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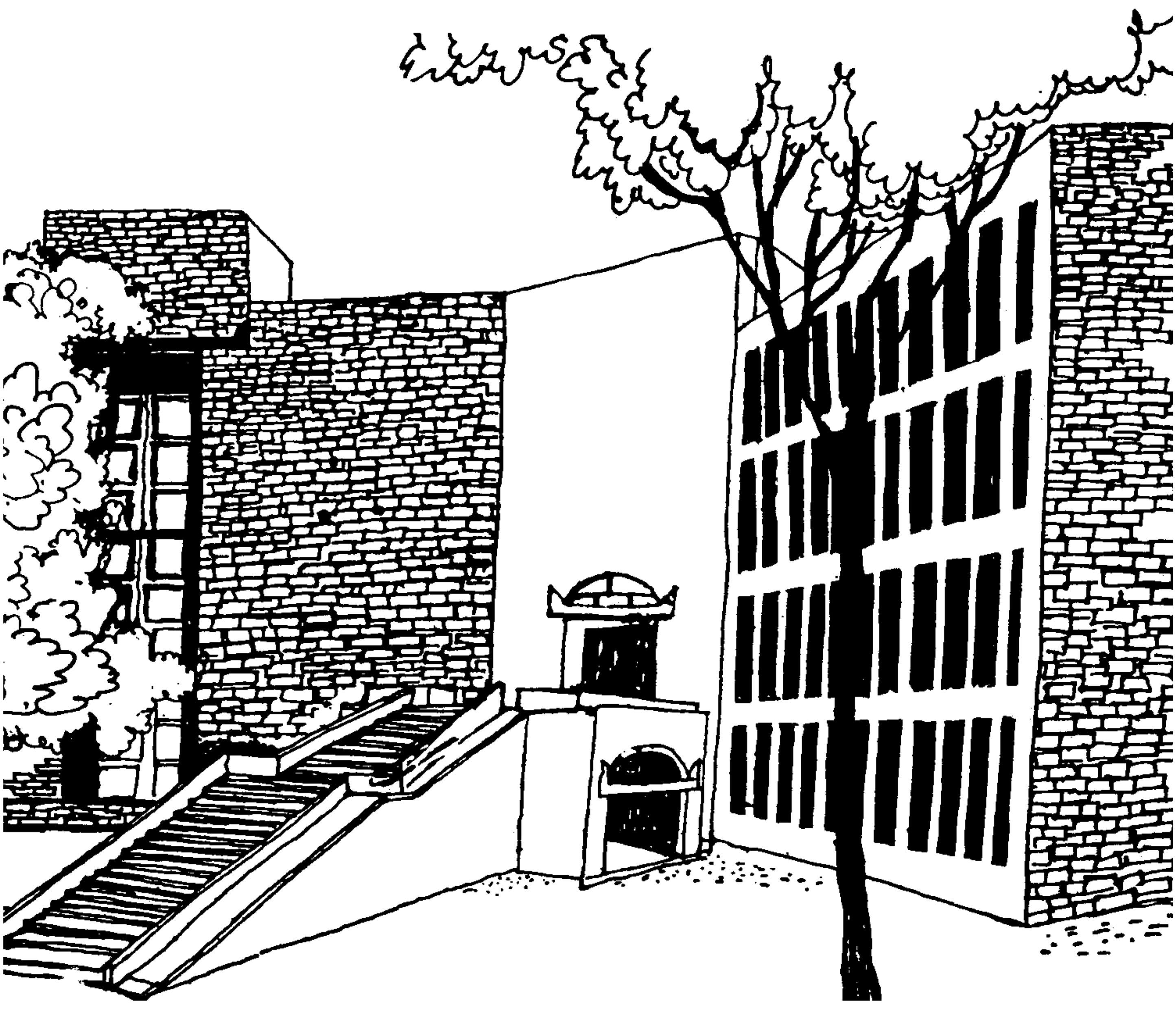

REGIONALIZING A NATIONAL INPUT-OUTPUT TABLE

By

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Abstract

In the present paper it is argued that the most popular among the non-survey methods to estimate regional input-output tables, namely the RAS method is not a consistent and efficient method to regionalise the national input-output tables. An alternative non-survey method is, therefore, proposed as a solution to the problem when data on industry/commodity-specific value added as well as final demand are available besides the gross outputs at the regional level.

