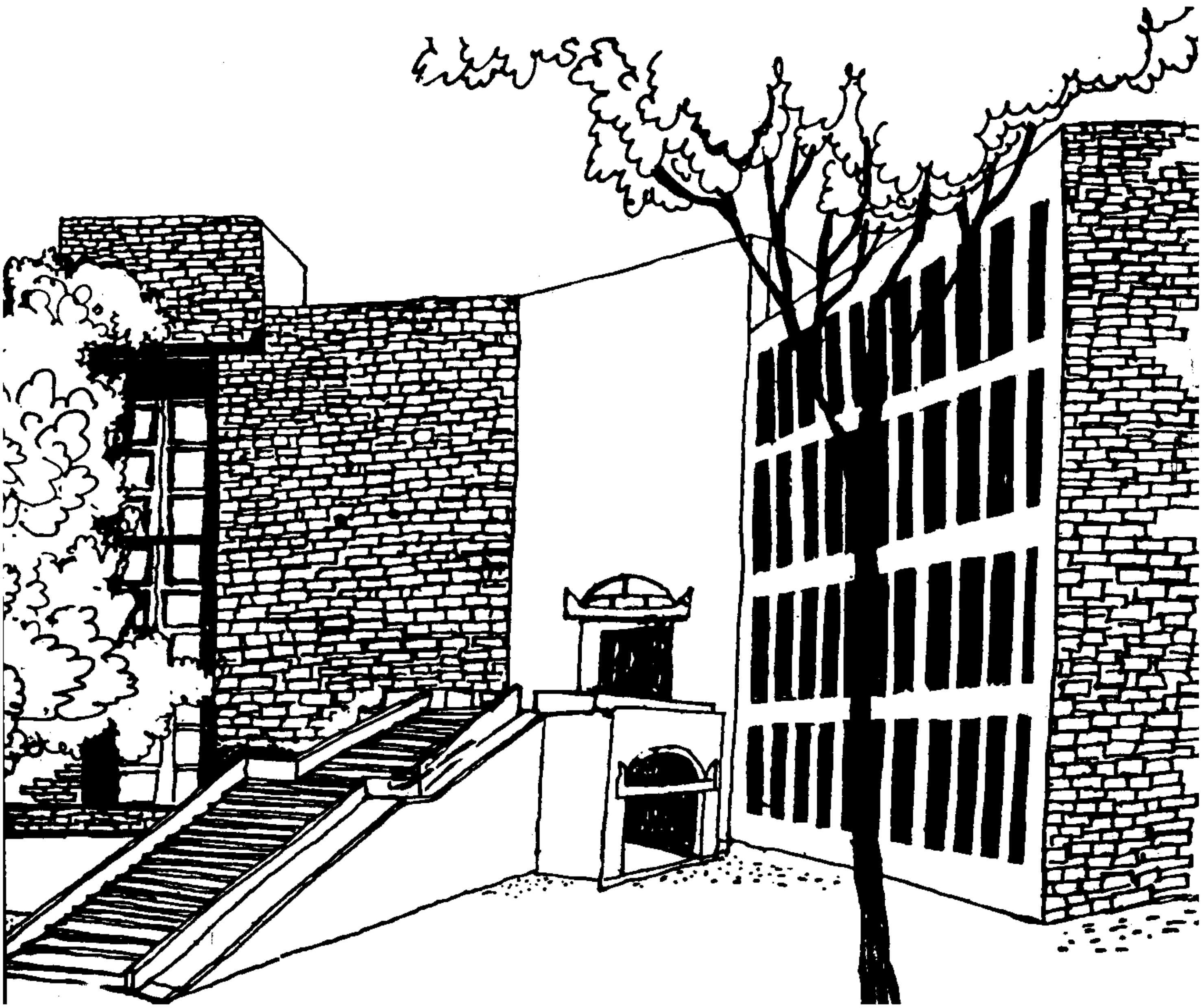
Working Paper



ORDER LEVEL LOT-SIZE INVENTORY MODEL WITH PERMISSIBLE DELAY IN PAYMENTS FOR A SYSTEM WITH TWO STORAGE FACILITIES

By
Nitin Shah
&
Y. K. Shah

W P No. 1029
May 1992

The main objective of the working paper series of the IIMA is to help faculty members to test out their research findings at the pre-publication stage.

INDIAN INSTITUTE OF MANAGEMENT
AHMEDABAD-380 015
INDIA

ORDER LEVEL LOT-SIZE INVENTORY MODEL WITH PERMISSIBLE DELAY IN PAYMENTS FOR A SYSTEM WITH TWO STORAGE FACILITIES

NITIN SHAH
Centre for Management in Agriculture
Indian Institute of Management
Vastrapur
Ahmedabad 380 015
India

Y.K.SHAH
Department of Statistics
School of Sciences
Gujarat University
Ahmedabad 380 009
India

ABSTRACT

In this paper a mathematical model of order level lot-size inventory model under permissible delay in payment is considered with two storage facilities. The supplier allows time for settling payments for no interest charges are payable. Here an inventory system under consideration does not have sufficient capacity to store the on-hand inventory in own warehouse after meeting the back-logged demand. In such a case W units are stored in own warehouse and rest in a rented warehouse. This paper an attempt is made to analyses : a) when the system has both warehouse facilities to accomodate the lot-size; b) own warehouse has large capacity to store the on-hand inventory; and c) maximum is stored in own warehouse. The system suggests when to hire rented warehouse for more profitability among the given alternatives. Expressions for optimal order quantity and order level are devloped for all the cases with an example.

INTRODUCTION

The problem of the economic ordering policy for the supplier or the buyer has been extensively discussed in OR literature. However, not much published material is available on the interaction between buyer and supplier. Normally, payment is made after goods are received. In practice, the supplier allows some time in settling the account and no interest is payable as long as the account is settled within the specified period. When the amount is settled beyond the specified period the supplier interest on the outstanding. These are some economic advantages to the buyer who would try to earn some interest from the revenue received during the specified period. Goyal [2] has studied an EOQ model under this situation. Shah, Patel and Shah [13] and Mandal and Phaujdar [4] have studied the same model by allowing shortages.

When the inventory system does not have sufficient storage facility to accomodate the on-hand inventory in own warehouse (OW), the excess quantity is required to be kept in a rented warehouse (RW). The inventory holding cost in RW is higher as compared to that in OW. Therefore, the stocks in RW should be cleared first to reduce the holding cost. In this paper some modifications to the models [2,4,13] are considered, where the system is buying quantity $Q = DT$ in bulk, which is larger than the capacity W of OW. Consequently, W units are stored at OW and rest are stored at RW after meeting the back-logged demand. The capacity of RW is assumed to be sufficiently large. Such systems have been studied by Hartley [3], Sarma [8,9,10], Murdeshwar and Sathe [5], Dave [1], and Shah and Shah [11,12].

