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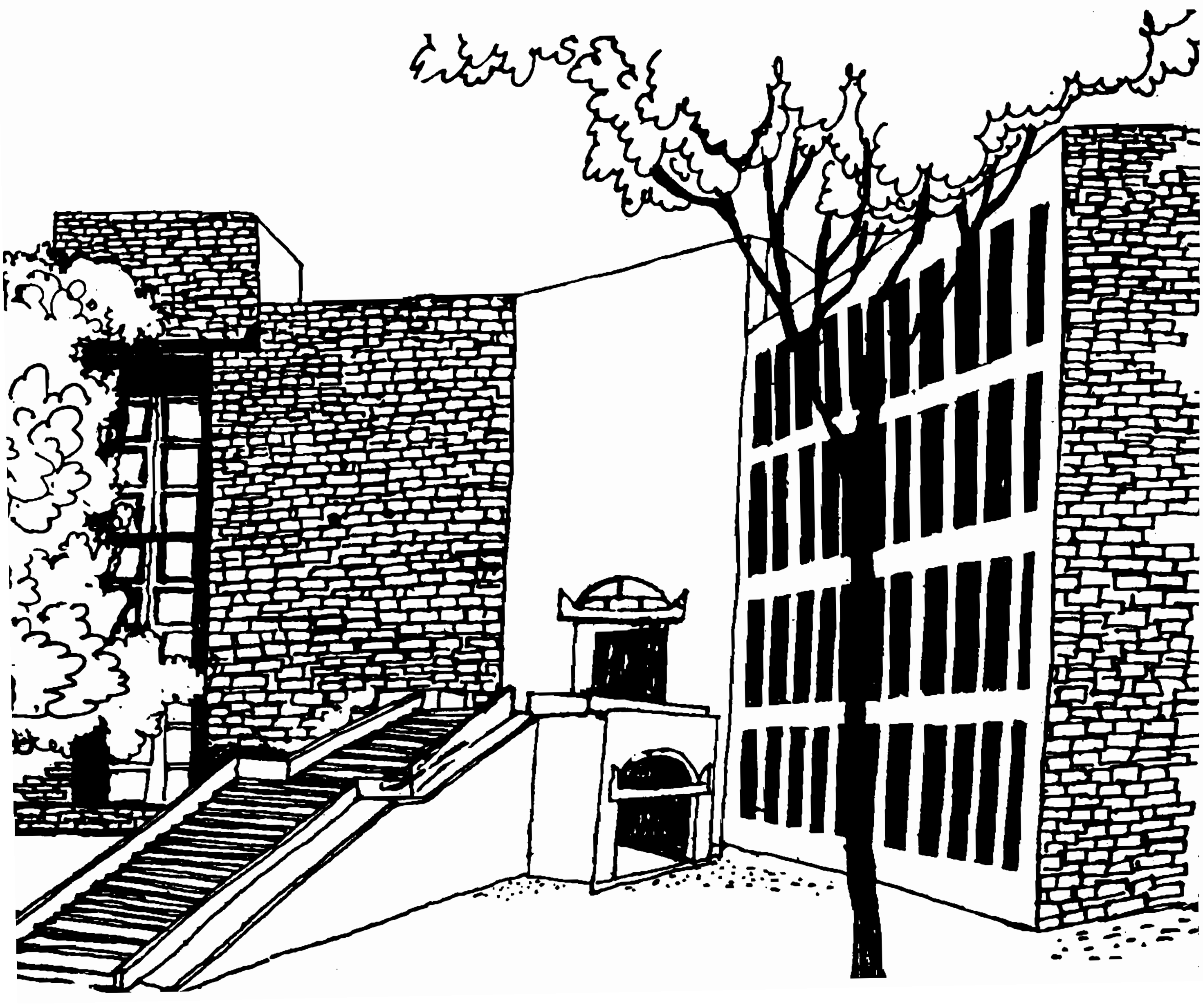

THE NASH-BARGAINING SOLUTION FOR PUBLIC SECTOR PRICING PROBLEMS

By

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Abstract

In this paper we study some issues in positive second-best theory, specifically the theory of optimal pricing of private goods produced by public firms; that is, firms whose objective departs from profit maximization.

The approach in this paper differs from earlier analysis of the so-called Ramsey pricing problem, in that we embed our problem in a bargaining theory framework.

