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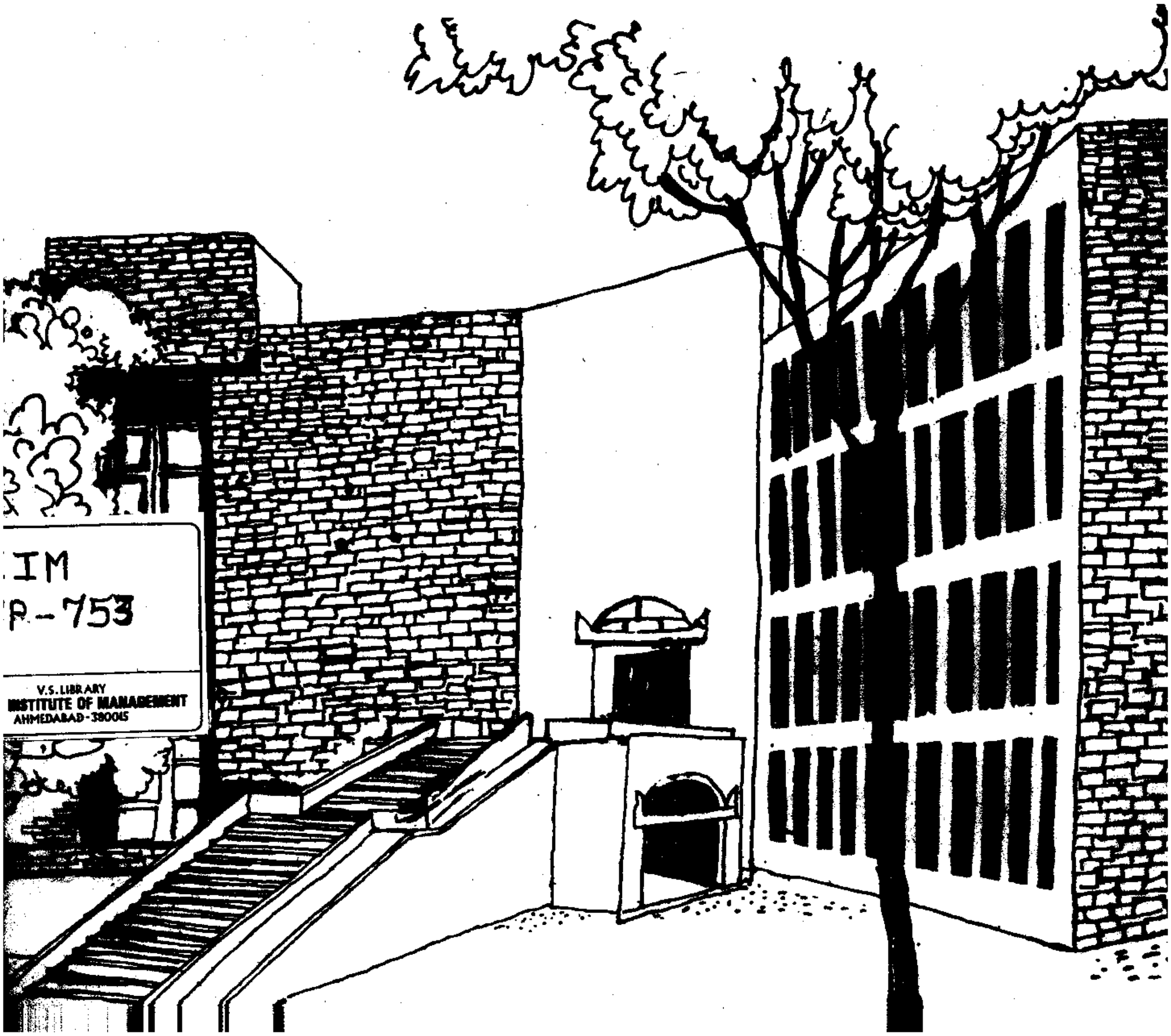

