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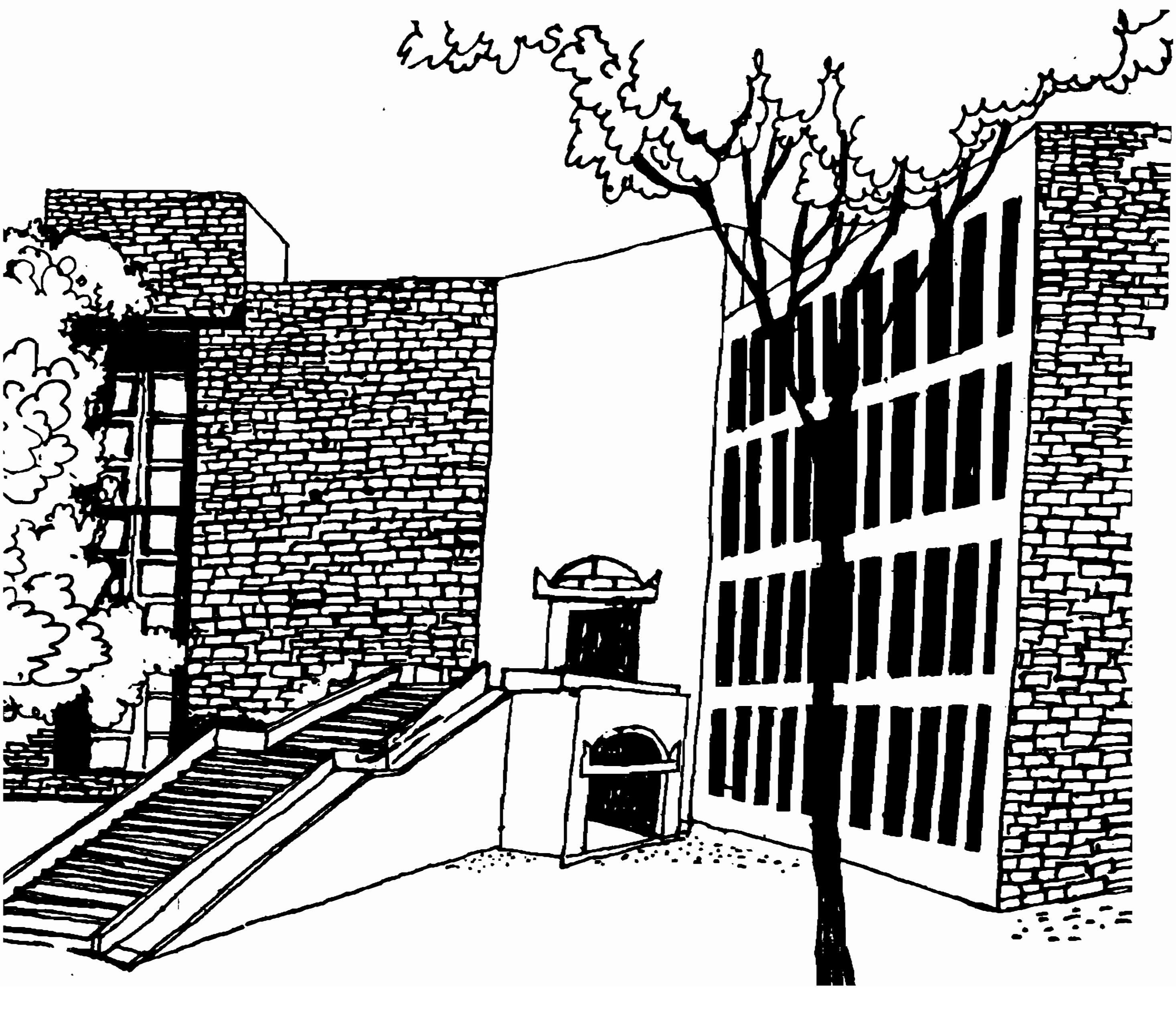

EQUAL INCOMES AND A CONTINUUM OF TASTES IN AN ECONOMY WITH PUBLIC GOODS

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Abstract

In this paper we show that for a continuum economy with public goods, every fair allocation corresponds to an Equal Income Lindahl Equilibrium under mild assumptions.

