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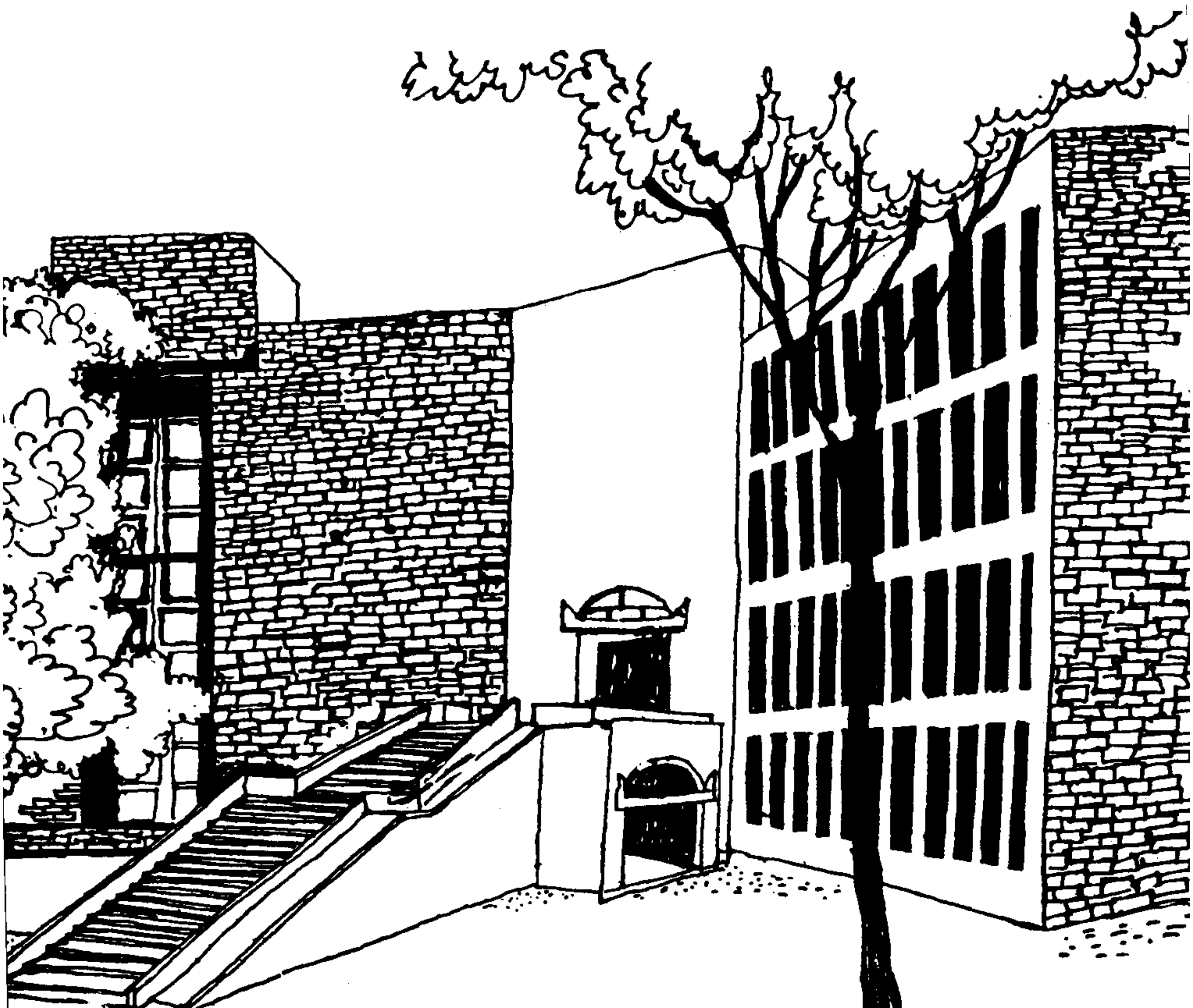

WHEN DOES THE EQUAL INCOME LINDAHL EQUILIBRIUM SOLUTION SATISFY RESOURCE MONOTONICITY?

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Abstract

In this paper we show that the Equal Income Lindahl Equilibrium Solution Function satisfies resource monotonicity when preferences are quasi-linear and there is a constant returns to scale technology converting private good into public good.

