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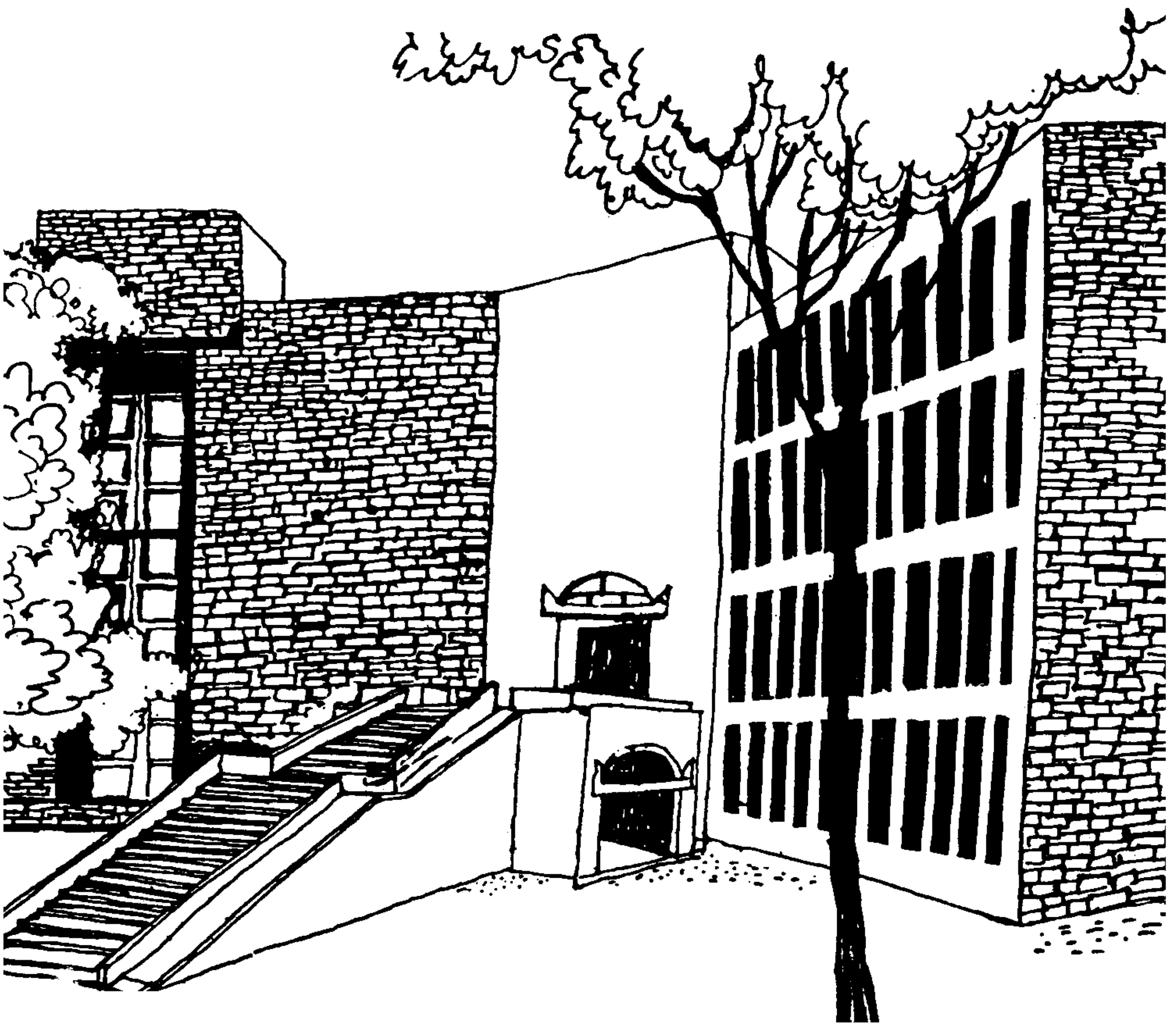