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A LOT SIZE MODEL WITH DISCRETE TRANSPORTATION COSTS

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A LOT SIZE MODEL WITH DISCRETE TRANSPORATION COSTS

ABSTRACT: The classical Harris-Wilson inventory model does not explicitly account for the costs incurred in transporting goods from the supplier to the buyer. Either such costs are assumed to be fixed and considered part of the ordering costs or they are assumed to be variable and are included in the item costs. In many situations, however, it is observed that a fixed cost is incurred for a transport mode, (of a given capacity), such as a truck or wagon. The very nature of this type of transport mode requires hiring of an integer number of trucks or wagons. Therefore the transportation cost function becomes a discrete function. In this paper we develop an inventory model with discrete transportation costs, and present an algorithm for the optimal lot size. Finally an example is given to illustrate the methodology.

INTRODUCTION

Economic lot size models have been studied extensively and one of the earliest model is due to Harris[1] which has provided well-known Wilson's EOQ formula. In the simplest possible case the optimal lot size Q^*, is given by:

$$Q^* = \sqrt{2AD/iC_I} \qquad \ldots(1)$$

where A is the ordering cost, D the demand rate, i is the inventory carrying rate, and C_I is the item cost. This model is valid under certain assumptions such as the lead time is constant, the demand rate is constant and uniform, item cost is constant, and no stockouts are allowed, etc. Many extensions to [1] are cited in the literature. These extensions, for example, allow for stochastic demand and leadtime, backlogging, variable item cost, etc. In fact, an indepth analysis of Harris-Wilson model with varying assumptions has been a subject of study of great many researchers.

Harris-Wilson model does not explicitly account for the cost incurred in transporting goods from the supplier to the buyer. Either such costs are assumed as variable costs and therefore added to the item costs or else they are assumed to be fixed (irrespective of order quantity) and are considered as part of the ordering costs. In many practical situations, however, such costs do not fall in either category. In this paper we investigate an extension of [1] where transportation cost structure is such that a fixed cost(C_2) is incurred for a transport mode of a given capacity (say, K units) such as a truck or wagon. This cost C_2 is incurred whether the capacity is fully utilised or not. If the order quantity exceeds one truck, one would have to hire two, three,.... trucks depending on the actual quantity. Therefore the transport cost would be a discrete function of the order quantity Q. Let f(Q) represent the total annual relevant cost (sum of ordering costs, inventory carrying costs, and transportation costs), we have:

$$f(Q) = AD/Q + QiC_1/2 + (D/Q)C_2(\lceil Q/K \rceil) \quad ...(2)$$

In this paper we investigate such an inventory model with discrete transportation costs, and develop an algorithm for the optimal lot size. We assume that all other assumptions of the classical Harris-Wilson model hold valid.

MODEL ANALYSIS

It is obvious that if the order quantity is Q, and the number of trucks used for delivering this order quantity is m, then

$$(m - 1)K < Q \leq mK \quad ...(3)$$

If m trucks are used to deliver the quantity Q (we will call it an m-truck policy), the total cost function can be expressed as:

$$f_m(Q) = AD/Q + QiC_1/2 + (D/Q)(C_2)m, \quad (m - 1)K < Q \leq mK ...(4)$$

It may be easily seen that f_m is a sequence of strictly increasing functions as shown in Figure 1. $f(Q)$ is a discontinuous function shown by the solid curve. The function $f_m(Q)$ attains the unconstrained minimum at

$$Q_m = \sqrt{2(A + C_2m)D/iC_1} \quad ...(5)$$

The value of $f_m(Q)$ at Q_m will be

$$\sqrt{2(A + C_2m)DiC_1} \quad ...(6)$$

Therefore,

$$f_m(Q) \geqslant \sqrt{2(A + C_2 m)DiC_1}, \qquad \forall Q. \qquad \ldots(7)$$

Now, suppose we consider ordering with one-truck(i.e., m=1),

$$f_1(Q) = AD/Q + QiC_1/2 + (D/Q)C_2, \quad 0 < Q \leq K \qquad \ldots(8)$$

f(Q) has the unconstrained minimum at

$$Q_1 = \sqrt{2(A + C_2)D/iC_1} \qquad \ldots(9)$$

If $Q_1 \leq K$, the optimal lot size Q^* is obviously Q_1. If $Q_1 > K$, $f_1(Q)$ will have the constrained minimum exactly at K, and the corresponding total annual relevant cost will be

$$f_1(K) = AD/K + KiC_1/2 + (D/K)C_2, \qquad \ldots(10)$$

which is an upper bound, say UB, on the optimal annual relevant costs.

