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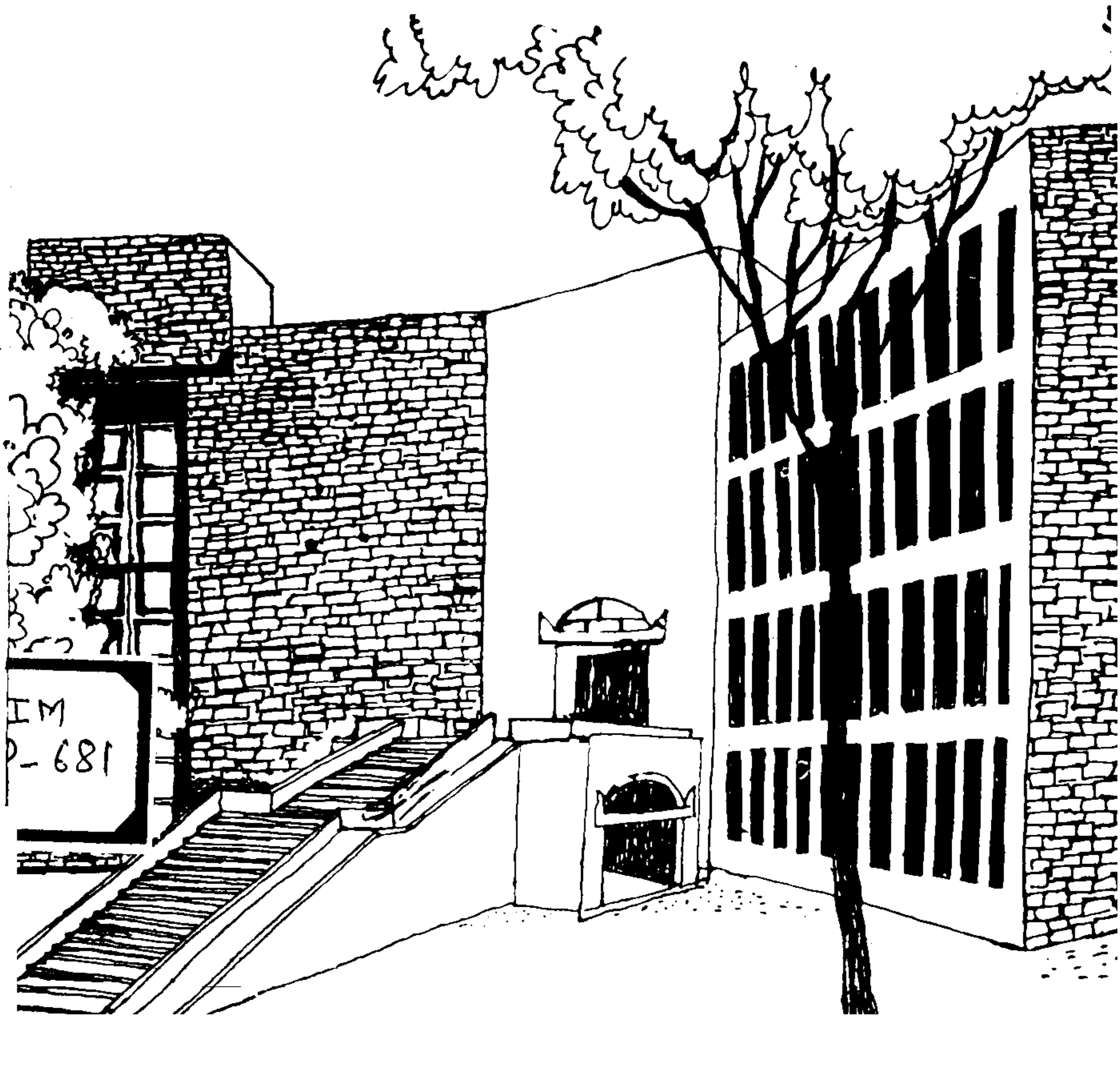

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APPLICATION OF MARKOV CHAINS TO MANAGEMENT OF LEASING

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APPLICATION OF MARKOV CHAINS TO MANAGEMENT OF LEASING

ABSTRACT

The paper presents application of Markov Chains to management of leasing. The paper demonstrates that, depite low percentage of bad debts, there could be a significant reduction in the returns earned by a lessor because of delays in payment. Since a lessor typically operates with a very small spread between returns earned and the cost of funds, a reduction in returns could jeopardise the very viability of the business. Markov Chains could be useful in assessing the impact on the rate of return because of the quality of accounts a lessor has, as reflected by the prevailing transition probability matrix. The model could in addition be used for assessing working capital needs, arriving at the age distribution of accounts and predicting the incidence of bad debts.