1. Introduction : In this paper we study some issues in positive second-best theory, specifically the theory of optimal pricing of private goods produced by public firms; that is, firms whose objective departs from profit maximization. It is assumed that these firms, characteristically, display increasing returns to scale. In these situations, first-best optima may require lump-sum taxes and subsidies. In view of the size of the public sector in most industrialized countries it is difficult to imagine that public activities can be financed without distortionary effects elsewhere in the economy.

Our purpose here is to pose and answer the following question : Given certain structural facts which prevent prices from being set equal to marginal costs everywhere in the economy, what would then be the optimal rules to follow for public production and prices under public control? These price distortions may be due to the infeasibility of lump-sum taxation, i.e. price distortion created by the government, or they may arise due to monopolistic pricing in the private sector which may have to be accepted for political reasons. Clearly this does not imply that monopoly in some industry <u>ought</u> to be dealt with by pricing policy by public firms.

The analysis below merely shows what factors would be relevant to take into consideration if price under public control were to be set on the basis of economic efficiency, if the inviolability of monopoly has to be accepted for some reason.

The approach in this paper differs from earlier analysis of the so-called Ramsey pricing problem, in that we embed our problem in a bargaining theory framework as developed by Nash

(1950) and dealt with extensively in Thomson (forthcoming). In this framework we obtain first-order necessary conditions for an optimal pricing policy.

2. Ramsey Pricing : Following Bos (1980) we consider an economy consisting of I individuals labelled $h \in \{1,\ldots,I\}$, m private firms indexed by $j \in \{1,\ldots,m\}$, and a public sector. For simplicity, the public sector is assumed to consist of one public firm. There are n+1 private goods indexed $i=0,1,\ldots,n$, consumed or supplied by the individuals, the private firms and the public sector.

We use the following notations :

$x^h \equiv (x^h_0, x^h_1, \ldots, x^h_n)$ = (n+1) - dimensional commodity vector representing consumer h's consumption plan,

$y^j \equiv (y^j_0, y^j_1, \ldots, y^j_n)$ = (n+1) - dimensional commodity vector representing private firm j's production plan,

$z \equiv (z_0, z_1, \ldots, z_n)$ = (n+1) - dimensional commodity vector representing the net production plan for the public sector, i.e. total public supply minus public consumption.

We apply the sign convention that negative components in the consumption plans represent the net supply of services, while net demand is measured in positive quantities. As for production plans, output is measured in positive quantities, while input is measured in negative quantities. In the subsequent analysis the commodity with index zero will be used as a numeraire good.

The index set E represents the set of goods whose prices are subject to public regulation. Goods of which the public sector is the sole supplier or consumer clearly belong to this set, although in order to control the price of a commodity, it is not necessary for the government to have complete control of its supply or demand

Production in the public sector takes place using labor and other private goods supplied by individuals and private firms as inputs. The public sector supplies consumer goods and intermediate goods to private firms. We assume that technically efficient production plans for the public sector are defined by the implicit production function

$$g(z) = 0.$$

Moreover, we assume that for structural reasons which are exogenous to this model, the activity in the public sector is subject to a budget constraint given by

$$b - \sum_i p_i z_i = 0,$$

where $p \equiv (p_0, p_1, \ldots, p_n)$ is an $(n+1)$ - dimensional price vector with $p_0 = 1$.

A binding budget constraint for the public sector can of course be motivated by increasing returns to scale in the public sector such that the amount of lump-sum financing (b) is insufficient to cover the public deficit at a first-best optimum. If b is equal to zero we are imposing a zero profit constraint on the public firm operating in the markets for private goods. This would mean that the public firm has to be financed entirely through distortionary commodity taxation.

It may also be noted that we treat the public sector as an aggregate. In a model with a disaggregated specification of the public sector, we would have many public firms with different production technologies. However, in assuming one overall budget constraint for the public sector, we must clearly have production efficiency in the public sector, at a second-best optimum. Hence, the optimal pricing and production rules must be the same for all public firms so that the method of treating the public sector in an aggrgate fashion entails no loss of generality.

For convenience of analysis and to focus on the efficiency aspect of optimal pricing, personal incomes are assumed to be perfectly redistributable through lump-sum transfers.

By means of this assumption, we do not have to specify how the public budget b is financed (if $b<0$), and we do not have to specify the distribution of profits in the private sector.

