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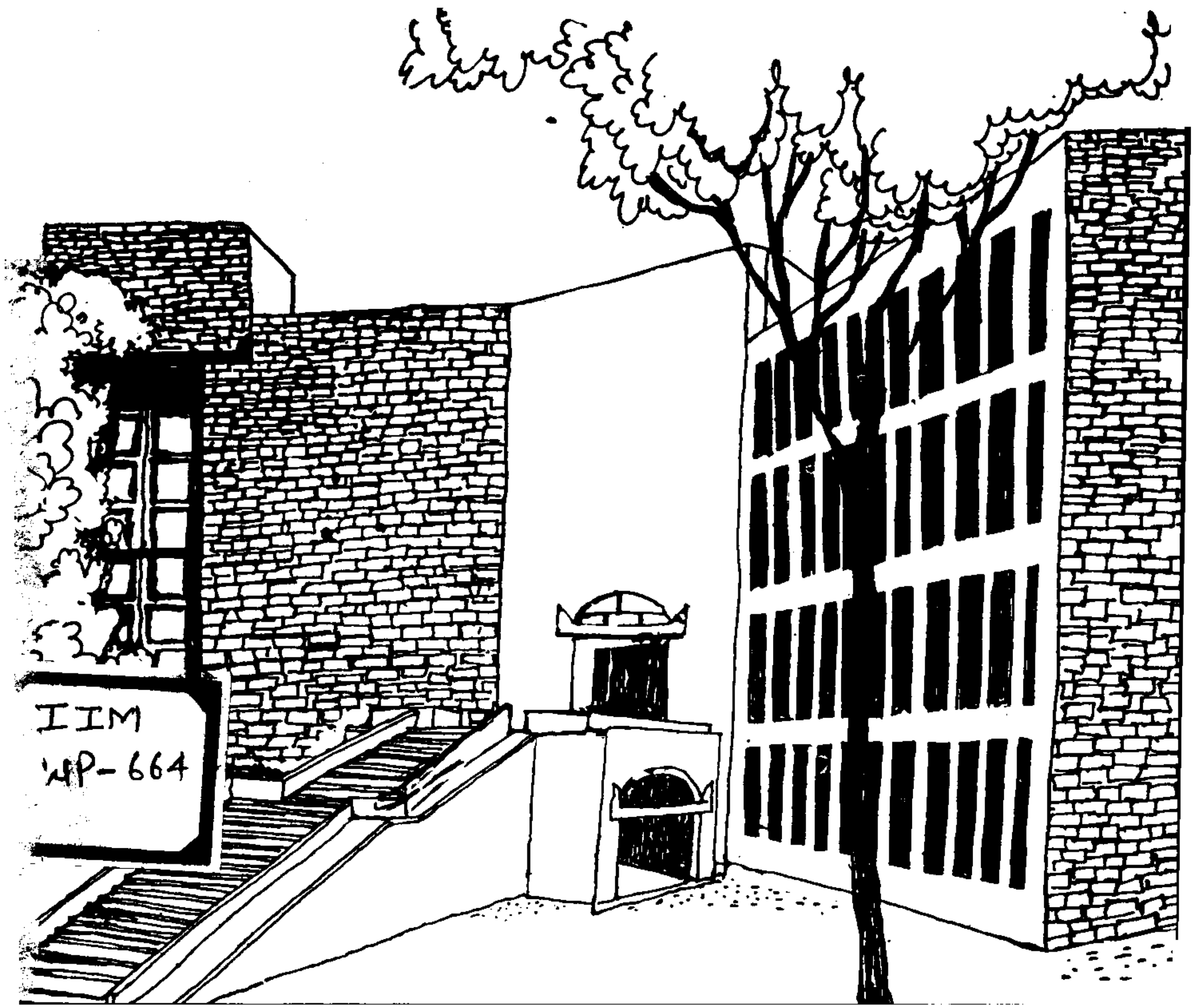

IMPLICATIONS OF CHANGES IN THE HOLDING PERIOD AND OTHER PARAMETERS ON SYSTEMATIC RISK AND PERFORMANCE OF A SECURITY

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"IMPLICATIONS OF CHANGES IN THE HOLDING PERIOD AND OTHER PARAMETERS ON SYSTEMATIC RISK AND PERFORMANCE OF A SECURITY"

