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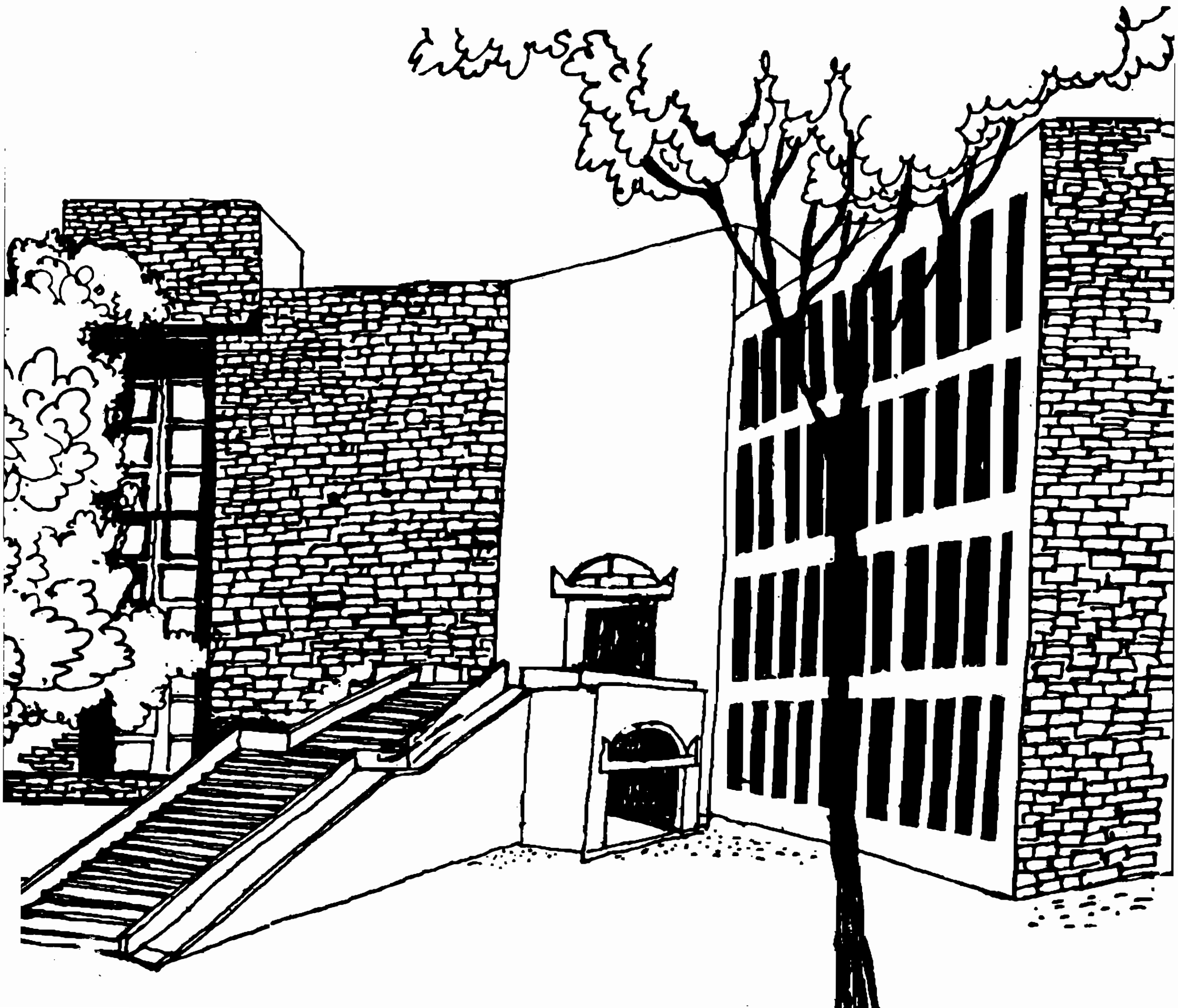

THE EGALITARIAN EQUIVALENT SOLUTION TO BARGAINING PROBLEMS IN ECONOMIC ENVIRONMENTS

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ABSTRACT

In this paper we set up an analytical framework to study bargaining (or object division) problems in economic environments, propose some new solutions and study the egalitarian equivalent bargaining solution. This analysis extends bargaining solutions defined on a restricted set of environments (corresponding to equal initial endowments) to a more general class of barginig problems.