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1. Introduction:

Need for reliable and relevant regional input-output tables is well recognised for efficient resource allocation, elaborate regional planning, transport planning, policies regarding commerce and infrastructure development, etc. Updated regional input-output tables compatible with the national I-O tables are, therefore, required to be constructed from time to time. These tables moreover should be for various regional units. The cost of constructing input-output tables based on primary data with survey method is very large in terms of both resources and time. Less developed countries cannot afford comprehensive revisions of even the national I-O tables very often. It is usually done only once in a decade or a quinquannium. A parallel effort at regional level to cover the whole system of regional I-O tables is bound to be prohibitively costly. A reliable method which yields regional I-O tables without recourse to primary survey data is not only very useful but also inevitable. Such a non-survey method has to be the one which uses all readily available information and satisfies all theoretical consistency conditions. In the present paper, we discuss a non-survey method to regionalize the national I-O table when we assume the ready availability of industry/commodity specific information at the required regional level. The main contension of the present paper is that the oft-used RAS method is neither efficient nor consistent method of regionalizing the national I-O tables. In the next section, we examine the limitations of the RAS method to regionalize I-O tables. In the third section, we examine the total number of unknowns and constraints in a complete regional I-O system. The fourth section is then devoted to propose an alternative non-survey method to regionalise I-O table when both the value added and final demand vectors at regional levels are known along with the gross outputs. In the final section concluding remarks are made. Before, we pass on to the next section, it may be noted that the discussion in this paper applies equally well to all types of I-O tables - whether they represent commodity x commodity; industry x industry or commodity x industry classifications.

2. Limitations of the RAS Method:

The system of regional I-O transaction tables must fulfill the following three sets of constraints:

i) The column totals of I-O transaction tables in each region must match the difference between gross output and value added in the respective commodity or industry if available;

ii) The row totals of the I-O transaction tables in each region must match the difference between the gross output and the final demand in the respective commodity or industry if available, and

iii) The totals of all the respective cells over regions must match the corresponding cells in the national I-O transaction table.

The main limitation of the RAS method is that it ensures fulfilment of only first two out of the three sets of constraints mentioned above. The third set of constraints, viz. element by element consistency of the system of regional I-O tables with national table is not met by the RAS method. This can be shown with the help of a simple numerical example:

Let us consider a simple 2 x 2 national I-O transaction table and basic data on Region A and Region B as given by:

	National Table			Region A			Region B		
Sector	1	2	Total	1	2	Total	1	2	Total
1	600	1400	2000			100			1900
2	500	1500	2000			200			1800
Total	1100	2900		160	140		940	2760	

When we apply the popular RAS method to the above case (See Kundu et al., 1976), we get the following 'solution' for Region A and Region B:

RAS Solution (S1):

	Region A			Region B			Nation = Region A+ Region B		
Sector	1	2	Total	1	2	Total	1	2	Total
1	57	43	100	527	1373	1900	584	1416	2000
2	103	97	200	413	1387	1800	516	1484	2000
Total	160	140		940	2760		1100	2900	

It can be seen from this illustration that the application of RAS method to regionalise the national I-O transaction table is not efficient since it leads to violation of the third set of constraints mentioned above. As a result, the RAS 'solution' may not be considered as regionalisation of the national I-O table at all. It is an inefficient method because it ignores very important a priori information on a set of constraints most relevant for regionalising the national I-O tables.