From (7), and above discussion, procuring with m-trucks will not be economical if

$$\sqrt{2(A + C_2 m)DiC_1} > UB. \qquad \ldots(11)$$

Simplifying,

$$m > [\{(UB)^2/2DiC_1\} - A]/C_2 \qquad \ldots(12)$$

$$= r, \text{ say}$$

We, therefore, can discard all m-truck policies where m exceeds r. We apply the above analysis and suggest the following algorithm for determining the optimal lot size.

ALGORITHM

1. Compute the unconstrained minimum as

$$Q_1 = \sqrt{2(A + C_2)D/iC_1}$$

If Q_1 is also the constrained minimum (i.e., if $Q_1 \leq K$), the optimal lot size is Q_1. Otherwise, set $Q^* = K$, and compute an upper bound UB as $f_1(K)$.

2. Compute the ratio r as:

$$r = [\{(UB)^2 /2DiC_1\} - A]/C_2$$

3. Ignore ordering with m-trucks if $m > r$.

4. If all truck-policies have been considered, stop. Otherwise, move to the next higher number of truck level policy, and compute the unconstrained optimal lot size Q (by 5). If Q is also the constrained optimal, compute f(Q). If $f(Q) < f(Q^*)$, set $Q^* = Q$ (If $f(Q) = f(Q^*)$, Q and Q^* are both optimal lot sizes), and stop.

If Q is not a constrained optimal, replace Q by the highest feasible value of Q, and compute f(Q). If $f(Q) < f(Q^*)$, set $Q^* = Q$, and $UB = f(Q^*)$. Recompute ratio r with the updated bound UB. Go to step 3.

AN EXAMPLE

Suppose:

$A = \$20$ $\quad$ $D = 400$ units per year

$i = 10\%$ per year $\quad$ $C_1 = \$20$

$C_2 = \$50$ $\quad$ $K = 50$ units

1. Compute Q_1.

$$Q_1 = \sqrt{2(20 + 50)400/(0.10)(20)} = 167.33$$

Since $Q_1 > 50$, it is not feasible.

Therefore, $Q^* = 50$.

$$f(Q) = 20(400)/50 + 50(0.10)(20)/2 + (400/50)50$$

$$= 160 + 50 + 400 = 610$$

Therefore UB = 610.

2. Compute r.

$$r = \{(610)^2/2(400)(0.10)(20)-20\}/50$$

$$= 4.25125$$

3. Ignore ordering with 5 or more trucks.

4. Consider next truck-policy

$$Q_2 = \sqrt{2(20 + 100)400/(0.10)(20)} = 219.10$$

Since $Q_2 > 2(50)$, it is infeasible. Replace it by $Q_2 = 100$.

$$f(Q_2) = 20(400)/100 + 100(0.10)(20)/2 + (400/100)(50)(2)$$

$$= 80 + 100 + 400 = 580$$

Since $f(Q_2) < f(Q^*)$, replace Q^* by Q_2.

Therefore, update $Q^* = 100$, and UB = 580.

5. Recompute r.

$$r = ((580)^2/2(400)(0.10)(20)-20)/50$$

$$= 3.805$$

6. Ignore ordering with 4 or more trucks.

7. Consider next truck-policy

$$Q_3 = \sqrt{2(20 + 150)400/(0.10)(20)} = 260.77$$

which is also infeasible. Replace Q_3 by 150.

