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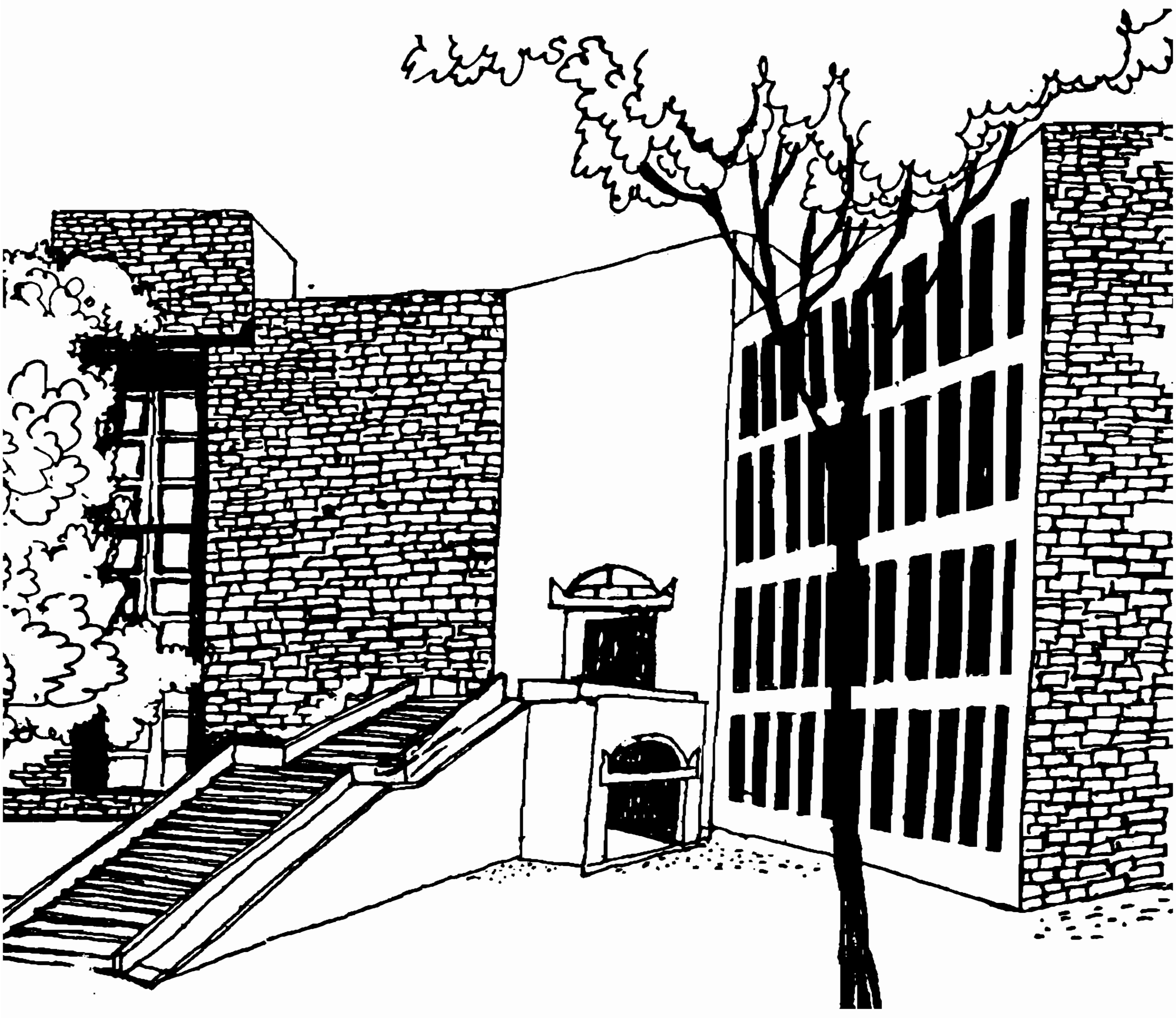

OPPORTUNITY FAIRNESS AND EQUAL INCOME LINDAHL EQUILIBRIUM

By
Somdeb Lahiri

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INDIAN INSTITUTE OF MANAGEMENT
AHMEDABAD-380 015
INDIA

Abstract

In this paper we propose a concept of opportunity fairness for economies in which there are public goods and establish its equivalence with the concept of an equal income Lindahl equilibrium.

1. Introduction :- Concepts of distributive justice that have been proposed and applied in normative economics. have been usually developed in the context of a private ownership economy (see Thomson and Varian (1985) for a survey). Only a few works. such as Moulin (1987). Sato (1985. 1987) and Otsuki (1992) have incorporated public goods explicitly in problems of distributive justice. From the stand point of welfare economics. studying problems of distributive justice with public goods is important. since public goods form an undeniable component of economic reality.

The problem of distributive justice is one of dividing a fixed amount of goods among a finite number of agents in a manner consistent with equity and justice. There are various ways of doing this. the variation being due to different perceptions of what is just.