It is also interesting to note that the performance of the RAS method does not improve but most likely to deteriorate further when we consider more number of regions. Again we show this with the help of a simple numerical illustration. Let us continue with our earlier illustration, but now we split Region B into two regions - Region B1 and Region B2 as follows:

	Region B			Region B1			Region B2		
Sector	1	2	Total	1	2	Total	1	2	Total
1			1900			500			1400
2			1800			600			1200
Total	940	2760		400	700		540	2060	

When we apply the popular RAS method to these two new regions – B1 and B2, the 'solution' is: RAS Solution (S2):

	Region B1			Region B2			Nation = Region A + Region B1 + Region B2		
Sector	1	2	Total	1	2	Total	1	2	Total
1	198	302	500	317	1083	1400	572	1428	2000
2	202	398	600	223	978	1200	528	1472	2000
Total	400	700		540	2060		1100	2900	

When we compare the solution (S2) with the solution (S1), we find that the RAS 'solution' to the system of regional I-O tables further diverges from the set of conditions regarding the equality of the cell by cell summation of the regional I-O tables with corresponding elements of national I-O transaction table. Thus, increasing the number of regions does not lead to reduction in the divergence but an increase in the divergence. The RAS method to regionalise the national I-O tables is, therefore, not only inefficient but also inconsistent.

The other limitation of the RAS method is its data requirements. In order to apply the RAS method, we must have the row totals as well as the column totals of all sectors at the regional level. This implies that not only the value added estimates but the final demand estimates are also required for every sector at the regional level along with the sectoral gross output. At a regional level, however, the estimates of row totals, i.e. final demands along with gross outputs for different sectors are extremely difficult to estimate given the present statistical network in the developing countries like India. (See Committee on Regional Accounts, 1976). Even at the state level, such estimates are not likely to be highly reliable. Moreover, because of the high degree of 'openness' of state economies, such estimates by commodities or industries are not likely to be very stable at all. The RAS method which puts heavy emphasis on fulfilment of conditions pertaining to such aggregates ignoring and violating in the process very relevant and important constraints is likely to generate unreliable, unstable and irrelevant set of I-O estimates at the regional level.

3. Unknowns and Constraints in Regional I-O System:

When the national I-O transaction table is known, the corresponding compatible regional I-O transaction tables have to be derived or estimated based on the information on industry/ commodity specific gross output, value added and final demand at regional level. First of all, it is important to pose this problem in a proper framework so that we can ensure consistency and efficiency of the proposed solution. If we consider a general I-O table in the form of commodity x industry transactions in different regions, we can count the number of unknowns to be estimated. Let the national I-O transaction table have n commodities and m industries. Therefore the national I-O table has n x m cells which are assumed to be known. Since the regional I-O tables have to be compatible to the national table, each of the K regions should also have n x m cells in its I-O table which are unknowns.

The total number of unknowns in
the system of regional I-O table $= n \cdot m \cdot K$ ..... (1)

The number of constraints on the other hand, is given by the following table:

<u>Table 1</u> : <u>Constraints on the System of Regional I-O Transaction Table</u>.

<u>Equation of Constraints</u>	<u>No.of Constraints</u>
1) $\sum_{r=1}^{K} A^r_{ij} = A_{ij}$; $i=1 \ldots n$; $j=1 \ldots m$	n x m
2) $\sum_{j=1}^{m} A^r_{ij} = X_i^r - F_i^r$; $r=1 \ldots K$; $i=1 \ldots n$.	K x n
3) $\sum_{i=1}^{n} A^r_{ij} = X_j^r - V_j^r$; $r=1 \ldots K$; $j=1 \ldots m$	K x m
4) $A^r_{ij} \geqslant 0$; $r=1 \ldots K$; $i=1 \ldots n$; $j=1 \ldots m$	K x m x n

Note:

A^{r}_{ij}	is the element of I-O transaction matrix in i-th commodity; j-th industry; and r-th region.
X_i^r	is the gross output of i-th commodity in r-th region.
X_j^r	is the gross output of j-th industry in r-th region.
F_i^r	is the final demand of i-th commodity in r-th region.
V_j^r	is the value added of j-th industry in r-th region