1. <u>Introduction</u> : In this paper we develop a framework for analysing bargaining problems in economic environments and propose a solution to such problems.

The general bargaining problem we analyse presupposes an economic environment with a finite number of agents each possessing an initial endowment of a finite number of commodities and each agent's preference being representable by a utility function on the commodity space. A bargaining solution is a trading rule or an allocation mechanism which assigns to each bargaining problem a feasible allocation of the aggregate commodity bundle, the idea being that trade takes place only if every agent agrees to the final allocation of resources. In the absence of agreement each agent receives his initial endowment which may thus be referred to as the disagreement point. This is the basic framework of general competitive analysis (see Arrow and Hahn (1970)), motivated as a bargaining problem in the sense of Nash (1950), by Friedman (1987).

Following Paznerand Schmeidler (1974), we consider the egalitarian equivalent solution for bargaining problems as the appropriate solution concept. We introduce other solution concepts as well, which may form the subject matter of future research. The equity problem posed by Pazner and Schmeidler (1974) turns out to be the special case of equal initial endowments in our framework. A detailed investigation and properties of the egalitarian equivalent solution are established in this paper.

2. Definitions : We consider private ownership exchange economies consisting of a finite set of agents and a finite set of commodities. The agents have preferences on the set of commodity bundles and have initial endowments. Let the total number of commodities in the economy be l indexed by $k, k=1,\dots,l$ and the total number of agents be n, indexed by $i, i=1,\dots,n$. The commodity space is $\mathbb{R}^l_+$. For each trader i, his initial endowment w_i, $w_i \in \mathbb{R}^l_+$, and his preference relation $\geq_i$ on $\mathbb{R}^l_+$ are specified. We assume that the preference relation $\geq_i$ on $\mathbb{R}^l_+$ of agent i is representable by a utility function $u_i : \mathbb{R}^l_+ \to \mathbb{R}$. An economic environment or a bargaining problem is a vector $\langle w_1,\dots,w_n; u_1,\dots,u_n\rangle = \mathcal{E}$ where each component of $\mathcal{E}$ has the interpretation given above.

Let Γ denote a collection of economic environments. For each $\mathcal{E} \in \Gamma$, let

$$V(\mathcal{E}) = \left\{(x_1,\dots,x_n) \in (\mathbb{R}^l)^n \,/\, \sum_{\substack{j=1 \\ j \neq i}}^{n} w_j^k \geq x_i^k \geq w_i^k, i=1,\dots,n; k=1,\dots,l \text{ and } \sum_{i=1}^{n} x_i \leq 0\right\}$$

$V(\mathcal{E})$ is called the set of feasible net trades (or feasible allocations) for the economic environment $\mathcal{E}$.

Our task is to study mechanisms F which map economic environments into feasible allocations of these environments. An allocation mechanism or a bargaining solution $F: \Gamma \to (\mathbb{R}^l)^n$ is a correspondence that associates to each economic environment $\mathcal{E}$ a subset of $V(\mathcal{E})$, the set of feasible allocations; i.e. $F(\mathcal{E}) \subset V(\mathcal{E})\ \forall \mathcal{E} \in \Gamma$.

Let $\bar{\Gamma}$ be the set of all economic environments for which the initial endowments of the agents are equal; i.e.

$\bar{\Gamma} = \{\mathcal{E} \in \Gamma / \mathcal{E} = \langle w_1, \ldots, w_n; u_1, \ldots, u_n \rangle$ and $w_1 = \ldots = w_n\}$.

For $S \subseteq \{1, \ldots, n\}$ and $x \in V(\mathcal{E})$ i.e. x a feasible net trade, $\sum_{i \in S} x_i$ is the net trade of coalition S. The net trade of the empty set is, by convention, zero.