The decision variables under public control are the prices under public control p_e, $e \in E$, the net production plan for the public firm denoted z, and the income distribution $\{r^h\}$, where $\{r^h\}$ denotes the share of individual h, in the aggregate non-labour income of the economy. For the sake of technical ease in exposition we assume $0<r^h<1$ for all $h \in \{1,\ldots,l\}$.

Assuming that individual consumption plans are ranked according to the strictly increasing and strictly quasi-concave utility function $u^h(x^h)$ and that only utility maximizing values are relevant we obtain

$V^h(p,r^h)=\max\ \{u^h(x^h)/p.x^h=r^hM\}$, $h=1,\ldots,l$

where M is the aggregate non-labour income of the economy, which is assumed given. Further owing to the strict quasi-concavity of u^h, there exists a function $x^h(p,r^h)$ such that

$V^h(p,r^h) = u^h[x^h(p,r^h)]$

or all $p>>0$, $r^h \in (0,1)$.

We further assume that the public authority is under a social obligation to ensure that consumer h has utility level of at least d^h in the arbitrated outcome. Let $d = (d^1, \ldots, d^h, \ldots d^l)$ be the vector of <u>minimum</u> <u>subsistence</u> <u>utility</u> <u>levels</u> which the public authority must ensure. This vector is analytically analogous to the vector of disagreement payoffs in bargaining theory (see Nash (1950)).

Efficient rules for pricing and production in the public sector and an optimal income distribution are obtained through solving the following constrained maximization problem :

$$(*) \qquad \max_{(p_e, \{r^h\}, z)} \sum_{h=1}^{l} \lambda^h \log [u^h(x^h) - d^h], \quad \lambda^h > 0, \forall h \in \{1, \ldots, l\}$$

Subject to

$$(1) \quad \sum_{h=1}^{l} x_i^h - z_i - \sum_j y^j = 0, \quad (i = 0, 1, \ldots, n) \qquad (P)$$

$$(2) \quad g(z) = 0$$

$$(3) \quad b - \sum_i p_i z_i = 0$$

We assume that problem (P) admits a solution, i.e. there exists $x = (x^1, \ldots, x^h, \ldots, x^l)$ satisfying (1), (2) and (3) such that $u^h(x^h) > d^h \forall h \in \{1, \ldots, l\}$.

To solve the problem we form the Lagrangian

$$L = \sum_h \lambda^h \log[u^h(x^h) - d^h] - \sum_i \alpha_i \{ \sum_h x_i^h - z_i - \sum_j y_i^h \} - \beta g(z)$$
$$- \gamma [b - \sum_i p_i z_i]$$

The necessary maximum conditions are :

$$\frac{L}{p_e} \equiv \sum_h \sum_i \lambda^h \frac{u_i^h}{u^h - d^h} \frac{\partial x_i^h}{\partial p_e} - \sum_i \alpha_i \left(\sum_h \frac{\partial x_i^h}{\partial p_e} - \sum_j \frac{\partial y_i^j}{\partial p_e} \right) + \gamma z_e = 0, e \in E, \quad (4)$$

$$\frac{L}{r^h} \equiv \lambda^h \sum_i \frac{u_i^h}{u^h - d^h} \frac{\partial x_i^h}{\partial r^h} - \sum_i \alpha_i \frac{\partial x_i^h}{\partial r^h} = 0, \quad h = 1, \ldots, l, \quad (5)$$

$$\frac{\partial L}{\partial z_i} \equiv \alpha_i - \beta g_i + \gamma p_i = 0, \quad i=0,1,\ldots,n \tag{6}$$

where u_i^h and g_i denote partial derivatives of the functions $u^h(.)$ and $g(.)$ with respect to the i^{th} argument.

Under the assumption that b exceeds the unconstrained profits (possibly negative as is the case under increasing retruns to scale in the public firm), $\gamma > 0$. Write $V(-b)$ for the maximum value of

$\sum_h \lambda^h \log [u^h(x^h)-d^h]$. Then we have

$$\gamma = \lim_{0 \to 0} [V(-b+s) - V(-b)]/s$$

Hence γ measures the value of marginally relaxing the constraint.