SPATIAL OPTIMISATION OF THE FUEL PURCHASE DECISION FOR ROAD TRANSPORT UNDERTAKINGS

By

G. Raghuram

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INDIAN INSTITUTE OF MANAGEMENT
AHMEDABAD-380 015
INDIA

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G. RAGHURAM
INDIAN INSTITUTE OF MANAGEMENT
AHMEDABAD 380 015

ABSTRACT

Road Transport Undertakings in India have been spending approximately between 15 - 20% of their total expenses on fuel. Given the fact that they visit different towns and sometimes even different states, the fuel which is available at different prices could be purchased in a way that the fuel cost is optimised. A network model, algorithms and linear programming formulations are presented for this problem in order to decide how much fuel should be purchased at the different towns that a bus visits as part of its vehicle duty schedule. The model presented minimises the fuel costs for a given amount of fuel to be consumed, taking advantage of spatial price differences.

1. THE PROBLEM:

A road transport undertaking provides passenger transport service. In order to do this economically and efficiently, operations planning should be done optimally. There is a hierarchy of operations planning decisions for a road transport undertaking. The first decision is to arrive at a route structure (origin, destination, stops, and timing/frequency), given the time dependent demand pattern and road network. The next decision is to arrive at the vehicle duty schedule, i.e. what each vehicle would do over the period of a day, given the route network. Given the vehicle duty schedule, various other decision areas can be tackled, like crew scheduling, allocation of vehicles to depots, etc.

One such area is to decide on the fueling plan, given a vehicle duty schedule circuit, i.e. the schedule which the vehicle repeats. The specific decision that needs to be taken is how much fuel to pick up at the various fueling points on a vehicle duty schedule circuit. This problem assumes significance since the fuel prices vary from location to location. The choices increase when the circuit is long, requiring fueling at more than one point, with many potential fueling points.

Based on 1989-90 statistics of the public sector road transport undertakings in India, material costs account for 30% of the total costs (personnel, interest, taxes and depreciation being the others). Fuel accounts for 55% of the material costs. The absolute amount spent on fuel is over Rs. 850 crores. One of the likely areas of cost saving could be in the purchase of fuel, especially since the fuel price has been rising steadily over the years.

While this problem does not address the more important dimension of fuel conservation, it does indirectly have a bearing on economic gains to the extent that fuel prices reflect the economic costs of making fuel available at the various points of purchase.

A vehicle duty schedule (VDS) consists of the set of trips that a vehicle performs in a day. This is usually represented as a 'link diagram' (figures 1 and 2). A vehicle duty schedule circuit (VDSC), is the schedule that a given vehicle performs repetitively. This could either be smaller or larger than a VDS, i.e. less than or more than a day. For example, in figure 1, the VDSC is less than a day, while in figure 2, it is more than a day.

VDS A B B A A B B A

VDSC: A-B-A (less than a day)

Figure 1

VDS-1 A C C D

VDS-2 D C C A

VDSC: A-C-D-C-A (more than a day)

Figure 2

Fueling is usually done only at the depots, which are the operational units of the road transport undertakings. This facilitates accounting and financial control. Some undertakings do not consider fueling at the depots of the other undertakings, even though their vehicles visit the associated terminals. But this is an administrative issue, which is overcome when the vehicles penetrate deep into the territory of the other undertaking. Further, with the intention of reducing the number of fueling points, the present procedure is to fill the tank to capacity at the starting point and if needed, again fill it to capacity at other predetermined depots.

Thus the basic decision problem can be stated as:

Given a VDSC, the potential fueling points on the VDSC, the fuel prices at these points, the vehicle fuel consumption between these points and the tank capacity of the vehicle, to determine the amount of fuel to be purchased at the various fueling points with the objective of minimising total purchase cost.

The solution procedure for this problem is discussed in the following sections. A greedy like optimal algorithm is presented in section 2, while a more flexible linear programming based approach is given in section 3. Section 4 discusses a specific example. The conclusions are given in section 5.

2. GREEDY-LIKE ALGORITHM

Given $i = 1...n$ be the n possible fueling points, appearing in sequence on a VDSC. Thus 1 follows n.

p_i = the price at fueling point i

c_i = the fuel consumption between i and i+1, (c_n is the fuel consumption between n and 1).

