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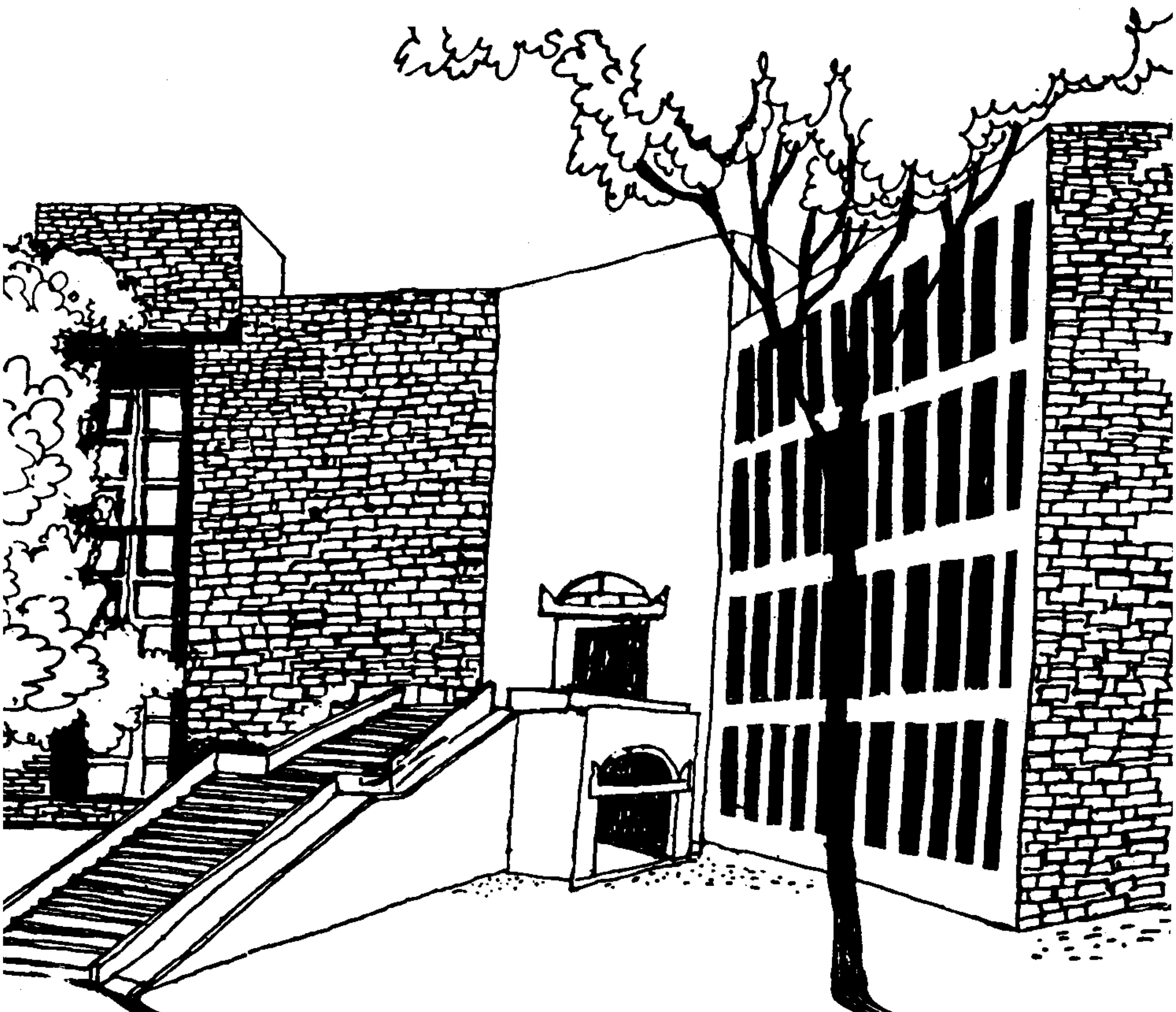

OPTIMAL LINEAR INCOME TAXATION: A NEW APPROACH

By

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Abstract

In this paper we propose a new approach to the theory of optimal taxation. We present the outlines of a theory which combines distributive justice will allocative efficiency. This is aimed to be a mere starting point of a potentially rich area of study.

1. <u>Introduction</u> :- In this paper we propose a new approach to the theory of optimal taxation. The convential theory discussed as for instance in Young (1987a, 1987b, 1988), Richter (1983) focuses almost entirely on the redistributive aspects of just taxation. Similar in spirit is the recent work of Berliant (1992). However, taxation has an additional purpose as far as society is concerned - namely the financing of social projects.

Our objective in this paper is to present the outlines of a theory which combines distributive justice with the equally pertinent issue of allocative efficiency. We consider a two good model, consisting of a private and a public good, where one of the goals of taxation is to cover the costs of production of the public good. Thereafter we proceed towards a characterization of the linear income tax profile.

It should be mentioned that this analysis is aimed to be a mere starting point of a potentially rich area of study.

2. <u>The Model</u> :- We consider a simple economy consisting of two goods - a private consumption good, and a public good. We assume that there are a finite number of agents in the economy, indexed by i=1,...,n. The consumption set of each agent is the nonnegative orthant of Euclidean two dimensional space, $\mathbb{R}^2_+$; i.e. $(x,g)\in\mathbb{R}^2_+$ denotes x units of consumption of the private good and g units of consumption of the public good.

The preferences of each agent i, is represented by a twice continuously differentiable, quasi-concave and strictly monotonically increasing utility function $u^i: \mathbb{R}^2_+ \to \mathbb{R}$. Each agent is assumed to be initially endowed with a positive amount of the private good. Let $w^i > 0$ be the initial endowment of agent i.

