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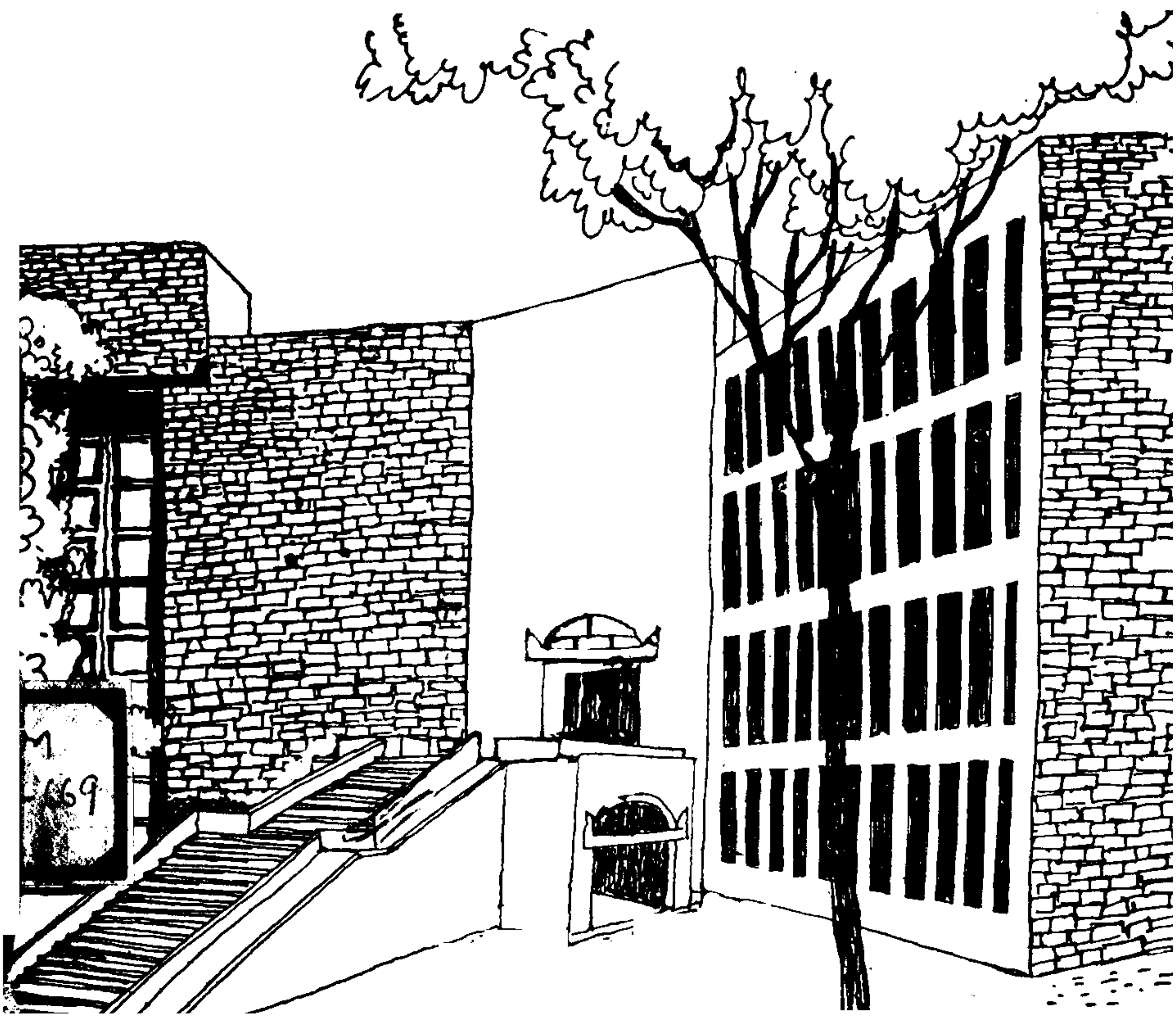

TARGET DEBT MAINTENANCE UNDER ALTERNATIVE NET PRESENT VALUE SPECIFICATIONS AND IMPLICATIONS FOR INVESTMENT AND FINANCE DECISIONS

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ABSTRACT

Two alternative specifications of weighted average cost of capital are prevalent in finance literature. Though both the specifications result in consistant accept/reject or ranking decisions the net present value arrived is different under each method. This paper traces the origin of this difference and resolves the same. It is shown that if projects are funded in such a way that resulting debt to equity is the optimal ~~target~~ Leverage then both specifications will yield identical net present values. In cases where such capital structure maintenance is not feasible there is loss of value due to unused debt capacity. We arrive at a lower bound for such a loss and also show that in such situations there exists a possibility of a synergy between projects which are otherwise independent.

