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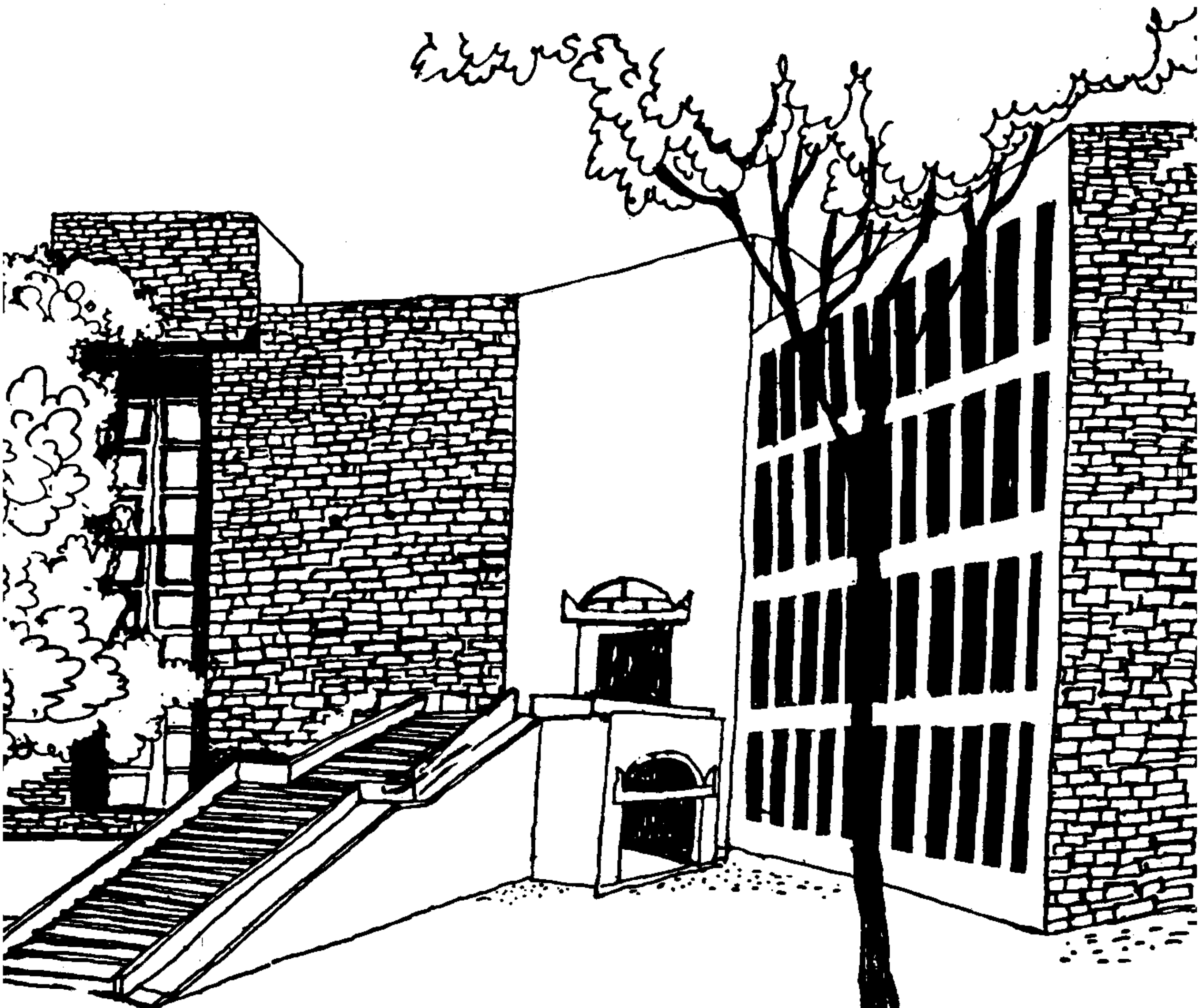

NATIONAL INCOME AND SOCIAL WELFARE IN A PUBLIC GOOD ECONOMY

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Abstract

In this paper we extend the relationship between social welfare and national income from an economy consisting solely of private goods to an economy consisting of public goods as well.

1. Introduction :- Consider as in Laffont (1988) two states of the economy, state 1 and state 2. Either the **Pareto** criterion allows us to rank these two states or it does not. In the former case, we are allowed to choose unambiguously between the two states (if we accept the Pareto criterion). In the latter case, in order to rank the two states, it is necessary to make interpersonal utility comparison.

The standard theory in this respect, which is surveyed in Varian (1984), Laffont (1988) amongst others, is developed in a pure exchange economy with private goods and asserts that starting from a competitive equilibrium, the variation in national income evaluated at the initial prices provides a criterion for evaluating the change in social welfare if we consider that the observed competitive equilibrium corresponds to an optimal allocation of resources (from the perspective of this social welfare function).

Our purpose in this paper is to extend the above result to an economy consisting of both private and public goods. This adds a measure of realism to the existing literature on social welfare.

