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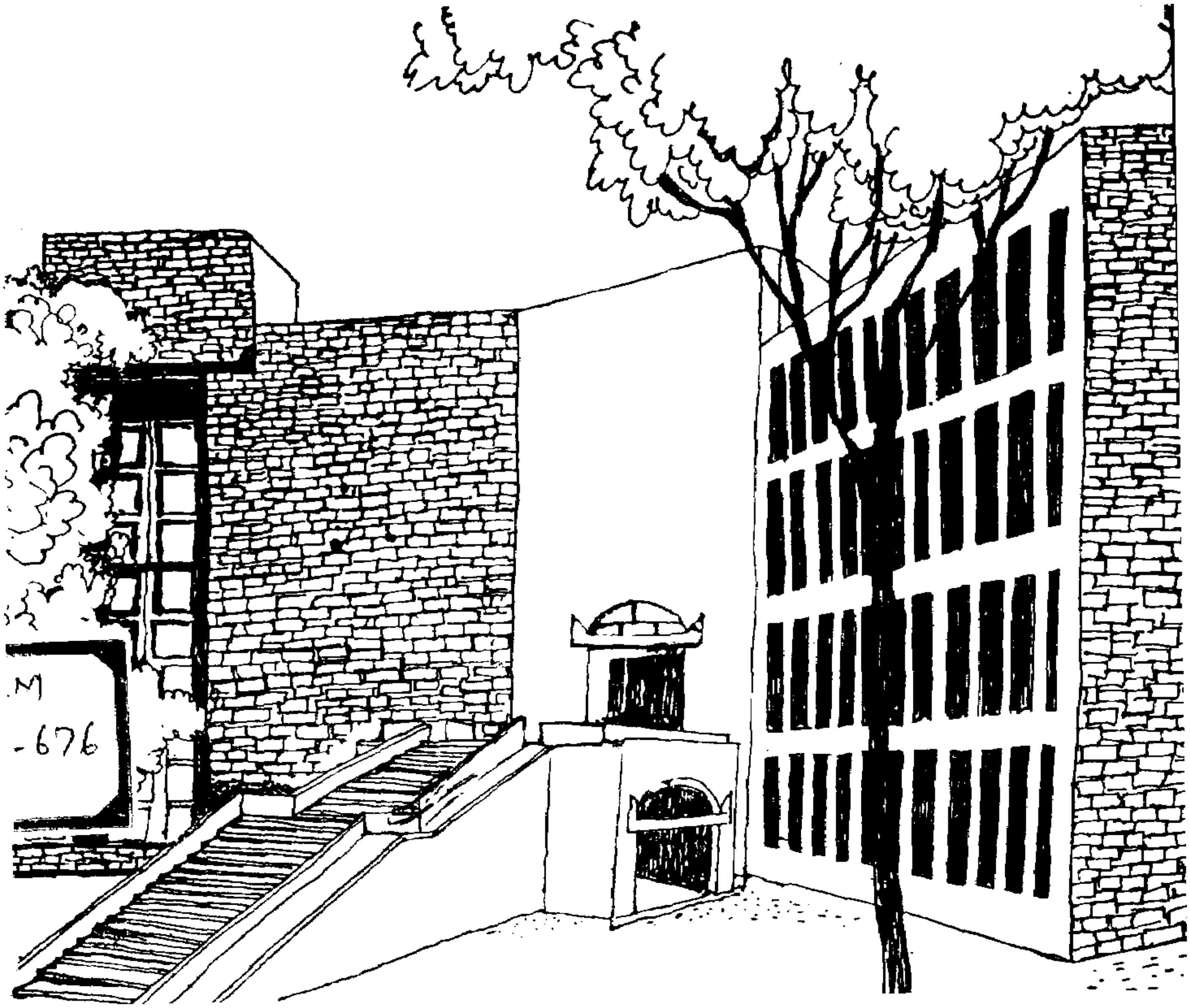

