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A Multi-Period Two Stage Stochastic Programming Based Decision Support System for Strategic Planning in Process Industries:
A Case of an Integrated Iron and Steel Company

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Abstract

The paper introduces the application of a generic, multiple period, two stage stochastic programming based Decision Support System (DSS) in an integrated steel company. We demonstrate that a generic, user friendly stochastic optimization based DSS can be used for planning in a probabilistic demand situation. We conduct a set of experiments based on the stochastic variability of the demand of finished steel. A two stage stochastic programming with recourse model is implemented in the DSS, and tested with real data from a steel company in North America. This application demonstrates the need for stochastic optimization in the process industry. The value of stochastic solution resulted from the implementation of steel company real data in the DSS is 1.61%, which is equivalent to USD 24.61 million.

Keywords: Decision support system, process industries, optimization, stochastic programming.

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1.0 Introduction and Motivation

This research is primarily motivated by earlier publications Fourer (1997), “Dutta & Fourer (2004)”, and Dutta, Gupta, & Fourer (2011) where a multi-period optimization based Decision Support System (DSS) was developed for process industries. “Fourer (1997)” described the fundamental principles of relational database construction to represent a linear program. This work was carried forward to develop single period deterministic optimization. The application ranges from a steel company in North America “(Dutta & Fourer, 2004)”, to a pharmaceutical company in Western India “(Dutta *et al.*, 2007)” and even further to an aluminum company “(Dutta, Gupta, & Fourer, 2011)” in Eastern India. The DSS was customized for an integrated steel plant in North-America, and demonstrated a potential increase of 16-17% in the bottom line of the company (Dutta, 1996), (Dutta & Fourer, 2004) for a single period optimization. Similarly, the DSS was further customized for an integrated aluminum company and a pharmaceutical company, and demonstrated a potential of 6.72% and 12.46% respectively. This high impact demonstrated by differences in the objective function motivated us to study the database construction principles and application of stochastic optimization based DSS for the process industry.

The database design principles and optimization based DSS have been introduced and discussed by Gupta *et al.*, (2014 in review). In this research, we discuss how complex production systems like integrated iron and steel manufacturing can be modeled in a user friendly generic, Stochastic Linear Programming (SLP) based DSS. We formulate a process industry mathematical model using two stage stochastic programming with recourse. In this paper, the DSS has been tested with real data from a steel company in North America. However, since it is generic, it can be used to model any other similar process industry. This research primarily focuses on modeling uncertainty in the demand for finished goods. However the multi-scenario, multi-period DSS is capable of simultaneously modeling uncertainty in a number of parameters of the model including demand for finished goods, cost of purchase of raw material, sell price of finished goods, supply of raw materials, etc. This is probably the first attempt when a multiple period stochastic programming based DSS has been created and tested with real data from a process industry.

The application of the stochastic programming based DSS demonstrated a significant potential impact on the bottom line of the steel company. The results of the application are discussed using stochastic programming's key performance parameters like Value of Stochastic Solution (VSS), Expected Value of Perfect Information (EVPI), and Expected Value of Expected Solution (EEV). We investigate the optimization results, and performance parameters to diagnose the reasons for the change in the value of VSS and EVPI, and the way they change with reference to the scenarios. We draw inferences from the patterns of the solutions of different stochastic optimization models from the aforesaid experiments. The concluding remarks discuss the impact of modeling uncertainty using stochastic programming and the challenges in practical application. This research is an attempt to demonstrate the significant impact of optimization under probabilistic demand, and the way the stochastic optimization can be done in a generic and user friendly manner.

The stochastic optimization based DSS is capable of addressing the following questions related to strategic planning in a steel company:

1. How does the value of stochastic solution (VSS) change with changes in demand variability?
2. How does the expected value of perfect information (EVPI) change with the increase in demand variability?
3. How does the VSS change with the change in the discrete probability distribution of demand?
4. How does the EVPI change with the change in the discrete probability distribution of demand?

This paper along with the earlier work by Gupta *et al.*, (2014 in review) is meant for two audiences. One is the set of researchers who are trying to develop fundamental principles for database construction for stochastic optimization. The second is the set of researchers who are trying to apply the SLP based DSS in a complex industry like the integrated iron and steel manufacturing industry.

1.1 Outline of the Paper

The paper introduces the need for an optimization based DSS, and its historical development process by current researchers in the first section, and reviews the literature on the application of modeling demand uncertainty using stochastic programming in the second section. We also discuss the basic principles of stochastic optimization. The fundamental elements of process industry modeling and the assumptions are discussed in a subsequent section (section 3). The optimization steps for the DSS are discussed in section 4. In section 5, we discuss how we model demand variability in this research. In section 6 we discuss the results of the experiments. The application of the two-stage SLP with recourse model and DSS in a steel company is presented and generic inferences are drawn. We conclude the paper with the future scope of the research.

2.0 Literature Review

A survey of stochastic programming by Birge, (1997) argues that using stochastic programming models, it is possible to make flexible and robust "near optimal" decisions for allocating resources when faced with an uncertain future. A collection of recent SLP test problems for eleven different families of contexts are discussed in the literature by Ariyawansa (2004). Another recent survey of using stochastic programming in supply chain for modeling demand uncertainty is presented by Sodhi & Tang (2009). The survey was motivated by stochastic programming applications in asset liability management problems. The study also presented an instance of a stochastic programming model to manage the risk pertaining to unmet demand. The focus of the study is more on discrete production and supply chain planning in contrast to the research presented in this paper for stochastic programming application in continuous production of finished steel.

The stochastic models are developed for a variety of fields including air fleet management by Ferguson (1955), electrical power generations by Sherali, (1984), reservoir water management, telecommunication network planning by Sen (1994), financial planning by Mulvey (1991), and Mulvey & Vladimirou (1992). A complete review of the extensive literature on stochastic programming and its applications in general and in the context of process industries in particular is beyond the scope of this paper. Extensive literature on probabilistic modeling and stochastic programming can be referred to; these include Birge (1997), Raghunathan (1992), Frauendorfer (1992), Marti & Kall (1997).

A recent application of a stochastic quadratic programming model and a decomposition algorithm to compute an optimal sales policy in dairy farms of Fonterra, New Zealand has been reported by Guan & Philpott (2011). The sales policy developed was later tested using simulation against a deterministic policy. The model captures uncertainty in the milk supply, price-demand curves and contracting. The focus of the research presented in this paper is an application of a two stage stochastic programming in an integrated steel plant; we looked for reported literature in this area. The application Summerfield & Dror (2013) is in the context of the biform game, where a single firm or a number of firms choose their production capacities as the game's strategy in the first stage, and form coalitions in the second stage to deliver the best value among them. Though the application is of a two stage stochastic programming, it is quite different from the process industry production planning reported in our study.

Fourer (1983) describes the algebraic formulation of a single period deterministic model for process industry planning. In a further extension, Dutta & Fourer (2004 and 2008) present a multi period deterministic model. Dutta & Fourer (2004) tested the multi-period model and the optimization based DSS with a set of real data for a single period. Dutta *et al.* (2008), and Dutta *et al.* (2011) demonstrated a significant impact using a multi-period planning model in a pharmaceutical and an aluminum company respectively with multi-period data.

As discussed in the review of literature, there are publications on modeling uncertainty using SLP in the airline industry, electric power generation, telecommunication network planning, financial planning etc. The literature reveals that there is little or no work published on modeling uncertainty in process industries using stochastic programming. This research realizes the need to address uncertainty in model parameters, and extend the multi period optimization model in Dutta & Fourer (2004) to develop and implement a two stage SLP in a DSS.

We find that not much work has been reported on modeling uncertainty using a two stage stochastic programming based DSS in a process industry. The principles of database construction and the design of DSS have already been explained in our earlier paper Gupta *et al.* (2014 in review). In this research, we introduce how the uncertainty in model parameters can be modeled using a user friendly generic, multi-period, multi-scenario optimization based DSS. The focus of the paper is to discuss the application of the DSS with real data from a process industry. The study primarily focuses on modeling market demand (upper bounds on the units of finished goods sold) as an uncertain parameter. A set of experiments have been designed to test the multi-scenario optimization based DSS with a set of real data from a steel company from North America. The experiments are designed by varying volatility in the market demand and probability of occurrence of the most possible economic scenarios.