Here an order level lot-size inventory system under permissible delay in payments with two storage facilities is discussed. The follwing assumptions are made for the development of the mathematical model.

i) Demand rate is known and constant.

ii) Shortages are allowed and are made up when the next procurement arrives.

iii) Lead time is zero, and time horizon is infinite.

iv) The storage capacity of OW is W, and that of RW is sufficiently large. If the order quantity exceeds W, the excess units are kept in RW.

v) During the permissible interval when the account is not settled, the sales revenue is deposited in the interest bearing account. At the end of this period the account is settled and payment for interest charges on the items in the stock is made.

NOTATIONS

Following notations are used:

D = Demand rate per time unit.

F = Unit stock holding cost at RW excluding interest charges.

H = Unit stock holding cost at OW excluding interest charges.

I_c = Interest charges per rupee per year.

I_e = Interest that can be earned on the sales revenue of units sold during the permissible delay period ($I_e < I_c$).

A = Ordering cost per order

T^* = Permissible delay period in settling the accounts.

T = Cycle time.

W = Capacity to store number of units in own warehouse.

C = Purchase cost per unit.

π = Unit shortage cost OW.

t_1 = Time period during which there is a positive inventory balance.

CONSTRUCTION OF MODEL

Total cost function is constructed by taking into account set up cost, inventory holding cost, shortage cost, interest charged on the investment for the items in the stock after the permissible delay and the interest earned on the generated sales revenue during the permissible delay period and before the settlement of account. In this situation , two cases may arise; (i) $0 \le T^* \le t_1$; (ii) $t_1 \le T^* \le T$ or $T \le T^*$. The situation is best depicted in figure 1(a) & 1(b).

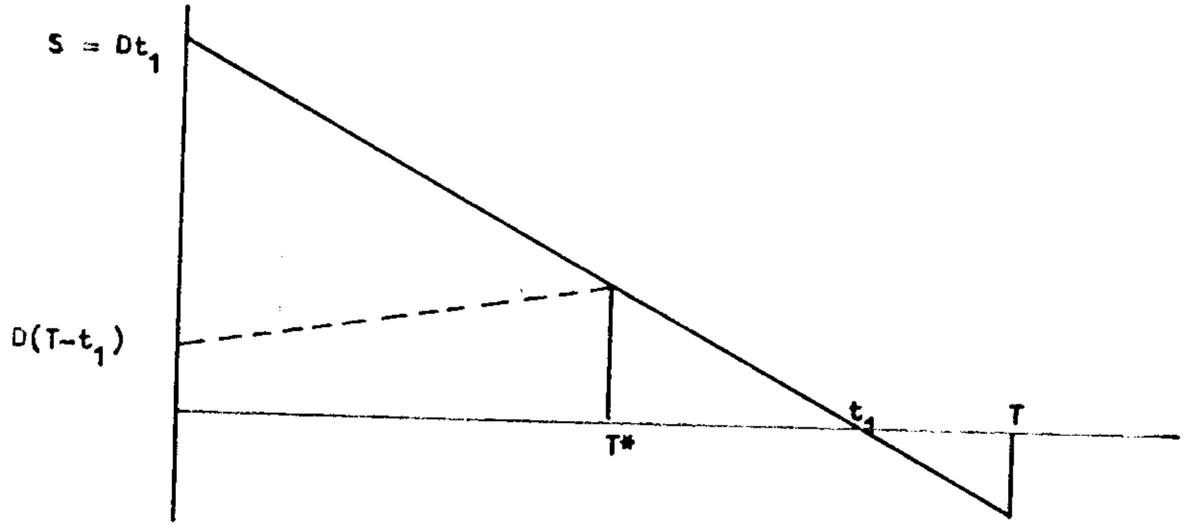


Figure 1(a) : Time weighted inventory level with planned shortage when $T^* \le t_1$

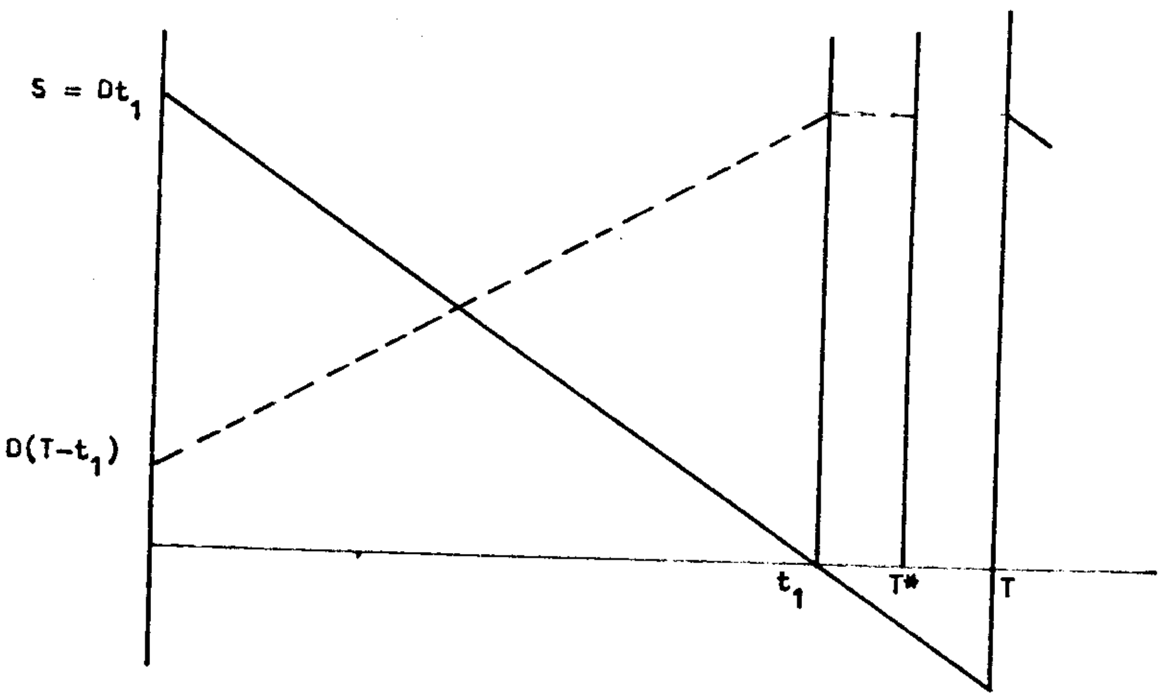


Figure 1(b) : Time weighted inventory level with planned shortage when $T^* > t_1$

Per time unit cost consists of the follwing variable cost.

i) Cost of placing an order per time unit is A/T

ii) Total holding cost at RW per time unit is

$F(DT - W)^2/2DT$

iii) Total holding cost at OW per time unit is

$HW\ (1 - W/2DT)$

iv) Total shortage cost per time unit is

$\pi\ D(T - t_1)^2\ /2T$

v) Interest payable per time unit is

$DCI_c(T - T^*)^2/2T$, if $T^* \leq T$, and is zero if $T^* > T$.

vi) Interest earned per time unit is

$DCI_eT^{*2}/2T$, if $T^* \leq T$

and $DCI_e(T^* - T/2)$ if $T^* > T$.

