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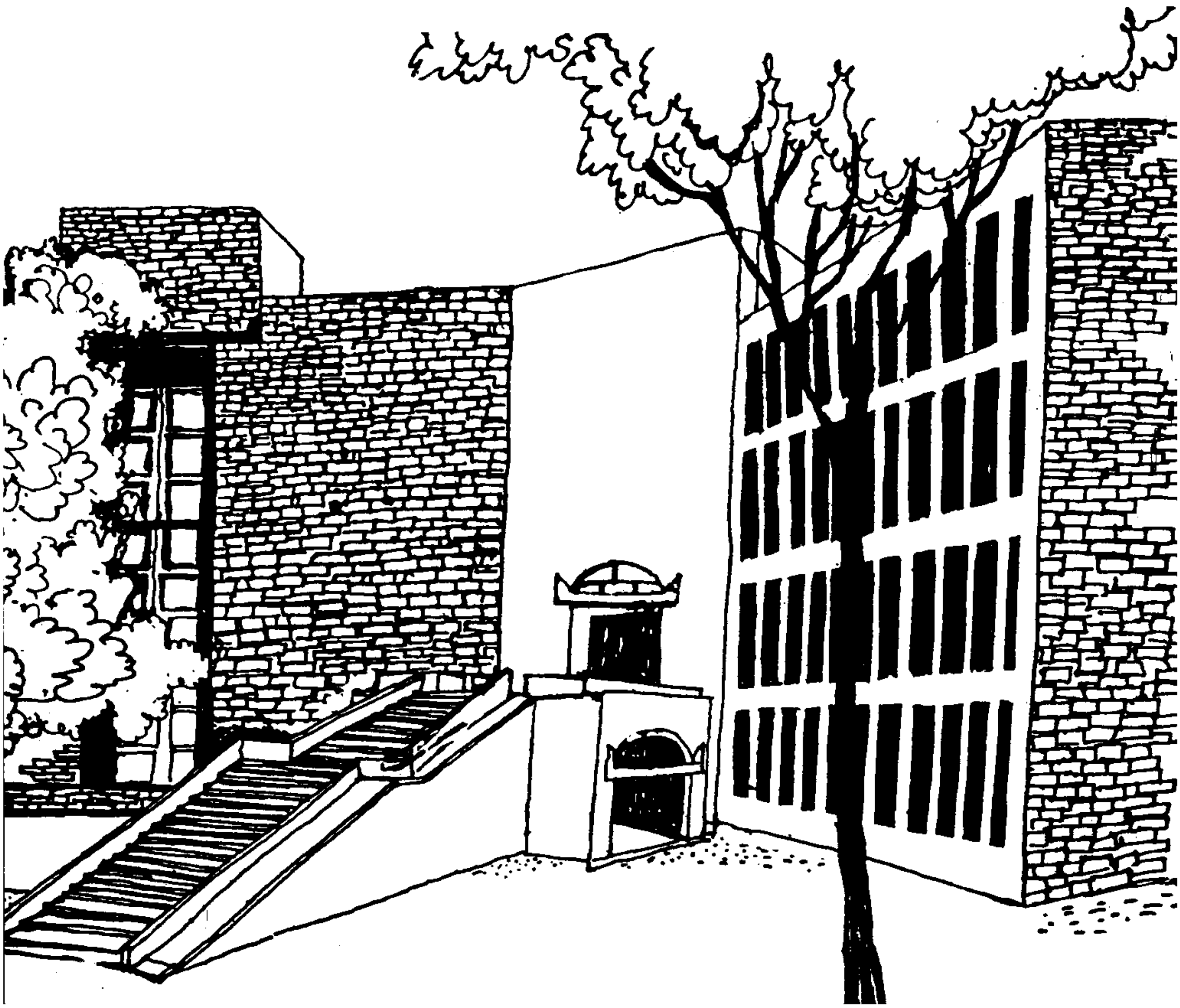

PUBLIC SERVICES SUBJECT TO CONGESTION: A REEXAMINATION

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Abstract

In this paper we reformulate the problem posed by public services subject to congestion in a manner consistent with economic theory and obtain first and second best solutions to the problem.

1. Introduction :- The theory of public services with congestion. as developed in Malinvaud (1985) is not correct. both in its analysis as well as in its prescription. Below we present a development of the first and second best solutions. in a decentralized economy. The model and its solution is analogous to the theory of consumption exernalities, although there are significant differences between the two.

We shall study "a public service involving a good that can be used privately but whose quality depends on the global demand to be satisfied, which is typical of situations of congestion such as arise more and more frequently in urbanised communities". We shall obtain first and second best solutions for the above problem.