1. Introduction :- In an economy consisting solely of private goods, the most popular "method of fairly dividing a bundle of infinitely divisible goods among a group of agents with equal claims on the goods is the method consisting of first dividing that bundle equally among the agents and then, operating the Walrasian mechanism." (Chichilinsky and Thomson (1987)). However this mechanism suffers from certain deficiencies; for instance, given a fixed supply of resources, it is conceivable that some of the existing agents benefit, when the population of the economy increases. This has been called the population paradox. It is also conceivable, that as the available resources in the economy increase, some amongst a fixed set of agents may actually be worse off. (See Chun and Thomson (1988), Moulin and Thomson (1988)). The presence of such a phenomenon is a violation of the property of resource monotonicity.

Resource monotonicity indicates that for a population of fixed size, an increase in the available resources, should lead to the detriment of none. In Lahiri (1992), we show that in a two person, one good economy, the conventional bargaining solutions satisfy resource monotonicity, even when preferences display consumption externalities, under assumptions made explicit in that paper. The definition of resource monotonicity there is a variant of the conventional definition of resource monotonicity. In this paper we consider a two good economy, comprising a fixed population. Of the two goods one is a private good and the other is a public good. Resource monotonicity in this context would mean that if the aggregate available resources of the economy increases, then every agent's utility increases. We proceed to *show in* this paper that when preferences are quasi-linear the method consisting of first dividing the aggregate resources equally among the agents and then, operating the Lindahl mechanism to obtain what is heretofore referred to as an Equal Income Lindahl Equilibrium (EILE) does satisfy resource monotonicity. In this changed context our answer to the question "can everybody benefit from growth" is in the affirmative.

2. The Model :- We consider (as for instance in Laffont (1988)) an economy consisting of I agents, indexed by $i=1,\ldots,I$; there are two goods in the economy, quantities of which are denoted by x and y. The first good is a private good and the second is a public good. Let x^i denote the consumption of the private good by agent $i=1,\ldots,I$; let y denote the consumption of the public good in the economy. The preferences of agent i are assumed to be represented by a twice continuously differentiable, quasi-linear utility function $u^i : \mathbb{R}_+^2 \to \mathbb{R}$, and

$$u^i(x^i,y) = x^i + v^i(y) \; \forall \; (x^i,y) \in \mathbb{R}_+^2$$

where $v^i : \mathbb{R}_+ \to \mathbb{R}$ satisfies the following:

(i) v^i is twice continuously differentiable, strictly increasing and strictly concave with $\dfrac{dv^i}{dy} > 0$ and $\dfrac{d^2v^i}{dy^2} < 0$ for all non-negativ reals;

(ii) $v^i(0) = 0$.

Quasi-linear utility functions and their applications to economic theory has been discussed in detail in Varian (1984). Their use in the context of a public good economy is quite common.

We assume as in Lahiri (1991), that the economy has a cost-function $c: \mathbb{R}_+ \to \mathbb{R}$, which indicates the cost in terms of the private good of producing the public good. We shall assume an especially simple cost-function of the form $c(y) \equiv y$, although any linear cost-function would serve equally well.

Let $W>0$ be the intial endowment of the private good available in the economy. Assuming that preferences remain fixed,

$$F(w) = \{(x^1,\ldots,x^I,y) \in \mathbb{R}_+^{I+1} / \sum_{i=1}^{I} x^i + y \leqq w\}$$

is the set of all feasible allocations for the economy whose initial endowment is w. Here $(x^1,\ldots,x^I,y) \in \mathbb{R}_+^{I+1}$ is a typical allocation. A feasible allocation $(x^1,\ldots,x^I,y)$ is said to be <u>Pareto optimal</u>, if there does not exist any other feasible allocation $(\bar{x}^1,\ldots,\bar{x}^I,\bar{y})$ such that $u^i(\bar{x}^i,y) \geqq u^i(x^i,y) \forall i=1,\ldots,I$ and $u^i(\bar{x}^i,y) > u^i(x^i,y)$ for some $i \in \{1,\ldots,I\}$.

A <u>solution function</u> is a mapping $S: \mathbb{R}_{++} \to \mathbb{R}^{I+1}$ such that $S(w) \in F(w) \forall W \in \mathbb{R}_{++}$, and S is continuous from the right.