ECONOMIC ORDER QUANTITY WHEN ORDERING COSTS ARE LOT SIZE DEPENDENT

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ECONOMIC ORDER QUANTITY WHEN ORDERING COSTS ARE LOT SIZE DEPENDENT

Classical EOQ model assumes that the ordering cost A is constant and indepedent of lot size. This paper deals with the case when A depends on the lot size and increases at a decreasing rate. Firstly A is assumed to increase in steps as a function of lot size. Secondly A is assumed to have a learning effect of the lot size. In both cases models are analysed and procedures are given for determining the optimal lot size.

INTRODUCTION

Wilson's classical EOQ model assumes that the ordering cost A is a constant and is independent of lot size. (Hadley and whitin 1963). This paper deals with the case when A is not constant and depends on the lot size. This may be a reasonable assumption in many cases where the ordering cost per order will increase with the quantity ordered. We will consider two cases. Firstly we will deal with the case when ordering cost A is a step function of lot size, and next we will take up the case where ordering cost is subject to a learning effect of the lot size. All other assumptions of classical EOQ model are assumed to hold valid.

CASE 1 : ORDERING COST IS A STEP FUNCTION OF LOT SIZE

This model assumes that ordering cost will increase step-wise as lot size increases. Let us use the following notations:

D = Annual demand

i = Inventory carrying charge per dollar per year

C = Unit cost

A_j = Order cost for ordering Q if

$Q \in (q_{j-1}, q_j]$ for $j = 1, 2, \ldots, m$;

where $q_0 = 0$, and $q_m = \infty$.

Assume that $A_1 < A_2 < \ldots < A_m$.

Q_j = EOQ with order cost A_j using Wilson's formula;

i.e.

$$Q_j = \sqrt{2 A_j D / iC}$$

$$Q_j^* = q_j$$

$$TC(Q_j) = \sqrt{2 A_j DiC}$$

$$TC(Q_j^*) = A_j D / Q_j^* + Q_j^* iC/2$$

Q^* = Optimal lot size

Since A_j is a strictly increasing scequence, one would like to order with as small a value of A_j as possible. However, the corresponding EOQ may not be order-feasible. Once an order-feasible lot size Q is determind, it can be used to eliminate high-valued A_j's from further consideration. Suppose Q is an order-feasible lot. Let TC(Q) be the sum of annual ordering costs and inventory carrying costs. Now if ordering with some ordering cost A_k is to be considered, it is necessary that

$$\sqrt{2 A_k iCD} < TC(Q)$$

Therefore,

$$A_k < (TC(Q))^2 / (2iCD)$$

Therefore one can eliminate from further consideration all those order levels where ordering cost exceeds $(TC(Q))^2 / (2iCD)$. Moreover this bound on A_j's can be further tightened as we improve upon the bound on the total annual relevant costs. Based on these observations, we give following procedure:

1) Set $j = 1$.

2) Compute Q_j.

   If $Q_j \in (q_{j-1}, q_j]$, Set $Q^* = Q_j$, and stop.

   Otherwise compute Q_j^*, $TC(Q_j^*)$, $TC(Q_j)$.

   Set $Q^* = Q_j^*$, $LB = TC(Q_j)$, and $UB = TC(Q_j^*)$.

3) Set $j = j + 1$; if $j > m$, stop. Otherwise, go to step 4.

4) Compute ratio $r_j = (UB)^2 / (2DiC)$. If $A_j > r_j$,

   Q^* is the optimal lot size. Otherwise go to step 5.

5) Replace m by $\max_{A_j \leq r_j} \{j\}$.

6) Compute Q_j. If $Q_j \notin (q_{j-1}, q_j]$, go to step 7.

   Otherwise compute $TC(Q_j)$. If $TC(Q_j) > UB$, Q^* is the optimal lot size. If $TC(Q_j) = UB$, Q_j and Q^* are both optimal lot sizes.

