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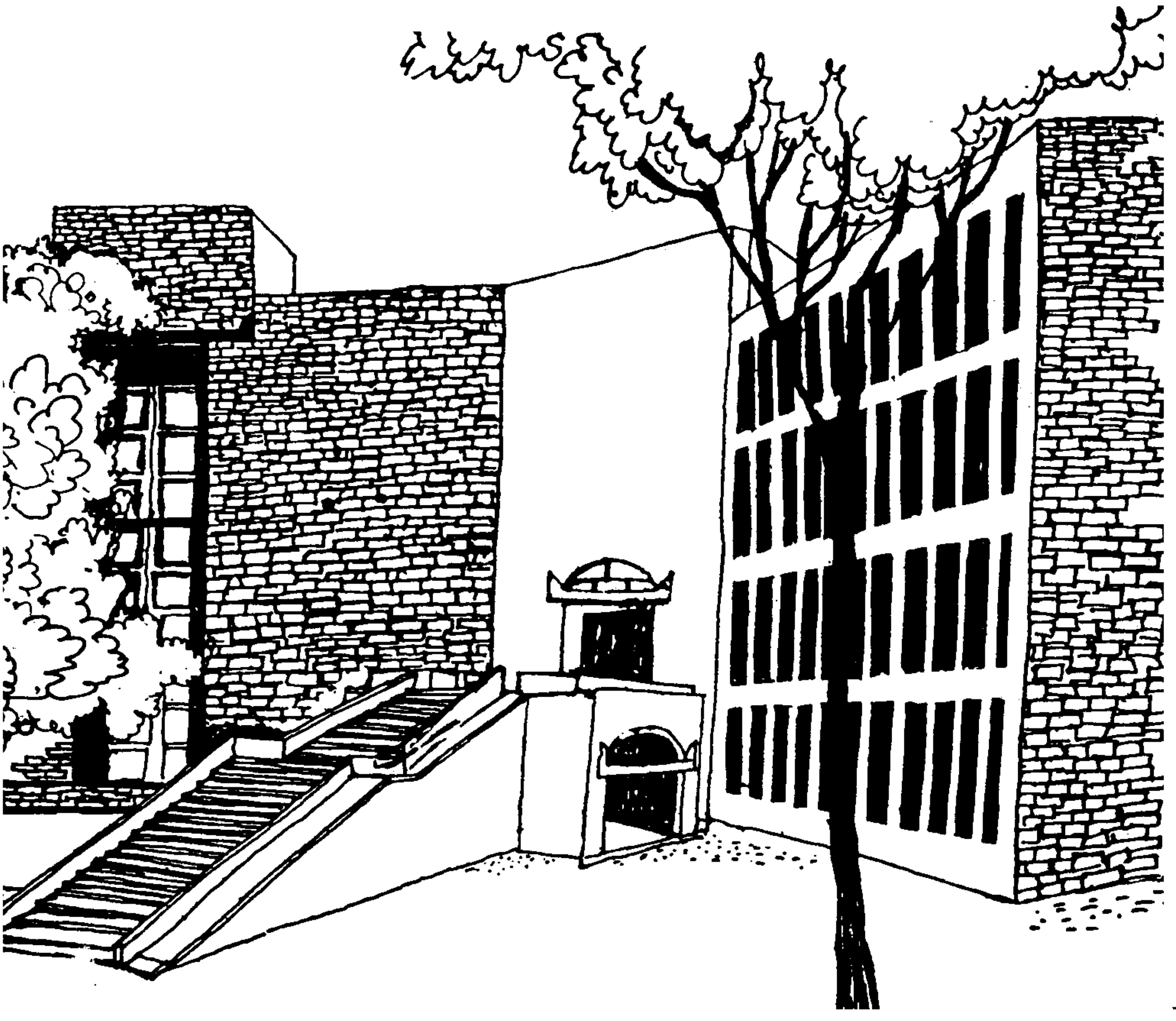

Pricing Economic Inequality

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Abstract

In this paper we propose a mechanism which decentralizes the creation of economic inequality. A personalized price for the public good/bad (i.e. economic inequality), determines the choice of disposable income by an individual. A perfect foresight equilibrium is defined, and a vector of prices which supports a utilitarian optimal solution is obtained. A by product of our analysis, is an income tax profile for the individuals which is compatible with decentralized distributive justice.