Note that the interest earned should be subtracted from other variable costs in order to get the net total variable costs per time unit.

In order to obtain the total cost equation, two cases are discussed below:

CASE I: DETERMINATION OF ORDER LEVEL LOT-SIZE MODEL WITH PLANNED SHORTAGES WHEN $0 \leq T^* \leq t_1$

An order for DT units is placed at $T = 0$ from which $D(T - t_1)$ units are delivered towards back orders leaving a balance of Dt_1 units. Thus the system will carry inventories during $(0,t_1)$ and will run with shortages during (t_1,T).

In this case the total costs per time unit is denoted by $Z_1(t_1,T)$, and is obtained as

$$Z_1(t_1,T) = (1/2DT)[2AD + (F - H)W^2 + D^2CT^{*2}(I_c - I_e) + D^2\{F_1t_1^2 - 2t_1((F - H)W/D + CI_cT^* - CI_cT^*/I_e)\}] + \pi D(T/2 - t_1) - CDT^*I_e/2 \quad ......(1)$$

where

$$F_1 = F + \pi + CI_c \quad ....(2)$$

For the optimum value of $t_1 = t_{1o}$ and $T = T_{1o}$

$dZ_1(T,t_1)/dt_1 = 0$, and $dZ(T,t_1)/dT = 0$, which give

$$t_{1o} = (1/F_1)[\pi T_{1o} + (F - H)W/D + CT^*(I_c - I_e/2)] \quad ....(3)$$

and

$$T_{1o} = [(F_1/F_2 D^2)\{2AD + (F - H)W^2 + D^2CT^{*2}(I_c - I_e) - (1/F_1)((F - H)W + DCT^*(I_c - I_e))^2\}]^{1/2} \quad .....(4)$$

where

$$F_2 = \pi(F + CI_c) \quad ....(5)$$

Total optimum cost $Z_1(t_{1o},T_{1o})$ can be obtained by substituting the values of equations (3) and (4) in (2), which gives

$$Z_{1o}(t_{1o},T_{1o}) = [(F_2/F_1)\{2AD + (F - H)W^2 + CD^2T^{*2}(I_c - I_e) - (1/F_1)((F - H)W + DCT^*(I_c - I_e))^2\}]^{1/2} - (\pi/F_1)[(F - H)W + DCT^*(I_c - I_e/2)] - CDT^*I_e/2 \quad ....(6)$$

Substituting the value of (4) in (3), optimum t_{1o} can be obtained.

Let $S = Dt_{1o}$, and $Q = DT_{1o}$ then

$$S_1 = Dt_{1o} = (1/F_1)[D\pi T_{1o} + (F - H)W + DCT^*(I_c - I_e)] \quad ....(7)$$

and

$$Q_1(T_{1o}) = DT_{1o} = [(F_1/F_2)\{2AD + (F - H)W^2 + D^2CT^{*2}(I_c - I_e) - (1/F_1)((F - H)W + DCT^*(I_c - I_e)^2\}]^{1/2} \quad ....(8)$$

It is interesting to note that when $F = H$, $I_c = I_e = 0$, and $T^* = 0$, then equations (6), (7) and (8) reduce to order level inventory model with planned shortages for a single storage model.

$$Z_1(t_{1o}, T_{1o}) = [2ADH\pi/(H + \pi)]^{1/2} \quad \ldots\ldots\ldots\ldots (9)$$

$$S_{1o} = Dt_{1o} = D\pi T_{1o}/(H + \pi) = \pi Q_{1o}/(H + \pi) \quad \ldots\ldots (10)$$

and

$$Q_{1o}(T_{1o}) = DT_{1o} = [2AD(H + \pi)/H\pi]^{1/2} \quad \ldots\ldots\ldots\ldots (11)$$

Equations (9), (10) and (11) are same as those of Naddor [6] for a single storage model.

If one does not wish to avail of facility of RW then the order quantity $Dt_1 = W$. In this case denoting the cycle time by T', the average total cost of the system is

$$Z_1(T') = (1/2DT')[2AD + H_1W^2 - DWCT^*(I_c - I_e/2) + CDT^{*2}(I_c - I_e)] + \pi DT'/2 - \pi W - CDT^* I_e/2 \quad \ldots\ldots\ldots\ldots (12)$$

where

$$H_1 = H + \pi + CI_c \quad \ldots\ldots\ldots\ldots (13)$$

For optimal time period T, $dZ_1(T)/dT = 0$, gives

$$T'_{1o} = [(1/D^2\pi)\{2AD + D^2CT^{*2}(I_c - I_e) + W(H_1W - 2DCT^*(I_c - I_e/2))\}]^{1/2} \quad \ldots\ldots\ldots\ldots (14)$$

Substituting the value of T'_{1o} in (12) optimum total cost is obtained as

$$Z_{1o}(T'_{1o}) = [\pi\{2AD + CD^2T^{*2}(I_c - I_e) + W(H_1W - 2DCT^*(I_c - I_e/2))\}]^{1/2} - \pi W - CDT^*I_e/2 \quad \ldots (15)$$

Then the optimal order quantity $Q_{1o} = DT'_{1o}$,

$$Q_{1o}(T'_{1o}) = [(1/\pi)\{2AD + CD^2T^{*2}(I_c - I_e) + W(H_2W - 2DCT^*(I_c - I_e))\}]^{1/2} \quad \ldots\ldots\ldots\ldots (16)$$

If $Z_1(t_{1o}, T_{1o}) < Z_1(T'_{1o})$ then storage facility at RW should be used.