If $TC(Q_j) < UB$, Q_j is the optimal lot size.

7) Compute $TC(Q_j)$, and set $LB = TC(Q_j)$.

Compute Q_j^*, and $TC(Q_j^*)$. If $TC(Q_j^*) < UB$, set $UB = TC(Q_j^*)$, and $Q^* = Q_j^*$, go to step 3.

A NUMERICAL EXAMPLE:

Suppose

D = 1000 units

C = \$ 1000

i = 20% per year, and

Ordering costs are given as follows:

j	Lot size (Q_j)	Ordering Cost (A_j)
1	1 - 20	\$ 100
2	21 - 30	\$ 110
3	31 - 40	\$ 120
4	41 - 50	\$ 130
5	51 or more	\$ 150

For A_1 = \$ 100, Q_1 = 31.62 which is not order-feasible.

Therefore Q_1^* = 20, $TC(Q_1^*)$ = 7000, $TC(Q_1)$ = 6324.

Therefore, Q^* = 20, LB = 6324, and UB = 7000.

Next take A_2 = \$ 110.

Compute r_2.

$$r_2 = (7000)^2 / 2\ (1000)(.20)(1000) = 122.5$$

Since $A_2 < r_2$, continue. However, we ignore A_4 and A_5 from further consideration.

Compute Q_2.

$Q_2 = 33.17$, which is also order-infeasible.

$TC(Q_2) = 6633$. Therefore LB updated to 6633.

$Q_2^* = 30$, $TC(Q_2^*) = 6667$.

Therefore $Q^* = 30$, LB = 6633, UB = 6667.

Next take $A_3 = \$\ 120$

Compute r_3.

$$r_3 = (6667)^2/2(1000)(.20)(1000) = 111.11$$

Since $A_3 > r_3$, stop.

$Q^* = 30$ is the optimal lot size.

CASE 2 : ORDERING COST IS UNDER LEARNING EFFECT OF LOT SIZE

In this case we assume that the ordering cost is an increasing function of lot size Q at a decreasing rate and is subject to a learning effect of lot size. In particular, the order cost A(Q), as a function of lot size Q, is estimated by following expression:

$$A(Q) = aQ^b \text{ where } a > 0, \quad 0 \leq b < 1.$$

Note that when $b = 0$, there is no learning effect, and the model reduces to the classical EOQ model. The parameters a and b can be evaluated if we have estimates of ordering costs at different order quantities.

Total annual relevant costs = Total annual ordering costs

+ Total annual inventory carrying costs

$$= A(Q)D/Q + QiC/2$$

$$= aQ^{b}D/Q + QiC/2$$

$$TC(Q) = aDQ^{b-1} + QiC/2$$

For minimization:

$$\frac{dTC(Q)}{dQ} = 0;$$

Therefore,

$$a(b-1)DQ^{b-2} + iC/2 = 0$$

$$Q = (rc/2a(1-b)D)^{\frac{1}{b-2}}$$

Since,

$$\frac{d^2TC(Q)}{dQ^2} = aD(b-1)(b-2)Q^{b-3} > 0,$$

Q^* is the optimal order quantity.

A NUMERICAL EXAMPLE:

Suppose, D = 1000 units

i = 20% per year

C = $ 1000, and

ordering cost for 10 units = $ 100

ordering cost for 20 units = $ 160

If $A(Q) = aQ^b$

$$100 = a(10)^b, \text{ and } 160 = a(20)^b$$

Solving these two equations, we get a=20.99; b=0.678.

Therefore,

$$A(Q) = 20.99\, Q^{0.678}$$

Therefore, the optimal lot size is

$$Q^* = 24.78 \text{ units.}$$

REFERENCE

Hadley, G. ,and Whitin, T. , 1963, Analysis Of Inventory Systems (Englewood Cliffs, N.J. ,Prentice Hall).