A <u>solution function</u> $S: \mathbb{R}_{++} \to \mathbb{R}_+^{I+1}$ is said to satisfy <u>resource monotonicity</u> if $w' > w$ implies $u^i(S^i(w'),S^{I+1}(w')) \geqq u^i(S^i(w),S^{I+1}(w)) \forall i=1,\ldots,I$, where $S \equiv (S^1,\ldots,S^I,S^{I+1})$. $S^{I+1}(w)$ indicates the consumption of the public good in the economy; $S^i(w)$, $1 \leqq i \leqq I$, indicates the consumption of the private good by agent i.

The <u>Equal Income Lindahl Equilibrium</u> (EILE) solution function denoted $L: \mathbb{R}_{++} \to \mathbb{R}_+^{I+1}$ is defined as follows: for all $w>0$,

(i) and, $\forall i \in \{1,\ldots,I\}$, $(L^i(w),L^{I+1}(w))$ solves

$x^i + v^i(y) \to \max$

subject to $x^i + t^i y \leq \frac{W}{I}$

$x^i \geqq 0$, $y \geqq 0$, for some $t^i \geqq 0$;

(ii) $y\left(\sum_{i=1}^{I} t_i\right) = y$

(iii) $\sum_{i=1}^{I} x^i + y = w$

(iv) $L(w)$ is a Pareto-optimal allocation.

In other words given $w>0$, $L(w)$ is an EILE if there exists a vector (of personalized prices) $(t^1, \ldots, t^I)$ such that $(L^i(w), L^{I+1}(w))$ maximizes agent i's preferences when he/she is faced with a price t^i(per unit) of the public good; the total revenue to the producer of the public good is equal to the total cost; and the allocation is itself feasible.

It is easily checked that the EILE solution function is well defined. Before we proceed to show that L satisfies resource monotonicity, it will be useful to characterize an EILE.

The Results :- On our way to the main result we present the following lemmas:

Lemma 1 :- Suppose there exists $\bar{y}>0$ such that $\sum_{i=1}^{I} \frac{dv^i(\bar{y})}{dy} = 1$

Then (i) such a $\bar{y}$ is unique

(ii) if $W<\bar{y}$, then $L^i(w) = \frac{w}{I}$, $i=1,\ldots,I$; $L^{I+1}(w)=0$

(iii) if $w \geq \bar{y}$, $L^{I+1}(w)=\bar{y}$ if and only if for all $i=1,\ldots,I$,

$$\frac{w}{I} - \left(\frac{dv^i(\bar{y})}{dy}\right)\bar{y} + v(\bar{y}) \geq \frac{w}{I}; \quad \text{otherwise} \quad L^{I+1}(w)=0.$$

If $L^{I+1}(w)=0$, then $L^i(w)=\frac{w}{I}$, $i=1,\dots,I$; if $L^{I+1}(w)=\bar{y}$,

then $L^i(w)=\frac{w}{I}-\left(\frac{dv^I(\bar{y})}{dy}\right)\bar{y}+v(\bar{y})$, $i=1,\dots,I$.

Proof :- (i) Follows from strict concavity of the v^i's; (ii) and (iii) are verified by referring to the definition of L.

Lemma 2 :- Suppose there does not exist $\bar{y}>0$ such that $\sum_{i=1}^{I}\frac{dv^i(\bar{y})}{dy}=1$.

Then (i) $\sum_{i=1}^{I}\frac{dv^i(\bar{y})}{dy}<1$ for all $\bar{y}>0$

(ii) $L^{I+1}(w)=0$, $L^i(w)=\frac{w}{I}$, $i=1,\dots,I$ for all $w>0$.

Proof :- (i) Follows once again from the strict concavity of the v^i's;

(ii) follows by referring to the definition of L.

In the above lemmas, the personalized prices associated with $L^{I+1}(w)=0$ is given by an I-vector $(t^1,\dots,t^I)$ such that $t^i\geq\frac{dv^i(o)}{dy}$ $\forall i\in\{1,\dots,I\}$. Whenever $L^{I+1}(w)=\bar{y}$, we have $t^i=\frac{dv^i}{dy}(\bar{y})$.