CASE - II: DETERMINATION ORDER LEVEL LOT-SIZE WITH PLANNED SHORTAGES WHEN $T < T^*$

In this case no interest is payable and the interest earned should be subtracted from other variable costs in order to get the net total cost per time unit, which is given by

$$Z_2(t_1, T) = (1/2DT)[2AD + F_3D^2t_1^2 - 2(F - H)WDt_1 + (F - H)W^2] - DCI_eT^*] + DT\pi/2 \quad \ldots.(17)$$

where

$$F_3 = F + \pi + CI_e \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(18)$$

For the optimum values of $t_1 = t_{2o}$ and $T_o = T_{2o}$,

$dZ_2(t_1,T)/dt_1 = 0$, and $dZ_2(t_1, T)/dT = 0$, give

$$t_{2o} = (1/F_3)[\pi T_{2o} + (F - H)W/D] \quad \ldots\ldots\ldots\ldots(19)$$

and

$$T_{2o} = [(F_3/D^2 F_4)\{2AD + (F - H)H_2W^2/F_3\}]^{1/2} \quad \ldots.(20)$$

where

$F_4 = \pi(F + CI_e)$ and $H_2 = H + \pi + CI_e$

Substituting the values of (19) and (20) in (17) the optimum cost is

$$Z_2(t_{2o}, T_{2o}) = [(F_4/F_3)\{2AD + (F - H)\pi W^2/F_3\}]^{1/2} - (F - H)\pi W/F_3 \quad \ldots\ldots\ldots\ldots\ldots\ldots(21)$$

Now, in order to obtain the order level and optimum order quantity;

$$S_{2o} = Dt_{2o} = (1/F_3)[\pi DT_{2o} + (F - H)W] \quad \ldots..(22)$$

and

$$Q_{2o}(T_{2o}) = DT_{2o} = [(F_3/F_4)\{2AD + (F - H)H_2W^2/F_3\}]^{1/2} \quad \ldots\ldots\ldots\ldots(23)$$

If we consider $I_c = 0$, and $F = H$, in equations (21), (22) and (23) then they are reduced to equations (9), (10) and (11) respectively.

If one does not wish to avail of facility of RW then $Dt_1 = W$. In this case denoting the cycle time by T', the average total cost per time unit for the system is

$$Z_2(T') = (1/2DT')[2AD + H_2W^2] - D\, CI_e\, T^* + \pi DT/2 - \pi W \qquad \text{..........(24)}$$

where

$H_2 = H + \pi + CI_e$

The optimum value of time period T' can be obtained by $dZ_2(T')/dT' = 0$.

This gives

$$T'_{2o} = [(1/\pi D^2)\{ 2AD + H_2 W^2 \}]^{1/2} \qquad \text{......(25)}$$

Total minimum cost $Z_{2o}(T'_{2o})$ can be obtained from (24), after substituting the value of (25) in (24).

$$Z_{2o}(T'_{2o}) = [\pi \{ 2AD + H_2W^2 \}]^{1/2} - DCI_eT^* - \pi W \qquad \text{.....(26)}$$

Optimal order quantity $Q_{2o}(T'_{2o}) = DT'_{2o}$ can be obtained as

$$Q_{2o}(T'_{2o}) = [(1/\pi)\{ 2AD + H_2 W^2 \}]^{1/2} \qquad \text{.......(27)}$$

If $Z_2(t_{2o}, T_{2o}) > Z_1(T'_{2o})$, storage facility at RW should not be utilized.

Example 1: A system has F = Rs.2, H = Rs.1, π = Rs.10, C = Rs.15, A = Rs.300(50)550, $I_c = 0.25$, $I_e = 0.10$, W = 900 units, and D = 10000 units a year, where the prescribed delay periods are 0.0833, 0.1550, and 0.2500 year. Calculate the order level Dt_1, ordering quantity DT_{1o}, and total minimum cost, with all the systems having two storage facilities, EOQ model and if one does not wish to use RW, then what will be Dt_{1o}, DT_{1o} and $Z(T_{1o})$?

Table 1 presents parameter values of t_{1o}, Dt_{1o}, T_{1o}, DT_{1o}, and $Z(T_{1o})$ for i) two storage facilities system , ii) EOQ of single storage system, and iii) for the system when inventory level $Dt_{1o} = W$, with increase in ordering cost and prescribed permissible delay periods. For any system, we find that t_{1o}, Dt_{1o}, T_{1o}, DT_{1o}, and $Z(T_{1o})$ increase as A increases. Obviously, for the corresponding single storage model, the total costs are lower, but when $Dt_{1o} = W$, the total cost is higher than that of the system with two storage facilities.

When $t_{1o} < T^* < T_{1o}$, we find that when RW is used the optimum cost is more than the optimum cost when RW is not used. This is perhaps due to the fact that when RW is not used, the optimum T_{1o} turns out to be smaller than T^*. This case it indicates that hiring RW is uneconomical and hence not advisable.

When $T^* > T$, the optimum cost when RW is used is always smaller than the minimum cost when RW is not used. Thus, hiring RW is profitable.

The impact of change in rate of interest on t_{1o}, Dt_{1o}, T_{1o}, DT_{1o}, and $Z_{1o}(T_{1o})$ can be visualized in Table 2. However, hiring RW is economical when $T^* < t_{1o}$ or $T_* > T$. But, in case, where $t_{1o} < T^* < T$, $Dt_{1o} = W$, gives a more economic optimum cost rather than other systems.

Table 3 all the parameter values are same as those of Table 1, except $I_e = 0.16$. Increase in rate of interest earned has direct influence on values of t_{1o}, Dt_{1o}, T_{1o}, DT_{1o}, and $Z(T_{1o})$. In the case of $T^* < t_{1o}$ or $T_* > T$, hiring RW is more advantageous as compared to not hiring it, while keeping goods only in OW is preferable when $t_{1o} < T^* < T$.

Table 1: Simulated values of time interval, order quantity, and total minimum cost with prescribed permissible delay period and increasing ordering cost.