2.1 Two-stage Stochastic Program with Recourse

The two stages of the stochastic program are defined by a set of decisions taken in those stages. The decisions taken in the first stage are the decisions which are implemented before the realization of the randomness in the system. The second stage decisions are the ones which are implemented after the realization of the randomness. The decisions taken in the first stage are non-anticipative in nature, and do not depend on the outcome of the randomness. The focus of the stochastic programming is to rectify the decision taken for the first stage well in advance such that the solution remains the same regardless of the outcome of the random realization. Readers may note that the profit from the SLP solution is a long run expected profit, and in the short run, the profit may be a little different from the profit resulting from the SLP solution. To simplify the understanding of the two-stage stochastic programming with recourse, we discuss an example, which is the first SLP with recourse, formulated by Dantzig (1955). The term recourse is defined by Fragniere (2002) as the decision variables adapting to the different outcomes of the random parameters at each time period. In a stochastic program with recourse, the response of the randomness of the model is corrected as a part of the model. We introduce SLP using the deterministic equivalent linear program developed by Dantzig (1955). It is a generalized two-stage program.

c_1= The cost vector of the first stage

c_2=The cost vector of the second stage

X_1= The first stage decision vector

X_2=The second stage decision vector

X_1 and X_2 are nonnegative decision vectors for all scenarios

p_1= The likelihood probability of the occurrence of scenario 1

p_2= The likelihood probability of the occurrence of scenario 2

p_3=The likelihood probability of the occurrence of scenario 3

Aij =The matrix representing technological coefficients (assumed to be deterministic)

B1

b= The random vector that varies in different scenarios is denoted as

Minimize Expected Contribution $Z = (c_1X_1 + p_1c_2X_2^{(1)} + p_2c_2X_2^{(2)} + p_3c_2X_2^{(3)})$ (2.1)

Subject to

$A_{11}X_1 = b_1$ (2.2)

$A_{21}X_1 + A_{22}X^{(1)}_2 = b^{(1)}_2$ (2.3)

$A_{21}X_1 + A_{22}X^{(2)}_2 = b^{(2)}_2$ (2.4)

$A_{21}X_1 + A_{22}X^{(3)}_2 = b^{(3)}_2$ (2.5)

A stochastic program with three scenarios is presented above. The superscript of the second stage decision vector denotes the decisions in each of the scenarios.

3.0 Process Flow of an Integrated Iron and Steel Company

The material flow diagram of the integrated steel plant is presented in Figure 1. The integrated plant is made of a network of facilities (or plants), running in sequence and in parallel. The raw materials are coal, iron ore, limestone, dolomite etc. Normally, raw materials (such as coal, ore, and limestone) can only be purchased, while finished products (bars, billets, plates, sheets axles, wheels) can only be sold in the market. Intermediates can often neither be bought nor sold. To keep our model general, we define three limits – on amounts bought, sold, inventoried — for each material, and allow the appropriate limits to be set to zero where no buying, selling or inventorying is possible.

The raw material is transformed at a collection of facilities such as steel melting shops, continuous caster, electric arc furnace finishing mills etc. A very small number of raw materials is transformed into a large range of finished steel through the chemical, heating, and fabrication process. The finished steel produced varies in shape, size, and composition which enhances their sell price with a very wide variation, and thereby the scope of optimization.

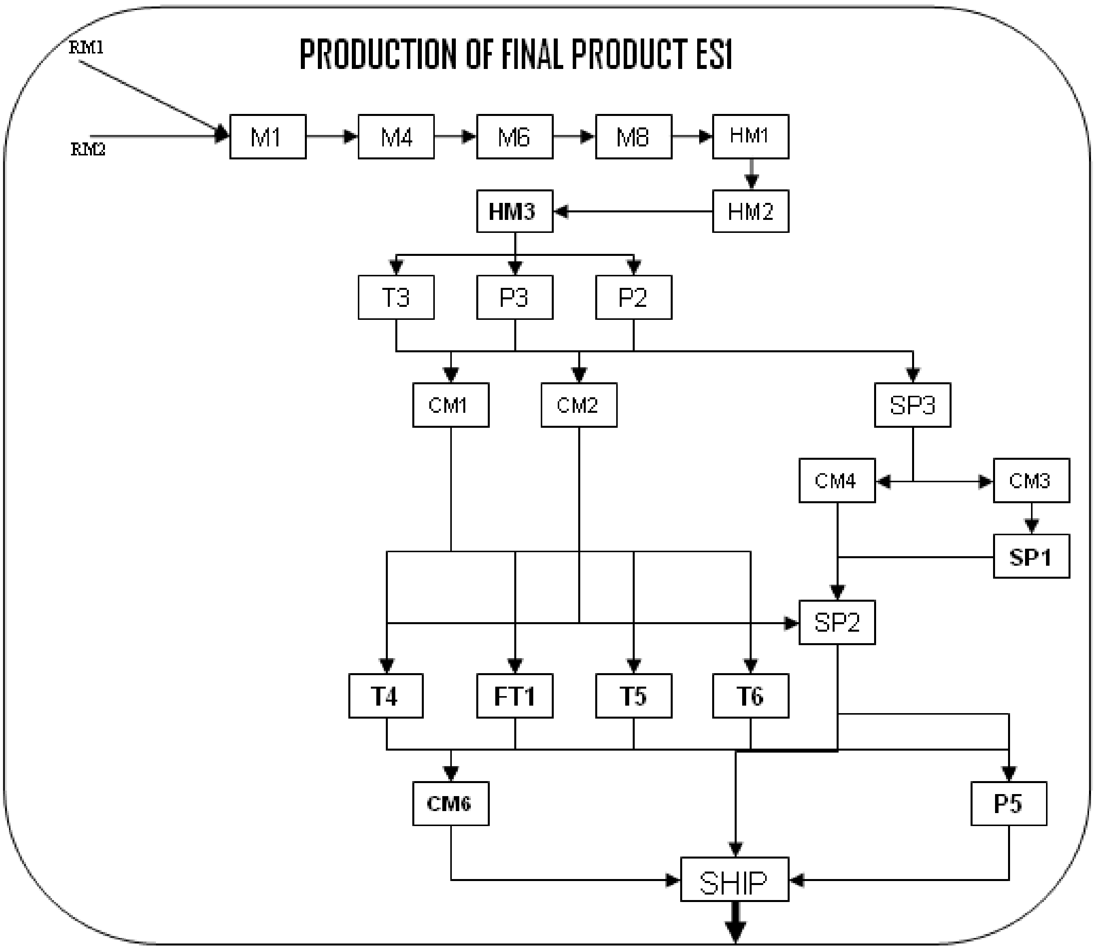

Figure 1: Typical Material Flow for Production of a Final Product in Steel Plant

3.1 Definitions

The SLP model has six fundamental elements. We describe the model elements in brief as follows:

Times are the periods of planning horizon, represented by discrete numbers (1, 2, 3 …).

Scenarios are the possible outline of a hypothesized chain of events.

Materials are the physical items that figure in any of the production stages. A material can be an input, intermediate, or finished product,

Facilities are the collection of machines that produce one or more materials from the other. For example a Hot Mill that produces sheets from slabs is a facility.

Activities are the productive transformation of the materials. Each facility houses one or more activities, which uses one or more input and produces materials in certain proportions. In each activity at each time, we have one or more input materials being transformed into various output materials. Production of hot metal at a blast furnace and production of billets at a rolling mill are examples of activities.

Storage-Areas are the places where raw materials, intermediates and finished goods can be stored.

In line with our earlier research, Dutta (1996), and Dutta & Fourer (2008), we have added Scenarios as an additional fundamental element as described in Gupta *et al.* (2014 in review).

3.2 Modeling Assumptions

The following assumptions are applicable to the model:

1. There are several facilities, which are in series, in parallel, or in a combination of series and parallel.
2. In each facility, there is either one or more than one activity.
3. There can be purchase, sale and storage of materials at the raw materials stage, intermediate processing stages, and finishing stage.
4. The purchase price of raw materials, the selling price of finished goods, and the inventory carrying costs vary over time and may also vary with the stages of production such as raw material supply, intermediate, and finishing.
5. At any time, a facility may use one or more materials as input and output. Generally more than one material is used to produce one product. The relative proportion of various inputs and outputs (generally called technological coefficients) in an activity remains the same in a period. Technological coefficients may vary with time.