TC = the tank capacity

To determine

x_i = the quantity of fuel to be purchased at fueling point i

The following are the steps of the algorithm:

Step 1: Sort all the fueling points in ascending order of price. The consideration of points for purchase of fuel will be in this order.

At the kth iteration:

Let r be the fueling point under consideration (if there is a tie, the choice can be arbitrary).

Let q be the fueling point where fuel was purchased earlier on the VDSC.

Similarly, let s be the fueling point where fuel will be purchased on the VDSC.

The points q and s would have been decided in an earlier iteration.

Let $FUEL_{k-1}$ be the total fuel purchased in the previous iterations.

Let $k = 1$, $q = s = r$ and $FUEL_0 = 0$.

Step 2: Let

$$PREV = \sum_{i=q}^{r-1} c_i \quad \text{if } r-1 >= q$$

$$= \sum_{i=q}^{n} c_i + \sum_{i=1}^{r-1} c_i \quad \text{otherwise}$$

i.e. the fuel consumed from the previous point (as determined so far) of fuel purchase on the VDSC.

$$NEXT = \sum_{i=r}^{s-1} c_i \qquad \text{if } s-1 >= r$$

$$= \sum_{i=r}^{n} c_i + \sum_{i=1}^{s-1} c_i \qquad \text{otherwise}$$

i.e. the fuel that will be consumed upto the next point (as determined so far) of fuel purchase on the VDSC.

$$REQT = \sum_{i=1}^{n} c_i - FUEL_{k-1}$$

i.e. the amount of fuel required as yet for completing the VDSC.

The quantity of fuel to be purchased at r is given as

$$x_r = MIN\ [PREV,\ NEXT,\ REQT,\ TC]$$

Also $FUEL_k = FUEL_{k-1} + x_r$

Step 3: If $FUEL_k < \sum_{i=1}^{n} c_i$

then k = k+1, determine r, q and s and go to step 2

else stop.

The optimality of the algorithm is quite obvious, since at a given best price, the maximum possible fuel is purchased. The maximum is constrained by either one of the amount of fuel consumed from the previous point of purchase (where the tank would have been filled up, unless a lesser amount only was required), the amount of fuel that will be consumed till the next, but cheaper point of purchase, the amount of fuel required to complete the VDSC or the tank capacity.

3. LINEAR PROGRAMMING FORMULATIONS

This problem can also be formulated as a linear program. The advantage of this approach is that it lends itself to extending the scope of the problem, like i) filling to full tank capacity whenever fuel is purchased, ii) filling a minimum amount whenever fuel is purchased, and iii) optimising the total cost of fuel purchase and an imputed fixed cost for each filling. This approach also lends itself to sensitivity analysis of the various parameters.

Basic Problem Formulation

Formulation 1 (F1)

Parameter definitions:

i = 1...n be the n possible fueling points, appearing in sequence on a VDSC.

p_i = the price of fuel at i

c_i = the fuel consumption between i and i+1, (c_n is the fuel consumption between n and 1).

TC = the tank capacity

Variable definitions:

x_i = the quantity of fuel to be purchased at i

z_i = the quantity of fuel in the tank at i on arrival

Objective function:

$$\text{MIN} \sum_{i=1}^{n} p_i * x_i$$

Constraints:

(Fuel conservation)

$x_i + z_i - z_{i+1} = c_i$ for i = 1..n

(For i = n, i+1 = 1)

(Tank capacity)

$x_i + z_i <= TC$ for i = 1..n

(Non-negativity)

All x_i, $z_i >= 0$

Extended Problem Formulations

Whenever fuel is purchased, the usual practice is to fill the tank to capacity. This can be accommodated in the formulation by using zero-one integer programming variables.