	With two storage facility					F = H and $I_c = I_e = 0$					When $Dt_1 = W$		
A	t_{1o}	Dt_{1o}	T_{1o}	DT_{1o}	$Z_{1o}(T_{1o})$	t_{1o}	Dt_{1o}	T_{1o}	DT_{1o}	$Z(T_{1o})$	T_{1o}	DT_{1o}	$Z_{1o}(T_{1o})$
When $T^* = 0.0833$ year $< T$													
300	0.1134	1134.27	0.1446	1446.48	2497.06	0.2335	2335.50	0.2569	2569.05	2335.50	0.1225	1225.15	2626.53
350	0.1193	1192.54	0.1538	1538.25	2832.10	0.2523	2522.62	0.2775	2774.89	2522.62	0.1265	1265.31	3028.06
400	0.1248	1247.52	0.1625	1624.84	3148.24	0.2697	2696.80	0.2966	2966.48	2696.80	0.1304	1304.22	3417.24
450	0.1300	1299.72	0.1707	1707.05	3448.37	0.2860	2860.39	0.3146	3146.43	2860.39	0.1342	1342.01	3795.13
500	0.1350	1349.51	0.1785	1785.48	3734.70	0.3015	3015.11	0.3317	3316.62	3015.11	0.1379	1378.77	4162.68
550	0.1397	1397.21	0.1861	1860.61	4008.96	0.3162	3162.28	0.3479	3478.51	3162.28	0.1415	1414.57	4520.67
When $T^* = 0.1550$ year $< T$													
300	0.1262	1261.90	0.1614	1613.56	2524.97	0.2335	2335.50	0.2569	2569.05	2335.50	0.1147	1147.20	1309.48
350	0.1347	1347.37	0.1729	1728.96	2824.14	0.2523	2522.62	0.2775	2774.89	2522.62	0.1190	1189.98	1737.34
400	0.1427	1427.49	0.1837	1837.12	3104.56	0.2697	2696.80	0.2966	2966.48	2696.80	0.1231	1231.28	2150.35
450	0.1503	1503.15	0.1939	1939.26	3369.37	0.2860	2860.39	0.3146	3146.43	2860.39	0.1271	1271.24	2549.94
500	0.1575	1575.02	0.2036	2036.28	3620.91	0.3015	3015.11	0.3317	3316.62	3015.11	0.1310	1309.99	2937.36
550	0.2320	2319.91	0.2577	2576.88	5518.30	0.3162	3162.28	0.3479	3478.51	3162.28	0.1348	1347.61	3313.64
When $T^* = 0.2500$ year $> T$													
300	0.1262	1261.90	0.1614	1613.56	1099.97	0.2335	2335.50	0.2569	2569.05	2335.50	0.1292	1291.80	2043.01
350	0.1347	1347.37	0.1729	1728.96	1399.14	0.2523	2522.62	0.2775	2774.89	2522.62	0.1330	1329.94	2424.44
400	0.1427	1427.49	0.1837	1837.12	1679.56	0.2697	2696.80	0.2966	2966.48	2696.80	0.1367	1367.02	2795.22
450	0.1503	1503.15	0.1939	1939.26	1944.37	0.2860	2860.39	0.3146	3146.43	2860.39	0.1403	1403.12	3156.22
500	0.1575	1575.02	0.2036	2036.28	2195.91	0.3015	3015.11	0.3317	3316.62	3015.11	0.1438	1438.31	3508.15
550	0.1644	1643.62	0.2129	2128.88	2435.99	0.3162	3162.28	0.3479	3478.51	3162.28	0.1473	1472.67	3851.68

Where F = Rs.2, H = Rs.1, π = Rs.10, C = Rs.15, A = Rs.300(50) 550, Ic = 0.25, Ie = 0.10, W = 900 units and D = 10000 units.

Table 2: Simulated values of time interval, order quantity, and total minimum cost with prescribed permissible delay period, change of interest charged and increasing ordering cost.

	With two storage facility					$F = H$ and $I_c = I_e = 0$					When $Dt_1 = W$		
A	t_{1*}	Dt_{1*}	T_{1*}	DT_{1*}	$Z_{1*}(T_{1*})$	t_{1*}	Dt_{1*}	T_{1*}	DT_{1*}	$Z(T_{1*})$	T_{1*}	DT_{1*}	$Z_{1*}(T_{1*})$
						When $T^* = 0.0833$ year $< T$							
300	0.1174	1174.16	0.1484	1483.74	2470.80	0.2335	2335.50	0.2569	2569.05	2335.50	0.1225	1225.02	2625.17
350	0.1239	1239.41	0.1582	1581.61	2797.03	0.2523	2522.62	0.2775	2774.89	2522.62	0.1265	1265.17	3026.75
400	0.1301	1300.84	0.1674	1673.76	3104.21	0.2697	2696.80	0.2966	2966.48	2696.80	0.1304	1304.10	3415.96
450	0.1359	1359.07	0.1761	1761.10	3395.35	0.2860	2860.39	0.3146	3146.43	2860.39	0.1342	1341.89	3793.89
500	0.1415	1414.54	0.1844	1844.31	3672.71	0.3015	3015.11	0.3317	3316.62	3015.11	0.1379	1378.65	4161.47
550	0.1468	1467.62	0.1924	1923.93	3938.08	0.3162	3162.28	0.3479	3478.51	3162.28	0.1414	1414.45	4519.49
						When $T^* = 0.1550$ year $< T$							
300	0.1262	1261.90	0.1614	1613.56	2524.97	0.2335	2335.50	0.2569	2569.05	2335.50	0.1160	1159.80	1435.45
350	0.1347	1347.37	0.1729	1728.96	2824.14	0.2523	2522.62	0.2775	2774.89	2522.62	0.1202	1202.13	1858.84
400	0.1427	1427.49	0.1837	1837.12	3104.56	0.2697	2696.80	0.2966	2966.48	2696.80	0.1243	1243.03	2267.81
450	0.1503	1503.15	0.1939	1939.26	3369.37	0.2860	2860.39	0.3146	3146.43	2860.39	0.1283	1282.62	2663.74
500	0.1575	1575.02	0.2036	2036.28	3620.91	0.3015	3015.11	0.3317	3316.62	3015.11	0.1321	1321.03	3047.82
550	0.2131	2131.15	0.2438	2438.30	5159.02	0.3162	3162.28	0.3479	3478.51	3162.28	0.1358	1358.35	3421.04
						When $T^* = 0.2500$ year $> T$							
300	0.1262	1261.90	0.1614	1613.56	1099.97	0.2335	2335.50	0.2569	2569.05	2335.50	0.1240	1239.96	1524.60
350	0.1347	1347.37	0.1729	1728.96	1399.14	0.2523	2522.62	0.2775	2774.89	2522.62	0.1280	1279.65	1921.48
400	0.1427	1427.49	0.1837	1837.12	1679.56	0.2697	2696.80	0.2966	2966.48	2696.80	0.1318	1318.14	2306.43
450	0.1503	1503.15	0.1939	1939.26	1944.37	0.2860	2860.39	0.3146	3146.43	2860.39	0.1356	1355.54	2680.44
500	0.1575	1575.02	0.2036	2036.28	2195.91	0.3015	3015.11	0.3317	3316.62	3015.11	0.1392	1391.94	3044.41
550	0.1644	1643.62	0.2129	2128.88	2435.99	0.3162	3162.28	0.3479	3478.51	3162.28	0.1427	1427.41	3399.10

Where F = Rs.2, H = Rs.1, π = Rs.10, C = Rs.15, A = Rs.300(50)550, Ic = 0.20, Ie = 0.10, W = 900 units and D = 10000 units.

Table 3: Simulated values of time interval, order quantity, and total minimum cost with prescribed permissible delay period, change of interest charged and increasing ordering cost. .