ABSTRACT

The Capital Asset Pricing Model is a single period model which specifies a linear relationship between return on an asset and return on the entire market. The model is widely used in literature as if a portfolio of securities can be designed based on a unique value of systematic risk. In this paper it is shown that in reality it is not possible to design a portfolio based on a unique value of systematic risk and performance index of securities, since both these measures are a function of not only the holding period, but also the values of expected market return and the risk-free rate of return likely to prevail for the period under consideration. Further, using computer simulation the paper captures the extent of impact of the holding period, expected market return, risk-free rate of return and the interaction of the holding period and expected market return together, on the single period measure of systematic risk and performance index of a security. The simulation results also show that other parameters such as the variance of market return, variance of the error term and other combinations of interaction terms do not have any significant impact on the single period measure of systematic risk.

IMPLICATIONS OF CHANGES IN THE HOLDING PERIOD AND OTHER PARAMETERS ON SYSTEMATIC RISK AND PERFORMANCE OF A SECURITY

Introduction

The capital asset pricing model (CAPM) developed by Sharpe (1964) Lintner (1965) and Mossin (1966) specifies a linear relationship between return on an asset and return on the entire market comprising all the assets available for investment. It is essentially a single period model which can be derived on the assumption that the one-period return on each asset follows a normal distribution. All investors are assumed to agree on the parameters of these distributions. In addition, they are expected to have identical investment horizon, as specified by the length of one period. In reality though, even if investors agree on the parameters of the one-period return distributions, the investment horizon is likely to vary across investors, and an extension of the model to multi-period situation becomes necessary. Such an extension poses difficulties because of the possible dependence between returns over time and because of the non-linear relationship between multi-period and one-period returns.

The existence of long term dependence in security returns was established by Greene and Fielitz (1977). In the presence of long term dependence, the appropriate model for return distributions would be normal with drift, which is also known as fractional Gaussian noise. In such a situation, Greene and

Fielitz (1979) established that the risk rankings of stocks or portfolios would vary with the length of the period chosen for computing returns. These inferences also implied that the composition of efficient portfolio would depend on the length of the investment horizon. The usefulness of CAPM, which assumes that investors have identical investment horizon for portfolio decisions, in reality, is therefore questionable.

Levy (1972) explored the impact of investment horizon in a different manner. He showed that even if it is assumed that the return distributions are normal and independent from period to period, the reward to variability ratio of efficient portfolios would be a function of the length of the investment horizon, because of the non-linear relationship between multi-period and one-period returns. Subsequently, Levhari and Levy (1977) showed that even for a single security, any deviation in the length of investment horizon from the length of the "true" investment horizon, causes a consistent bias both in the measure of the systematic risk, β, as well as in the index of performance measured by the ratio between the expected return and β of the security. They derived the direction of the bias, depending on the value of β for the true horizon and the relationship between the lengths of the true and the actual horizons. Another interesting feature observed by them was that for any given value of β for the true horizon, the higher the length of the investment horizon, the higher would be the index of performance.

This paper is an extension of the work done by Levhari and Levy. It reports the direction as well as the extent of impact on β and the index of performance of a security because of changes not only in the holding period but also in the other parameters used in deriving the CAPM. Also, the approach adopted in this paper completely dispenses with the need for assuming any "true" investment horizon, unlike Levhari and Levy. The impact of the length of the investment horizon (n), the expected market return (E_m), the variance of market return (V_m), the risk-free rate of return (r_f) and the variance of the error term (V_e), on β and the performance index of a security, has been assessed using computer simulation.

Generation of Data

The one-period market return for period t, r_{mt}, was randomly generated from a normal distribution with parameters E_m and V_m. The error term r_{et}, for the period was generated from a normal distribution with parameters zero and V_e. The one-period return on a security i, for period t, r_{it}, for specified values of single period measure of systematic risk, β_1, and the risk-free rate, r_f, was computed as follows:

$$r_{it} = r_f + \beta_1 (r_{mt} - r_f) + r_{et}$$

The values of the parameters used for generating different sets of one-period returns are presented in Table 1. As can be seen from the table, r_f is specified at two levels, E_m and β_1 are specified at five levels each, and V_m and V_e are specified at three levels each, resulting in a total of 450 different sets of values of parameters. These sets capture a wide spectrum of conditions that are likely to prevail in the market for a weekly holding period.