According to the first-order conditions for consumer optima, $u_i^h/u_o^h = p_i$, and $\sum_i p_i(\partial x_i^h/\partial r^h) = M$. Hence the necessary condition (5) for an optimal income distribution simplifies to

$$\lambda^h \sum_i \frac{p_i u^h}{u^h-d^h} \frac{\partial x_i^h}{\partial r^h} = \sum_i \alpha_i \frac{\partial x^h}{\partial r^h}$$

$$\Rightarrow \quad \frac{\lambda^h \, M \, u_o^h}{(u^h-d^h)} = \sum_i \alpha_i \cdot \frac{\partial x_i^h}{\partial r^h}, \quad h=1,\ldots,l. \tag{7}$$

Substituting (7) into (4) and using the fact that for each h, $\sum_i p_i(\partial x^h/\partial p_e) = -x^h$, the necessary conditions for an optimal price structure p_e simplify to

$$\sum_i \alpha_i \left(\sum_h \frac{\partial x_i^h}{\partial p_e} \right) - \sum_i \alpha_i \sum_j \frac{\partial y_i^j}{\partial p_e} - \sum_h \sum_i \lambda^h \frac{u_i^h}{u^h-d^h} \frac{\partial x_i^h}{\partial p_e} = \gamma z_e$$

$$\Rightarrow \sum_i \alpha_i \left(\sum_h \frac{\partial x_i^h}{\partial p_e} \right) - \sum_i \alpha_i \sum_j \partial y_i^j / \partial p_e - \sum_i \left[\sum_h \frac{u^h - d^h}{M u_o^h} \frac{u_i^h}{u^h - d^h} \left(\sum_i \alpha_i \frac{\partial x_i^h}{\partial r^h} \right) \frac{\partial x_i^h}{\partial p_e} \right] = \gamma' z_e$$

$$\Rightarrow \sum_i \alpha_i \sum_h \frac{\partial x_i^h}{\partial p_e} - \sum_i \alpha_i \sum_j \frac{\partial y_i^j}{\partial p_e} - \frac{1}{M} \sum_h \left(\sum_i \alpha_i \frac{\partial x_i^h}{\partial r^h} \right) \sum_i p_i \frac{\partial x_i^h}{\partial p_e} = \gamma' z_e$$

$$\Rightarrow \sum_i \alpha_i \sum_h \frac{\partial x_i^h}{\partial p_e} - \sum_i \alpha_i \sum_j \frac{\partial y_i^j}{\partial p_e} + \frac{1}{M} \sum_h \left(\sum_i \alpha_i \frac{\partial x_i^h}{\partial r^h} \right) x_e^h = \gamma' z_e$$

$$\Rightarrow \sum_i \alpha_i \left[\sum_h \left(\frac{\partial x_i^h}{\partial p_e} + \frac{x_e^h}{M} \frac{\partial x_i^h}{\partial r^h} \right) - \sum_j \frac{\partial y_i^j}{\partial p_e} \right] = \gamma' z_e, \quad e \in E$$

We define,

$$\frac{\partial \hat{x}_i}{\partial p_e} \equiv \sum_h \left(\frac{\partial x_i^h}{\partial p_e} + \frac{x_e^h}{M} \frac{\partial x_i^h}{\partial r^h} \right),$$

representing compensated price derivatives of the private consumption demand for commodity i. Similarly,

$$\frac{\partial \hat{z}_i}{\partial p_e} \equiv \frac{\partial \hat{x}_i}{\partial p_e} - \sum_j \frac{\partial y_i^j}{\partial p_e}$$

represents compensated price derivatives of the net market demand for commodity i.

We eliminate the dual variable through the normalizations $\delta_i \equiv \alpha_i / \beta g_o$ and $\mu \equiv \gamma' / g_o$. Then we define $c_i^o \equiv g_i / g_o$. Hence, c_i^o denotes the marginal cost of producing commodity i in the public sector if $z_i > 0$, or the marginal technical rate of substitution between input i and the numeraire if $z_i < 0$.

With these defnintions, conditions (8) and (6) can be rewriteen as

$$\sum_i \delta_i \frac{\partial \hat{z}_i}{\partial p_e} = \mu z_e, \quad e \in E \tag{9}$$

$$\delta_i = c_i^o - \mu p_i, \quad i = 0, 1, \ldots, n \tag{10}$$

We note from (10), $\delta_o + \mu = 1 - \mu p_o + \mu = 1 - \mu + \mu = 1$, and since $\delta_o > 0$, from strict monotonicity of the utility functions, we must have that $0<\mu<1$.