1. Introduction :- The theory of the measurement of economic inequality, has grown impressively both in volume as well as in technical complexity, since the seminal work of Atkinson (1970) (see for instance the survey by Barooah (1992) and the references therein). Few studies however, emphasize the need to use these measures, in order to facilitate redistributive taxation.

In this paper, we take the view as in Orr (1976), Morawetz (1977) and in particular Thurow (1971), that inequality in the distribution of income is a public good. Ng (1973) proposes a theory of the paradox of redistribution, from a similar standpoint (see Ng (1979) as well). As far as inequality goes ~~exclusion is~~ impossible; each individual must consume the same quantity. Thus it is a public good.

Our objective in this paper is to decentralize the consumption of economic inequality in such a way that a set of personalized prices for inequality in income distribution guides each agent in his/her choice of final income. A by-product of our analysis is an income tax profile which corrects the externality generated by disparities in income distribution.

2. The Model :- We consider an economy consisting of I individuals, indexed by i=1,...,I. The initial endowment of the ith individual is a real number $w^i > 0$, which denotes the quantity of his initial income or ability to spend. The inequality perceptions of the economy are summarized by a continuously differentiable function $M:\mathbb{R}^I_+ \to \mathbb{R}_+$ where for each final distribution of income $x=(x^1, \ldots, x^I) \in \mathbb{R}^I_+$, $M(x)$ is a measure of the inequality inherent in the income distribution.

The preferences of the ith individual over alternative income and inequality pairs are summarized by a quasi-linear utility function. Thus if (x^i, M) is the ordered fair of final income and economic inequality then the utility derived from this consumption opportunity to individual i is given by $x^i + v^i(M)$, where $v^i : \mathbb{R} \to \mathbb{R}$ is a strictly quasi-concave, continuously differentiable function. $v^i(M)$ is the income equivalent of M units of economic inequality (a public good or bad as the case may be) to individual i.

Let $t^i \in \mathbb{R}$ be the personalized price faced by individual 'i' for one unit of economic inequality. We assume that agent 'i' is capable of forming expectations about the final income of the remaining agents in the economy, on the basis of this price information. Let $y^j_i : \mathbb{R} \to \mathbb{R}$ be the function which summarizes individual i's beliefs about individual j's final income when individual i is confronted with alternative prices for economic inequality. We assume, $\{y^j_i \,/ i=1,\ldots,I; j\neq i, j=1,\ldots,I\}$ is a family of continuously differentiable functions.

Confronted with a price t^i for economic inequality, agent i chooses $x^i(t^i) \in \mathbb{R}$ so as to solve the following problem:

$$(*) \begin{cases} \text{Maximize } [x^i + v^i(M((y^j_i(t^i))_{j\neq i}, x^i))] & (1) \\ \text{s.t. } x^i + t^i M = w^i. & (2) \end{cases}$$

A first order necessary condition for $x^i(t^i)$ to solve this problem. is that

$$t^i \frac{\delta M}{\delta x^i} = \frac{dv^i}{dM} \frac{\delta M}{\delta x^i} \qquad (3)$$

where $\frac{\delta M}{\delta x^i}$ stands for $\frac{\delta M}{\delta x^i}((y^j_i(t^i))_{j\neq i}, x^i(t^i))$

and $\frac{dv^i}{dM}$ stand for $\frac{dv^i(M((y^j_i(t^i))_{j\neq i}, x^i(t^i)))}{dM}$.

Condition (3) is obtained by substituting $x^i = w^i - t^i M$ from (2) into (1) and then maximizing the resulting objective function.