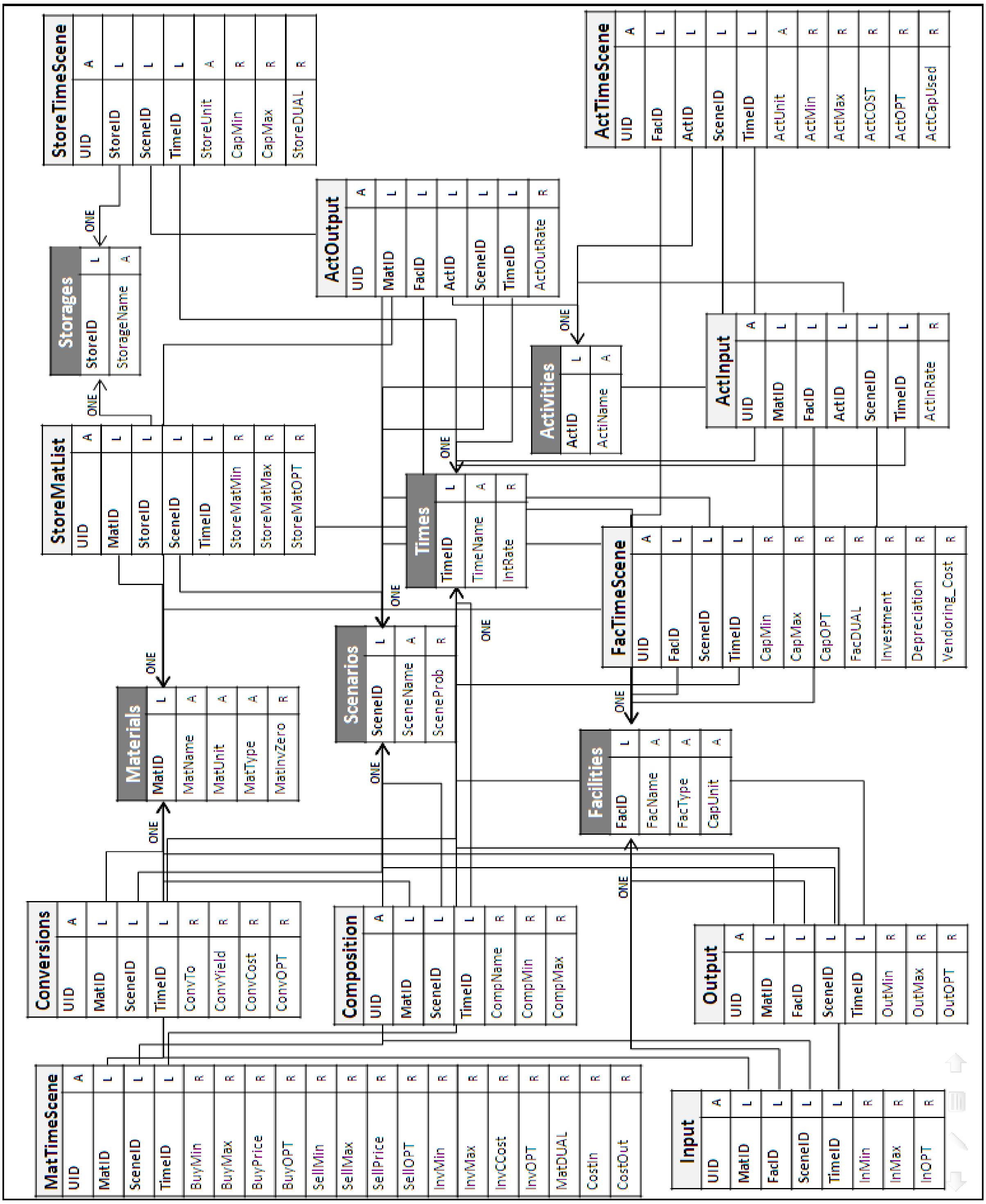

Figure 2: Database Structure of a SLP based DSS

6. The capacity of each facility, each storage-area is finite.
7. As the facilities may have different patterns of preventive maintenance schedules, the capacity of the machines may vary over a period of time.
8. The demand variation of the final finished products is represented by a discrete probability

distribution.

4.0 Optimization

The end-to-end optimization process requires generation of the stochastic optimization model in the form of variables, constraints, and coefficients files using a matrix generator, which is then converted to a MPS (CPLEX compatible input) format. The output of the optimizer is updated in the appropriate fields of the database using the report writer. The steps of optimization are described below.

4.1 Optimization Steps

A typical process of optimization is described in a step by step procedure. The principle steps are as follows; see Figure 3.

1. Data describing manufacturing operation at different time periods and potential scenarios is collected and entered at appropriate files in the different files in the DSS.
2. Constraints of the SLP are generated in the [Constraints] file. The constant terms of the equations, inequalities, LoRHS, HiRHS (ranges) are extracted from the database and stored in the [Constraints] file.
3. Variables of the associated SLP are generated in the [Variables] file in the DSS along with their coefficients in the constraints. The data values for the lower bound, upper bound and objective coefficient associated with the decision variables, and the coefficients, are extracted from the company's data [entered in step 1 above] and stored in the [Variables], and [Coefficients] files respectively.
4. The [Constraints] and [Variables] files are scanned and all of the essential information about the linear program (LP) is written to an ordinary text file in a compact format. This text file is the input file to our solver.
5. The optimizer reads the LP from the input text file generated in the MPS format (a CPLEX compatible format). An optimal solution is generated and the output of the optimizer is written in another ordinary text file.
6. The output text file is read and the optimal values are written at appropriate fields in the database tables.

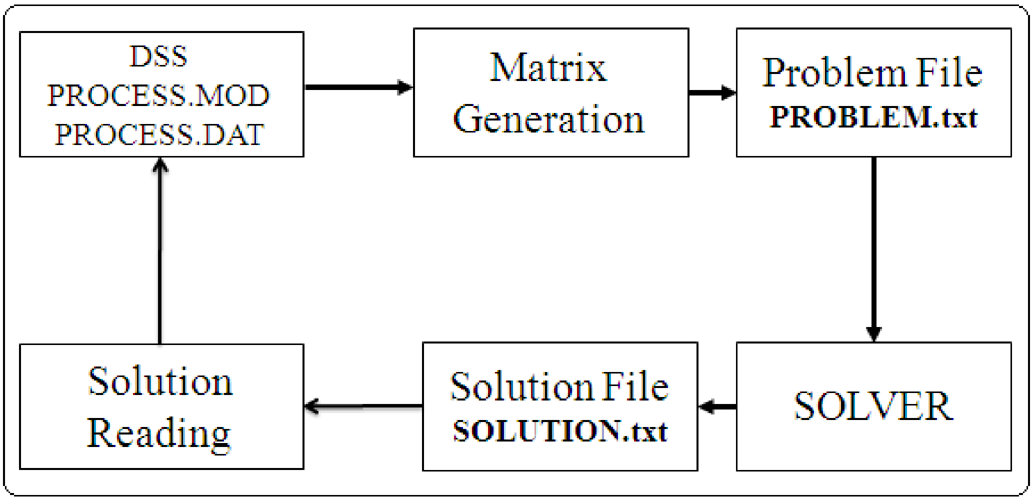

Figure 3: Steps of Optimization

4.2 Features of the Multi-Period, Multi-Scenario DSS

A detailed discussion on database construction principles for SLP based DSS, and the features of the DSS are discussed in our earlier publication Gupta *et al.* (2014 in review). In this research, we state the features of the SLP based DSS in brief.

1. The SLP model and SLP based DSS is so generic that it allows modeling: single scenario, single period; single scenario, multiple period; multiple scenario, multiple periods; and in different process industries by changing the industry real data.
2. The key strategic decisions that the DSS can address are impact of prices and costs parameters on the final optimal product mix, identification of bottleneck processes, diversification decisions, and the economic viability of a product promotion campaign.
3. The DSS is equipped with a set of diagnostic rules. The diagnostic rules were designed to ensure that the optimization data entered in the DSS is completely error free before it is processed by the CPLEX optimizer.
4. Data retrieval and storages procedures are the critical features of this DSS.
5. The core task of this DSS are generation of the data to the [Constraints], [Variables], and [Coefficients] file as mentioned in section 4.
6. The DSS is operated in three different modes - Data, Update, and Optimal.
    a. The *Data* mode is used for entering and loading the company specific data, and scenario specific data.

b. The *Update* mode is used to update the parameter values directly to the variables, constraints, and coefficients files. The feature of the *Update* mode saves the total time required to regenerate the SLP model; see Figure 4.

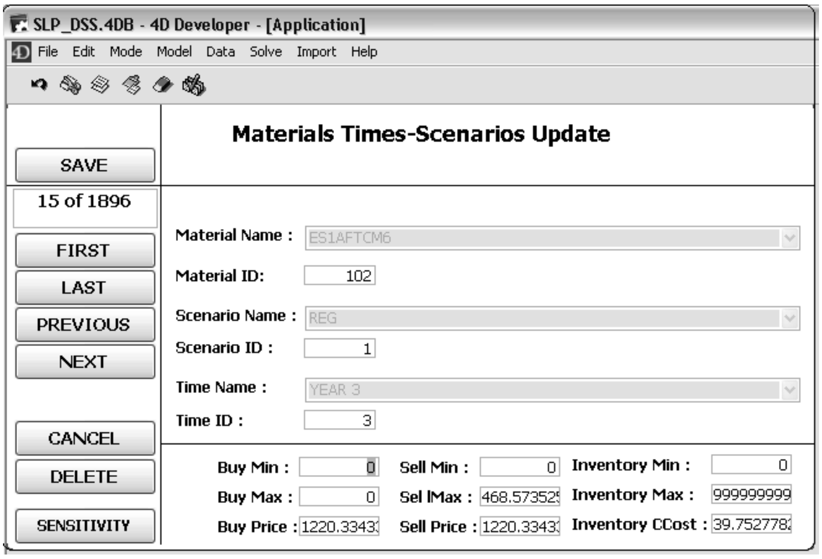

Figure 4: Materials Time Scenario Update Layout

c. In the *Optimal* mode, a user can see the optimal solution and the optimal summary of the cash flows. We report cash flows as nominal and discounted cash flows. The issues related to data reporting, data loading, and data updates can be referred to in detail in the authors' earlier publication Dutta & Fourer (2008); see Figure 5.