Target Debt Maintenance Under Alternative Net Present Value Specifications and Implications for Investment and Financing Decisions

Introduction:

Considerable literature in finance is devoted to the weighted average cost of capital (WCOC). The issues range from whether or not the tax shield on interest should be incorporated in the cost of capital, to the suitability of one or the other specification of WCOC for investment and financing decisions. Nantell and Carlson [3] show that each specification of WCOC can be used as a cut-off rate for project evaluation, provided the cash flows discounted are suitably specified. While this contention of Nantell and Carlson is true, the fact remains that the net present values (NPVs) for a project obtained by using different specifications of WCOC are different, except in case of a value maintaining project (NPV = o).

For non-value maintaining projects (NPV ≠ 0), Greenfield, Randall and Woods [(GRW), 2] show that in order to maintain the target debt to equity ratio of a firm, the actual financing mix of the incremental project will have to be different from the target. In this paper we show that the NPVs obtained from the two specifications of WCOC will be identical, if the financing of the projects is done as suggested by GRW, so that the firm's target debt to equity ratio is maintained. Next we show that only one of the two WCOC specifications can be used for arriving at the appropriate project financing decision. We further show that there are situations where it may not be possible to fund a

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project by GRW formula. Under such conditions if unutilised debt capacity is created in the firm we show that a project with negative NPV which might ordinarily have been rejected becomes acceptable, if such a project uses some of the unutilised debt capacity.

<u>Valuation Under Alternative Specifications of Weighted Average Cost of Capital</u>:

The two major specifications of WCOC for a firm are:

$$K = K_e \frac{S}{D+E} + r\,(1-T)\,\frac{D}{D+E} \qquad \ldots (1)$$

and

$$K' = K_e \frac{S}{D+E} + r\,\frac{D}{D+E} \qquad \ldots (2)$$

where K and K' are alternative specifications of WCOC,

K_e = Cost of Equity,

r = Cost of Debt,

T = Tax Rate,

E = Market Value of Equity,

D = Market Value of Debt, and

V = D+E, is the value of the firm.

The cash flow specifications (assuming an infinite stream) consistent with K and K' are X_t and X_t' respectively, specified as:

$$X_t = X\,(1 - T) \qquad \ldots (3)$$

and

$$X_t' = X\,(1 - T) + IT \qquad \ldots (4)$$

Where X = The Operating Cash flows, and

I = rD, is the Interest Bill .

Nantell and Carlson show that these two specification types, viz. (K, X_t) and (K', X'_t) result in the same accept/reject or ranking decisions. However, it turns out that the NPVs resulting from these two specifications are not equal, whenever the project's NPV is non zero. In fact, for all projects with positive/negative NPV, the NPV obtained from specification (K, X_t) is always higher/ more negative than the NPV obtained from specification (K', X'_t). This is proved as under.

Let us consider a firm undertaking an incremental project with investment I. Let the firm's market value be V, consisting of debt valued at D and equity valued at E so that $V = D+E$. Let the firm's target debt to equity ratio be D:E. Further let us assume that the project is financed with debt B and equity S, so that $B + S = I$, and $B:S = D:E$. Let the project's operating earnings be X and the corporate tax rate be T.

Then in accordance with the two alternate specification types (K, X_t) and (K', X_t'), we have:

$$PV_1 = \frac{X(1-T)}{K}, \text{ and } PV_2 = \frac{X(1-T) + RT}{K'}$$

Where $R = rB$.