TABLE 1

Using each set of parameter values, N one period market and security returns were generated; N defining the length of run for the simulation performed subsequently. The value of N itself was specified at three different levels of 780, 1560 and 3120. The n-period returns, corresponding to any set of single period returns, were computed by taking the product of sets of n successive one-period returns. The number of n-period returns in one set would be N/n. The values of n used were 2, 4, 13, 26 and 52, representing fortnightly, monthly, quarterly, half-yearly and yearly holding periods.

Computer Simulation

The n-period market and security returns generated for various combination of parameters were used to estimate the value of β_n, the n-period measure of the systematic risk, by regressing the security returns with the market returns, using the CAPM specification. The average value of the normalised measure of the systematic risk, $\hat{\beta}_n / \beta_1$, for different initial values of β_1 and for a given value of N were computed. To find out whether the results were affected by the initial values (seeds) used for generating the random numbers, ten different sets of seeds were used resulting in ten different average values of $\hat{\beta}_n$ / β_1 . The average values of these normalised measures of n-period systematic risk for various combinations of initial conditions are presented in Table 2. As is apparent from the values, the initial condition had little impact on the values, and lengths of run of 780 observations produced results which were very close to the results produced with 3120 observations. Similar results were obtained when the normalised risk measure was averaged over E_m and r_f . This indicated that 3120 observations would certainly be adequate to provide stable results. Hence, the analysis was done only of the results obtained with 3120 observations.

TABLE 2

Analysis of Variance

The variability observed in the average value of normalised risk was sought to be explained by the variations in the five parameters as well as the variation resulting from interactions among these parameters. The five-way classification model to capture the main, as well as the interaction effects was specified as follows:

$$y_{ijklmp} = \mu + S_1 + S_2 + S_3 + S_4 + S_5 + u_{ijklmp}$$

Where y_{ijklmp} is the observed value of normalized risk corresponding to

i^{th} level of E_m,

j^{th} level of σ_m,

k^{th} level of σ_e,

l^{th} level of r_f,

m^{th} level of n, and

p^{th} level of β_1 (not consiered an assignable factor).

μ is the average response

S_1 is the sum of responses of factors taken one at a time, i.e., $S_1 = a_i + b_j + c_k + d_l + e_m$, where a_i is the average response due to i^{th} level of E_m and so on .

S_2 is the sum of responses of factors taken two at a time, that is, $S_2 = (ab)_{ij} + \ldots + (de)_{lm}$, where $(ab)_{ij}$ is the average response due to i^{th} level of E_m and j^{th} level of σ_m and in general

S_w is the sum of responses of factors taken w at a time, and

u_{ijklmp} is the error term.

The results of the analyses of variance, in terms of F-values, for a typical set is presented in Table 3. It is apparent from the table that the length of the holding period had the most significant impact on the normalized value of systematic risk, being significant at $\alpha = .001$. The impact of the expected market return was significant at $\alpha = .005$ and that of the risk free rate at $\alpha = .05$. Of the interaction terms, the second level interaction between the holding period and the expected market return was significat at $\alpha = .05$. The other F-values were not significant even at $\alpha = .1$.

TABLE 3

The analysis of variance was done for all the ten sets of initial conditions. The F-values for the four variables mentioned in the earlier paragraph, for each run, are presented in Table 4. As is apparent the conclusion reached earlier is largely borne out.

TABLE 4

To determine the direction of impact of these variables, the average values of normalised measures of systematic risk for various levels of these variables are presented in Table 5. It was observed that the normalized risk increases with increase in the levels of holding period and expected market return, and it decreases with increase in the risk-free rate.

TABLE 5

Homogenising on β_1

The mean effects of the three parameters, n, r_f , and E_m and the interaction term (n,E_m) were further analysed by computing the average values of normalised measures of systematic risk for various initial values of β_1 . These results are presenteed in Tables 6a and 6b. The results revealed several interesting features. If the value of β_1 was less than one, then normalised measure of systematic risk decreased with increase in the length of the holding period, while if the value of β_1 exceeded one, it increased with increase in the length of the holding period. Thus, the direction of the impact of holding period depended on the value of β_1 . A similar impact on the normalised measure of risk was also observed with respect to the increase in the expected market return. However the impact on normalised risk was opposite with respect to increase in the risk-free rate. In other words, the value of normalised risk increased with increase in risk-free rate, when β_1 was less than one and the value decreased with increase in risk-free rate, when β_1 was greater than 1 .