We assume that the economy is equipped with a simple linear technology which converts private good into public good. Without loss of generality (and purely for the sake of notational economy) let us assume that to produce one unit of the public good the economy requires one unit of the public good.

In what follows we shall assume that the preferences of the agents are fixed. Thus $(w^1,\dots,w^n) \in \mathbb{R}^n_{++}$ (the strictly positive orthant of Euclidean n-space) corresponds to a realization of the economic environment.

Given $(w^1,\dots,w^n) \in \mathbb{R}^n_{++}$, the set of feasible levels of consumption good is given by $F(w^1,\dots,w^n) = \left\{ g \geqq 0 / g \leqq \sum_{i=1}^{n} w^i \right\} \equiv \left[0, \sum_{i=1}^{n} w^i\right]$.

A tax <u>profile</u> is a function $T : \mathbb{R}^n_{++} \longrightarrow \mathbb{R}^n$ such that

(i) for all $(w^1,\dots,w^n) \in \mathbb{R}^n_{++}$, $T_i(w^1,\dots,w^n) \leqq w^i, i=1,\dots,n$

(ii) $\sum_{i=1}^{n} T_i(w^1,\dots,w^n) \geqq 0 \; \forall \, (w^1,\dots,w^n) \in \mathbb{R}^n_{++}$.

i.e. agent i cannot be taxed more than what he can pay in terms of the private good, and, the total tax should coverthe cost of production of the public good which is nonnegative.

A <u>Linear tax profile</u> is a tax profile $T : \mathbb{R}^n_{++} \rightarrow \mathbb{R}^n$ such that there exists two functions $a : \mathbb{R}^n_{++} \rightarrow \mathbb{R}$ and $b : \mathbb{R}^n_{++} \rightarrow \mathbb{R}$ satisfying $T_i(w^1,\dots,w^n) = a(w^1,\dots,w^n)\, w^i + b(w^1,\dots,w^n)$, for all i, and for all $(w^1,\dots,w^n) \in \mathbb{R}^n_{++}$.

3. <u>Optimal Linear Tax Profile</u> :- As in Heady (1988), let us assume that the preferences of all the agents are identical and representable by a log-linear utility function where for $0 < c < 1$

$$u(x,g) = c \log x + (1-c) \log g, \; x > 0, \; g > 0 \qquad (1)$$

$$= -\infty \qquad \text{, otherwise}$$

The most commonly used social welfare function is utilitarian and has the form :

$$W(u^1,\ldots,w^n)= \sum_i (u^i) \qquad (2)$$

where u^i is the utility of the ith agent.

The problem fased by the economy is the following

$$\left.\begin{array}{l}\max\limits_{(a,b)} \left[c \sum_{i=1}^{n} (\log (w^i - a\, w^i - b) + (1-c) \log (an\bar{w} + nb)\right] \\ \qquad \text{s.t. } aw^i + b \leqq w^i,\ i=1,\ldots,n.\end{array}\right\} \quad (3)$$

From the nature of the problem, it clear that for optimal choices of a,b, $aw^i+b < w^i$, otherwise utility of the i agent and hence social welfare becomes $-\infty$. Thus the constraints can be ignored. Let a^*, b^* be the optimal choices. Then they satisfy the equation system:

$$-c \sum_{i=1}^{n} \left(\frac{w^i}{w^i - a^* w^i - b^*} \right) + \frac{(1-c)n\bar{w}}{a^* n\bar{w} + nb^*} = 0 \qquad (4)$$

$$-c \sum_{i=1}^{n} \frac{1}{w^i - a^* w^i - b^*} + \frac{(1-c)n}{a^* n\bar{w} + nb^*} = 0 \qquad (5)$$

From (4) and (5),

$$\sum_{i=1}^{n} \left(\frac{w^i}{w^i - a^* w^i - b^*} \right) \Bigg/ \sum_{i=1}^{n} \frac{1}{(w^i - a^* w^i - b^*)} = \bar{w}$$

or
$$\sum_{i=1}^{n} \left(\frac{w^i}{w^i - a^* w^i - b^*} \right) = \bar{w} \sum_{i=1}^{n} \frac{1}{(w^i - a^* w^i - b^*)} \qquad (6)$$

Equations (4) and (6) or Equations (5) and (6) determine a^*, b^* where $\bar{w} = \frac{1}{n} \sum_{i=1}^{n} w^i$.

If as a special case we require $b(w^1,\ldots,w^n) \equiv 0$, we get $a^*(w^1,\ldots,w^n) = \frac{(1-c)}{nc + (1-c)}$, which is independent of the initial endowments. An explicit solution is in general not available in the general case and one must resort to computational techniques to obtain approximate solutions. Even at this level of simplicity, explicit solutions are difficult to obtain.

4. Conclusion :- There are two points which deserve mentioning. The first point is that, unlike in the conventional literature on distributive justice with taxation, the total amount of taxes is determined endogenously in our model. In fact our analysis, is an answer to the question : "What to we need the taxes for?", which is not addressed to in the existing literature.

Our analysis parallels the analysis of income taxation from a micro-economic point of view, in terms of methodology. However, the objectives and framework are entirely different. In the micro-economic framework as in Atkinson and Stiglitz (1980), tax is determined as a function of wage income, whereas we determine taxes from distributions of initial endowments. Our analysis actually focuses on optimal cost-sharing in the production of a public good, where the level of production is determined endogenously.

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