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ECONOMIC ORDER QUANTITY WHEN A DISCOUNT IS OFFERED ON CASH PURCHASES

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ECONOMIC ORDER QUANTITY WHEN A DISCOUNT IS OFFERED ON CASH PURCHASE

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Classical Wilson's EOQ model assumes that the payments for goods are made at the time of delivery and therefore a sufficient portion of carrying cost is incurred due to money blocked in inventories. In many situations, however, it is observed that the supplier allows delay in making payments of goods and offers a unit discount on cash purchases. The delay offered may be of various types such as delay of a fixed duration or a delay of an inventory cycle. In this paper, mathematical models have been developed to analyze when, if at all, purchasing on cash is preferable over delayed payment options. A total of four delay options are considered. Finally a numerical example is given to illustrate the methodology.

ECONOMIC ORDER QUANTITY WHEN A DISCOUNT IS OFFERED ON CASH PURCHASES

1. INTRODUCTION

Classical Wilson's EOQ model assumes that the payment for goods are made at the time of delivery. In many situations, however, it is a common practice to provide a certain delay in making payments in one form or another and a unit discount is given when buyer purchases on cash. Goyal (1985), S. Chand and J. Ward (1987) have considered models where a delay is allowed for a fixed period. In this paper we consider purchasing with cash discounts and compare the same with purchasing without cash discounts but with delayed payments.

Let us use the following notations:

- A = Ordering cost per order
- D = Annual demand
- C = Unit cost charged when payments are delayed
- r = Discount per dollar of cash purchase
- i_1 = Interest charge per dollar of investment in inventory per year
- i_2 = Other inventory carrying charges per dollar per year

Suppose the buyer has option of either purchasing on cash or delaying payments. Payments can be delayed in one of the following manners:

1. Payments are due in the middle of the inventory cycle.
2. Payments are due at the end of the inventory cycle.
3. An Interest free period P is provided by the supplier, and, thereafter interest i_1 is charged.
4. An interest free period P is provided by the supplier, but if the payment is not made in the period, interest i_1 is charged from the delivery date.

We will compare purchasing with cash discounts with each of the four options of delayed payments and each case will be analysed to decide when, if at all, it would be advisable to avail cash discounts.

2. MODEL ANALYSIS

Let us first consider purchasing with cash discounts. When items are purchased on cash, unit price charged is C(1-r), and therefore, the optimal order quantity is

$$Q_0^* = \sqrt{2AD/(i_1+i_2)C(1-r)} \qquad \ldots(1)$$

The total annual relevant costs will include ;

a) total unit costs,

b) total interest charges,

c) total other holding charges, and

d) total ordering costs.

Total annual relevant costs, therefore, are:

$$TC(Q_0^*) = DC(1-r) + \sqrt{2\ AD(i_1+i_2)C(1-r)} \qquad \ldots(2)$$

2.1 Credit Purchase-Option 1

When purchases are made on credit with this option, the payments are required to be made in the middle of the inventory cycles. Therefore the buyer, on an average, neither makes any invesments in stocks nor does he earn any interest thereon. Therefore, only other inventory carrying charges would be applicable.

The optimal order quantity, therefore, is:

$$Q_1^* = \sqrt{2AD/i_2C} \qquad \ldots(3)$$

And, the total annual relevant costs are:

$$TC(Q_1^*) = DC + \sqrt{2ADi_2C}$$

Availing cash discounts will be better than delaying the payments (Option 1) if:

$$TC(Q_0^*) < TC(Q_1^*)$$

i.e., $DC(1-r)+\sqrt{2AD(i_1+i_2)C(1-r)} < DC+\sqrt{2ADi_2C}$

i.e., $DCr/\sqrt{2ADC} > \sqrt{(i_1+i_2)(1-r)} - \sqrt{i_2}$

$$\doteq \sqrt{i_1+i_2}\ (1-r/2) - \sqrt{i_2}$$

Simplifying, we get

$$r > \frac{i_2}{(\sqrt{DC/2A}+1/2\sqrt{i_1+i_2})\ (\sqrt{i_1+i_2}+\sqrt{i_2})} \qquad \ldots(4)$$

This provides a lower bound on the cash discount rate r for purchasing on cash to be an attractive proposition over Option 1.