Now, let us define various equitable net trades existing in the literature :

(a) A <u>net trade</u> x is <u>envy free</u> for $= \langle w_1, \ldots, w_n; u_1, \ldots u_n \rangle \in \Gamma$ if

(i) $x \in V(\mathcal{E})$

(ii) for all i and $j \in \{1, \ldots, n\}$, $u_i(w_i + x_i) \geq u_i(w_i + x_j)$

(b) A <u>net trade</u> x is <u>strongly</u> (or additively) <u>envy free</u> for $\mathcal{E} = \langle w_1, \ldots, w_n; u_1, \ldots, u_n \rangle \in \Gamma$ if

(i) $x \in V(\mathcal{E})$

(ii) for all $i \in \{1, \ldots, n\}$ and for all nonnegative integers n_j, $j = 1, \ldots, n$, $u_i(w_i + x_i) \geq u_i(w_i + \sum_{j=1}^{n} n_j x_j)$ whenever

$$-w_i^k \leq \sum_{j=1}^{n} n_j \; x_j^k \leq \sum_{\substack{j=1 \\ j \neq i}}^{n} w_j^k, \quad k = 1, \ldots, l.$$

(c) A <u>net trade</u> x is said to be <u>coalitionally envy free</u> for $\mathcal{E} = \langle w_1, \ldots, w_n; u_1, \ldots, u_n \rangle \in \Gamma$ if

(i) $x \in V(\mathcal{E})$

(ii) there does not exist $S_1, S_2 \subseteq \{1, \ldots, n\}$, $S_1, S_2 \neq \emptyset, S_1 \cap S_2 = \emptyset$, $|S_1| \geq |S_2|$ and $y \in V(\mathcal{E})$, such that $u_i(w_i + y_i) > u_i(w_i + x_i)$, $\forall i \in S_1$ with $\sum_{i \in S_1} y_i = \sum_{i \in S_2} x_i$.

(d) A net trade x is said to be <u>egalitarian equivalent</u> for $\mathcal{E} = \langle w_1, \ldots, w_n; u_1, \ldots, u_n \rangle \in \Gamma$ if

(i) $x \in V(\mathcal{E})$

and (ii) there is a vector $t \in \mathbb{R}^l$ such that for all $i \in \{1, \ldots, n\}$, $u_i(w_i + x_i) = u_i(w_i + t)$; such a vector t is called an <u>egalitarian reference trade</u>.

(e) A <u>net trade</u> x is said to be <u>A-envy free</u> for $\mathcal{E} = \langle w_1,\ldots,w_n; u_1,\ldots,u_n\rangle \in \Gamma$ if

(i) $x \in V(\mathcal{E})$

and (ii) $u_i(w_i+x_i) \geq u_i(w_i+a_i(x))$ for all $i \in \{1,\ldots,n\}$ where $a \equiv (a_1,\ldots,a_n) : (\mathbb{R}^1)^n \to (\mathbb{R}^1)^n$ called the averaging operation is defined thus :

$$a_i(z) = \left(\sum_{j\neq i} z_j/(n-1)\right) \forall\, z \in (\mathbb{R}^1)^n$$

(f) A <u>net trade</u> x is said to be <u>individually rational</u> for $\mathcal{E} = \langle w_1,\ldots,w_n; u_1,\ldots,u_n\rangle \in \Gamma$ if

(i) $x \in V(\mathcal{E})$

and (ii) $u_i(w_i+x_i) \geq u_i(w_i) \forall\, i \in \{1,\ldots,n\}$

(g) A <u>net trade</u> x is said to be <u>Pareto Optimal</u> for $\mathcal{E} = \langle w_1,\ldots,w_n; u_1,\ldots,u_n\rangle \in \Gamma$ if

(i) $x \in V(\mathcal{E})$

and (ii) if $y \in V(\mathcal{E})$ and $u_i(w_i+y_i) \geq u_i(w_i+x_i) \forall\, i \in \{1,\ldots,n\}$, then $u_i(w_i+y_i) = u_i(w_i+x_i)$ for $\forall\, i \in \{1,\ldots,n\}$