A	With two storage facility					F = H and $I_c = I_e = 0$					When $Dt_1 = W$		
	t_{1e}	Dt_{1e}	T_{1e}	DT_{1e}	$Z_{1e}(T_{1e})$	t_{1e}	Dt_{1e}	T_{1e}	DT_{1e}	$Z(T_{1e})$	T_{1e}	DT_{1e}	$Z_{1e}(T_{1e})$
							When T^* = 0.0833 year < T						
300	0.1082	1081.63	0.1401	1401.07	2194.39	0.2335	2335.50	0.2569	2569.05	2335.50	0.1227	1227.19	2271.92
350	0.1142	1141.67	0.1496	1495.63	2539.61	0.2523	2522.62	0.2775	2774.89	2522.62	0.1267	1267.28	2672.81
400	0.1198	1198.13	0.1585	1584.56	2864.26	0.2697	2696.80	0.2966	2966.48	2696.80	0.1306	1306.14	3061.39
450	0.1252	1251.59	0.1669	1668.75	3171.64	0.2860	2860.39	0.3146	3146.43	2860.39	0.1344	1343.87	3438.75
500	0.1302	1302.48	0.1749	1748.90	3464.24	0.3015	3015.11	0.3317	3316.62	3015.11	0.1381	1380.58	3805.80
550	0.1351	1351.13	0.1826	1825.53	3744.00	0.3162	3162.28	0.3479	3478.51	3162.28	0.1416	1416.33	4163.33
							When T^* = 0.1550 year < T						
300	0.1095	1094.94	0.1487	1486.71	1447.74	0.2335	2335.50	0.2569	2569.05	2335.50	0.1139	1139.42	534.24
350	0.1169	1168.74	0.1593	1592.98	1772.44	0.2523	2522.62	0.2775	2774.89	2522.62	0.1182	1182.49	964.92
400	0.1238	1237.91	0.1693	1692.59	2076.81	0.2697	2696.80	0.2966	2966.48	2696.80	0.1224	1224.05	1380.46
450	0.1303	1303.23	0.1787	1786.66	2364.22	0.2860	2860.39	0.3146	3146.43	2860.39	0.1264	1264.23	1782.34
500	0.1365	1365.28	0.1876	1876.01	2637.25	0.3015	3015.11	0.3317	3316.62	3015.11	0.1303	1303.18	2171.84
550	0.1425	1424.51	0.1961	1961.30	2897.85	0.3162	3162.28	0.3479	3478.51	3162.28	0.1341	1341.00	2550.02
							When T^* = 0.2500 year > T						
300	0.1095	1094.94	0.1487	1486.71	-832.26	0.2335	2335.50	0.2569	2569.05	2335.50	0.1175	1175.44	-245.64
350	0.1169	1168.74	0.1593	1592.98	-507.56	0.2523	2522.62	0.2775	2774.89	2522.62	0.1217	1217.23	172.30
400	0.1238	1237.91	0.1693	1692.59	-203.19	0.2697	2696.80	0.2966	2966.48	2696.80	0.1258	1257.64	576.37
450	0.1303	1303.23	0.1787	1786.66	84.22	0.2860	2860.39	0.3146	3146.43	2860.39	0.1297	1296.78	967.84
500	0.1365	1365.28	0.1876	1876.01	357.25	0.3015	3015.11	0.3317	3316.62	3015.11	0.1335	1334.78	1347.85
550	0.1425	1424.51	0.1961	1961.30	617.85	0.3162	3162.28	0.3479	3478.51	3162.28	0.1372	1371.73	1717.32

Where F = Rs.2, H = Rs.1, π = Rs.10, C = Rs.15, A = Rs.300(50) 500, I_c = 0.25, I_e = 0.16, W = 900 units and D = 10000 units.

Table 4: Simulated values of time interval, order quantity, and total minimum cost with prescribed permissible delay period, and increasing demand.

	With two storage facility					F = H and $I_e = I_c = 0$					When $Dt_1 = W$		
D	t_{1*}	Dt_{1*}	T_{1*}	DT_{1*}	$Z_{1*}(T_{1*})$	t_{1*}	Dt_{1*}	T_{1*}	DT_{1*}	$Z(T_{1*})$	T_{1*}	DT_{1*}	$Z_{1*}(T_{1*})$
							When $T^* = 0.0833$ year < T						
10000	0.1134	1134.27	0.1446	1446.48	2497.06	0.2335	2335.50	0.2569	2569.05	2335.50	0.1225	1225.15	2626.53
11000	0.1089	1197.97	0.1383	1521.80	2550.81	0.2227	2449.49	0.2449	2694.44	2449.49	0.1131	1244.51	2757.63
12000	0.1051	1260.68	0.1330	1595.56	2598.89	0.2132	2558.41	0.2345	2814.25	2558.41	0.1054	1264.81	2898.12
13000	0.1017	1322.53	0.1283	1667.98	2642.05	0.2048	2662.88	0.2253	2929.16	2662.88	0.0989	1286.01	3047.56
14000	0.0988	1383.63	0.1242	1739.22	2680.90	0.1974	2763.40	0.2171	3039.74	2763.40	0.0934	1308.05	3205.52
15000	0.0963	1444.08	0.1206	1809.43	2715.96	0.1907	2860.39	0.2098	3146.43	2860.39	0.0887	1330.91	3371.57
							When $T^* = 0.1550$ year < T						
10000	0.1262	1261.90	0.1614	1613.56	2824.97	0.2335	2335.50	0.2569	2569.05	2335.50	0.1137	1137.35	1360.99
11000	0.1194	1313.89	0.1531	1683.75	2804.43	0.2227	2449.49	0.2449	2694.44	2449.49	0.1063	1169.09	1577.17
12000	0.1136	1363.79	0.1459	1751.12	2776.61	0.2132	2558.41	0.2345	2814.25	2558.41	0.1003	1203.41	1819.08
13000	0.1086	1411.85	0.1397	1816.00	2742.32	0.2048	2662.88	0.2253	2929.16	2662.88	0.0954	1240.08	2084.58
14000	0.1042	1458.25	0.1342	1878.64	2702.21	0.1974	2763.40	0.2171	3039.74	2763.40	0.0914	1278.91	2371.65
15000	0.1002	1503.15	0.1293	1939.26	2656.87	0.1907	2860.39	0.2098	3146.43	2860.39	0.0880	1319.71	2678.37
							When $T^* = 0.2500$ year > T						
10000	0.1262	1261.90	0.1614	1613.56	1099.97	0.2335	2335.50	0.2569	2569.05	2335.50	0.1292	1291.80	2043.01
11000	0.1194	1313.89	0.1531	1683.75	906.93	0.2227	2449.49	0.2449	2694.44	2449.49	0.1249	1374.43	2681.82
12000	0.1136	1363.79	0.1459	1751.12	706.61	0.2132	2558.41	0.2345	2814.25	2558.41	0.1218	1462.02	3370.19
13000	0.1086	1411.85	0.1397	1816.00	499.82	0.2048	2662.88	0.2253	2929.16	2662.88	0.1195	1553.73	4099.75
14000	0.1042	1458.25	0.1342	1878.64	287.21	0.1974	2763.40	0.2171	3039.74	2763.40	0.1178	1648.86	4863.63
15000	0.1002	1503.15	0.1293	1939.26	69.37	0.1907	2860.39	0.2098	3146.43	2860.39	0.1165	1746.87	5656.22

Where F = Rs.2, H = Rs.1, π = Rs.10, C = Rs.15, A = Rs.300, $I_c = 0.25$, $I_e = 0.16$, W = 900 units and D = 10000(1000)15000 units.