TABLE 6

Impact on Performance Measure

The performance index of a security for n-period holding (P_n) could be represented by the following expression:

$$P_n = \frac{(1+E_i)^n - (1+r_f)^n}{\hat{\beta}_n}$$

Where E_i is the one period expected return on security i.

The average values of the performance index for the ten runs, for various levels of and various values of n, r_f, E_m and the interaction team (E_m, n) is summarized in Tables 7a and 7b. It is easily observed from Table 7a that the performance measure increases with increase in the length of the holding period, for all values of β_i, though for higher values of β_i, the increase is relatively smaller compared to that of the lower values of β_i The impact of the expected market return on the performance measure is also similar. However the effect of increasing the risk-free rate is understandably the opposite, i.e., the performance measure decreases with increase in the risk-free rate.

Conclusion

The basic findings of the paper may briefly be summed up as follows:

a. Holding period has the most significant impact on the measure of systematic risk β, especially when the value of one-period measure of systematic risk, β_i, is substantially different from one. When it is smaller than one, the estimated measure of risk decreases with increase in the holding period, while when it is greater than one, the estimated measure of risk increases with increase in the holding period. β shows a similar behaviour with respect to the expected market return E_m as well as the interaction term (n, E_m). However, impact of the risk free return r_f on β though significant, is opposite. That is, an increase in r_f causes a rise in the risk measure when β_1 is less than one and causes a drop, when it is greater than one.

b. Further, the results show that the performance measure of a security increases with increase in the holding period. The results also indicate that irrespective of the values of β_1, the performance measure of a security improves with increase in expected market return and the interaction term and worsens with increase in the risk-free return.

In practical terms the results in Table 6a may be interpreted to imply that holding an aggressive security ($\beta_1 > 1$) longer is riskier, whereas holding a defensive security ($\beta_1 < 1$) longer reduces the risk. Further, if the market return is expected to be high in the period under consideration, aggressive stocks would become more risky and defensive stocks less risky. An increase in the risk-free rate has an opposite effect on the riskiness of securities. Also, when the market return is expected to be high during the period, if securities are held longer, both E_m and n would be higher so that the aggressive stocks would become more risky, while the defensive stocks would become less risky. However, when the market return is expected to be low, if the securities are held longer, E_m is lower whereas n increases, so that the two effects neutralise each other's impact considerably, though the resultant impact reflects th relative dominance of increasing the holding period over th reduction in the expected market return (see Table 6b).

Another important implication of the results concerns studie on market efficiency using comparisons of trading rules with bu and hold strategy. The two strategies cannot be compared becaus returns are measured over different holding periods, and holdin period itself has a significant impact on the measure of risk an performance. Thus, to be able to compare the performance of trading strategy with buy and hold strategy one must have performance measure which is free from the influence of th holding period.

Finally, the results underscore the fact that in reality portfolio cannot be designed on the basis of a unique value of systematic risk and measure of performance of securities, because both these measures are a function of not only the holding period but also the values of expected market return and the risk-free rate of return likely to prevail for the period under consideration.

TABLE 1

Parameter Values

Parameter	Levels				
	1	2	3	4	5
r_f	0.0015	0.003			
E_m	0.005	0.0075	0.01	0.0125	0.015
E_m / σ_m	0.25	0.50	0.75		
σ_e^2 / σ_m^2	0.50	0.75	0.85		
β_1	0.75	1.00	1.25	1.50	1.75

TABLE 2

Normalized Risk for Various Simulation Runs

Length of run, N = 780

Initial Condition

n	1	2	3	4	5	6	7	8	9	10
2	1.0011	1.0035	1.0041	1.0019	1.0027	1.0015	1.0012	1.0012	1.0026	1.0021
4	1.0071	1.0077	1.0077	1.0059	1.0055	1.0091	1.0047	1.0070	1.0079	1.0042
13	1.0240	1.0276	1.0216	1.0239	1.0247	1.0257	1.0240	1.0203	1.0274	1.0221
26	1.0644	1.0553	1.0454	1.0527	1.0544	1.0520	1.0490	1.0456	1.0548	1.0501
52	1.1333	1.1027	1.1141	1.1172	1.1231	1.1097	1.1112	1.1196	1.1033	1.1202