(h) A <u>net trade</u> x is said to be competitive for $\mathcal{E} = \langle w_1,\ldots,w_n; u_1,\ldots,u_n\rangle \in \Gamma$ if

(i) $x \in V(\mathcal{E})$

and (ii) there exists a (price) vector p, $p \neq 0$, $p \in \mathbb{R}^1_+$, such that $p.x_i \leq 0 \;\forall\, i \in \{1,\ldots,n\}$ and $u_i(w_i+x_i) \geqslant u_i(w_i+y_i)$ whenever $p.y_i \leq 0$ and $-w_i^k \leq y_i^k \leq \sum_{\substack{j=1 \\ j\neq i}}^{n} w_j^k$, $k \in \{1,\ldots,l\}$

(i) A <u>net trade</u> x is said to be <u>egalitarian</u> for $\mathcal{E} = \langle w_1,\ldots,w_n; u_1,\ldots,u_n\rangle \in \Gamma$ if

(i) $x \in V(\mathcal{E})$

and (ii) $u_i(w_i+x_i) = u_j(w_j+x_j)$, for all $i,j \in \{1,\ldots,n\}$

(j) A net trade x is said to be weakly Pareto optimal for $\mathcal{E} = \langle w_1,\dots,w_n; u_1,\dots,u_n\rangle \in \Gamma$ if

(i) $x \in V(\mathcal{E})$

and (ii) there does not exist $y \in V(\mathcal{E})$ such that $u_i(w_i+y_i) > u_i(w_i+x_i)$ for all $i \in \{1,\dots,n\}$.

(k) A net trade x is said to be coalitionally A-envy free for $\mathcal{E} = \langle w_1,\dots,w_n; u_1,\dots,u_n\rangle \in \Gamma$ if

(i) $x \in V(\mathcal{E})$

and (ii) there does not exist $i \in \{1,\dots,n\}$ and $S \subseteq \{1,\dots,n\}$, $S \neq \emptyset$, $i \notin S$ such that $u_i\left(w_i + \left(\sum_{j \in S_2} x_j / |S|\right)\right) > u_i(w_i+x_i)$

For $\mathcal{E} \in \bar{\Gamma}$, (a) was defined by Foley (1967) and Varian (1974) and for $\mathcal{E} \in \Gamma$ was defined by Schmeidler and Vind (1972), Feldman and Kirman (1974), (b) was defined by Schmeidler and Vind (1972) on Γ; (c) was defined by Varian (1974) on $\bar{\Gamma}$ and a variant of it extended to Γ by Jaskold-Gabszewicz (1975); (d) was defined on $\bar{\bar{\Gamma}}$ by Pazner and Schmeidler (1974); (e) was defined on $\bar{\bar{\Gamma}}$ by Thomson (1982); (i) was defined on Γ by Kalai (1977) with a modification discussed in Roemer ((1986), (1988)); (f) was defined on Γ by Pazner (1979).

A net trade x is said to be fair for $\mathcal{E} = \langle w_1,\dots,w_n; u_1,\dots,u_n\rangle \in \Gamma$ if x satisfies (a) and (g); a net trade x is said to be coalitionally fair for $\mathcal{E} = \langle w_1,\dots,w_n; u_1,\dots,u_n\rangle \in \Gamma$ if x satisfies (c) and (g); a net trade x is said to be A-fair for $\mathcal{E} = \langle w_1,\dots,w_n; u_1,\dots,u_n\rangle \in \Gamma$ if x satisfies (e) and (g). Let us agree to call a net trade x coalitionally A-fair for $\mathcal{E} = \langle w_1,\dots,w_n; u_1,\dots,u_n\rangle \in \Gamma$ if x satisfies (k) and (g).

These do not exhaust all possible equity criteria. For instance the Nash (1950) bargaining solution and the Kalai-Smorodinsky (1975) bargaining solution are significant equity criteria not mentioned above. The one we intend to discuss in this paper is (d), of which no comparable analysis exists in the literature.