One of the earliest concept of equity is due to Foley (1987) which has been analysed in great detail in the context of a private good economy by Schmeidler and Yaari (1978). Kolm (1972). Schmeidler and Vind (1972). Varian (1972). Variations occur in Goldman and Sussangkarn (1978). Thomson (1982). Feldman and Weiman (1979). The extension of this concept to an economy with public goods can be found in Sato (1987).

In Varian (1976) can be found the concept of opportunity fairness which extends in the context of a private good economy. the equity concept introduced by Foley. In this paper we first propose to reintroduce this context. in an economy with public goods. Then propose to establish the equivalence of opportunity fa[illegible] in an economy with public goods to the concept of an equal income Lindahl equilibrium as defined in Sato (1987) or Lahiri (1992).

2. The Model :- We assume a simplified framework (largely for expositional purposes) of an economy consisting of I agents (or consumers). Let $\mathbb{R}^2_+$ denote the nonnegative quadrant of the Euclidean space of dimension 2, the set of commodity bundles. Thus we assume that there are two goods in the economy - a private good and a public good.

Each agent in the economy has preferences on consumption bundles, which are represented by a twice continuously differentiable, quasi-concave strictly monotonic increasing utility function. Thus, agent i's preferences are represented by a twice continuously differentiable function $u^i : \mathbb{R}^2_+ \to \mathbb{R}$, such that $\{(x',g') \in \mathbb{R}^2_+ / u^i(x',g') \geq u^i(x,g)\}$ is a strictly convex set for each $(x,g) \in \mathbb{R}^2_+$, $i=1,\ldots,I$; and for each $i=1,\ldots,I$ $(x,g) \in \mathbb{R}^2_+$, $(x',g') \in \mathbb{R}^2_+$, $(x',g') \neq (x,g)$, $(x',g') \geq (x,g)$ implies $u^i(x',g') > u^i(x,g)$. Here the first coordinate of the ordered pair $(x,g) \in \mathbb{R}^2_+$ represents the level of consumption of the private good and the second coordinate represents the level of consumption of the public good.

We assume that the aggregate initial endowment of the private good in the economy is given by the positive real number W(>0) and that there is a simple linear technology which converts private good into public good i.e. there exists a constant p>0, such that to produce one unit of the public good, we require p units of the private good.

<u>A feasible allocation</u> is a vector $(x^1,\ldots,x^I,g) \in \mathbb{R}^{I+1}_+$ such that $\Sigma^I_{i=1} x^i + pg \leq W$. Let F be the set of feasible allocations.

A feasible allocation $(x^1 \ldots\ldots x^I, g)$ is said to be <u>Pareto-efficient</u>, if there does not exist $(x^{1'}, \ldots\ldots x^{I'}, g') \in F$ with $u^i(x^{i'}, g') \geq u^i(x^i, g)\ \forall\ i=1,\ldots,I$ and $u^i(x^{i'}, g') > u^i(x^i, g)$ for some $i \in \{1,\ldots\ldots I\}$.

Let $(x^1 \ldots\ldots x^I, g) \in F$. We introduce the following concepts and notations :

$$\pi^i \equiv \pi^i(x^i, g) \equiv \frac{\partial u^i(x^i, g)/\partial g}{\partial u^i(x^i, g)/\partial x}, \quad i=1,\ldots\ldots I$$

$$A(\pi^i, \pi^j) \equiv \pi^j/\pi^i, \quad i, j=1,\ldots\ldots I$$

Owing to the monotonicity assumption, $\pi^i > 0, i=1,\ldots\ldots I$; $A(\pi^i, \pi^j) > 0, i, j=1,\ldots\ldots I$. Sato (1987) provides a detailed explanation of these concepts and their application to the theory of fairness.

We say that a feasible allocation $(x^1,\ldots x^I, g)$ is <u>L-equitable</u> if $\forall\ i, j \in \{1,\ldots\ldots I\}, u^i(x^i, g) \geq u^i(x^j, A(\pi^i, \pi^j) g)$. An L-equitable allocation is said to be L-fair, if it satisfies Pareto efficiency.

We say that a feasible allocation $(x^1,\ldots, x^I g)$ is opportunity fair if

(i) $(x^1,\ldots\ldots x^I, g)$ is Pareto-efficient

(ii) $\forall\ i, j \in \{1,\ldots\ldots I\}, u^i(x^i g) \geq u^i(x, A(\pi^i, \pi^j)\bar{g})$ whenever, $(\bar{x}, \bar{g}) \in {}^2$ and $\bar{x} + \pi^j \bar{g} \leq x^j + \pi^j g$.

In other words a feasible allocation is opportunity fair if it is Pareto-efficient and no agent "envies" a consumption bundle belonging to the "budget set" of some other agent, at the efficiency prices.