Length of run, N = 1560

2	1.0020	1.0023	1.0026	1.0019	1.0026	1.0021	1.0013	1.0012	1.0037	1.0025
4	1.0066	1.0050	1.0070	1.0063	1.0060	1.0088	1.0052	1.0064	1.0095	1.0044
13	1.0218	1.0227	1.0229	1.0246	1.0247	1.0287	1.0270	1.0210	1.0299	1.0230
26	1.0577	1.0479	1.0521	1.0512	1.0551	1.0537	1.0524	1.0462	1.0578	1.0537
52	1.1191	1.1118	1.1135	1.1206	1.1155	1.1200	1.1152	1.1143	1.1119	1.1247

Length of run, N = 3120

2	1.0017	1.0030	1.0021	1.0015	1.0026	1.0018	1.0015	1.0020	1.0021	1.0020
4	1.0065	1.0066	1.0062	1.0071	1.0071	1.0074	1.0044	1.0062	1.0066	1.0044
13	1.0234	1.0259	1.0213	1.0252	1.0255	1.0263	1.0256	1.0224	1.0237	1.0239
26	1.0547	1.0524	1.0494	1.0524	1.0551	1.0528	1.0536	1.0458	1.0522	1.0556
52	1.1168	1.1189	1.1149	1.1200	1.1192	1.1218	1.1174	1.1109	1.1107	1.1239

TABLE 3

Analysis of Variance : A Typical Result

Term	F-Value
n	89.9323
E_m	17.2058
$\sigma_{\bar{m}}$	0.2047
$\sigma_{\bar{e}}$	1.2618
r_f	4.5904
n, E_m	4.4716
$n, \sigma_{\bar{m}}$	0.0450
$n, \sigma_{\bar{e}}$	0.1294
n, r_f	1.3039
$E_m, \sigma_{\bar{m}}$	1.0764
$E_m, \sigma_{\bar{e}}$	0.4777
E_m, r_f	0.2264
$\sigma_{\bar{m}}, \sigma_{\bar{e}}$	0.1553
$\sigma_{\bar{m}}, r_f$	0.3499
$\sigma_{\bar{e}}, r_f$	0.5251

Term	F-Value
$n, E_m, \sigma_{\bar{m}}$	0.2144
$n, E_m, \sigma_{\bar{e}}$	0.1431
n, E_m, r_f	0.2076
$n, \sigma_{\bar{m}}, \sigma_{\bar{e}}$	0.0625
$n, \sigma_{\bar{m}}, r_f$	0.1184
$n, \sigma_{\bar{e}}, r_f$	0.0655
$E_m, \sigma_{\bar{m}}, \sigma_{\bar{e}}$	0.3797
$E_m, \sigma_{\bar{m}}, r_f$	0.8122
$E_m, \sigma_{\bar{e}}, r_f$	0.6577
$\sigma_{\bar{m}}, \sigma_{\bar{e}}, r_f$	0.4205
$n, E_m, \sigma_{\bar{m}}, \sigma_{\bar{e}}$	0.0876
$n, E_m, \sigma_{\bar{m}}, r_f$	0.2710
$n, E_m, \sigma_{\bar{e}}, r_f$	0.1100
$n, \sigma_{\bar{m}}, \sigma_{\bar{e}}, r_f$	0.1261
$E_m, \sigma_{\bar{m}}, \sigma_{\bar{e}}, r_f$	0.5696
$n, E_m, \sigma_{\bar{m}}, \sigma_{\bar{e}}, r_f$	0.1181

TABLE 4

F-Values for Significant Terms

Term	Run Number									
	1	2	3	4	5	6	7	8	9	10
n	84.46	79.30	80.31	72.44	86.86	87.65	86.68	68.53	70.79	89.93
E_m	17.09	18.47	23.67	11.94	23.77	20.12	22.39	20.33	15.31	17.21
r_f	1.90	4.35	4.03	7.69	1.59	2.86	1.33	2.83	5.89	4.59
n,E_m	4.06	5.13	5.94	4.22	6.79	6.50	6.16	4.82	4.85	4.47