3. Egalitarian Equivalent Net Trades :- A utility function $u_i : \mathbb{R}^l_+ \to \mathbb{R}$ is said to be :

(a) weakly monotonic if $x_i \geqq y_i$, $x_i, y_i \in \mathbb{R}^l_+$ implies $u_i(x_i) \geq u_i(y_i)$;

(b) monotonic if in addition $x_i > y_i, y_i \in \mathbb{R}^l_+$ implies $u_i(x_i) \quad u_i(y_i)$;

(c) weakly convex if $\forall \lambda \in (0,1)$, $u_i(x_i) \geq u_i(x_i') \Rightarrow u_i(\lambda x_i + (1-\lambda) x_i) \geq u_i(x_i')$;

(d) convex if $\forall \lambda \in (0,1)$, $u_i(x_i) > u_i(x_i') \Rightarrow u_i(\lambda x_i + (1-\lambda) x_i') > u_i(x_i')$;

and (e) continuous if $u_i : \mathbb{R}^l_+ \to \mathbb{R}$ is a continuous function.

To begin with let us note that the no trade option is always egalitarian equivalent for all $\xi \in \Gamma$.

We assume that Γ consists of all economic environments ξ satisfying (a) to (e) above. In addition to avoid unnecessary complication we assume that if $\xi = \langle w_1, \ldots, w_n ; u_1, \ldots, u_n \rangle$ then $w_i \in \mathbb{R}^l_{++} \forall i \in \{1, \ldots, n\}$.

First the following results are established.

Proposition 1 : For every $\bar{t} \in \mathbb{R}^l_{++}$ there is a nonnegative real number $\bar{r}$ so that there exists a Pareto efficient egalitarian equivalent trade x with $\bar{r}\,\bar{t}$ being the egalitarian reference bundle (i.e. for all $i \in \{1,\dots,n\}$, $u_i(W_i+x_i) = u_i(W_i+\bar{r}\,\bar{t})$) for each $\mathcal{E} \in \Gamma$).

Proof of Proposition 1 :- Let $\mathcal{E} \in \Gamma$ be given and let $C = \{r \geq 0 /$ there is an $r\,\bar{t}$ - equivalent trade$\}$. The set C is non-empty, since $0 \in C$. It is bounded, since monotonicity of preferences implies that there is no $r\,\bar{t}$ - equivalent trade when $r\,\bar{t} > \max\{\sum_{j\neq 1} W_j, \dots, \sum_{j\neq n} W_j\}$. (We require $\bar{t} \in \mathbb{R}^1_{++}$ for this step in the proof). Let $\bar{r}$ be the least upper bound of C(sup C). Because of the compactness of the set of allocations and the continuity of preferences $\bar{r} \in C$.

To complete the proof, one has to show that an $\bar{r}\,\bar{t}$ -equivalent trade is Pareto-efficient. Denote by x an $\bar{r}\,\bar{t}$ -equivalent trade, and suppose, per absurdum, that there is another feasible net trade y (i.e. $y \in V(\mathcal{E})$) with $u_i(W_i+y_i) \geq u_i(W_i+x_i)$ for all i and $u_i(W_i+y_i) > u_i(W_i+x_i)$ for some i. Because of our monotonicity and continuity assumption there is another feasible trade z, so that $u_i(W_i+z_i) \quad u_i(W_i+x_i)$ for all $i \in \{1,\dots,n\}$. By continuity (and monotonicity) there is, for each $i \in \{1,\dots,n\}$ a positive number s_i so that $u_i(W_i+z_i) = u_i(W_i+(\bar{r}+s_i'')t_i)$. Setting $\bar{s} = \min_i s_i$ and applying once again monotonicity and continuity, we get an $(\bar{r}+\bar{s})\,\bar{t}$ - equivalent trade- which is a contradiction.

Q.E.D.

For the next result the <u>strict convexity</u> assumption is used : for all i in $\{1,\dots,n\}$, all $z_i, z_i' \in \mathbb{R}^1_+$ and all $\alpha \in (0,1)$, if $z_i \neq z_i'$ and $u_i(z_i) \geq u_i(z_i')$, then $u_i(\alpha z_i + (a-\alpha) z_i') > u_i(z_i')$.