APPLICATION OF MARKOV CHAINS TO MANAGEMENT OF LEASING

INTRODUCTION

The pioneering work on use of Markov Chain to forecast account receivable flows was done by Cyert, Davidson and Thompson (2). Their famous CDT model used the oldest balance to decide the age of an account, and then assuming that the resulting transition probability matrix was stable, forecast the steady state distribution of accounts receivables. Subsequently, Van Kuelen, Sprork and Corcoran (3) pointed out that the oldest balance method would not provide a correct picture about the age distribution of the accounts receivables. Instead, they used the method of partial balances, which kept track of the actual age of various amounts outstanding, to provide a better assessment of the steady state distribution. In another work Corcoran (1) also suggests the use of exponential smoothing to alter the transition probability matrix over time. This method is also based on partial balances but by its very nature does not focus attention on the steady state distribution of accounts receivables but attempts to track the age distribution of receivables over time.

This paper reports an application of Markov Chain in a different area, namely, leasing. The lease rentals receivable by a lessor have characteristics which are very similar to accounts receivables for the manufacturing industry. The major difference is in the time span under consideration. While accounts receivables are payable within a short period of time, lease rentals are payable over a much longer period of several years. The longer time span involved in itself increases the possibility of a default either in the form of non-payment or delays in payment of rentals. Besides, since a lessor typically has very high percentage of debt in his capital employed, and invariably operates on very low margins, such defaults by lessees may jeopardise the very viability of the business.

In this paper we demonstrate the use of Markov Chain to model the behaviour of rentals receivable from a set of accounts. The cashflow pattern predicted by the model has been subsequently used to compute the rate of return earned by a lessor. Three hypothetical transition probability matrices representing a variety of conditions likely to prevail in reality have been used to show that the rate of return is not only affected by the occurence of bad debts but is significantly influenced by delays in payment of rentals. We have also shown how the model could be used for assessing the working capital needed, obtaining the age distribution of accounts and assessing the extent of bad debts in the long run.

THE MARKOV CHAIN MODEL

We assumed that an account could be in six different states; from being fully paid-up, indicated by state 1, to being delayed in payment by five months, indicated by state six. A delay of five months was assumed to imply bad debt. Therefore, state 6 was an absorbing state.
An account in state j in period t could move to any state from 1,...., (j+1) in period (t+1), depending on the number of instalments paid. No payment would move it to (j+1), payment of one instalment would keep

it in j, and payment of j instalments would move it to state 1. Thus, the jth row (except row 6) of the transition probability matrix would have (j+1) non-zero values. State six, being an absorbing state, would have only one non-zero value.

The above implies that the transition probability matrix would in all have 21 non-zero values. Since the possible sets of values of these probabilities are infinite, we focussed attention on three different patterns, specified by the value of p_{11} and a formula for determination of the other p_{ij}'s. The patterns reflected a wide variety of values that are likely to prevail in reality. The formulae used for the three patterns were as follows:

Pattern 1

$p_{11} = K,\ p_{12} = (1-K)$

$p_{ij} = 2\,(i+2-j)/(i+1)\,(i+2)$

for all $j \leq (i+1)$ and $i = 2, \ldots, 5$

$p_{66} = 1$

$p_{ij} = 0$ for all other (i,j)

Pattern 2

$p_{11} = K,\ p_{12} = (1+K)$

$p_{ij} = 1/(i+1)$

for all $j \leq (i+1)$ and $i = 2, \ldots, 5$

$p_{66} = 1$

$p_{ij} = 0$ for all other (i,j)

Pattern 3

$p_{11} = K,\ p_{12} = (1-K)$

$p_{ij} = 2j/(i+1)\ (i+2)$

for all $j \leq (i+1)$ and $i = 2, \ldots 5$

$p_{66} = 1$

$p_{ij} = 0$ for all other (i,j)

The value of p_{11} specifies the chance that an up-to-date account would go into default in any period. The three patterns capture a variety of possible behaviour of accounts which enter the default category. Pattern 1 assumes that from any state the probability of full payment is the highest. The probability of moving to other states decreases, becoming the lowest for a further delay in payment. The values for each row were generated by using the sum of digits method. Pattern 2 is less optimistic compated to pattern 1. It assumes that there is an equal chance of moving to any of the possible states from a given state. Pattern 3 is the most pessimistic pattern. It assumes that from any state, the probability of a further dela in payment is the highest. The probability of moving to other states decre ases, becoming the lowest for full payment. The values for each row were generated using sum of digits method in the reverse order. To further clarify these patterns, the numerical values of probabilities for a specified value of p_{11}, are presented in Appendix I.