Formulation 2 (F2)

Parameter definitions:

Same as F1

Variable definitions:

In addition to the variables in F1, we define

y_i = 1 if fuel is purchased at i
= 0 otherwise

Objective function:

Same as F1

Constraints:

In addition to the constraints in F1, we have

(Recognition of fuel purchase point)

$x_i <= TC * y_i$ for i = 1..n

(Fill to tank capacity)

$x_i + z_i >= TC * y_i$ for i = 1..n

In addition to filling the tank to capacity, if the requirement is to purchase a minimum quantity of fuel, the same can be considered with the following modification to formulation F2.

Formulation 3 (F3)

Parameter definitions:

In addition to the parameters in F2, we define

m_i = minimum quantity of fuel to be purchased at i, if at all

Variable definitions and objective function:
Same as F2

Constraints:

In addition to the constraints in F2, we have

(Minimum fuel purchase)

$x_i >= m_i * y_i$ for i = 1..n

The filling to capacity and/or the minimum fuel purchase requirements are essentially there to ensure fueling does not take place in too many points. This is because there is a perceived cost at each point of fuel purchase due to delay, administrative work etc. This can be taken care of by imputing a fixed cost, whenever fuel is purchased. The following modifications to F1 are needed.

ation 4 (F4)

Parameter definitions:

In addition to the parameters in F1, we define

f_i = fixed cost of fueling at i

Variable definitions:

In addition to the variables in F1, we define

y_i = 1 if fuel is purchased at i
= 0 otherwise

Objective function:

$$\text{MIN} \sum_{i=1}^{n} p_i * x_i + \sum_{i=1}^{n} f_i * y_i$$

Constraints:

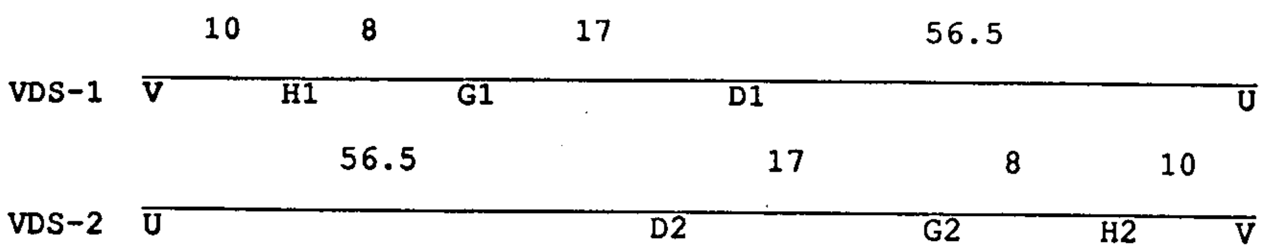
In addition to the constraints in F1, we have

(Recognition of fuel purchase point)

$x_i <= TC * y_i$ for i = 1..n

4. EXAMPLE

To demonstrate the application of the algorithm and the linear programming formulation, we consider an example. Two VDSs constitute one VDSC, as shown in figure 3. There are seven fueling points on the VDSC. The numbers in between the fueling points represent the fuel consumption. H1 and H2, G1 and G2 and D1 and D2 are the same fueling points, except that they are at different points on the VDSC. The distinction is important in the fueling decision.



Since U is an out of state point, fueling is not done there. Hence, the VDSC with the relevant fueling points are

VDSC: V-H1-G1-D1-D2-G2-H2-V (more than a day)

Figure 3

The tank capacity has been standardized at 160 liters.

The details of the fuel rates at depots are given in Table 1.

Table 1

Fueling Point	Fuel Rates (Rs./1000 lts.)
V	3158
H	3162
G	3166
D	3202

Greedy-like Algorithm

Applying the greedy like algorithm, the fueling plan can be generated as follows.

Step 1: The fueling points, sorted in the ascending order of price are V, H1, H2, G1, G2, D1 and D2.

For the first iteration, we determine the purchase quantity at V.

Step 2:

PREV = 10 + 8 + 17 + (56.5 + 56.5) + 17 + 8 + 10 = 183

NEXT = PREV = 183

REQT = 183 - 0 = 183

TC = 160

Quantity of fuel to be purchased at V

= MIN [183,183,183,160] = 160 litres

Step 3:
Fuel purchased so far is 160 litres. Since total fuel purchased is less than the circuit requirement of 183 litres, we continue. The next point considered for purchase is H1 (H2 could also have been considered, without affecting the total cost). V is both the previous and the next fueling point on the circuit.