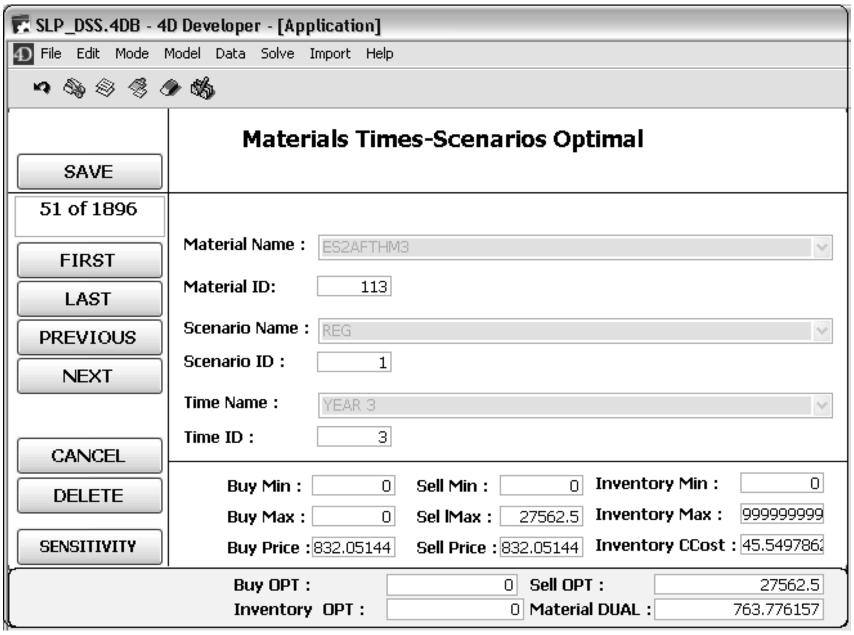

Figure 5: Optimal Solution Reporting Layout of Material Time Scenario Layout

5.0 Defining Models and Designing Demand Scenarios

In this section, we define the instances of the optimization model, and the model's nomenclature. We also design different demand scenarios based on the volatility in demand and change in probability of demand scenario occurrence.

5.1 Instances of Stochastic Optimization Model

To measure the performance of the solution of the stochastic optimization model, we compare four solutions - Mean Value solution (Z_{MV}), Perfect Information solution (Z_{PI}), SLP solution (Z_{SLP}) and Expectation of Expected Value solution (Z_{EEV}). They are described below:

1. Perfect Information Solution (Z_{PI}): This is the expected value of the solution from each scenario planning model individually solved as a multiple-period optimization model. The expected value of the solution from each scenario planning model is determined as the weighted average of the solution of the individual scenario model with their corresponding probabilities as the weights . In stochastic programming literature the solution is also known as a '*wait and see*' solution. This solution may not be implementable.

2. **Mean Value Solution (Z_{MV})**: This is the solution of the multi-period optimization model in which uncertain demand parameters are replaced by the expected value of the demand. The mean value solution may neither be feasible nor be achievable in practice. In most of the instances it would be far from the realized solution.

3. Stochastic Linear Programming Solution (Z_{SLP}): To capture the uncertainty in demand of finished steel, a deterministic equivalent (DE) linear program is generated and solved. The solution of this model is called a stochastic solution. The SLP solution in stochastic programming literature is known as the '*here and now*' solution.

4. Expectation of Expected Value Solution (Z_{EEV}): This is the solution of the SLP where first stage decisions are fixed and replaced with the optimal value of the first stage decision variables of the mean value model.

5.2 Modeling Uncertainty

The modeling is done based on the assumption that the future is unknown and uncertain. The model sensitive parameters like demand and sell prices of the finished steel may attain a different value based on the economy of the local geography.

We assume three economic situations for the purpose of modeling in this study. First, there is an economic crisis, and therefore the demand of the finished steel is poor in the market. We call this a poor (Low) economy. Second, there is not much variation in the economy. The demand of the finished steel is as regular as expected. We call this situation a regular (Reg) economy. Third, there is a boom in the economy therefore the demand for finished steel is strong in the market. We call this situation a strong (High) economy. While modeling, all the three economic situations need to be considered, because any of these may occur with different probabilities.

To conduct the experiments on stochastic programming, and optimization based DSS, we design a set of scenarios based on two primary assumptions, in line with Leung *et al.* (2006). The Cartesian product of the three cases in Table 1 and Table 2 each result in a total of nine scenarios (Table 1 X Table 2). Each scenario captures the three economic situations named as Low, Regular , and High. The following are the assumptions behind generating these scenarios:

1. **Assumption 1**: The probability of occurrence of economic situations comes from a probability distribution. We assume three probability distributions. A pre-determined probability value is assigned to each economic situation based on the assumed probability distribution; see Table 1, Figure 6.
2. **Assumption 2**: The demand magnitude corresponding to the Low and High economic situation is evenly defined around the regular economic situation; see Table 2, Figure 7.

Scenario Cases	Economic Situation →	LOW	REG	HIGH
Case i (Right Skewed)		0.75	0.15	0.10
Case ii (Equally Likely)		0.33	0.33	0.33
Case iii (Left Skewed)		0.10	0.15	0.75

Table 1: Definition of Scenario Designs Generated

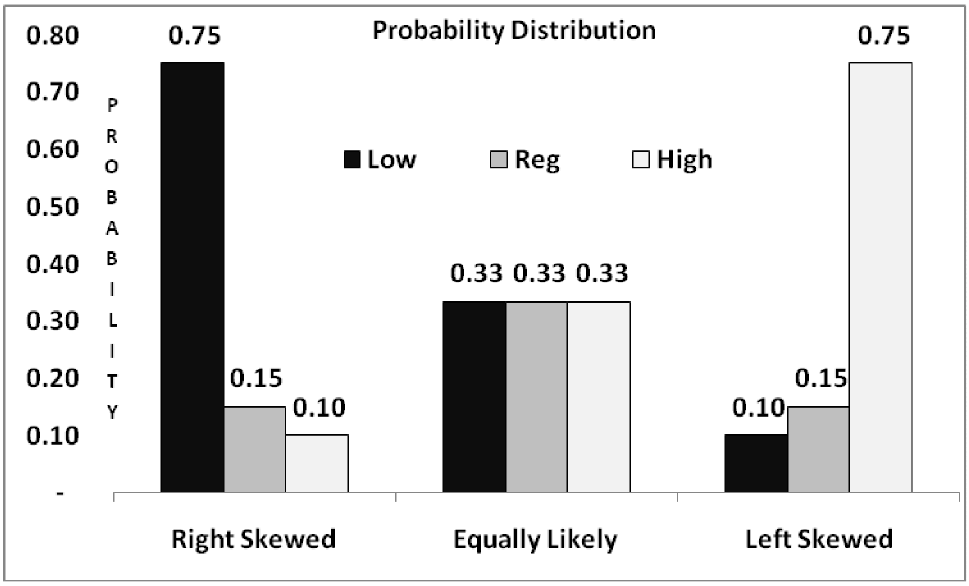

Figure 6: Three Different Probability Distributions of Demand

Demand Cases	Economic Situation →	LOW	REG	HIGH
Case 1: 20 % Demand Volatility		80% of D	D	120% of D
Case 2: 30 % Demand Volatility		70% of D	D	130% of D
Case 3: 40 % Demand Volatility		60% of D	D	140% of D

Table 2: Definition of Random Demand Occurrence Cases Corresponding to Scenarios

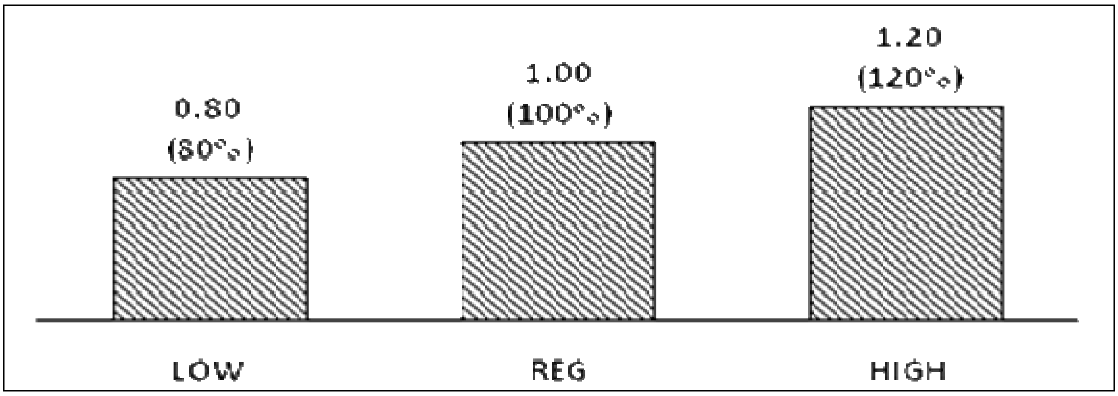

Figure 7: Demand Distribution at 20% Demand Volatility

5.3 Important Parameters of the Stochastic Solution

The impact of optimization under uncertain demand is measured using the key performance parameters of stochastic programming:

1. **Value of Stochastic Solution (VSS):** VSS is equal to the difference between the Z_{SLP} and Z_{EEV}. VSS is the impact on net (contribution to) profit when the available information is neglected.
2. **% Improvement in NP:** This signifies the percentage improvement in net (contribution to) profit from SLP. This is the value of VSS in terms of percentage of Z_{EEV}.
3. **Expected Value of Perfect Information (EVPI):** This is equal to the difference between the Z_{PI} and Z_{SLP}.

6.0 Application of the DSS in the Steel Company

We describe the application of the SLP model in a steel company. The company is an integrated steel plant with an annual turnover of USD 1400 million located in North America. The company produced 104 final products. To demonstrate the scope of optimization, we describe the range of different parameters (See Table 3).