1. Introduction :- The equal-income Walrasian allocations find a distinctive place amongst allocations that are considered just. In the context of a economy consisting solely of private goods, "equal-income Walrasian allocations are necessarily envy free and efficient under very mild assumptions". (See Thomson and Varian (1985) and references therein). Under certain assumptions discussed in Varian (1976), we have the stronger result that equal-income Walrasian allocations are the only envy-free allocations.

Public goods form an undeniable part of social and economic reality and theories of distributive justice should naturally be extended to include public goods. Such attempts occur in Moulin (1987), Sato (1985,1987), Otskuki (1992), Lahiri (1992a, 1992b).

In this paper, we show that in a continuum economy, equal income Lindahl allocations are the only equitable allocations.

2. The Model :- We shall draw on Varian (1976), Sato (1987) and Lahiri (1992b) to postulate the following model :

The set of agents is regarded as the open unit interval I and there are two goods in the economy, a private good and a public good. Thus $(x,g) \in \mathbb{R}^2_+$ denotes a level of consumption of the private good followed by a level of consumption of the public good. $\mathbb{R}^2_+$ is the consumption set of each agent.

Agent t in I will be endowed with a twice-continuously differentiable strictly quasi-concave utility function $u_t : \mathbb{R}^2_+ \to \mathbb{R}$. Further the utility functions are assumed to be strictly monotonically increasing. As in Varian (1976), we want nearby agents to have similar tastes; thus we assume :

Assumption 1 :- $u : I \times \mathbb{R}^2_+ \to \mathbb{R}$ defined by $u(t,x,g) = u_t(x,g)$ is a continuous function.

The economy is assumed to be endowed with a simple linear technology which converts private good into a public good i.e. there exists a real number p>0, such that to produce one unit of the public good we require 'p' units of the private good.

A feasible allocation is defined to be an ordered pair (x(.),g) where

(i) $x(.) \to [o,w]$ is a Reimann integrable function;

(ii) $g \in \mathbb{R}_+$

(iii) $\int_0^1 x(t)dt + pg = w$.

The following assumption will be found crucial for subsequent analysis :

Assumption 2 :- $(x^*(.), g^*)$ is a Lindahl equilibrium for some Reimann integrable price function $p^* : I \to \mathbb{R}$ and some Reimann integrable income distribution $y^* : I \to [0,w]$ i.e.

(i) $\forall t \in I$, $(x^*(t), g^*)$ maximizes $u_t(.)$ on $B_t = \{(z,g) \in \mathbb{R}^2_+ /$
$z + p(t)g = y^*(t)\}$

(ii) $\int_0^1 y^*(t)\, dt = w$

(iii) $g^* \int_0^1 p(t)dt = pg^*$

(iv) $(x^*(.), g^*)$ is a feasible allocation.

Following Sato (1987), we extend the concept of L-equitability to a continuum economy :

Given a feasible allocation $(x^*(.), g^*)$, define

$$\pi^*(t) \equiv \frac{\delta u_t (x^*(t), g)/\delta g}{\delta u_t (x^*(t), g)/\delta x} \quad \forall t \in I$$

By strong monotonicity, $\pi^*(t) > 0 \; \forall t \in I$

Define, $$A(\pi^*(t), \pi^*(s)) \equiv \frac{\pi^*(s)}{\pi^*(t)} > 0 \; \forall s, t \in I.$$

We shall say that a feasible allocation $(x^*(.), g^*)$ is L-equitable if and only if $\forall s, t \in I$,

$u_t(x^*(t), g^*) \geq u_t(x^*(s), A(\pi^*(t), \pi^*(s))g^*)$.

We shall say that a feasible allocation $(x^*(.), g^*)$ is fair if and only if

(i) $(x^*(.), g^*)$ is L-equitable

(ii) $\exists$ a Reimann integrable price function $p^* : I \to \mathbb{R}$ and a Reimann integrable income distribution $y^* : I \to [0,w]$ with respect to which $(x^*(.), g^*)$ is a Lindahl equilibrium.

It should be noted that our concept of fairness has been stated differently from that of Sato (1987). This highlights a major difference between a finite economy and a continuum economy.

3. Results :-

In this section we draw heavily on Varian (1976), to arrive at the desired conclusions.