Substituting (10) into (7) and (9), we get

$$\frac{\dfrac{\lambda^h M u_o^h}{(u^h-d^h)}}{\dfrac{\lambda^l M u_o^l}{(u^l-d^l)}} = \frac{\sum_i \alpha_i \dfrac{\partial x_i^h}{\partial r^h}}{\sum_i \alpha_i \dfrac{\partial x_i^l}{\partial r^l}} = \frac{\sum_i \delta_i \dfrac{\partial x_i^h}{\partial r^h}}{\sum_i \delta_i \dfrac{\partial x_i^l}{\partial r^l}}$$

$$= \frac{\sum_i (c_i^o - \mu p_i)\, \partial x_i^h / \partial r^h}{\sum_i (c_i^o - \mu p_i)\, \partial x_i^l / \partial r^l}$$

$$= \frac{\sum_i c_i^o \dfrac{\partial x_i^h}{\partial r^h} - \mu M}{\sum_i c_i^o \dfrac{\partial x_i^l}{\partial r^l} - \mu M}$$

$$\Rightarrow \frac{\lambda^h u_o^h}{\lambda^l u_o^l} \frac{(u^l-d^l)}{(u^h-d^h)} = \frac{\sum_i c_i^o \dfrac{\partial x_i^h}{\partial r^h} - \mu M}{\sum_i c_i^o \dfrac{\partial x_i^l}{\partial r^l} - \mu M} \qquad (11)$$

Further,

$$\sum_i (c_i^o - \mu p_i) \frac{\partial \hat{z}_i}{\partial p_e} = \mu z_e, \quad e \in E \qquad (12)$$

If prices deviate from marginal costs in the public sector and Engel elasticities differ among consumers, the percentage social costs of marginal income transfers will be different for different consumers, which calls for setting $\dfrac{\lambda^h u_o^h}{u^h-d^h} \neq \dfrac{\lambda^l u_o^l}{u^l-d^l}$ for $h \neq l$.

We define $c_i^j \equiv -\partial y_o^j / \partial y_i^j$ as the marginal cost of producing commodity i in firm j if $y_i^j>0$ and if $y_i^j<0$, c_i^j denotes the marginal rate of substitution between input i and <u>numeraire</u> in firm j.

Subtracting $(1-\mu)\sum_i p_i(\partial \hat{z}'_i/\partial p_e)$ on both sides of (12) yields

$$\sum_i (c_i^o - p_i)\frac{\partial \hat{z}_i}{\partial p_e} = \mu z_e - (1-\mu)\sum_i p_i(\partial \hat{z}_i/\partial p_e). \qquad (13)$$

Letting t_i^j denote the mark-up on the price of commodity i in firm j (which may be negative), that is $t_i^j \equiv p_i - c_i^j$, and writing $x_i \equiv \sum_h x_i^h$, from well known properties of compensated market demand function we have

$$\sum_i p_i \frac{\partial \hat{z}_i}{\partial p_e} = \sum_i p_i \frac{\partial \hat{x}_i}{\partial p_e} - \sum_j \sum_i t_i^j \frac{\partial y_i^j}{\partial p_e} - \sum_j \sum_i c_i^j \frac{\partial y_i^j}{\partial p_e}$$

$$= -\sum_j \sum_i t_i^j \frac{\partial y_i^j}{\partial p_e} = -\sum_j \sum_i (p_i - c_i^j)\frac{\partial y_i^j}{\partial p_e}$$

and substituting into (13) we obtain our central condition for optimal pricing of private goods in the public sector

$$\sum_i (c_i^o - p_i)\frac{\partial \hat{z}_i}{\partial p_e} = \mu z_e + (1-\mu)\sum_j \sum_i (p_i - c_i^j)\frac{\partial y_i^j}{\partial p_e}, \quad e \in E \qquad (14)$$

3. Conclusion : In this paper we obtain necessary conditions which characterize the Nash bargaining solution for the public sector pricing problem. Condition (11), which is the crucial result of our paper explains the dependence of the solution on our specific objectives and tells the public authority how to price its products.

Reference :

(1) D. Bos (1986) : "Public Enterprise Economics", North-Holland

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(3) W. Thomson (forthcoming) : "Bargaining Theory : The Axiomatic Approach".