Production Parameters		**Model Parameters**	
Annual Turnover (Million USD)	1,400	**Number of Variables**	44100
Annual Production (Tons)	860,000	**Number of Constraints**	40472
Sell Price Ratio	7.38	**Number of Coefficients (Non zeros)**	168600
Market Demand Ratio	1,841.12	**Sparseness (LP Density – Non zeros)**	0.0094%
Buy Price Ratio	147.99	**Number of Materials**	632
Facility Activity Ratio (T/H)	3,240.83	**Number of Facilities**	56
Activity Cost Ratio (US USD /Ton)	178.57	**Number of Activities**	1286
		Number of Planning Periods	3
		Number of Scenarios	3

Table 3: Industry Characteristics and Optimization Variability

6.1 Impact of Stochastic Programming in Steel Company

To illustrate the impact of SLP in the steel company, we discuss the results of the scenarios (20% demand volatility with equally likely occurrence of each scenario); see Table 4, Figure 9. The results of SLP for the above mentioned scenario are presented in the column chart below; see Figure 8:

Planning Periods	Total Revenue	Cost of Purchases	Cost of Inv Carrying	Cost of Activities	Net Profit	Total Steel
Unit	Million USD	Million USD	Million USD	Million USD	Million USD	Tons
Grand Total	3,295.4	801.3	121.1	742.4	**1,630.5**	2,182,111
Unit Basis	1510.2	367.2	55.5	340.2	747.2	
Scenario: L	1,080.4	266.5	40.4	244.2	529.3	2,190,040
Scenario: R	1,102.5	266.8	40.2	248.9	546.5	2,177,207
Scenario: H	1,112.5	268.0	40.5	249.3	554.6	2,179,085

Table 4: Equally Likely Scenario, 20% Demand Volatility

Note: The above table should be replicated for a Cartesian product of demand variability of 20%, 30% and 40%, and the three probability distributions like left skewed, equally likely, and right skewed

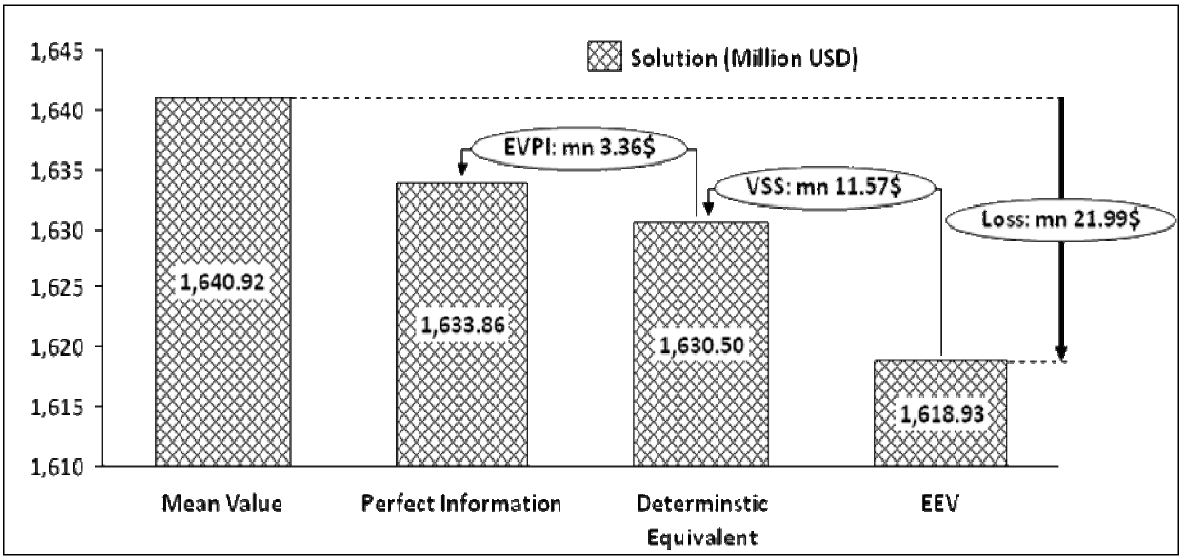

Figure 8: Results of SLP Model for an Equally Likely Occurrence and 20% Demand Volatility

1. One would only solve a MV problem, when the information about the future is not available, and expect to achieve Z_{MV} (USD 1,640.9 million), while once the planning horizon is over, one would end up realizing only Z_{EEV} (USD 1,618.9 million). A loss of ($Z_{MV} - Z_{EEV}$) USD 21.99 million is incurred due to the non availability of any information about the future.

2. The Z_{MV} is highest among all the solutions namely Z_{MV}, Z_{PI}, Z_{SLP}, and Z_{EEV}. This indicates that practically Z_{MV} (USD 1640.9 million) can never be realized. In the long run, one can only realize the Z_{PI} (USD 1633.9 million) even with the availability of perfect information.
3. The impact of stochastic programming is measured in terms of VSS (USD 11.6 million), the improvement in net (contribution to) profit by Z_{SLP} compared to Z_{EEV}. The VSS as a percentage of Z_{EEV} is 0.71%.
4. There are instances when it is possible to get perfect information about the future and know with certainty which scenario would occur. In these situations we are expected to achieve a long run solution as Z_{PI} (USD 1,633.9 million). When we use the partially available information about the future scenario occurrence, we are able to achieve Z_{SLP} (USD 1,630.5 million). The expected value of perfect information is the difference between Z_{PI} and Z_{SLP} (USD 3.4 million).
5. In most practical situations, buying partial information (scenario forecasts) about the future with a significantly low investment is a feasible option, while the value derived using the partial information is significant i.e. VSS (USD 11.6 million).
6. The opportunity for buying perfect information from the market is practically close to zero. Alternatively, one needs to incur an infinite amount of money to buy perfect information, whereas the marginal value derived using such information compared to partial information is very small, that is, EVPI (USD 3.4 million).

6.2 SLP Model Validation

According to the principles of stochastic programming "(Birge, 1997)", the order of solutions should follow a decreasing trend as follows ($Z_{MV} >= Z_{PI} >= Z_{SLP} >= Z_{EEV}$) for a SLP with maximization objective. The optimization results from the application in a steel company in this research confirm the results in line with the principles of SLP. The order of solution reverses for a minimization objective SLP. It can be inferred from the above analysis that the PI solution is a long run expected profit and is a 'wait and see' type of solution. Under an immediate solution implementation situation, one may like to obtain a solution which takes all the expected scenarios into account, and provides a single solution for the first stage i.e. a 'here and now' solution. The SLP provides a single implementable solution for the first stage; in addition, it also ensures the maximization of total profit under the occurrence of any scenario.

6.3 Trend Analysis with Volatility in Market Demand

Probability Distribution		**Right Skewed (R)**			**Equally Likely (E)**			**Left Skewed (L)**		
Demand Volatility Cases →		**20%**	**30%**	**40%**	**20%**	**30%**	**40%**	**20%**	**30%**	**40%**
Model Instances		**1**	**2**	**3**	**1**	**2**	**3**	**1**	**2**	**3**
PI	Mn $	1605.3	1579.5	1551.4	**1633.9**	**1626.3**	**1617.1**	1658.5	1665.0	1669.8
MV	Mn $	1610.3	1591.2	1567.8	1640.9	1640.9	1640.9	1661.2	1670.5	1679.4
SLP	Mn $	1603.8	1577.1	1548.5	**1630.5**	1621.3	1610.5	1654.0	1658.5	1661.0
EEV	Mn $	1593.2	1563.9	1523.9	**1618.9**	**1604.8**	**1587.1**	1646.4	1646.4	1644.8
VSS	Mn $	10.6	13.2	24.6	11.6	16.5	23.4	7.6	12.1	16.2
VSS (% of EEV)	%	0.67%	0.84%	1.61%	0.71%	1.03%	1.47%	0.46%	0.73%	0.98%
EVPI	Mn $	1.5	2.4	2.8	3.4	5.0	6.7	4.5	6.5	8.8

Table 5: Results of Experiments from Multi-Scenario Planning (Mn means million)

Note:

1. Demand variability 20%, 30%, and 40% is indexed as 1, 2, and 3
2. R, E, and L stands for Right Skewed, Equally Likely, and Left Skewed respectively
3. PI, MV, SLP, EEV stands for Perfect Information, Mean Value Solution, Stochastic Programming Solution, and Expectation of Expected Value

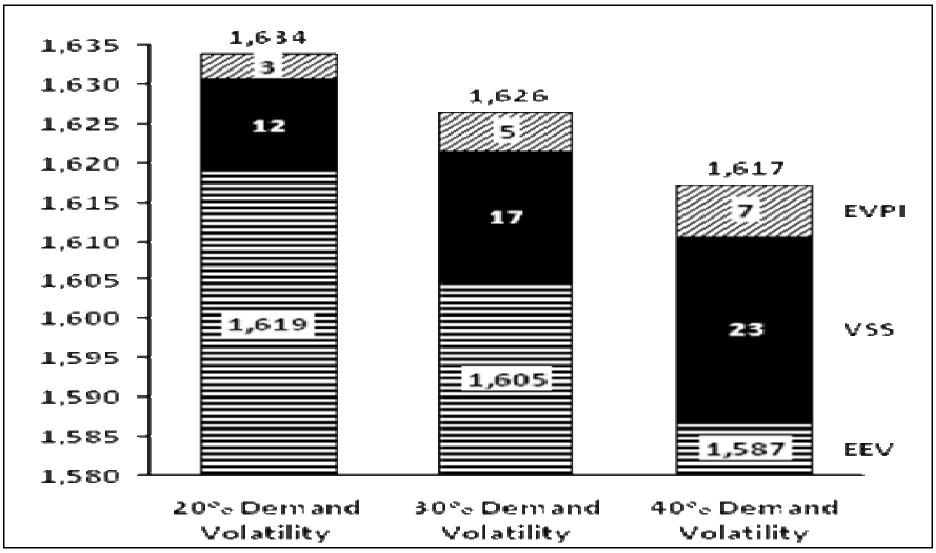

Figure 9: Stochastic Optimization Results over Increasing Volatility of Demand

We attempt to draw inferences from the incremental value of net (contribution to) profit when demand volatility increases from 20% to 40% (Table 5). The important inferences from different solutions due to a change in demand volatility from 20% to 40% are listed as follows; see Figure 10, and 11.