2. The Model :- In this paper we follow Mas-Colell (1980) or Lahiri (1991) and postulate the following model :

There are I agents in an economy. There are L private consumption goods and (purely for the sake of convenience) one pure public good, in the economy. The first private good, i.e. good 1, is a numeraire, and is the only good used to produce the public good. The input requirement for the production of the public good is summarized by a cost function $c : \mathbb{R}_+ \to \mathbb{R}$, where $c(y)$ is the total amount of good 1, required to produce y units of the public good. We assume that the aggregate initial endowment of privately consumable goods in the economy is given by the vector $w \in \mathbb{R}^L_+$. Agent i's preferences are summarized by a utility function $u^i : \mathbb{R}^L_+ \times \mathbb{R}_+ \to \mathbb{R}$, where $u^i(x^i, y)$ is the satisfaction derived by consumer i, from a consumption bundle consisting of y units of the public good and the vector

x^I of private goods. Here $x^i \in \mathbb{R}^L_+$, $y \in \mathbb{R}_+$.

Given $w \in \mathbb{R}^L_+$, the set

$$F(w)=\{((x^i)^I_{i=1}, y) \in (\mathbb{R}^L_+)^I \times \mathbb{R}_+ \,/\, \Sigma^I_{i=1} x^i_1 + c(y) \leq w_1,$$
$$\Sigma^I_{i=1} x^i_j \leq w_j, \; j=2,\ldots,L\}$$

is the set of all feasible allocations given initial resources w.

For the sake of simplicity we assume that the economic environment (i.e. all utility functions and the cost function) is twice continuously differentiable. We further assume that $u^i : \mathbb{R}^L_+ \times \mathbb{R}_+ \to \mathbb{R}$ is concave for each $i=1,\ldots,I$ and $c : \mathbb{R}_+ \to \mathbb{R}$ is convex. All functions are assumed to be increasing.

Finally we assume that there exists a twice continuously differentiable Bergson-Samuelson social welfare function $W : \mathbb{R}^I \to \mathbb{R}$, which evaluates social states. The welfare function is assumed to be increasing in the utility levels of each agents.

3. The Problem and the result :- Consider the problem

$$\left.\begin{array}{l} \text{Max } W(u^1(x^1,y),\ldots,u^I(x^I,y)) \\ \\ \quad \text{subject to } ((x^i),y) \in F(w) \end{array}\right\} \quad (*)$$

Consider the vector of prices $(p_1,\ldots,p_L, t^1,\ldots,t^I)$ where $p_1 = 1$ and $t^i \in \mathbb{R}$ is a personalized price of the public good for agent i, $i=1,\ldots,I$. We assume that those prices correspond to a Pareto-optimal solution to the above maximization problem(*). The above vector of prices corresponds to a <u>Lindahl equilibrium</u>, starting from suitable initial endowments in a private ownership economy. Thus p_j, $j=2,\ldots,L$ are proportional to the Lagrange multipliers associated with each constraint to the problem(*) and $\Sigma^I_{i=1} t^i$ is proportional to $c'(y)$.

Now if we imagine a change dw in the aggregate initial endowment vector, with its corresponding change in quantities consumed constrained by

$$\Sigma^I_{i=1} dx^i_j = dw_j, \; j=2,\ldots,L$$
$$\Sigma^I_{i=1} dx^i_1 + c'(y)dy = dw_1,$$

we obtain a variation in welfare of

$$dW = \Sigma^{I}_{i=1}\Sigma^{L}_{j=1}\frac{\partial W}{\partial u^{i}_{1}}\frac{\partial u^{i}}{\partial x^{i}_{1}}dx^{i}_{1} + \Sigma^{I}_{i=1}\frac{\partial W}{\partial u^{i}}\frac{\partial u^{i}}{\partial y}dy$$

$$\& \Sigma^{I}_{i=1}\Sigma^{L}_{j=1} p_j dx^{i}_{j} + c'(y')dy$$

$$= \Sigma^{L}_{j=2}\Sigma^{I}_{i=1} p_j dx^{i}_{j} + [\Sigma^{I}_{i=1} dx^{i}_{1} + c'(y)dy]$$

$$= \Sigma^{I}_{j=2} p_j \, dw_j + dw_1$$

Therefore, for a marginal change in initial endowments we have

$$dW \gtreqless 0 \iff \Sigma^{L}_{j=1} p_j dw_j + dw_1 \gtreqless 0.$$

Starting from a Lindahl equilibrium, the variation in national income evaluated at the initial prices provides a criterion for evaluating the change in social welfare if we consider that the observed Lindahl equilibrium corresponds to an optimal allocation of resources (from the perspective of this social welfare function).

4. The Conclusion :- This paper extends a result well known in the context of a private good economy to a mixed economy.

In an economy consisting of two goods, i.e. a private good and a public good if agents' preferences are quasi-linear (as in Gahvari (1992)) and the relevant cost function is linear, a separate endeavour (Lahiri (1992)) reveals that an increase in the availability of the private good leads to an increase in welfare for each individual agent, provided the solution to the resource allocation problem consists of first equally dividing the private good among all the agents and then operating the Lindahl mechanism. Thus if we were to use an arbitrary social welfare function to evaluate two different states of the economy, then this procedure would declare aggregate abundance as unequivocally superior to aggregate deprivation. Further the results concerning individual consumption of the goods and marginal utilities in Gahvari (1992), continue to hold.

In the general context as in this paper, the result is weaker. We cannot say whether each individual benefits from growth. We can only say that society as a whole does, provided resource allocation and evaluation of welfare use the same social welfare function.

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