TABLE 5

Direction of Impact on Normalised Risk

Impact of Holding Period		Impact of Risk-free Rate		Impact of Exp. Market Return	
Holding Period	Average Normalized risk	Risk-free rate*	Average Normalised risk	Exp. Mark Return*	Average Normalized Risk
2	1.0022	0.0015	1.0451	0.0050	1.0131
4	1.0065	0.0030	1.0362	0.0075	1.0265
13	1.0246			0.0100	1.0386
26	1.0526			0.0125	1.0558
52	1.1177			0.0150	1.0697

* Parameters specified on one-period basis.

TABLE 6a

Impact of β_1 on Normalized Risk for Various Parameters

β_1	Holding Period				
	2	4	13	26	52
0.75	0.9995	0.9959	0.9786	0.9516	0.9056
1.00	1.0001	1.0006	1.0003	1.0031	1.0016
1.25	1.0022	1.0059	1.0231	1.0480	1.1036
1.50	1.0034	1.0117	1.0479	1.1010	1.2217
1.75	1.0062	1.0184	1.0729	1.1594	1.3559

β_1	Risk-free rate*	
	0.0015	0.003
0.75	0.9657	0.9667
1.00	1.0004	1.0018
1.25	1.0401	1.0330
1.50	1.0855	1.0688
1.75	1.1340	1.1111

β_1	Expected Market Return*				
	0.005	0.0075	0.01	0.0125	0.015
0.75	0.9844	0.9748	0.9647	0.9611	0.9461
1.00	1.0029	1.0012	0.9978	1.0029	1.0008
1.25	1.0111	1.0234	1.0384	1.0465	1.0635
1.50	1.0264	1.0517	1.0705	1.1066	1.1305
1.75	1.0406	1.0813	1.1215	1.1619	1.2075

* Specified on one-period basis

TABLE 6b

Impact of β_i on Normalized Risk for Interaction Term

Market Return	Holding Period 2	4	13	26	52
	β_i = 0.75				
0.0050	1.0002	0.9959	0.9921	0.9780	0.9557
0.0075	0.9977	0.9957	0.9816	0.9641	0.9349
0.0100	1.0000	0.9994	0.9785	0.9481	0.8978
0.0125	0.9997	0.9965	0.9729	0.9437	0.8928
0.0150	0.9996	0.9918	0.9678	0.9245	0.8467
	β_i = 1.00				
0.0050	0.9997	1.0002	1.0006	1.0046	1.0095
0.0075	1.0002	1.0010	1.0002	1.0024	1.0024
0.0100	1.0004	1.0001	0.9988	0.9970	0.9927
0.0125	0.9999	1.0021	1.0024	1.0061	1.0041
0.0150	1.0005	0.9995	0.9994	1.0052	0.9993
	β_i = 1.25				
0.0050	0.9998	1.0015	1.0063	1.0135	1.0344
0.0075	1.0007	1.0025	1.0151	1.0336	1.0648
0.0100	1.0031	1.0062	1.0221	1.0497	1.1106
0.0125	1.0023	1.0077	1.0328	1.0613	1.1287
0.0150	1.0050	1.0117	1.0393	1.0819	1.1797
	β_i = 1.50				
0.0050	1.0007	1.0026	1.0163	1.0355	1.0770
0.0075	1.0032	1.0095	1.0358	1.0679	1.1419
0.0100	1.0015	1.0101	1.0441	1.0916	1.2049
0.0125	1.0053	1.0170	1.0652	1.1391	1.3065
0.0150	1.0061	1.0193	1.0779	1.1710	1.3782
	β_i = 1.75				
0.0050	1.0016	1.0063	1.0238	1.0543	1.1167
0.0075	1.0063	1.0153	1.0525	1.1075	1.2248
0.0100	1.0056	1.0180	1.0752	1.1599	1.3486
0.0125	1.0069	1.0230	1.0948	1.2114	1.4735
0.0150	1.0104	1.0291	1.1185	1.2638	1.6160