1. The Z_{PI} decreases from USD 1633.9 million to USD 1617.1 million, while Z_{SLP} decreases from USD 1630.5 million to USD 1610.5 million. The EVPI increases from USD 3.4 million to USD 6.7 million. The increase in EVPI is primarily due to a steep reduction in Z_{SLP} compared to the Z_{PI}.
2. The Z_{EEV} decreases from USD 1618.9 million to USD 1587.1 million, however the VSS increases from USD 11.6 million to USD 23.4 million. The increase in VSS is primarily due to the steep rate of reduction in Z_{EEV} compared to the Z_{SLP}.
3. The Z_{MV} remains unchanged while there is a significant decrease of USD 31.9 million in Z_{EEV}. One expects to obtain Z_{MV} (USD1640.9 million), but would end up achieving only Z_{EEV} (USD 1587.1 million); see 40% demand volatility case (Table 5).
4. The VSS, % improvement in NP, and EVPI shows an increase of USD 11.8 million, (0.76 equivalent points), and USD 3.3 million respectively.
5. The increase in VSS (USD 11.8 million) is significantly higher than the increase in EVPI (USD 3.3 million).

The experiments demonstrate that an increase in the volatility in demand of finished goods increases the VSS; see Figure 10. The Z_{EEV} decreases, while VSS and EVPI increase with the increase in demand volatility. It is also interesting to observe that the rate of increase of VSS is significantly steeper than EVPI with reference to the increase in demand volatility. The results are consistent in the remaining experiments of probability skewness, that is, right skewed, left skewed. As the volatility of demand of finished steel increases, the total contribution from SLP consistently decreases; see Figure 11.

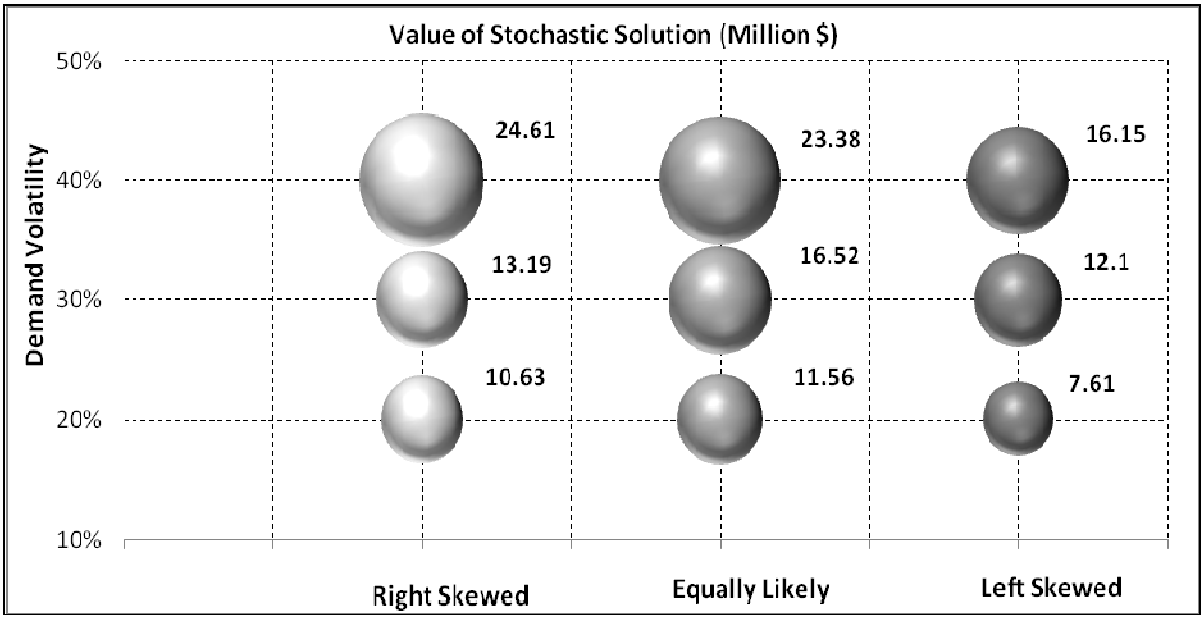

Figure 10: Trend in VSS over Increasing Volatility of Demand

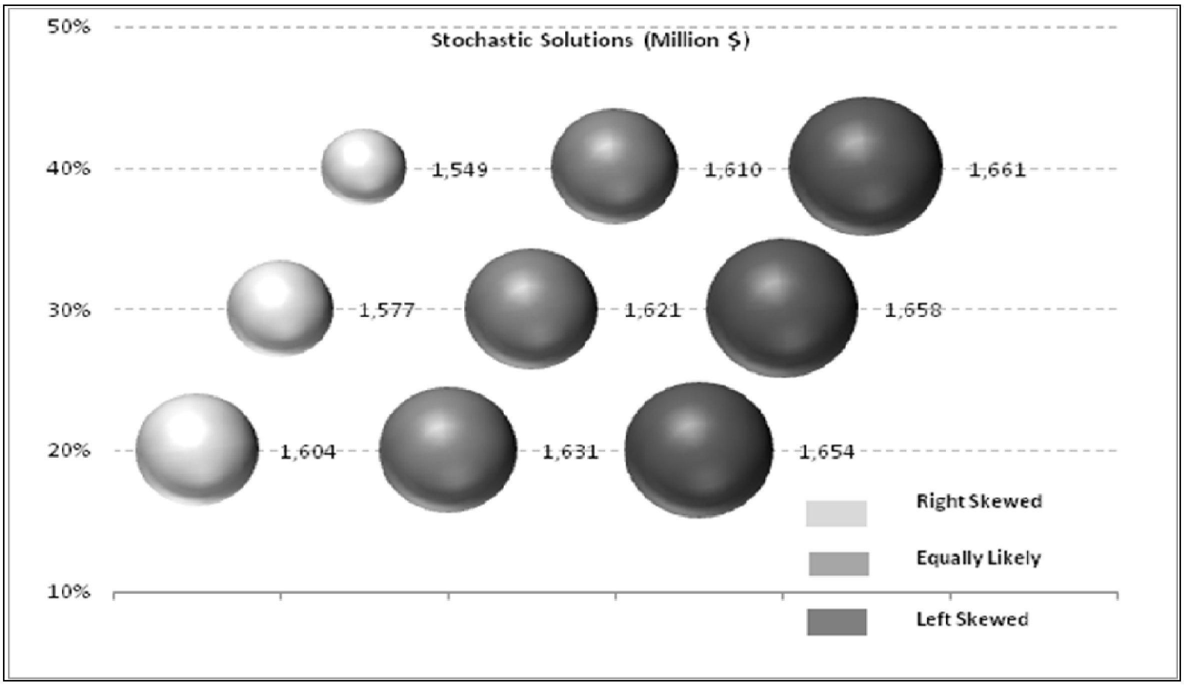

Figure 11: Decrease in Total Contribution from Stochastic Optimization over Increasing Demand Volatility

6.4 Trend Analysis over Probability Distribution

We observe that the VSS is the lowest when the probability of occurrence of a high demand situation is the highest, that is, left skewed probability distribution. This indicates that as the probability of occurrence of a lower demand situation increases, the VSS increases. The impact of optimization on the VSS using stochastic programming becomes more visible. Companies are expected to be more concerned about profits when the probability of occurrence of low demand is a little high. This indicates that it makes more sense to apply optimization when the probability of occurrence of a low demand situation is high.

We also attempt to identify a pattern in the VSS with a change in the discrete probability distribution. The VSS is highest in an equally likely scenario for the 20% and 30% demand volatility, but it is highest in the right skewed situation for the 40% demand volatility. We report that it is difficult to find any specific trend in the VSS when the probability distribution changes. To study the pattern, more experiments with different process industry real data may be required.

It is interesting to note that the EVPI consistently increases with a change in the discrete probability distribution from a right skewed to a left skewed demand situation. The EVPI is the highest with the left skewed probability situation (followed by equally likely and right skewed probability situations) in each of the three demand volatility cases, that is, 20%, 30%, and 40%. The investigation of this pattern reveals that the EVPI is high when the probability of occurrence of high demand volatility case is high. This reasoning is applicable to all the three demand volatility cases, that is, 20%, 30%, and 40%.