We have, $$\Delta PV = PV_1 - PV_2 = \frac{X(1-T)}{K} - \frac{X(1-T) + RT}{K'} \quad \ldots (5)$$

But $$K' = K + \frac{rBT}{B+S} = K + \frac{RT}{B+S}, \quad \ldots (6)$$

Substituting for K' in (5), we have

$$\Delta PV = \frac{X(1-T)}{K} - \frac{X(1-T) + RT}{K + \frac{RT}{B+S}}$$

or $$\Delta PV = X(1-T) - \frac{(X(1-T) + RT)(B+S)}{(B+S)K + RT}$$

or $$\Delta PV = \frac{RT(X(1-T) - K(B+S))}{K \quad (B+S)K + RT}$$

or $$\Delta PV = \frac{RT\left(\frac{X(1-T)}{B+S} - K\right)}{K \cdot K'} \quad \ldots (7)$$

Now for NPV of the project to be positive we must have,

$\frac{X(1-T)}{K} > B+S$, so that:

$$\frac{X(1-T)}{B+S} > K \quad \ldots (8)$$

Therefore, from (6) and (7), PV must be positive, which implies that $PV_1 > PV_2$.

This result is irritating in so far as one would like to be certain as to whether PV_1 or PV_2 yields the correct market value of the project.

<u>Origin and Resolution of Difference in Valuation Under Alternative Specifications of WCOC</u>:

At first glance the above difference in the two present values appears to be intrinic to the manner in which the two WCOC's and the associated cash flows are specified. This however is not true. The two specifications yield different NPV's only because when a project (NPV = 0) is financed as per the target debt to equity ratio, the resulting debt to equity ratio for the project based on market values is not equal to the target debt to equity ratio (see GRW). This difference between the target debt to equity ratio and the actual debt to equity ratio obtained for the project, introduces a bias in the cash flow when it is specified as X_t', through the R element. This bias in the X_t' disappears when the project is in such a way that the target dept to equity ratio is obtained.

GRW discuss the procedure for arriving at the amount of project investment which should be funded by debt so that the target debt to equity ratio is maintained. The logic of this procedure is briefly dealt with below.

When the project is funded in the ratio of D:E, the resulting debt to equity ratio is $(D+B) : (E+S+NPV_1)$ which is not equal to the target ratio of D:E. Thus, essentially, the firm's problem is to finance the project with such a mix of debt (B') and equity (S'), such that the following equations are satisfied:

$$B' + S' = I \qquad \ldots (9)$$

$$\frac{D + B'}{E+S'+NPV_1} = \frac{D}{E} \qquad \ldots (10)$$

where $NPV_1 = PV_1 - I$

Solving (8) and (9) yields:

$$B' = PV_1 \frac{D}{D+E}, \qquad \ldots (11)$$

Therefore $S' = I - B'$.

Now, if the incremental project under consideration is funded by debt equal to B' and equity equal to S', it can be seen that the present value PV_2 obtained by using the (K', X_t') type of specification exactly equals PV_1. Consider the following:

We have $PV_2 = \frac{X(1-T) + rB'T}{K'} \qquad \ldots (12)$

Where $K' = K_e \frac{E}{D+E} + r \frac{D}{D+E}$

Substituting for K' from Equation 6, and B' from Equation 11, into Equation 12, we have:

$$PV_2 = \frac{X(1-T) + r(PV_1)\left(\frac{D}{D+E}\right)T}{K + r\left(\frac{D}{D+E}\right)T}$$

But $$PV_1 = \frac{X(1-T)}{K}$$

Substituting for PV_1 in equation 13 and simplifying, we have:

$$PV_2 = \frac{X(1-T)}{K} = PV_1.$$

It is thus clear that when the incremental project is financed such that the resulting debt to equity ratio conforms to the target debt to equity ratio, both specifications result in identical present values.

<u>Project Financing Decisions and Alternative Specifications of WCCC:</u>

However there is a very major difference between the (K, X_t) type and (K', X_t') type specifications. While the former can be used to estimate the project financing pattern which would maintain the firm's target capital structure, the latter specification cannot be used for the purpose. This is because PV_2 itself is dependent on B' in the first place and B' cannot be determined without regard to PV_1, which in turn is obtained from the (K, X_t) type specification **In other words, while (K, X_t) type specification** is suitable for both investment and financing decisions, (K', X_t') specification cannot be used for financing decisions because of which its suitability for investment decisions also becomes less general[1].