Now we proceed to establish the main result.

Theorem :- L satisfies resource monotonicity.

Proof :- We divide the proof into two cases:

Case 1 :- There does not exist $\bar{y}>0$ such that $\sum_{i=1}^{I}\frac{dv(\bar{y})}{dy}=1$.

Then by Lemma 2, $L^i(w)=\frac{w}{I}$, $i=1,\dots,I$; $L^{I+1}(w)=0$.

Thus if $w'>w$, then

$$L^i(w')=\frac{w'}{I}>\frac{w}{I}=L^i(w),\ i=1,\dots,I;\ L^{I+1}(w)=L^{I+1}(w')=0.$$

Thus $u^i(L^i(w'), L^{I+1}(w')) = \frac{w'}{I} > \frac{w}{I} = u^i(L^i(w), L^{I+1}(w))$, $i=1,\ldots,I$.

Case 2 :- There does exist $\bar{y}>0$ such that $\sum_{i=1}^{I} \frac{dv(\bar{y})}{dy} = 1$.

Choose $0<w<w'$.

Possibility 1 :- $0<w<w'<\bar{y}$; then by Lemma 1(ii),

$$L^i(w')>L^i(w), i=1,\ldots,I; \; L^{I+1}(w') = L^{I+1}(w) = 0.$$

Possibility 2 :- $w<\bar{y}\leq w'$; then by Lemma 1(ii),

$$L^i(w)=\frac{w}{I}.$$

By Lemma 1(iii), $L^{I+1}(w')=\bar{y}$ if and only if $\frac{w'}{I} - \left(\frac{dv^i(\bar{y})}{dy}\right)\bar{y}+v(\bar{y}) \geq \frac{w'}{I}$ $\forall$ $i=1,\ldots,I$. Otherwise $L^{I+1}(w')=0$. In either case,

$u^i(L^i(w'),L^{I+1}(w'))>u^i(L^i(w),L^{I+1}(w))$ $\forall$ $i=1,\ldots,I$.

Possibility 3 :- $0<\bar{y}\leq w<w'$; once again it is easily verified that $u^i(L^i(w), L^{I+1}(w))<u^i(L^i(w'),L^{I+1}(w'))$ $\forall$ $i=1,\ldots,I$.

Q.E.D.

Conclusion :- In this paper we have proved that the EILE solution function satisfies resource monotonicity. Our context seems more appropriate than that of Thomson and Moulin (1988), to resolve questions relating to distributive justice in a growing economy, in so far as public goods are an acknowledged reality. Although we require quasi-linear preferences for our results to hold, the practice is common in the public goods literature.

The problems we confront by relaxing our assumption of linear cost functions are however insurmountable. This to my mind is the only caveat we need to furnish along with our analysis. A linear cost function is associated with a constant returns to scale technology, and a justification for one is a necessary justification for the other.

Finally, it should be noted that the Equal Income Lindahl Equilibrium Solution we consider satisfies Pareto-optimality. For the solution where $L^{l+1}(w)>0$, there exists another EILE with the consumption of public good being zero, which we do not consider, owing to its non optimality.

References :-

1. Chichilinsky, G. and W. Thomson (1987) : "The Walrasian Mechanisams from Equal Division is Not Monotonic With Respect To Variations In The Number of Consumers", Journal of Public Economics, 32, pp. 119-124.

2. Chun, Y. and W. Thomson (1988) : "Monotonicity Properties of Bargaining solutions when applied to Economics", Mathematical Social Sciences, 15, pp. 11-27.

3. Laffont, J.J. (1988) : "Fundamentals of Public Economics:, MIT Press.

4. Lahiri, S. (1991) : "Distributive Justice With Externalities And Public Goods", Indian Institute of Management, Ahmedabad, Working Paper No. 993.

5. Lahiri, S. (1992) : "Resource Monotonicity of Bargaining Solutions", Indian Institute of Management, Ahmedabad, mimeo.

6. Moulin, H. and W. Thomson (1988) : "Can Everyone Benefit from Growth?", Journal of Mathematical Economics 17, pp. 339-345.

7. Varian, H. (1984) : "Micro-Economic Analysis", 2nd Edition, W.W. Norton and Co., New York and London.