From Table-1, it can be seen that number of constraints on the regional I-O system incorporates some constraints which are not linearly independent of the rest of the constraints. If we consider only the linearly independent set of constraints on the regional I-O system, we have to recognize that (1) in each region the total number of effective constraints on account of the row totals and column totals is not (m+n) but (m+n-1); and (2) the total number of effective constraints on account of the regional cell totals is not (n x m) but (n-1) x (m-1). Thus, the total number of effective constraints on the regional I-O system excluding the non-negativity conditions, is:

$$(n-1)(m-1) + K(n+m-1) = \text{total effective constraints} \quad \text{....(2)}$$

Considering the number of unknowns in (1) above with the number of effective constraints in (2) which are also linear, we can readily see that a unique non-trivial solution to the system is not possible. This is because unique solution to the system would require equality between (1) and (2).

$$\text{i.e. } n.m.K = (n-1)(m-1) + K(n+m-1)$$

$$\text{i.e. } nmK = K(n+m-1) - (n-1)(m-1) = 0$$

$$\text{i.e. } K(nm-n-m+1) - (n-1)(m-1) = 0$$

$$\text{i.e. } (K-1)(m-1)(n-1) = 0$$

This is possible only under trivial solution of either K=1 or m=1 or n=1.

If K,m and n are greater than the trivial value of 1, we have the number of variables exceeding the number of effective linear constraints. This points to the possibility of multiple solution of the regional I-O system. However, several of these solutions may not fulfill the non-negativity conditions and are therefore economically meaningless.

4. Alternative Non-Survey Method:

As we have already seen in the second section above, the solution given by the RAS method is basically not a solution to the regional I-O system at all because it does not fulfill several effective constraints on the system. One of the possible non-survey methods which fulfills all the effective constraints on the regional I-O system and would therefore provide a solution to the system is proposed here.

$$\text{Let} \quad \sum_{j=1}^{m} A^r_{ij} = X_i^r - F_i^r = A_i^r \qquad \ldots\ldots \qquad (3)$$

$$\text{and} \quad \sum_{i=1}^{n} A^r_{ij} = X_j^r - V_j^r = A_j^r \qquad \ldots\ldots \qquad (4\ )$$

Let the terms without superscript r denote the corresponding aggregate at the national level.

It is also assumed that we have firm estimates about the following:

a) X_i for $i=1\ldots n$

b) X_j for $j=1\ldots m$

c) A_{ij} for $i=1\ldots n$ and $j=1\ldots m$

d) V_j for $j=1\ldots m$

e) F_i for $i=1\ldots n$

f) X_i^r for $r=1\ldots K$ and $i=1\ldots n$

g) X_j^r for $r=1\ldots K$ and $j=1\ldots m$

h) V_j^r for $r=1\ldots K$ and $j=1\ldots m$

i) F_i^r for $r=1\ldots K$ and $i=1\ldots n$

We can, therefore, get A_i; A_j; A_i^r and A_j^r for all r, i and j. Now, in order to regionalize the national I-O transaction table, we take the following steps:

<u>Step-1</u>: Generate $B^r_{ij} = A_{ij} * \dfrac{A_i^r}{A_i}$ for all i, j and

This is usually the first step in the RAS method if one follows the row allocation.

<u>Step-2</u>: Generate $C_{ij} = A_{ij} \cdot \frac{A_i^r}{A_j}$ for all i, j and r.

This is the first step in the RAS method if one follows the column allocation.

<u>Step-3</u>: Generate $\hat{A}^r_{ij} = \frac{1}{2}(B^r_{ij} + C^r_{ij})$ for all r but $i=1..(n-1)$, and $j=1...(m-1)$.