Table 5: Simulated values of time interval, order quantity, and total minimum cost with prescribed permissible delay period, and increasing ordering cost and demand.

		With two storage facility					$F = H$ and $I_c = I_e = 0$					When $Dt_1 = W$		
A	D	t_{1*}	Dt_{1*}	T_{1*}	DT_{1*}	$Z_{1*}(T_{1*})$	t_{1*}	Dt_{1*}	T_{1*}	DT_{1*}	$Z(T_{1*})$	T_{1*}	DT_{1*}	$Z_{1*}(T_{1*})$
							When $T^* = 0.0833$ year $< T$							
300	10000	0.1134	1134.27	0.1446	1446.48	2497.06	0.2335	2335.50	0.2569	2569.05	2335.50	0.1225	1225.15	2626.53
350	11000	0.1144	1258.90	0.1471	1617.77	2901.17	0.2405	2645.75	0.2646	2910.33	2645.75	0.1171	1287.95	3191.99
400	12000	0.1153	1383.97	0.1491	1789.75	3307.83	0.2462	2954.20	0.2708	3249.62	2954.20	0.1130	1356.37	3813.74
450	13000	0.1161	1509.37	0.1509	1962.25	3716.36	0.2509	3261.34	0.2760	3587.48	3261.34	0.1100	1429.62	4483.70
500	14000	0.1168	1635.01	0.1525	2135.14	4126.30	0.2548	3567.53	0.2803	3924.28	3567.53	0.1076	1506.98	5194.84
550	15000	0.1174	1760.84	0.1539	2308.33	4537.34	0.2582	3872.98	0.2840	4260.28	3872.98	0.1059	1587.86	5941.14
							When $T^* = 0.1550$ year $< T$							
300	10000	0.1262	1261.90	0.1614	1613.56	2524.97	0.2335	2335.50	0.2569	2569.05	2335.50	0.1147	1147.20	1309.48
350	11000	0.1276	1403.96	0.1641	1805.35	2789.70	0.2405	2645.75	0.2646	2910.33	2645.75	0.1119	1231.21	2033.36
400	12000	0.1289	1546.69	0.1665	1998.03	3056.76	0.2462	2954.20	0.2708	3249.62	2954.20	0.1101	1321.56	2820.56
450	13000	0.1300	1689.91	0.1686	2191.38	3325.52	0.2509	3261.34	0.2760	3587.48	3261.34	0.1090	1417.02	3658.97
500	14000	0.1310	1833.50	0.1704	2385.22	3595.58	0.2548	3567.53	0.2803	3924.28	3567.53	0.1083	1516.64	4538.92
550	15000	0.1318	1977.37	0.1720	2579.45	3866.63	0.2582	3872.98	0.2840	4260.28	3872.98	0.1080	1619.65	5452.75
							When $T^* = 0.2500$ year $> T$							
300	10000	0.1262	1261.90	0.1614	1613.56	1099.97	0.2335	2335.50	0.2569	2569.05	2335.50	0.1292	1291.80	2043.01
350	11000	0.1276	1403.96	0.1641	1805.35	1222.20	0.2405	2645.75	0.2646	2910.33	2645.75	0.1285	1413.88	3076.32
400	12000	0.1289	1546.69	0.1665	1998.03	1346.76	0.2462	2954.20	0.2708	3249.62	2954.20	0.1285	1541.91	4169.14
450	13000	0.1300	1689.91	0.1686	2191.38	1473.02	0.2509	3261.34	0.2760	3587.48	3261.34	0.1288	1674.53	5307.84
500	14000	0.1310	1833.50	0.1704	2385.22	1600.58	0.2548	3567.53	0.2803	3924.28	3567.53	0.1293	1810.73	6482.32
550	15000	0.1318	1977.37	0.1720	2579.45	1729.13	0.2582	3872.98	0.2840	4260.28	3872.98	0.1300	1949.76	7685.10

Where F = Rs.2, H = Rs.1, π = Rs.10, C = Rs.15, A = Rs.300(50)550, Ic = 0.25, Ie = 0.16, W = 900 units and D = 10000(1000)15000 units

Table 6: Simulated values of time interval, order quantity, and total minimum cost with prescribed permissible delay period, and increasing holding cost of RW & OW.