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GENERATION OF CASHFLOWS

The two basic matrices needed for generation of cashflow from lease rentals are the transition probability matrix P and the instalment matrix C. The elements in the matrix C denote the number of instalments when an account moves from state i to state j. For example, $c_{4,3} = 2$ means that an account which is in state 4 (that is three instalments are overdue) would pay two instalments comprising the current and one overdue instalment to move to state 3 in which two instalments are overdue. The C matrix is displayed in Table 1. The computations needed to arrive at the cashflows using these two matrices is captured in the following steps:

TABLE 1

Step 1: The expected number of instalments is computed through matrix D where $d_{ij} = p_{ij} \times c_{ij}$

Step 2: The expected number of instalments from an account in state i is computed through column Vector E, where $e_i = \sum_{j=1}^{6} d_{ij}$

Step 3: The expected number of accounts in various states at the beginning of each period, is captured through vector A_t, where

$A_1 = \{1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\}$ (the initial distribution)

$A_t = A_{t-1} \times p$ for $t > 2$

Step 4: The cash collection in a period t would be given by CC_t.

$CC_t = 1 \times A_t \times E$

Where 1 is the lease rent per period.

The cashflows thus generated are needed for computation of rate of return earned as well as for arriving at the funds needed by a lessor for sustaining his business.

COMPUTATION OF IRR

As mentioned in the introduction, the paper focusses on the viability of leasing and hence it is necessary to assess the impact of bad debts and delays in payment on the return earned. In addition to the cashflows from lease rentals generated though the model described in the previous section other cashflows such as the tax-shield on depreciation and bad debts, and tax on lease rentals would be relevant. The computations of these cashflows alongwith the necessary assumptions are given below:

a. Depreciation Tax-shield

It has been assumed that WDV method is used for computing annual depreciation and the tax-shield is assumed to be available at the end of each year. The salvage value at the end of the lease period has been assumed to be zero and therefore the terminal depreciation equals the WDV.

b. Tax on Lease-Rent

It has been assumed that the revenue is recognised on an accrual basis, therefore the tax has been computed on the basis of lease rent accrued rather than the collection during the year. It is assumed that the tax is paid at the end of the year.

e. Tax-shield on Bad-debts

As mentioned earlier all accounts which enter state 6 are assumed to have gone bad. Since the income is recognised on accrual basis, the uncollected amount because of the bad accounts would result in a reduction of tax. This reduction is again available at the end of each year.

For the computation of the IRR, the initial investment would be the cost of the asset leased less the first periods lease rent, it being assumed that the lease rentals were payable at the beginning of each month. The cashflows used also assumed that the tax-shields on depreciation and bad debts were available in the year of their occurence and that there was no need to postpone them in the absence of adequate profits.

AN ILLUSTRATION

The model was used for a hypothetical (but fairly common) example from the Indian leasing industry. The period of lease in the example was eight years and the lease rental was Rs. 25 per thousand rupees worth of asset leased. The rentals were payable at the beginning of each month, the first rental being paid at the time of signing the contract. The rate of depreciation was assumed to be $33\frac{1}{3}$% and the corporate tax rate was assumed to be 50%.

In the absence of any delay in payment or non-payment of lease rentals, the rate of return earned by the lessor from such accounts would be 20.5%. This rate has been arrived at by using a single discount rate for all the cashflow and finding out that value of discount rate which reduces the NPV to zero.

In the presence of delays and bad debts, resulting from the use of the transition probability matrices discussed earlier, the IRR would reduce. The expected cashflow from lease rentals for the eight year time span were computed by using the transition probability matrices. The aggregate cashflow was then arrived at by incorporating the tax implications and then the IRR was computed. The resulting rates of return are tabulated in Table 2.