$$f(Q_3) = 20(400)/150 + 150(0.10)(20)/2 + (400/150)(50)(3)$$

$$= 53.33 + 150 + 400$$

$$= 603.33$$

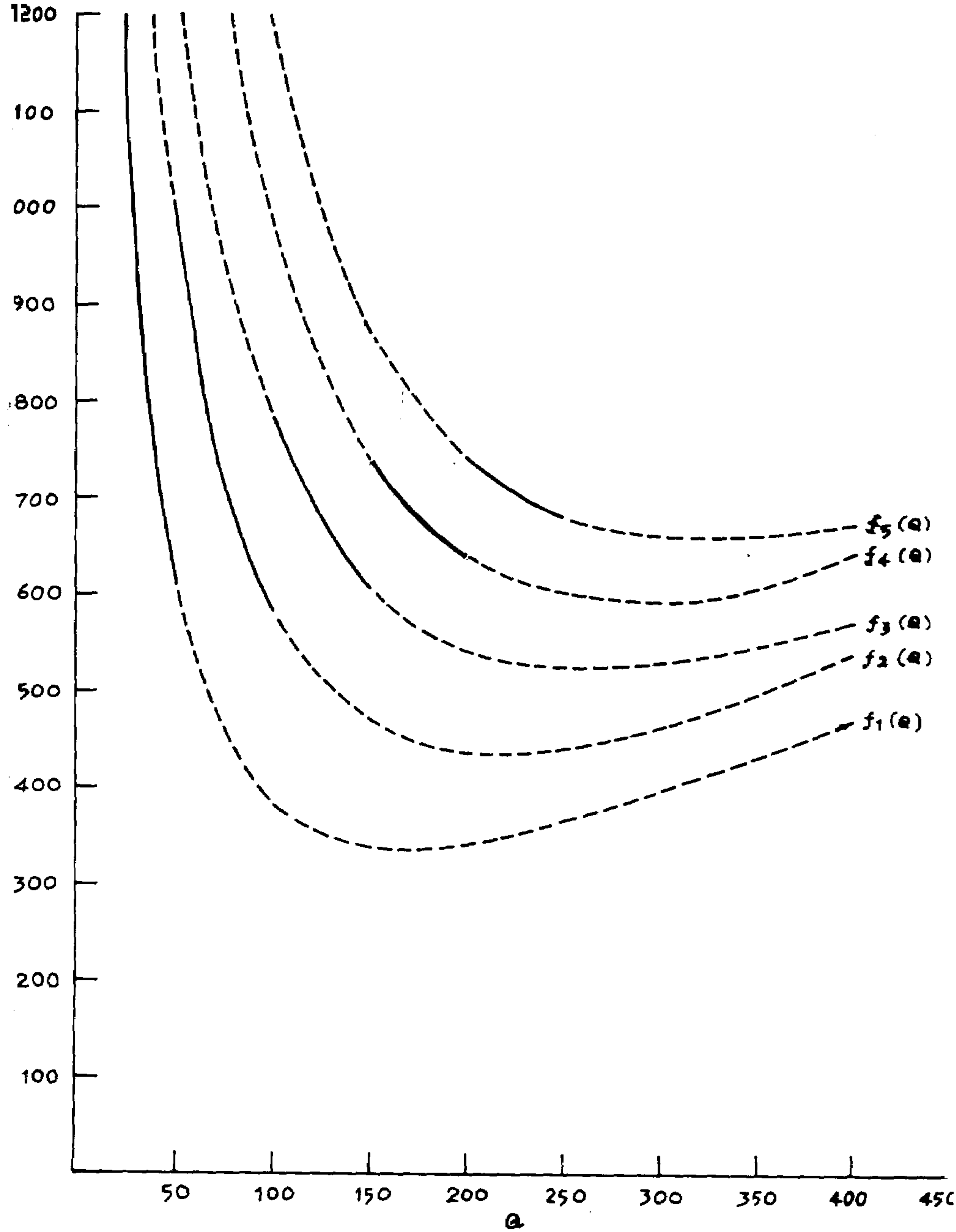
Since $f(Q_3) > 580$, we do not change Q^*.

We have exhausted all policies.

The optimal lot size is 100 units (2-trucks).

REFERENCE

1. Harris Ford, Operations and costs (Factory Management Series), A.W.Shaw Co., Chicago Ill. 48-52 (1915).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