps.<br>5 |                            |
| 1           | 22           | ---       | 35        | 25        | ---       | ---       | 25       | 107                        |
| 2           | 22           | 8         | 22        | 24        | ---       | ---       | 28       | 104                        |
| 3           | 22           | ---       | 35        | 25        | 5         | ---       | 5        | 92                         |
| 4           | 22           | ---       | 35        | 25        | ---       | 11        | 3        | 96                         |
| 5           | 21           | 10        | 22        | 24        | 2         | ---       | 20       | 99                         |
| 6           | 22           | 8         | 22        | 24        | ---       | 11        | 6        | 93                         |
| 7           | 22           | ---       | 30        | 24        | 11        | 7         | 2        | 96                         |
| 8           | 22           | 8         | 21        | 25        | 2         | 7         | 7        | 92                         |

TABLE 3  
COMPOSITION OF COUPON BOOK WITH 10 % COUPONS OF 5 PAISE

| COMBINATION | DENOMINATION |           |           |           |           |           |          | TOTAL             |
|-------------|--------------|-----------|-----------|-----------|-----------|-----------|----------|-------------------|
|             | Rs.<br>1     | Ps.<br>50 | Ps.<br>30 | Ps.<br>25 | Ps.<br>20 | Ps.<br>10 | Ps.<br>5 | NO. OF<br>COUPONS |
| 1           | 22           | ---       | 35        | 25        | ---       | ---       | 25       | 107               |
| 2           | 22           | 8         | 22        | 24        | ---       | ---       | 28       | 104               |
| 3           | 22           | ---       | 35        | 24        | 5         | ---       | 10       | 96                |
| 4           | 22           | ---       | 35        | 24        | ---       | 10        | 10       | 101               |
| 5           | 21           | 10        | 22        | 24        | 2         | ---       | 20       | 99                |
| 6           | 22           | 9         | 20        | 24        | ---       | 10        | 10       | 95                |
| 7           | 22           | ---       | 29        | 24        | 11        | 6         | 10       | 102               |
| 8           | 22           | 8         | 21        | 24        | 2         | 8         | 10       | 95                |



TABLE 4  
COMPOSITION OF COUPON BOOK WITH 15 % COUPONS OF 5 PAISE

| COMBINATION | DENOMINATION |           |           |           |           |           |          | TOTAL<br>NO. OF<br>COUPONS |
|-------------|--------------|-----------|-----------|-----------|-----------|-----------|----------|----------------------------|
|             | Rs.<br>1     | Ps.<br>50 | Ps.<br>30 | Ps.<br>25 | Ps.<br>20 | Ps.<br>10 | Ps.<br>5 |                            |
| 1           | 22           | ---       | 35        | 25        | ---       | ---       | 25       | 107                        |
| 2           | 22           | 8         | 22        | 24        | ---       | ---       | 28       | 104                        |
| 3           | 22           | ---       | 34        | 25        | 4         | ---       | 15       | 100                        |
| 4           | 22           | ---       | 36        | 21        | ---       | 12        | 15       | 106                        |
| 5           | 21           | 10        | 22        | 24        | 2         | ---       | 20       | 99                         |
| 6           | 22           | 8         | 22        | 23        | ---       | 10        | 15       | 100                        |
| 7           | 22           | ---       | 28        | 25        | 10        | 6         | 15       | 106                        |
| 8           | 22           | 8         | 20        | 25        | 2         | 6         | 15       | 98                         |

TABLE 5  
COMPOSITION OF COUPON BOOK WITH 20 % COUPONS OF 5 PAISE

| COMBINATION | DENOMINATION |           |           |           |           |           |          | TOTAL<br>NO. OF<br>COUPONS |
|-------------|--------------|-----------|-----------|-----------|-----------|-----------|----------|----------------------------|
|             | Rs.<br>1     | Ps.<br>50 | Ps.<br>30 | Ps.<br>25 | Ps.<br>20 | Ps.<br>10 | Ps.<br>5 |                            |
| 1           | 22           | ---       | 35        | 25        | ---       | ---       | 25       | 107                        |
| 2           | 22           | 8         | 22        | 24        | ---       | ---       | 28       | 104                        |
| 3           | 21           | ---       | 36        | 24        | 6         | ---       | 20       | 107                        |
| 4           | 22           | ---       | 33        | 24        | ---       | 11        | 20       | 110                        |
| 5           | 21           | 10        | 22        | 24        | 2         | ---       | 20       | 99                         |
| 6           | 21           | 9         | 23        | 22        | ---       | 11        | 20       | 106                        |
| 7           | 22           | ---       | 30        | 22        | 9         | 7         | 20       | 110                        |
| 8           | 22           | 8         | 22        | 22        | 1         | 7         | 20       | 102                        |