TABLE 2

DISCUSSION OF RESULTS

We have said that pattern 1 is the most optimistic, pattern 3 is the most pessimistic and pattern 2 falls between the two. To get an idea of what is implied by these patterns, the incidence of bad debt in the long run under the three patterns for the two extreme values of p_{11} are presented in Table 3. The bad debts range from 0.33 percent to 37.80 percent. While the latter percentage of bad debt may never be observed, the other figures may prevail for a lessor. For the same combination, Table 4 contains the reduction in the rate of return earned by the lessor from the 20.5% he would earn in the absence of default. The reduction in return ranges from a little over half a percent to a little over 12%. The interesting feature is that even if the incidence of bad debt is low, the drop in the return is substantial because of the delays in payment. Since the lessors operate with a spread of only a few percentage points, this drop may seriously affect the viability of their business.

TABLE 3 & 4

MONITERING AND PLANNING

The model outlined can be used for various other purposes such as predicting the age distribution, incidence of bad debt and aggregate cashflow over time. These predictions can be used for monitoring the actual performance as well as planning for future through introduction of corrective measures to influence the performance.

The age distribution of accounts at the begining of period t is straightaway given by the vector A_t computed in step three of the section on generation of cashflows. The element a_6^t of this vector would give the percentage bad debt at that point in time. The cashflow generated from one account in period t is given by the number CC_t computed in the last step of the same section. This value when aggregated over all the accounts would provide the total inflow of cash in period t in the form of lease rentals. Deducting the anticipated operating expenses for the period would indicate the surplus/deficit for the period. Such a prediction would be immensely useful in planning to deploy/raise resources.

CONCLUSION

The paper illustrates how a Markov Chain Model could be used for planning and monitoring the leasing business. It highlights the fact the rate of return is significantly affected not only by incidence of bad debts but also by delays in payment. Though the analysis was done only for one category of lease accounts, it is quite clear that the method could be easily extended to a situation where a lessor has several categories of accounts. The aggregate picture can be developed for the organisation, despite using different transition probability matrices for different categories of accounts with different pattern of rental payments. The approach would prove extremely useful.

TABLE 1

THE INSTALMENTS MATRIX C

	1	2	3	4	5	6
1	1	0	0	0	0	0
2	2	1	0	0	0	0
3	3	2	1	0	0	0
4	4	3	2	1	0	0
5	5	4	3	2	1	0
6	0	0	0	0	0	0

TABLE 2

RATES OF RETURN EARNED

(in percentage)

Chance of default of a good account	Transition Probability Matrix		
	Pattern 1	Pattern 2	Pattern 3
0.90	19.82	18.63	8.31
0.91	19.83	18.74	9.00
0.92	19.84	18.85	9.76
0.93	19.86	18.96	10.60
0.94	19.87	19.08	11.54
0.95	19.89	19.21	12.57
0.96	19.90	19.35	13.72
0.97	19.92	19.49	15.02
0.98	19.94	19.64	16.47
0.99	19.96	19.80	13.11

TABLE 3

INCIDENCE OF BAD DEBT

(in percentage)

Chance of default of a good account	Transition Probability Matrix		
	Pattern 1	Pattern 2	Pattern 3
0.90	0.27	4.68	37.80
0.99	0.03	0.61	7.17

TABLE 4

REDUCTION IN RATE OF RETURN

(in percentage)

Chance of default of a good account	Transition Probability Matrix		
	Pattern 1	Pattern 2	Pattern 3
0.90	0.68	1.87	12.19
0.99	0.54	0.70	2.39

APPENDIX I

TRANSITION PROBABILITY MATRICES

If $p_{11} = 0.90$

Pattern 1:

	1	2	3	4	5	6
1	0.9	0.1	0	0	0	0
2	3/6	2/6	1/6	0	0	0
3	4/10	3/10	2/10	1/10	0	0
4	5/15	4/15	3/15	2/15	1/15	0
5	6/21	5/21	4/21	3/21	2/21	1/21
6	0	0	0	0	0	1

Pattern 2:

	1	2	3	4	5	6
1	0.9	0.1	0	0	0	0
2	1/3	1/3	1/3	0	0	0
3	1/4	1/4	1/4	1/4	0	0
4	1/5	1/5	1/5	1/5	1/5	0
5	1/6	1/6	1/6	1/6	1/6	1/6
6	0	0	0	0	0	0

Pattern 3:

	1	2	3	4	5	6
1	0.9	0.1	0	0	0	0
2	1/6	2/6	3/6	0	0	0
3	1/10	2/10	3/10	4/10	0	0
4	1/15	2/15	3/15	4/15	5/15	0
5	1/21	2/21	3/21	4/21	5/21	6/21
6	0	0	0	0	0	0

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