2.2 Credit Purchase-Option 2

This case deals with the credit policy of allowing payments at the end of the cycle. Since payments are made only at the end of the cycle, the buyer earns i_1 interest on the cash realized after sales. Therefore, the total annual relevant costs, are:

$$TC(Q_2) = AD/Q_2 + Q_2(i_2 - i_1)C/2 + DC$$

Minimizing $TC(Q_2)$, we get :

$$Q_2^* = \sqrt{2AD/(i_2 - i_1)C} \quad \text{provided } i_2 > i_1 ; \qquad \ldots(5)$$

and

$$TC(Q_2^*) = DC + \sqrt{2AD(i_2 - i_1)C} \qquad \ldots(6)$$

Cash purchase will be preferable over Credit Option 2 if:

$$TC(Q_1^*) < TC(Q_2^*)$$

i.e., $DC(1-r) + \sqrt{2AD(i_1 + i_2)C(1-r)} < DC + \sqrt{2AD(i_2 - i_1)C}$

Simplifying,

$$> \frac{2i_1}{(\sqrt{DC/2A} + 1/2\sqrt{i_1 + i_2})\ (\sqrt{i_1 + i_2} - \sqrt{i_2 - i_1})} \qquad \ldots(7)$$

This provides a lower bound on the cash discount rate r for cash purchases to be preferable over Option 2.

In case of $i_2 \leqslant i_1$, it is obvious that buyer's ordering costs as well as inventory costs reduce as lot size increases, and therefore the optimal value of order quantity Q is infinite. This in turn would mean that the supplier will provide credit for ever which of course can never happen in practice. Therefore, it is reasonable to assume that Option 2 would be available only if $i_2 > i_1$.

2.3 Credit Purchase-Option 3

When the supplier allows for an interest free period P, and if the buyer wishes to delay the payments, it will obviously be advantageous to him to pay only after period P after the purchase has been made. Therefore, the buyer effectively has an opportunity of earning interest i_1 on the item costs. Since, over the year, the total item cost is DC, he earns an interest on this amount for period P per year. Therefore, his optimal policy would be to order:

$$Q_3^* = \sqrt{2AD/(i_1+i_2)C} \qquad \ldots(8)$$

and, the total annual relevant costs can be expressed as:

$$TC(Q_3^*) = CD + \sqrt{2AD(i_1+i_2)C} - PDCi_1 \qquad \ldots(9)$$

Therefore, cash purchases will be preferable over Option 3 if $TC(Q_0^*) < TC(Q_3^*)$.

Therefore,

$$DC(1-r) + \sqrt{2AD(i_1+i_2)C(1-r)} < CD + \sqrt{2AD(i_1+i_2)C} - PDCi_1$$

Simplifying, we get:

$$r > \frac{PDCi_1}{DC + 1/2\sqrt{2ADC(i_1 + i_2)}} \quad \ldots(10)$$

This provides a lower bound on the cash discount rate r for cash purchases to be preferable over Option 3.

2.4 Credit Purchase-Option 4

In this case an interest free period P is provided and an interest charge is levied from the delivery date if payments are not made within the allowed period P. Therefore, if the payments are delayed beyond period P, the effective inventory carrying costs are at the rate $(i_1 + i_2)$. Hence the optimal order quantity is:

$$Q_4^* = \sqrt{2AD/ \quad (i_1 + i_2)C} \quad \ldots(11)$$

and, the total annual relevant costs are:

$$TC(Q_4^*) = DC + \sqrt{2ADC(i_1 + i_2)} \quad \ldots(12)$$

Comparing with (2), it is clear that this policy is always worse than purchasing on cash.

3. A NUMERICAL EXAMPLE

Suppose

$A = \$\ 10$

$D = 360$ units per year

$C = \$\ 8$

$i_1 = 9\%$ per year

$i_2 = 16\%$ per year

If credit purchase with Option 1 is available, it will be better to purchase on cash if the cash discount rate

$$r > \frac{0.16}{\left(\sqrt{(360)(8)/2(10)} + 1/2\sqrt{.25}\right)\left(\sqrt{.25} + \sqrt{.16}\right)}$$

(from expression 4)

Simplifying, $r > 0.0145$.

If credit purchase with Option 2 is available, purchasing with discount will be preferable if the cash discount rate

$$r > \frac{2(0.09)}{\left(\sqrt{(360)(8)/2(10)} + 1/2\sqrt{.25}\right)\left(\sqrt{.25} - \sqrt{.07}\right)}$$

(from expression 7)

Simplifying, $r > 0.0624$.

Similarly, purchasing with cash discount will be preferable when compared with purchasing with Option 3 if,

$$r > \frac{P(360)(8)(.09)}{(360)(8) + 1/2\sqrt{2(10)(360)(8)(.25)}}$$

(from expression 10)

Simplifying, $r > 0.0882\ P$.

4. REFERENCES

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