1 Note that this conclusion is more or less similar to what Mantell and Carlson conclude, but the issues and arguments provided by us are quite different from theirs.

Infeasibility of Capital Structure Maintenance, Valuation and Financial Synergy:

We now consider a special case where the capital structure indicated for the project under the GRW framework for maintaining the target debt to equity ratio is not feasible to realise. This happens when the borrowing B' required to maintain the target debt to equity ratio for the firm exceeds the investment (I) required for the incremental project, under this situation, if the borrowing is limited to the investment required for the project the resulting debt to equity ratio becomes $(D+I)/(E+NPV_1)$, which is less than the target leverage of $D : E$.[2] This condition is obtained, whenever $NPV_1 > I \frac{E}{D}$. In such cases, the firm may borrow in excess of the required investment and redeem a part of the equity. This may however not be possible in cases where the project is new and also in cases where redemption of equity is prohibited by regulation (like in India, U.K. etc.). Under these conditions, the firm has to settle for borrowing less than the optimum level (assuming that the targeted leverage is optimal), and some unutilised debt capacity is created. This is also the case when a firm moves from a lower to a higher target debt to equity ratio.

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[2] This is easily seen as under:

$$B' = PV_1 \frac{D}{D+E} > I \Rightarrow NPV_1 > I \frac{E}{D} \quad (\text{Given } PV_1 - I = NPV_1)$$

$$\text{or} \quad \frac{I}{NPV_1} < \frac{D}{E}$$

$$\text{Therefore} \quad \frac{D+I}{E+NPV_1} < \frac{D}{E}$$

This debt capacity thus created by the sub-optimal leverage results in an appropriate loss in the market value of the firm, which is now below its optimum value. However, each such project which creates an unutilised debt capacity in the firm may improve the viability of the next project if the unused borrowing capacity created earli.r can be utilised while financing the next project. This results in a situation where a project with negative NPV which would ordinarily be rejected outright, may be accepted if it can absorb the existing unutilised debt capacity. Thus, even when the projects are economically independent, they are synergistically linked through the firm's optimal capital structure. We shall now illustrate the above discussion through an example.

Example:

Let us assume the following parameters:

Current Market Value of the firm (V)	= Rs. 30,000
Current Market Value of the Equity of the firm (E)	= Rs. 10,000
Current Market Value of the Debt of the firm (D)	= Rs. 20,000
Target Debt to Equity Ratio (D:E)	= 2 : 1
Cost of Equity (K_e)	= .30
Cost of Debt (r)	= .15
Tax Rate (T)	= 50%
Investment required for an incremental project (I) (in the same risk class as the firm).	= Rs. 1000
Before Tax operating cash inflow (X) from the project (in perpetuity)	= Rs. 900

Using (K, X_t) type specification, we have

$$X_t = X(1-T) = 450$$

$$K = K_e \frac{E}{D+E} + r(1-T)\frac{D}{D+E} = .15$$

∴ The present value of the incremental project (PV_1)

$$= \frac{X(1-T)}{K} = \frac{450}{.15} = \text{Rs. } 3000.$$

$$\therefore NPV_1 = PV_1 - I = \text{Rs. } 2000.$$

Thus, the implied borrowing B' for financing the project in order to maintain the target D:E ratio $= PV_1 \frac{D}{D+E} = 3000 \times \frac{2}{3} =$ Rs. 2000.

However, the investment required is only Rs. 1000. If the firm is in a position to redeem a part of its equity, the firm may still borrow Rs. 2000 and use Rs. 1000 for financing the incremental project and utilise the balance to redeem its equity. In such a situation, the firm's total borrowings would be valued at Rs. 22000 and its equity at Rs. 11,000, so that the target debt to equity is obtained.

On the other hand, if the firm is not allowed to redeem its equity as in India, U.K. etc., the firm's borrowing is restricted to Rs. 1000 only. Thus, even when the entire project is financed by debt, the firm's resulting debt to equity ratio falls short of the optimum leverage of 2:1, and unutilised debt capacity worth Rs. 1000 is created. Owing to this unutilised debt capacity, the firm's value remains short of its optimum high. In other words now, consequent to the acceptance of the above project, the firm's value will not increase by Rs. 2000, but by a lesser value.