2. The Model :- Suppose that there are only two goods 1 and 2 of which the second good is a public service. Assume a simple linear technology for the production of good 2 from good 1 : to produce one unit of the public service we require p>0 units of good 1.

Suppose there are I agents (consumers) and one firm (the producer of the public service) in the economy. The i^{th} consumer's preferences are represented by a twice continuously differentiable utility function $u^i : \mathbb{R}^3_+ \to \mathbb{R}$ where for each vector $(x^i_1, x^i_2, x_3) \in \mathbb{R}^3_+$, denoting his level of consumption of the first good, the second good and the aggregate consumption of the second good. $u^i(x^i_1, x^i_2, x_3)$ denotes his level of satisfaction. We assume that $\frac{\partial u^i}{\partial x_1} > 0$, $\frac{\partial u^i}{\partial x_2} \geq 0$ and $\frac{\partial u^i}{\partial x_3} \leq 0$, the latter being a symptom of congested public services. Let R^i be the initial endowment of good 1 with agent i. To consume the public service, each agent is required to make a voluntary contribution of the first good; in addition, we shall assume that agent i is taxed at the rate of t^i units of good 1, for each unit of good 2 that he consumes. Thus agent i solves the following maximization problem :

$$\left.\begin{array}{l} \text{Max } u^i \,(x^i_1 \,,\, x^i_2 \,,\, x^i_2 + \Sigma_{j \neq i} x^j_2) \\ \\ \text{s.t. } (p+t^i)x^i_2 + x^i_1 = R^i \end{array}\right\} \quad (1)$$

A necessary condition for $(x^i_1(t^i), x^i_2(t^i)) \in \mathbb{R}^2_+$, to solve this problem is that

$$\frac{\partial u^i/\partial x_2 + \partial u^i/\partial x_3}{\partial u^i/\partial x_1} = p + t^i \quad (2)$$

at these values.

3. The First Best Solution :- The first best solution corresponding to an utilitarian optimum is obtained by solving the following problem :

$$\left.\begin{array}{l} \underset{(t^i)}{\text{Max}}\, \Sigma^I_{i=1} \{u^i \,(x^i_1(t^i), x^i_2(t^i) + \Sigma_{j \neq i} x^i_2\,(t^j)\} \\ \\ \text{s.t. } p\Sigma^I_{i=1} x^i_2\,(t^i) + \Sigma^I_{i=1} x^i_1(t^i) = \Sigma^I_{i=1} R^i \end{array}\right\} \quad (3)$$

For the sake of simplicity let us assume that $u^i(x^i_1, x^i_2, x_3) = x^i_1 + v^i(x^i_2, x_3)$, $i=1,\ldots,I$, where $v^i : \mathbb{R}^2_+ \to \mathbb{R}$ is a twice continuously differentiable function satisfying $\partial v^i/\partial x_3 \geq 0$ and $\partial v^i/\partial x_3 \leq 0$, $i=1,\ldots,I$.

It is easily seen by applying the Lagrange multiplier technique, that

$$t^i = -\Sigma_{j \neq i} \partial v^j/\partial x_3, \quad i=1,\ldots,I$$

is a solution to the problem and hence a first-best solution. Thus for a first-best solution, the consumers require to be taxed since $\partial u^j/\partial x_3 \leq 0$, $j=1,\ldots,I$, as is customary for the case of congested public services.

3. The Second Best Solution :- Let us now assume that the tax levied on the consumers is uniform i.e. $t^i = t \;\forall\; i=1,\ldots,I$. A second best solution for the problem can be found as follows :

$$\left.\begin{array}{l} \underset{t}{\text{Max}}\, \Sigma^I_{i=1} u^i\,(x^i_1(t), x^i_2(t), x^i_2(t) + \Sigma_{j \neq i} x^j_2\,(t)) \\ \\ \text{s.t. } p\Sigma^I_{i=1} x^i_2\,(t) + \Sigma^I_{i=1} x^i_1\,(t) = \Sigma^I_{i=1} R^i \end{array}\right\} \quad (4)$$

By applying the Lagrange multiplier technique once again, and making the assumption of quasi-linear utilities as before we observe that

$$t = \frac{-\Sigma^{I}_{i=1} \frac{\partial dx^{i}_{2}}{\partial dt}(\Sigma_{j \neq i} \partial w^{j} / \partial x_3)}{\Sigma^{I}_{i=1} \frac{dx^{i}_{2}}{dt}}$$

solves the problem and is a second best solution. The second best solution is a convex combination of the sum of the marginal utilities of total consumption of the remaining agents.

4. Conclusion :- The above theory which differs from that of Malinvaud (1985). puts the theory of congested public services in its proper perspective.

References :-

1. E. Malinvaud (1985) : "Lectures on Micro-Economic Theory", North-Holland.