Thus, except for one column and one row, all other elements in each of the regional I-O table are estimated as the average of the values of the respective cells in <u>steps 1 and 2</u> above. For the remaining one column and one row in each of the regional I-O table, constraints (2) and (3) in <u>Table-1</u> are used to obtain consistent estimates. The system of regional I-O tables so estimated ensures that all the constraints given in <u>Table-1</u> are fulfilled and hence it becomes a solution. This is because constraints (2) and (3) are fulfilled by the very procedure and it can be shown that our estimate in <u>Step-3</u> above fulfills the first set of constraints in <u>Table-1</u> as under:

$$\sum_{j=1}^{m} B^r_{ij} = \sum_{j=1}^{m} A_{ij} \cdot \frac{A_i^r}{A_i} = \frac{A^r_i}{A_i} \sum_{j=1}^{m} A_{ij} = A^r_i \quad \ldots \qquad (5)$$

$$\text{and} \sum_{r=1}^{K} B^r_{ij} = \sum_{r=1}^{K} A_{ij} \cdot \frac{A^r_i}{A_i} = \frac{A_{ij}}{A_i} \sum_{r=1}^{K} A_i^r = A_{ij} \quad \ldots \quad (6)$$

$$\text{Similarly,} \sum_{i=1}^{n} C^r_{ij} = \sum_{i=1}^{n} A_{ij} \cdot \frac{A_j^r}{A_j} = \frac{A^r_j}{A_j} \sum_{i=1}^{n} A_{ij} = A^r_j \quad \ldots (7)$$

$$\text{and} \sum_{r=1}^{K} C^r_{ij} = \sum_{r=1}^{K} A_{ij} \cdot \frac{A^r_j}{A_j} = \frac{A_{ij}}{A_j} \sum_{r=1}^{K} A^r_j = A_{ij} \quad \ldots \quad (8)$$

$$\text{Now,} \sum_{r=1}^{K} \hat{A}^r_{ij} = \frac{1}{2} \sum_{r=1}^{K} (B^r_{ij} + C^r_{ij}) \quad \text{by } \underline{\text{Step-3}} \text{ above.}$$

$$= \frac{1}{2} \left[\sum_{r=1}^{K} B^r_{ij} + \sum_{r=1}^{K} C^r_{ij} \right]$$

$$= A_{ij}$$

Moreover, it is clear from <u>Steps 1 and 2</u> above that both B^r_{ij} and C^r_{ij} are non-negative and hence their average is also non-negative.

Finally, we may observe that while selecting the residual row and the residual column in the regional I-O table, choice should be made on the basis of the reliability of the estimates of the regional final demand in the commodity and of the regional value added in the industry concerned. The commodity/industry giving the least reliable estimates of the final demand/value added should be considered for the residual estimation.

5. <u>Concluding Remarks</u>:

It is shown in the present paper that RAS method does not provide a solution to the regional I-O system. It is, therefore, an inefficient and inconsistent method to regionalize the national I-O table. If the reliable industry/commodity-specific estimates of the value added, final demand and gross output are available at regional level, the method proposed in the present paper can yield estimates of elements of regional I-O table which would be consistent with all the constraints in the regional I-O system. Moreover, the application of our method does not require knowledge of the rest of the regions in the system. In other words, our method can be applied to a given region in isolation without sacrificing consistency.

Very often we find, however, that industry or commodity-specific estimates of final demand at the regional level are either not reliable or not simply available. In such cases, the regionalization of the national I-O table can be done by following the Dholakia-Choudhry (1986) method which can be considered as a special case of our more general method presented here. If, however, estimates of industry/commodity-specific final demand as well as value added are not available at regional level, but estimates of only industry/commodity-specific gross outputs are available, then, the Dalvi-Prasad (1981) method of generalised inverse solution can be used for regionalizing the national I-O table consistent with all the constraints. The Dalvi-Prasad method is not efficient when value added information is available at regional level. The Dholakia-Choudhry method is not efficient when both the value added and final demand information is available at the regional level.

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