		With two storage facility					F = H and $I_r = I_a = 0$					When $Dt_1 = W$		
F	H	t_{1e}	Dt_{1e}	T_{1e}	DT_{1e}	$Z_{1e}(T_{1e})$	t_{1e}	Dt_{1e}	T_{1e}	DT_{1e}	$Z(T_{1e})$	T_{1e}	DT_{1e}	$Z_{1e}(T_{1e})$
							When T^* = 0.0833 year < T							
2	1	0.1134	1134.27	0.1446	1446.48	2497.06	0.2335	2335.50	0.2569	2569.05	2335.50	0.1225	1225.15	2626.53
3	1	0.1102	1102.01	0.1416	1415.87	2513.59	0.2335	2335.50	0.2569	2569.05	2335.50	0.1225	1225.15	2626.53
4	1	0.1078	1077.60	0.1393	1392.73	2526.36	0.2335	2335.50	0.2569	2569.05	2335.50	0.1225	1225.15	2626.53
5	1	0.1058	1058.46	0.1375	1374.61	2536.53	0.2335	2335.50	0.2569	2569.05	2335.50	0.1225	1225.15	2626.53
3	2	0.1027	1027.29	0.1381	1380.71	2909.21	0.1581	1581.14	0.1897	1897.37	3162.28	0.1258	1257.78	2952.76
4	2	0.1012	1011.53	0.1365	1365.47	2914.38	0.1581	1581.14	0.1897	1897.37	3162.28	0.1258	1257.78	2952.76
5	2	0.0999	999.25	0.1354	1353.60	2918.45	0.1581	1581.14	0.1897	1897.37	3162.28	0.1258	1257.78	2952.76
							When T^* = 0.1550 year < T							
2	1	0.1262	1261.90	0.1614	1613.56	2824.97	0.2335	2335.50	0.2569	2569.05	2335.50	0.1137	1137.35	1360.99
3	1	0.1189	1188.83	0.1544	1543.80	4007.50	0.2335	2335.50	0.2569	2569.05	2335.50	0.1137	1137.35	1360.99
4	1	0.1140	1140.47	0.1498	1497.73	5031.45	0.2335	2335.50	0.2569	2569.05	2335.50	0.1137	1137.35	1360.99
5	1	0.1106	1106.05	0.1465	1464.98	5927.93	0.2335	2335.50	0.2569	2569.05	2335.50	0.1137	1137.35	1360.99
3	2	0.1079	1079.48	0.1475	1475.24	3174.02	0.1581	1581.14	0.1897	1897.37	3162.28	0.1172	1172.42	1711.67
4	2	0.1049	1048.50	0.1445	1445.18	4264.35	0.1581	1581.14	0.1897	1897.37	3162.28	0.1172	1172.42	1711.67
5	2	0.1027	1026.67	0.1424	1424.00	5221.05	0.1581	1581.14	0.1897	1897.37	3162.28	0.1172	1172.42	1711.67
							When T^* = 0.2500 year > T							
2	1	0.1262	1261.90	0.1614	1613.56	1099.97	0.2335	2335.50	0.2569	2569.05	2335.50	0.1292	1291.80	2043.01
3	1	0.1189	1188.83	0.1544	1543.80	2282.50	0.2335	2335.50	0.2569	2569.05	2335.50	0.1292	1291.80	2043.01
4	1	0.1140	1140.47	0.1498	1497.73	3306.45	0.2335	2335.50	0.2569	2569.05	2335.50	0.1292	1291.80	2043.01
5	1	0.1106	1106.05	0.1465	1464.98	4202.93	0.2335	2335.50	0.2569	2569.05	2335.50	0.1292	1291.80	2043.01
3	2	0.1079	1079.48	0.1475	1475.24	1449.02	0.1581	1581.14	0.1897	1897.37	3162.28	0.1323	1322.78	2352.81
4	2	0.1049	1048.50	0.1445	1445.18	2539.35	0.1581	1581.14	0.1897	1897.37	3162.28	0.1323	1322.78	2352.81
5	2	0.1027	1026.67	0.1424	1424.00	3496.05	0.1581	1581.14	0.1897	1897.37	3162.28	0.1323	1322.78	2352.81

Where F = Rs.2(1)5, H = Rs.1,2, π = Rs.10, C = Rs.15, A = Rs.300, I_c = 0.25, I_e = 0.16, W = 900 units and D = 10000 units.

It is evident from Table 4 that as demand increases t_{1o}, Dt_{1o}, T_{1o}, and DT_{1o} decrease and $Z(T_{1o})$ increases with constant ordering cost. In all the three cases hiring RW is more economical.

Table 5 is similar to Table 4, except that here demand and ordering costs are increasing. If the length of permissible delay period is more, t_{1o}, Dt_{1o}, T_{1o}, DT_{1o}, and $Z(T_{1o})$ are also increasing. Hiring RW is more economical, where RW has all the time minimum total cost. Here again increases in ordering cost and demand proportionally increases the values of t_{1o}, Dt_{1o}, T_{1o}, DT_{1o}, and $Z(T_{1o})$.

Table 6 shows that when ordering cost and demand are fixed, holding cost of RW and OW varies. Hiring RW is economical when $T^* < t_{1o}$. But with increase in RW and OW costs hiring RW is not economic in case $t_{1o} < T^* < T$ and $T^* > T$.

CONCLUSION : This study examined the conditions under which delay in payment is permissible, and its effect on optimal lot-size with planned shortages with two storage facilities. The numerical example results shows that there is a considerable change in EOQ with permissible delay period and planned shortage under the two storage facilities.

REFERENCES .

1. Dave, U.,(1988), "On the EOQ Models with Two Levels of Storage", OPSEARCH , 25(3), pp.190-196.

2. Goyal, S.K., (1985), "Economic Order Quantity Under Conditions of Permissible Delay in Payments," Journal of Operations Research Society, 36(4), pp.335-338.

3. Hartley, Ronald. V., (1976), **Operations Research - A Managerial Emphasis**, Good Year Publishing Company, California.

4. Mandal, B.N. and Phaujdar, S., (1989), " Some EOQ Models Under Permissible Delay in Payments," International Journal of Management and Systems, 5(2), pp. 99 - 108.

5. Murdeshwar, T. M. and Sathe, Y. S., (1985), "Some Aspects Lot-size Models with Two Levels of Storage," OPSEARCH, 22(4), PP.255-262.

6. Naddor, E., (1966), **Inventory Systems**, John Wiley & Sons, Inc., New York.

7. Pakkala, T.P.M.. and Achray, K.K., (1990), "Sensitivity Analysis in Perishable Inventory Model with Two Warehouses and Finite Replenishment Rate" in Proceedings of XIII Annual Convention of Operations Research Society of India on Operations Research, Computers and Decision Making, at Operations Research Society of India ,New Delhi on 14-16, December 1990.

8. Sarma, K.V.S., (1983), "A Deterministic Inventory Model with Two Levels Storage and Optimum Release Rule," OPSEARCH, 20(3), pp. 175 - 180.

9. ____________ and Sastry, M.P., (1988), " Optimum Inventory for Systems with Two Levels of Storage," Industrial Engineering Journal, 17(8), August, pp.12-19.

10. ____________ and Rao, K.S., (1989), "An Order Level Inventory Model Under L_2 - System with Power Pattern Demand" in Proceeding of XXII Annual Convention of Operations Research Society of India, on OR in the Management of Engineering Systems, at Indian Institute of Technology, Kharagpur, during 7-10 December.

11. Shah, Nitin, and Shah, Y.K., (1991), "An EOQ Model Under Price Change Anticipation for a System with Insuffucient Storage Capacity," International Journal of Management and Systems, 7(2), May-August,pp.

12. ____________,(1992), "Deterministic Lot-size Inventory Model When Delay in Payments are Permissible for a System with Two Storage Facilities," Industrial Engineering Journal, XXIII(3), March, pp. 5-11.

13. Shah, V.R., Patel, H.C., and Shah, D.K., (1988), "Economic Ordering Quantity when Delay in Payments of Order and Shortages are Permitted," Gujarat Statistical Review, 15(2), October, pp. 51-56.