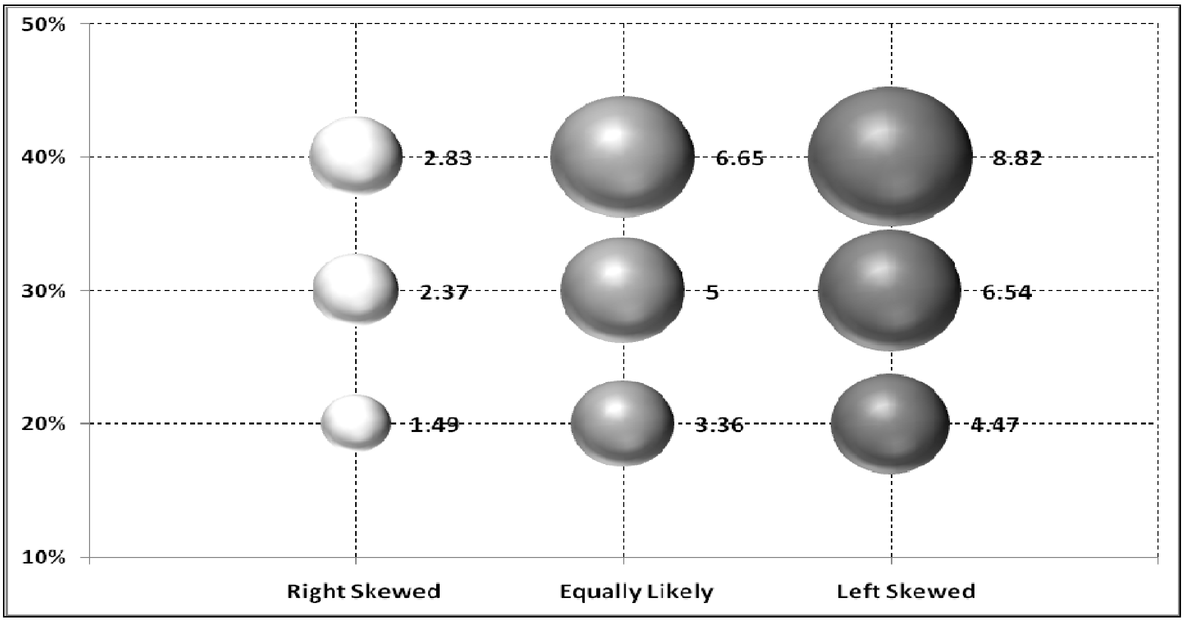

Figure 12: Changes in EVPI with the change in the discrete probability distribution

Figure 12 clearly shows that the Z_{SLP} increases with the increasing probability of high demand situations (Right skewed to Left skewed distribution). The Z_{EEV} increases at a steeper rate than Z_{SLP} with the increase in the probability of occurrence of high demand situations (i.e. Right skewed to left skewed distribution). One may notice that when the probability of occurrence of a low demand situation is high, one plans for a Z_{MV}, but realizes Z_{EEV}. The difference in Z_{MV} and Z_{EEV} happens to be more due to incorrect first stage decisions. The SLP corrects the first stage decisions in such a way that the losses due to Z_{MV} expectations are minimized and the VSS is maximized. When the high demand situation occurs with a high probability of occurrence (left skewed distribution), the Z_{EEV} starts approaching Z_{SLP} and thereby the VSS in this situation is lower than the VSS in the right skewed probability distribution.

7. 0 Conclusion and Limitations

The focus of the study was to model a process industry in a demand uncertain environment. The paper presented the application of a two stage stochastic programming with recourse model in a steel company in North America. While there have been several studies on SLP, we did not see much reported work on a SLP based DSS and its application with real data in industries in general, and process industries in particular. In this paper, we have made an attempt to bridge the gap, and demonstrate the impact of SLP using a user friendly DSS.

1. The uncertainty in multiple input parameters can be modeled using the stochastic programming based generic DSS. In this research, we designed a set of experiments by varying the volatility of the demand of finished goods by a pre-defined value and likelihood occurrence of the only demand scenarios.
2. The size of the model increases significantly with the increase in the number of uncertain parameters in the model. To deal with the speed of model generation, we designed and developed a relational database structure Gupta *et al.*, (2014, in review); see Figure 2.
3. The implementation of the multi-scenario model in the DSS demonstrated a significant potential to improve the company's (contribution to) profit.
4. A typical observation was that the profit from the application of the stochastic programming solution (in terms of VSS and EVPI) increases with the increase in the spread of demand distribution.
5. The optimization results of the application in the steel company conforms with the theory of two stage stochastic programming.
6. Another important inference is that the VSS is very high in a right skewed probability distribution as compared to the left skewed distribution (Table 5). This emphasizes the need for stochastic programming in an uncertain demand situation to improve profitability, especially when the probability of occurrence of the low demand situation is very high.
7. A consistent pattern in EVPI is observed, that is, EVPI is the highest in the situation when the probability of occurrence of high demand is high (left skewed) in each of the three demand cases, that is 20%, 30%, and 40%.

This research describes the inferences from the application of SLP in a steel company. All of the inferences cannot be generalized to the application of stochastic optimization in other industries and contexts. To generalize these inferences, one needs more evidence and results from multiple application instances. The results cannot be generalized because the optimization results are an outcome of a multiple set of varied inputs like facility capacity restrictions at many facilities and stages, supply, demand, and storage space restrictions in different scenarios etc. The spread of these model parameters may change in different contexts and so will the optimization results and inferences.

We demonstrate that the application of stochastic programming in process industries can result in a significant impact on the profits of a company, a successful use of the technique involves facing several challenges. The potential variability in model parameters is a known fact; however, finding multiple sets of reliable data corresponding to the individual economic situation and scenarios is one of the biggest challenges in modeling uncertainty using stochastic programming.

8.0 Extensions

The current research of multi-scenario, multi-period planning has a number of potential extensions.

1. The SLP based user friendly DSS has been currently tested for uncertainty in demand of finished steel. The DSS being generic, it may be interesting to test the DSS for modeling uncertainty in multiple input parameters simultaneously, for example, demand and sell price of finished steel, cost of purchase of raw materials and so on.
2. This research considers only three empirical probability distribution instances to test the DSS. A large number of different realistic probability distributions can be attempted, and a Monte Carlo simulation in the SLP based DSS may reveal interesting trends and inferences from the results.
3. The application of this SLP based DSS in multiple process industries for strategic planning can lead to the generalization of observations resulting from this research.

4. Uncertainty in multiple parameters of the model including supply of raw materials, costs of raw material purchase, and price of selling finished goods can be modeled using stochastic programming.
5. One may like to model the non-linear behavior of costs with an increasing scale of operation.
6. The non-linearity may further be extended to model under a demand uncertain environment.
7. The process industry modeling using fuzzy LP and fuzzy stochastic linear program can be a worthwhile extension of this research.
8. The development of a SLP based DSS for an integrated supply chain planning and its application with real data from a process industry can be another interesting extension to this research.
9. The SLP can be explored in asset liability modeling and capture the uncertainty in model parameters.

Appendix

Model Formulation

We define sets, parameters, variables, objective function and constraints of the SLP as follows.

Time Data: $T = \{1\ldots T\}$ is the set of time periods in the planning horizon, indexed by t

ρ_l is the interest rate per period in each of the scenario l, taken as zero if there is no discounting

Scenario Data: $L = \{1\ldots L\}$ is the set of scenarios in the planning horizon, indexed by l

p_l is the probability of occurrence of the scenario l

Materials Data:

***M* is the set of all materials, indexed by j**

$l_{jlt}^{buy}, u_{jlt}^{buy}, c_{jlt}^{buy}$ are lower limit, upper limit, and cost per unit of material j purchased, for each $j \in M, l \in L, and\ t \in T$ respectively

$l_{jlt}^{sell}, u_{jlt}^{sell}, c_{jlt}^{sell}$ are lower limit, upper limit, and revenue per unit of material j sold, for each $j \in M, l \in L, and\ t \in T$

$l_{jlt}^{inv}, u_{jlt}^{inv}, c_{jlt}^{inv}$ are lower limit, upper limit, and holding cost per unit of material j inventoried, for each $j \in M, l \in L, and\ t \in T$

v_{j0}^{inv} = initial inventory of material j, for each $j \in M$

$M^{conv} \subseteq \{j \in M, j' \in M : j \neq j'\}$ is the set of conversions:

$(j,j') \in M^{conv}$ means that material j can be converted to material j'

$\alpha_{jj'lt}^{conv}$ = number of units of material j' that result from converting one unit of material j, for each $(j, j') \in M^{conv}$, $l \in$ L, $t \in T$

$c_{jj'lt}^{conv}$ = cost per unit of material j of conversion from j to j', for each $(j, j') \in M^{conv}$, $l \in$ L, $t \in T$

***Facilities Data: F* is the set of facilities, indexed by i**

l_{ilt}^{cap}, u_{ilt}^{cap} are minimum, and maximum unit of capacity of facility i that must be used, for each $i \in F, l \in L, and\ t \in T$ respectively

c_{ilt}^{cap} = cost of outsourcing a unit of capacity at facility i, for each $i \in F, l \in L, and\ t \in T$

$F^{in} \subseteq F$ is the set of facility inputs: $(i, j) \in F^{in}$ that material j is used as an input at facility i

l_{ijlt}^{in}, and u_{ijlt}^{in} are the minimum, and maximum amount of material j that must be used as input to facility i, for each $(i, j) \in F^{in}$, $l \in L$, $t \in T$

$F^{out} \subseteq F$ is the set of facility outputs $(i, j) \in F^{out}$ that material j is produced as an output at facility i

l_{ijlt}^{out}, and u_{ijlt}^{out} are the minimum, and maximum amount of material j that must be produced as output at facility i, for each $(i, j) \in F^{out}$, $l \in L$, $t \in T$