For the second iteration, we determine the purchase quantity at H1.

Step 2:

PREV = 10

NEXT = 8 + 17 + 113 + 17 + 8 + 10 = 173

REQT = 183 - 160 = 23

TC = 160

Quantity of fuel to be purchased at H1

= MIN [10,173,23,160] = 10 litres

Step 3:

Fuel purchased so far is 170 litres. Since total fuel purchased is less than the circuit requirement of 183 litres, we continue. The next point considered for purchase is H2 H1 is the previous and V is the next fueling point on the circuit.

For the third iteration, we determine the purchase quantity at H2.

Step 2:

PREV = 8 + 17 + 113 + 17 + 8 = 163

NEXT = 10

REQT = 183 - 170 = 13

TC = 160

Quantity of fuel to be purchased at H1

= MIN [163,10,13,160] = 10 litres

Step 3:

Fuel purchased so far is 180 litres. Since total fuel purchased is less than the circuit requirement of 183 litres, we continue. The next point considered for purchase is G1 (could also have been G2). H1 is the previous and H2 is the next fueling point on the circuit.

For the fourth iteration, we determine the purchase quantity at G1.

Step 2:

PREV = 8

NEXT = 17 + 113 + 17 + 8 = 155

REQT = 183 - 180 = 3

TC = 160

Quantity of fuel to be purchased at H1

= MIN [8,155,3,160] = 3 litres

Step 3:

Fuel purchased so far is 183 litres. Since total fuel purchased is equal to the circuit requirement of 183 litres, we stop.

Thus the fueling plan is as given in Table 2.

Table 2

Fueling point	Qty. (Lts.)	Rate (Rs./1000 lts.)	Cost (Rs.)
V	160	3158	505.28
H1	10	3162	31.62
G1	3	3166	9.50
H2	10	3162	31.62
Total	183		578.02

Linear Programming

Formulation F1 obviously yields the result given in Table 2. The results based on formulation F2 are given in Table 3.

Table 3

Fueling point	Qty. (Lts.)	Rate (Rs./1000 lts.)	Cost (Rs.)
V	10	3158	31.58
H1	10	3162	31.62
G1	8	3166	25.33
H2	155	3162	490.11
Total	183		578.64

The increased cost due to the requirement of filling to capacity is Rs. 0.62, which is 0.1%. In the earlier solution, we are not filling to capacity at G1 and H2.

Existing Practice

Since these schedules are attached to the depot at V, this is a point of fuel purchase. Further, since D is the last point before the vehicle goes to a neighbouring state and is approximately in the middle of the schedule, fueling is done here in both the directions i.e. D1 and D2. The practice of filling the tank to capacity is adopted. Given that the same vehicle is on the circuit, the fueling plan is as given in Table 4.

Table 4

Fueling point	Qty. (Lts.)	Rate (Rs./1000 lts.)	Cost (Rs.)
V	35	3158	110.53
D1	35	3202	112.07
D2	113	3202	361.83
Total	183		584.43

The total cost here is more than the optimal by Rs. 6.41 i.e. about 1.1%. It should be noted that even though the schedules are attached to V, due to the practice of filling the tank to capacity, and purchasing fuel at both D1 and D2, the quantity of fuel purchased at V on a regular basis is only 35 litres. Thus, a major share of the fuel is purchased at a more expensive place. By avoiding fueling at D2, the revised plan could be as given in Table 5.

Table 5

Fueling point	Qty. (Lts.)	Rate (Rs./1000 lts.)	Cost (Rs.)
V	148	3158	467.38
D1	35	3202	112.07
Total	183		579.45

This cost is marginally more expensive than the optimal.

5. CONCLUSIONS

1. This problem, though academically an interesting application, has significant value if the fuel prices vary significantly from place to place.

2. The methodology used in this problem, on the one hand brings out the efficacy of greedy-like procedures, while on the other the flexibility of linear programming is highlighted.

Acknowledgements

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