It is difficult to estimate this loss of market value precisely, since a precise effect of deviation of leverage from optimum on the firm's cost of capital is difficult to estimate. However, it is quite possible to determine the lower bound for this loss of market value with precision. This is given by $PV_1 - PV_2$, where PV_2 is determined from (K', X_t') type specification.

We have:

$X_t = X(1-T) + rBT = 525$ [where B, the project debt = Rs. 1000]

and

$$K' = K_e \frac{E}{D+E} + r \frac{D}{D+E} = .20$$

$$\therefore \; PV_2 = X_t'/K' = \text{Rs. } 2625.$$

$$\therefore \; NPV_2 = PV_2 - I = \text{Rs. } (2625 - 1000) = \text{Rs. } 1625.$$

Thus the minimum loss in the present value of the project owing to the unutilised debt capacity created will be Rs. (3000 - 2625) = Rs. 375. Rs. 375 is the lower bound for the loss in value, because, K' is the optimum cost of capital, so that Rs. 2625 is the maximum value which PV_2 can take.

Alternatively, the minimum loss in the present value of the project may be viewed as the present value (at rate K) of the tax shield on the interest of the unutilised debt (B'-B). This is given by $\frac{r(B'-B)T}{K}$, which is Rs. 375.

After the firm finance the above project with debt, let us assume that another project enters its opportunity set. This project is identical to the above project except that the investment required is much higher, at say Rs. 3200. Thus for this project, the net present value with (K, X_t) type specification will be negative (- Rs. 200). Such a project is ordinatily rejected outright.

However, at the time this project enters the firm's opportunity set, the firm's financial leverage is less than optimal. If this opportunity is used to utilise the debt capacity fully, the above project yields a positive net present value. This may be seen from below.

The present value using (K, X_t) type specification for the second project is Rs. 3000. Assuming that the previous project did not leave any unutilised debt capacity in the firm, the borrowing required for the project in order to maintain the target debt to equity ratio of 2:1, comes to Rs. 2000. However, the previous project did leave an unutilised debt capacity worth Rs. 1000. Therefore in order to utilise this debt capacity fully, this project should be financed with a total debt of Rs. 3000 and equity of Rs. 200, so that the resultant leverage is again the target 2:1.

It can be seen now that the present value of this project with (K', X_t') type specification is now Rs. 3375. In other words, the loss of Rs. 375 owing to the unutilised debt capacity while accepting the first project is now made up. Thus, the project now yields a positive NPV of Rs. 175 (i.e., Rs. 3375 - Rs. 3200), instead of a negative NPV of Rs. 200.

Alternatively, taken jointly, the two projects involve an investment of Rs. 4200 and a present value of Rs. 6000, so that the joint net present value of the two projects together is Rs. 1800. However, taken independently of each other the first project yields an NPV of Rs. 1625, whereas the second project yields a negative NPV of Rs. 200, so that the firm obtains an overall NPV of Rs. 1425. The difference of Rs. 375 (1800-1425) is in fact the benefit of synergy arising from the maintenance of debt capacity of the firm when the two projects are viewed jointly. In other words, though economically independent (operating cash flows being un-correlated) the two projects are synergistically linked through the firm's optimal capital structure.

Conclusion:

In conclusion the discussion above may be summarised as follows:

a. The two commonly used specifications for WCOC do not yield identical NPVs for project evaluation.

b. The reason for above is not owing to any intrinsic difference between the two specifications, but due to the non maintenance of the target leverage of the firm while financing a project. When a project is financed such that the firm's target leverage is obtained, the two specifications yield identical NPVs.

c. The (K, X_t) type specification alone is useful for arriving at the project's financing scheme so as to obtain the firm's target leverage.

d. Under conditions when the firm's target leverage cannot be maintained by accepting a project, the value added by the project to the firm is less than optimal. It is difficult to estimate this loss in value precisely. However, the (K', X_t') type specification is useful in arriving at the lower bound for this loss.

e. When maintenance of target leverage is infeasible, the firm has some unutilised debt capacity. Under such a situation it is possible that two economically independent projects may together yield an NPV which is greater than the sum of the NPVs of the two projects obtained without regard to the unutilised debt capacity of the firm.

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