Activities Data: A is the set of activities, indexed by k

$F^{act} \subseteq \{(i, k): i \in F\}$ is the set of activities:

$(i, k) \in F^{act}$ means that k is an activity available at facility i

$l_{iklt}^{act}, u_{iklt}^{act}$ are the minimum, and maximum number of units of activity k that may be run at facility i, for each $(i, k) \in F^{act}$, $l \in L$, $t \in T$

c_{iklt}^{act} = the cost per unit of running activity k at facility i, for each $(i, k) \in F^{act}$, $l \in L$, $t \in T$

r_{iklt}^{act} = the number of units of activity that can be accommodated in one unit of capacity of facility i, for each $(i, k) \in F^{act}$, $l \in L$, $t \in T$

$A^{in} \subseteq \{(i, j, k, t): (i, j) \in F^{in}\ (i, k) \in F^{act}, t \in T\}$ is the set of activity inputs:

$(i, j, k, t) \in A^{in}$ means that input material j is used by activity k at facility i during time period t

α^{in}_{ijklt} = units of input material j required by one unit of activity k at facility i in time

Period t, in scenario l, for each $l \in L$, and $(i, j, k, t) \in A^{in}$

$A^{out} \subseteq \{(i, j, k, t)$: $(i, j) \in F^{out}$ $(i, k) \in F^{act}$, $t \in T\}$ is the set of activity outputs:

$(i, j, k, t) \in A^{out}$ means that output material j is produced by activity k at facility i

during time period t

α^{out}_{ijklt} = units of output material j produced by one unit of activity k at facility i in time

Period t, scenario l, for each, $l \in L$, and $(i, j, k, t) \in A^{out}$

Storage-areas Data: *S* is the set of storage areas, indexed by s.

l^{stor}_{slt} = lower limit on total material in storage area s, for each $s \in S, l \in L, and\ t \in T$

u^{stor}_{slt} = upper limit on total material in storage area s, for each $s \in S, l \in L, and\ t \in T$

Variables

x^{buy}_{jlt}, x^{sell}_{jlt}, and x^{inv}_{jlt} are the units of material j bought, sold, and inventoried for each $j \in M, l \in L, and\ t \in T$ respectively

x^{stor}_{jslt} = units of material j in storage area s, for each $j \in M, s \in S, l \in L, and\ t \in T$

x^{inv}_{j0} = initial inventory of material j, for each $j \in M$

$x^{conv}_{jj'lt}$ = units of material j converted to material j', for each $(j, j') \in M^{conv}$, $l \in L$, $t \in T$

x^{in}_{ijlt} = units of material j used as input by facility i, for each $(i, j) \in F^{in}$, $l \in L$, $t \in T$

x^{out}_{ijlt} = units of material j produced as output by facility i, for each $(i, j) \in F^{out}$, $l \in L$, $t \in T$

x^{act}_{iklt} = units of activity k operated at facility i, for each $(i, k) \in F^{act}$, $l \in L$, $t \in T$

x_{ilt}^{cap} = units of capacity vendored at facility i, for each $i \in F, l \in L, and\ t \in T$

First Stage Variables

x_{1j}^{buy} = units of material j bought, for each $j \in M$, in first period

x_{1j}^{sell} = units of material j sold, for each $j \in M$, in first period

x_{1js}^{stor} = units of material j in storage area s, for each $j \in M$, $s \in S$, in first period

x_{1j}^{inv} = total units of material j in inventory (storage), for each $j \in M$, in first period

x_{1jj}^{conv} = units of material j converted to material j', for each $(j, j') \in M^{conv}$, in first period

x_{1ij}^{in} = units of material j used as input by facility i, for each $(i, j) \in F^{in}$, in first period

x_{1ij}^{out} = units of material j produced as output by facility i, for each $(i, j) \in F^{out}$, in first period

x_{1ik}^{act} = units of activity k operated at facility i, for each $(i, k) \in F^{act}$, in first period

x_{1i}^{cap} = units of capacity outsourced at facility i, for each $i \in F$, in first period

Objective

Maximize the sum over all time periods of revenues from sales less costs of purchasing, holding inventories, converting, operating activities at facilities, and cost of operating facilities:

$$Z_N = \sum_{l \in L} p_l \sum_{t \in T} Z(l,t)$$

Objective function for nominal cash flows Eq. (1)

$$Z_D = \sum_{l \in L} p_l \left(\sum_{t \in T} (1+\rho_l)^{-t} Z(l,t) \right)$$

Objective function for discounted cash flows Eq. (2)

Where,

$$Z(l,t) = \sum_{j \in M} c_{jlt}^{sell} x_{jlt}^{sell} - \sum_{j \in M} c_{jlt}^{buy} x_{jlt}^{buy} - \sum_{j \in M} c_{jlt}^{inv} x_{jlt}^{inv} - \sum_{(j,j') \in M^{conv}} c_{jj'lt}^{conv} x_{jj'lt}^{conv} - \sum_{(i,k) \in F^{act}} c_{iklt}^{act} x_{iklt}^{act}$$

$$- \sum_{i \in F} c_{ilt}^{cap} x_{ilt}^{cap}$$

Eq. (3)

Constraints

For each $j \in M$, $l \in L$ and $t \in T$, the amount of material j made available by purchases, production, conversions and beginning inventory must equal the amount used for sales, production, conversions and ending inventory:

$$x_{jlt}^{buy} + \sum_{(i,j) \in F^{out}} x_{ijlt}^{out} + \sum_{(j',j) \in M^{conv}} \alpha_{j'jlt}^{conv} x_{j'jlt}^{conv} + x_{jlt-1}^{inv}$$

$$= x_{jlt}^{sell} + \sum_{(i,j) \in F^{in}} x_{ijlt}^{in} + \sum_{(j,j') \in M^{conv}} x_{jj'lt}^{conv} + x_{jlt}^{inv} \qquad \text{Eq. (4)}$$

For each $(i, j) \in F^{in}$, $l \in L$ and $t \in T$, the amount of input j used at facility i must equal the total consumption by all the activities at facility i:

$$x_{ijlt}^{in} = \sum_{(i,j,k,t) \in A^{in}} \alpha_{ijklt}^{in} x_{iklt}^{act} \qquad \text{Eq. (5)}$$

For each $(i, j) \in F^{out}$, $l \in L$ and $t \in T$, the amount of output j produced at facility i must equal the total production by all the activities at facility i:

$$x_{ijlt}^{out} = \sum_{(i,j,k,t) \in A^{out}} \alpha_{ijklt}^{out} x_{iklt}^{act} \qquad \text{Eq. (6)}$$

For each $i \in F$, $l \in L$ and $t \in T$, the capacity used by all activities at facility i must be within the range given by the lower limit and the upper limit plus the amount of capacity vendored:

$$l_{ilt}^{cap} \leq \sum_{(i,k) \in F^{act}} x_{iklt}^{act} / r_{iklt}^{act} \leq u_{ilt}^{cap} + x_{ilt}^{cap} \qquad \text{Eq. (7)}$$

For each $j \in M$, the amount of material inventoried in the plant before the first time period is defined to equal the specified initial inventory:

$$x_{j0}^{inv} = v_{j0}^{inv} \qquad \text{Eq. (8)}$$

For each $j \in M$, $l \in L$ and $t \in T$, the total amount of material j inventoried is defined as the sum of the inventories over all storage areas:

$$\sum_{s\in S} x^{stor}_{jslt} = x^{inv}_{jlt}$$ Eq. (9)

For each $s \in S$, $l \in$ L and $t \in T$, the total of all materials inventoried in storage area s must be within the specified limits:

$$l^{stor}_{slt} \leq \sum_{j\in M} x^{stor}_{jslt} \leq u^{stor}_{slt}$$ Eq. (10)

Implementability (Non-Anticipativity) Constraints

$x^{buy}_{jlt} = x^{buy}_{1j}$ for each of the $j \in M$, $l \in L$ and $t = 1$ Eq. (11)

$x^{sell}_{jlt} = x^{sell}_{1j}$ for each of the $j \in M$, $l \in L$ and $t = 1$ Eq. (12)

$x^{stor}_{jslt} = x^{stor}_{1js}$ for each of the $j \in M$, $s \in S, l \in L$ and $t = 1$ Eq. (13)

$x^{inv}_{jlt} = x^{inv}_{1j}$ for each of the $j \in M$, $l \in L$ and $t = 1$ Eq. (14)

$x^{conv}_{jj'lt} = x^{conv}_{1jj}$ for each $(j, j') \in M^{conv}$, $l \in L$, and $t = 1$ Eq. (15)

$x^{in}_{ijlt} = x^{in}_{1ij}$ for each $(i, j) \in F^{in}$, $l \in L$, and $t = 1$ Eq. (16)

$x^{out}_{ijlt} = x^{out}_{1ij}$ for each $(i, j) \in F^{in}$, $l \in L$, and $t = 1$ Eq. (17)

$x^{act}_{iklt} = x^{act}_{1ik}$ for each $(i, k) \in F^{act}$, $l \in L$, and $t = 1$ Eq. (18)

$x^{cap}_{ilt} = x^{cap}_{1i}$ for each $i \in F$, $l \in L$, $t = 1$ Eq. (19)

All variables must lie within the relevant limits (bounds) defined by their respective bounds.

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