## APPENDIX I

## FREQUENCY DISTRIBUTION OF TRANSACTIONS

| AMOUNT<br>(PAISE) | DINNER | BREAKFAST | LUNCH | TOTAL |
|-------------------|--------|-----------|-------|-------|
| 15                | 3      | 0         | 2     | 5     |
| 20                | 0      | 42        | 0     | 42    |
| 25                | 0      | 23        | 3     | 26    |
| 30                | 4      | 1         | 4     | 9     |
| 35                | 2      | 0         | 0     | 2     |
| 40                | 5      | 20        | 2     | 27    |
| 45                | 1      | 7         | 1     | 9     |
| 50                | 1      | 44        | 1     | 46    |
| 55                | 3      | 2         | 2     | 7     |
| 60                | 2      | 16        | 5     | 23    |
| 65                | 4      | 3         | 2     | 9     |
| 70                | 6      | 13        | 3     | 22    |
| 75                | 4      | 12        | 12    | 28    |
| 80                | 6      | 23        | 13    | 42    |
| 85                | 4      | 12        | 15    | 31    |
| 90                | 10     | 25        | 13    | 48    |
| 95                | 7      | 23        | 13    | 43    |
| 100               | 7      | 26        | 32    | 65    |
| 105               | 11     | 17        | 9     | 37    |
| 110               | 15     | 37        | 28    | 80    |
| 115               | 8      | 5         | 30    | 43    |
| 120               | 15     | 13        | 18    | 46    |
| 125               | 14     | 6         | 33    | 53    |
| 130               | 25     | 22        | 56    | 103   |
| 135               | 24     | 22        | 34    | 80    |
| 140               | 17     | 48        | 40    | 105   |
| 145               | 23     | 11        | 53    | 87    |
| 150               | 29     | 14        | 25    | 68    |
| 155               | 25     | 17        | 47    | 89    |
| 160               | 20     | 27        | 44    | 91    |
| 165               | 27     | 6         | 47    | 80    |
| 170               | 21     | 7         | 49    | 77    |
| 175               | 17     | 6         | 48    | 71    |
| 180               | 35     | 17        | 60    | 112   |
| 185               | 19     | 11        | 53    | 83    |
| 190               | 16     | 21        | 52    | 89    |
| 195               | 16     | 14        | 28    | 58    |
| 200               | 14     | 10        | 55    | 79    |
| 205               | 13     | 6         | 33    | 52    |
| 210               | 20     | 9         | 55    | 84    |
| 215               | 11     | 6         | 25    | 42    |
| 220               | 15     | 16        | 36    | 67    |
| 225               | 13     | 4         | 22    | 39    |
| 230               | 12     | 5         | 29    | 46    |
| 235               | 13     | 1         | 22    | 36    |
| 240               | 5      | 10        | 26    | 41    |
| 245               | 3      | 5         | 18    | 26    |
| 250               | 3      | 6         | 27    | 36    |

|                    |            |            |             |             |
|--------------------|------------|------------|-------------|-------------|
| 255                | 3          | 3          | 22          | 28          |
| 260                | 2          | 5          | 18          | 25          |
| 265                | 5          | 3          | 22          | 30          |
| 270                | 3          | 8          | 13          | 24          |
| 275                | 2          | 0          | 13          | 15          |
| 280                | 3          | 8          | 22          | 33          |
| 285                | 1          | 1          | 12          | 14          |
| 290                | 1          | 7          | 9           | 17          |
| 295                | 7          | 1          | 7           | 15          |
| 300                | 2          | 1          | 18          | 21          |
| 305                | 1          | 2          | 10          | 13          |
| 310                | 1          | 0          | 11          | 12          |
| 315                | 0          | 0          | 9           | 9           |
| 320                | 0          | 5          | 9           | 14          |
| 325                | 0          | 0          | 10          | 10          |
| 330                | 0          | 4          | 8           | 12          |
| 335                | 0          | 5          | 6           | 11          |
| 340                | 0          | 1          | 4           | 5           |
| 345                | 0          | 1          | 3           | 4           |
| 350                | 0          | 0          | 7           | 7           |
| 355                | 0          | 0          | 6           | 6           |
| 360                | 0          | 0          | 6           | 6           |
| 365                | 0          | 0          | 3           | 3           |
| 370                | 0          | 1          | 2           | 3           |
| 375                | 0          | 0          | 2           | 2           |
| 380                | 0          | 0          | 6           | 6           |
| 385                | 0          | 0          | 2           | 2           |
| 390                | 1          | 0          | 4           | 5           |
| 395                | 0          | 0          | 2           | 2           |
| 400                | 0          | 0          | 5           | 5           |
| 405                | 0          | 0          | 0           | 0           |
| 410                | 0          | 0          | 1           | 1           |
| 415                | 0          | 0          | 0           | 0           |
| 420                | 0          | 3          | 0           | 3           |
| 425                | 0          | 0          | 0           | 0           |
| 430                | 0          | 0          | 1           | 1           |
| 435                | 0          | 0          | 0           | 0           |
| 440                | 0          | 1          | 2           | 3           |
| 445                | 0          | 1          | 0           | 1           |
| 450                | 0          | 0          | 1           | 1           |
| 455                | 0          | 0          | 0           | 0           |
| 460                | 0          | 2          | 5           | 7           |
| 465                | 0          | 0          | 1           | 1           |
| 470                | 0          | 0          | 2           | 2           |
| 475                | 0          | 0          | 2           | 2           |
| 480                | 0          | 0          | 0           | 0           |
| 485                | 0          | 0          | 0           | 0           |
| 490                | 0          | 0          | 0           | 0           |
| 495                | 1          | 0          | 0           | 1           |
| 500 or more        | 0          | 7          | 1           | 8           |
| <b>GRAND TOTAL</b> | <b>601</b> | <b>761</b> | <b>1512</b> | <b>2874</b> |

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