An allocation $(x^1, x^2 \ldots\ldots x^I, g) \in F$ is said to be an <u>Equal Income Lindahl Equilibrium</u> (EILE) allocation if there exists a vector $(p^1 \ldots\ldots p^I) \in \mathbb{R}^I_{++}$ for which the following is true :

(i) $\bar{x}^i + p^i \bar{g} = W/I$ and $u^i(\bar{x}^i, \bar{g}) \geq u^i(x, g)$ whenever $(x, g) \in \mathbb{R}^2_+$ and $x + p^i g \leq W/I$;

(ii) $(\Sigma^I_{i=1} p^i)\bar{g} = p\bar{g}$

The above definition can be found in Sato (1987) or Lahiri (1992).

3. The Main Results :- In this section we establish the main results which establishes an equivalence between the set of opportunity fair allocations and the set of EILE allocations for the economy described in Section 2. Before we proceed, let us note the following : if $(p^1 \ldots\ldots p^I) \in \mathbb{R}^I_{++}$ are the personalized prices associated to an EILE allocation $(\bar{x}^1 \ldots\ldots \bar{x}^I, \bar{g})$ then $p^i = \pi^i \; \forall\, i \in \{1, \ldots\ldots I\}$ where

$$\pi^i = \frac{\partial u^i(\bar{x}^i, \bar{g})/\partial g}{\partial u^i(\bar{x}^i, \bar{g})/\partial x}, \quad i = 1, \ldots\ldots, I.$$

Theorem 1 - Let $(\bar{x}^1 \ldots\ldots \bar{x}^I, \bar{g}$ be an EILE allocation. Then $(\bar{x}^1 \ldots\ldots \bar{x}^I, \bar{g})$ is opportunity fair.

Proof :- Since an EILE allocation is Pareto-efficient, we only need to show that the second part of the definition of opportunity fairness is satisfied. Thus, let $(x, g) \in \mathbb{R}^2_+$ satisfy

$x + \pi^j g \leq W/I$.

Then $x + \pi^i A(\pi^i, \pi^j) g = x + \pi^j g \leq W/I$.

$\therefore (x, A(\pi^i \pi^i, \pi^j) g) \in \mathbb{R}^2_+$ and belongs to the budget set of consumer i. Thus by part (i) of the definition of an EILE allocation,

$u^i(\bar{x}^i, \bar{g}) \geq u^i(x, A(\pi^i, \pi^j) g)$.

Thus $(\bar{x}^1 \ldots\ldots \bar{x}^I, g)$ is opportunity fair.

Q.E.D.

The next theorem establishes the converse to the above proposition.

Theorem 2 :- Let $(\bar{x}^1 \ldots\ldots \bar{x}^I, \bar{g})$ be an opportunity fair allocation. Then it is also an EILE allocation.

Proof :- By part (i) of the definition of an opportunity fair allocation $(\bar{x}^1 \ldots\ldots \bar{x}^I, \bar{g})$ is a Pareto-efficient allocation. Thus the following is satisfied for $p^i = \pi^i$:

(i) $u^i(x, g) \leq u^i(\bar{x}^i, \bar{g}) \; \forall\, (x, g) \in \mathbb{R}^2_+$ whenever, $x + p^i g \leq \bar{x} + p^i \bar{g}$:

(ii) $(\Sigma^I_{i=1} p^i) \bar{g} = p\bar{g}$

Thus if we can establish that $\forall\, i, j \in \{1, \ldots\ldots I\}$, $p^i \bar{g} + \bar{x}^i = p^j \bar{g} + \bar{x}^j$, we can then appeal to the monotonicity of preferences and assert that $p^i \bar{g} + \bar{x}^i = W/I \;\; \forall\, i = 1, \ldots\ldots, I$.

Suppose towards a contradiction that there exists $i, j \in \{1, \ldots\ldots I\}$ with $p^j \bar{g} + \bar{x}^j > p^i \bar{g} + \bar{x}^i$.

Therefore,

$p^i A(\pi^i, \pi^j)\bar{g}+\bar{x}^j > p^i \bar{g}+\bar{x}^i$.

By the strict quasi-concavity of preferences, there exists $(x,g) \in \mathbb{R}^2_+$ with $p^i g+x = p^i A(\pi^i, \pi^j)\bar{g}+\bar{x}^j$ and

$u^i(x,g) > u^i(x^i, g)$.

Let $g' = g/A(\pi^i, \pi^j) = A(\pi^j, \pi^i)g$.

$\therefore\ p^j g'+x = p^j A(\pi^j, \pi^i)\ g + x = p^i g + x = p^i A(\pi^i, \pi^j)\bar{g}+\bar{x}^j$

$= p^j \bar{g}+\bar{x}^j$.

and $u^i(x, A(\pi^i, \pi^j)g') = u^i(x,g) > u^i(\bar{x}^i, \bar{g})$.

Thus, there exists a consumption bundle in the budget set of consumer j, which consumer i "prefers" to his own. This contradicts opportunity fairness and proves the theorem.

Q.E.D.

4. Conclusion :- In Sato (1987) an equivalence between EILE allocations and what he calls strongly L-fair allocations can be found. In this paper we have established the equivalence between EILE allcoations and opportunity fair allocations. The latter concept is simpler than strong L-fairness, at least in its conceptualization. This should enhance the intuitive appeal of the EILE resource allocation mechanism.

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