TABLE 7a

Impact of β_1 on Performance Measure for Various Parameters

β_1	Holding Period				
	2	4	13	26	52
0.75	0.0156	0.0317	0.1111	0.2489	0.6239
1.00	0.0156	0.0317	0.1097	0.2403	0.5852
1.25	0.0156	0.0315	0.1081	0.2343	0.5523
1.50	0.0156	0.0314	0.1066	0.2276	0.5223
1.75	0.0155	0.0313	0.1051	0.2210	0.4957

β_1	Risk-free Rate*	
	0.0015	0.003
0.75	0.2224	0.1901
1.00	0.2125	0.1804
1.25	0.2032	0.1735
1.50	0.1942	0.1672
1.75	0.1870	0.1604

β_1	Expected Market Return*				
	0.005	0.0075	0.01	0.0125	0.015
0.75	0.0626	0.1262	0.1990	0.2746	0.3689
1.00	0.0607	0.1221	0.1909	0.2629	0.3457
1.25	0.0603	0.1196	0.1823	0.2536	0.3260
1.50	0.0592	0.1163	0.1783	0.2397	0.3100
1.75	0.0583	0.1134	0.1705	0.2313	0.2951

* Specified on one-period basis

TABLE 7b

Impact of β_1 on Performance Measure for Interaction Term

Market Return Level	Holding Period 2	4	13	26	52
	β_1 = 0.75				
0.0050	0.0055	0.0112	0.0376	0.0799	0.1790
0.0075	0.0106	0.0214	0.0733	0.1582	0.3674
0.0100	0.0156	0.0315	0.1099	0.2437	0.5943
0.0125	0.0207	0.0419	0.1479	0.3316	0.8309
0.0150	0.0257	0.0526	0.1869	0.4312	1.1480
	β_1 = 1.00				
0.0050	0.0055	0.0111	0.0373	0.0782	0.1715
0.0075	0.0106	0.0213	0.0725	0.1542	0.3521
0.0100	0.0156	0.0316	0.1087	0.2367	0.5620
0.0125	0.0207	0.0419	0.1454	0.3203	0.7863
0.0150	0.0257	0.0524	0.1844	0.4118	0.0540
	β_1 = 1.25				
0.0050	0.0055	0.0111	0.0373	0.0780	0.1694
0.0075	0.0105	0.0213	0.0719	0.1520	0.3422
0.0100	0.0155	0.0315	0.1074	0.2299	0.5273
0.0125	0.0206	0.0418	0.1433	0.3133	0.7490
0.0150	0.0256	0.0520	0.1806	0.3983	0.9735
	β_1 = 1.50				
0.0050	0.0055	0.0111	0.0370	0.0768	0.1654
0.0075	0.0105	0.0212	0.0710	0.1493	0.3293
0.0100	0.0156	0.0314	0.1063	0.2265	0.5117
0.0125	0.0206	0.0415	0.1411	0.3016	0.6937
0.0150	0.0257	0.0518	0.1775	0.3835	0.9113
	β_1 = 1.75				
0.0050	0.0055	0.0111	0.0369	0.0760	0.1621
0.0075	0.0105	0.0211	0.0704	0.1464	0.3184
0.0100	0.0155	0.0313	0.1045	0.2186	0.4827
0.0125	0.0206	0.0415	0.1394	0.2932	0.6617
0.0150	0.0256	0.0516	0.1744	0.3707	0.8534

REFERENCES

1. GREENE M.T. & FIELITZ B.D., "The Effect of Long Term Dependence on Risk-Return Models of Common Stocks", Op. Res., 27(5) (September-October 1979), 944-951.

2. LEVY H., "Portfolio Performance and Investment Horizon", Mgt. Sci., 18(12) (August 1972), B 645-653.

3. LEVHARI D. & LEVY H., "The Capital Asset Pricing Model and the Investment Horizon", R. Econ. & Stat., 59(1) (February 1977), 92-104.

4. LINTNER J., "Security Prices Risk and Maximal Gains from Diversification", J. Fin., 20(4) (December 1965), 587-616.

5. MOSSIN J., "Equilibrium in Capital Asset Market", Econometrica, 34(4) (1966), 768-783.

6. SHARPE W.F., "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk", J. Fin., 19(3) (September 1964), 425-442.