Lemma 1 :- Let $f:X \to Y$ be a function from a metric space to a compact metric space. Let (x_i) be a sequence in X converging to x_0, and let $(f(x_i))$ be the corresponding sequence in Y. If every convergent subsequence of $(f(x_i))$ converges to $f(x_0)$, then the function f is continuous at x_0.

Proof :- See Varian (1976).

Lemma 2 :- Let $(x^*(.), g^*)$ be an L-equitable allocation: then $x^*(.)$ is a continuous function.

Proof :- Let (t_i) be a sequence in I converging to t_0, and let $(x^*(t_i))$ be the corresponding sequence in $[o,w]$. We will show that every convergent subsequence of $(x^*(t_i))$ converges to $(x^*(t_0))$.

We first note that by compactness $(x^*(t_i))$ has a convergent subsequence that converges to some to some point x^* $\in[o,w]$. Suppose $u_{t0}(x^*, g^*) > u_{t0}(x^*(t_0), g^*)$. Then for $x^*(t_i)$ close to x^*, $u_{t0}(x^*(t_i), A(\pi^*(t_0), \pi^*(t_i))) > u_{t0}(x^*(t_0), g^*)$. (since $A(\pi^*(t_0), \pi^*(t_i)) \to 1$ as $t_i \to t_0$), and this contradicts L-equity.

Similarly, suppose $u_{t0}(x^*, g^*) < u_{t0}(x^*(t_{t0}), g^*)$. Then for t_i close to t_0 and $x(t_i)$ close to x^*, we have $u_{ti}(x^*(t_i), A(\pi^*(t_i), \pi^*(t_0))g^*) < u_{t0}(x^*(t_0), g^*)$, (once again since $A(\pi^*(t_i), \pi^*(t_0)) \to 1$ as $t_i \to t_0$), and this again contradicts L-equity.

Thus $u_{t0}(x^*, g^*) = u_{t0}(x^*(t_0), g^*)$. By strict monotonicity $x^* = x^*(t_0)$.

Q.E.D.

Unfortunately, continuity is not enough and to get the main result we need to specialize a bit more.

Theorem 1 :- Let $(x^*(.), g^*)$ be a fair allocation which is differentiable; then $y(t) = x^*(t) + p^*(t)g^*$ is a constant function.

Proof :- Choose an agent t_0 and define a differentiable function $v: I \to \mathbb{R}$ by $v(t) = u_{t0}(x^*(t), A(\pi^*(t_0), \pi^*(t))g^*)$. Since $(x^*(.), g^*)$ is a fair allocation, $v(t)$ reaches a maximum at t_0

and therefore its derivative must vanish. This implies that

$$Dv(t_0)=D_t u_{t_0}(x^*(t_0),g^*)=D_x u_{t_0}(x^*(t_0),g^*) \cdot D_t x^*(t_0) + \frac{D_g u_{t_0}(x^*(t_0),g^*) D_t \pi^*(t_0) g^*}{\pi^*(t_0)} = 0$$

$$\text{or } D_x u_{t_0}(x^*(t_0),g^*) D_t x^*(t_0) + \frac{D_g u_{t_0}(x^*(t_0),g^*) D_t[\pi^*(t_0)g^*]}{\pi^*(t_0)} = 0.$$

But $(x^*(.),g^*)$ is a Lindahl equilibrium, so agent t_0 is maximized on his budget set, which means

$$\frac{D_g u_{t_0}(x^*(t_0),g^*)}{D_x u_{t_0}(x^*(t_0),g^*)} = \pi^*(t_0)$$

This follows from $\pi^*(t)=p^*(t)\ \forall t \in I$, which is necessary at a Lindahl equilibrium.

$$\therefore D_t x^*(t_0) + D_t[\pi^*(t_0)g^*] = D_t[x^*(t)+\pi^*(t)g^*]\Big|_{t=t_0} = 0.$$

The choice of t_0 was arbitrary, which implies y^* is a differentiable function with everywhere a zero derivative. It must therefore be a constant function.

Q.E.D.

Conclusion :- In this paper we have established, that in the case of a continuum economy, under suitable assumptions all fair allocations correspond to Lindahl equilibria from equal income. The converse, i.e. an Equal Income Lindahl equilibrium is fair is immediate. This result therefore emphasizes the intrinsic justice involved in implementing equal income Lindahl Equilibrium allocations in a mixed economy.

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