We shall now define a perfect foresight equilibrium for the above problem. A vector $(t^1, \ldots, t^I) \in \mathbb{R}^I$ is a <u>perfect foresight equilibrium</u> if the following conditions are satisfied:

(a) $y^j_i(\hat{t}^i) = x^j(\hat{t}^j)$, $\forall j \neq i, i=1,\ldots,I$.

(b) $\Sigma^I_{i=1} \hat{t}^i = 0$.

3. The Utilitarian Optimal Perfect Foresight Equilibrium :- In what follows we will need to assume the following:

<u>Assumption</u> :- The function $x^i : \mathbb{R} \to \mathbb{R}$, which for each $t^i \in \mathbb{R}$, solves (*) above, is a continuously differentiable function, for $i=1,\ldots,I$.

It should be noted that this does not follow from the assumptions we have made. However with additional regularity assumptions on v^i and M and $\{y^j_i\}$ differentiability of x^i would follow as a necessary consequence of the implicit function theorem.

We seek to choose $(\hat{t}^1,\ldots,\hat{t}^I) \in \mathbb{R}^I$ so as to maximize:

$$\Sigma^I_{i=1}\, \alpha^i\, [x^i + v^i(M((x^j - t^j))^I_{j=1}))]$$

$$\text{s.t. } \Sigma^I_{i=1}\, t^i = 0$$

and the first-order condition (3).

Here $\alpha^i > 0, i=1,\ldots,I$, are the weights assigned to the utilities of the respective agents.

A solution $(\hat{t}^1,\ldots,\hat{t}^I)$ to this problem must satisfy

$$\hat{t}^i \frac{\partial M}{\partial x^i} = -1 - \frac{\left[\lambda + \left(\Sigma_{j \neq i}\, \alpha_j \frac{dv^j}{dM}\right) \frac{\partial M}{\partial x_i} \frac{dx^i}{dt^i}\right]}{\alpha^i \frac{dx^i}{dt^i}}$$

where λ is the Lagrange multiplier associated with the social planner's problem.

It may be reasonable to assume that $\hat{\beta}^j = \frac{dv^j}{dM} \frac{\partial M}{\partial x^j} \gtrless 0$,

for all $j=1,\ldots,I$. This amounts to assuming that, if an increase in disposable income increases inequality (as in the case of a rich man), then the agent prefers increasing inequality i.e. economic inequality is a public "good" for the agent: if an increase in disposable income decreases inequality (as in the case of a poor man), then the agent prefers decreasing inequality i.e. economic inequality is a public "bad" for the agent.

Since, $\hat{\beta}^i = \hat{t}^i \frac{\delta M}{\delta x^i} \gtreqless 0$

it is necessary that

$$\frac{\left[\lambda + \left(\Sigma_{j \neq i}\, \alpha_j \frac{dv^j}{dM}\right) \frac{\delta M}{\delta x^i} \frac{dx^i}{dt^i}\right]}{\alpha^i \frac{dx^i}{dt^i}} \lesseqgtr -1$$

holds true for an optimal solution. It may be observed that t^i is directly related to $(\Sigma_{j \neq i}\, \alpha^j \frac{dv^j}{dM})$ if $\frac{\delta M}{\delta x^i} \neq 0$ i.e. $\hat{t}^i$ depends on a weighted linear combination of the marginal utility of economic inequality to the other agents.

The contribution of agent i is $\hat{t}^i\_M$, which can be considered to be an income tax, derived from the above exercise. Further, if for an agent, increase in income contributes to increase in economic inequality (i.e. the agent is rich) he should pay a positive price for economic inequality. A poor agent should be subsidized i.e. pay a negative price for inequality.

3. Conclusion :- We have succeeded in characterizing an income tax policy which corrects the economic inequality inherent in an income distribution, by allowing decentralised choice of disposable income and economic inequality. The idea is to make agents pay a price for generating more inequality, by creating a market for this public good/or bad as the case may be.

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