<u>Proposition 2</u> :- Under the assumption of strict convexity, for every egalitarian - reference trade there is atmost one Pareto-efficient trade for each $\xi \in \Gamma$.

<u>Proof</u> :- Suppose x and y are two different Pareto-efficient trades corresponding to the same egalitarian reference trade $\bar{t}$. Clearly $\frac{1}{2}x + \frac{1}{2}y$ is a feasible trade which Pareto dominates both x and y, contradicting the Pareto optimality of x and/or y.

<u>Q.E.D.</u>

<u>Proposition 3</u> :- Let there be given a convergent sequence $\{\bar{t}_K\}$ in $\mathbb{R}^l_{++}$ with limit $\bar{t} \in \mathbb{R}^l_{++}$. Let

$$\bar{r}_K = \sup\{r \geq 0 / r\ \bar{t}^K \text{ is an egalitarian reference trade}\}$$

$$\bar{r} = \sup\{r \geq 0 / r\ \bar{t} \text{ is an egalitarian reference trade}\}$$

for $\xi \in \Gamma$. Then $\bar{r}_K \to \bar{r}$.

<u>Proof</u> :- For all $i \in \{1,\ldots,n\}$,

$$u_i(w_i + \bar{r}_K\ \bar{t}^K) = u_i(w_i + x_i^K),\ x^K = (x_1^K,\ldots,x_n^K) \in V(\xi)$$

and x^K satisfies (g) for $\xi \in \Gamma\ \forall\ K = 1,2,\ldots$

Since the set $V(\xi)$ is compact, there exists a subsequence $\{x^{q_K}\}_{K \in \mathbb{N}}$ of $\{x^K\}_{K \in \mathbb{N}}$ and a $\bar{x} \in V(\xi)$ such that

$$w_i + x_i^{q_K} \to w_i + \bar{x}_i,\ i \in \{1,\ldots,n\}.$$

Further the set of Pareto optimal trades for being closed, and x^K being Pareto optimal $\forall K$, we have $\bar{x}$ is Pareto optimal. $\{\bar{r}_K\ \bar{t}^K\}_{K \in \mathbb{N}}$ moves in a compact set. So does $\{\bar{r}_{q_K}\ \bar{t}^{q_K}\}_{K \in \mathbb{N}}$. Hence it has a limit point. Hence $\bar{x} \in V(\xi)$ is also an egalitarian equivalent trade by continuity of preferences.

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$\therefore \; u_i(W_i + \lim_{K\to\infty}(\bar{r}_{q_K} \; \bar{t}^{q_K})) = u_i(W_i + \bar{x}_i) \; \forall i \in \{1,\ldots,n\}.$

Since $\bar{r}^k_{q_K} = \dfrac{(\bar{r}_{q_K} \; \bar{t}^{q_K})^k}{(\bar{t}^{q_K})^k} \; \forall \; k \in \{1,\ldots,l\}$

and since $\lim_{\to\infty} \bar{t}^{q_K} = \bar{t} > 0$, we get,

$\lim_{K\to\infty} \bar{r}^k_{q_K} = \dfrac{\lim_{K\to\infty} (\bar{r}_{q_K} \; \bar{t}^{q_K})^k}{\lim_{K\to\infty} (\bar{t}^{q_K})^k}$ exists

$\therefore \; u_i(W_i + (\lim_{K\to\infty} \bar{r}_{q_K})(\lim_{K\to\infty} \bar{t}^{q_K})) = u_i(W_i + \bar{x}_i)$

or $u_i(W_i + (\lim_{K\to\infty} \bar{r}_{q_K}) \; \bar{t}) = u_i(W_i + \bar{x}_i)$.

Since preferences are monotonic neither $\bar{r} > \lim_{K\to\infty} \bar{r}_{q_K}$ nor $\bar{r} < \lim_{K\to\infty} \bar{r}_{q_K}$

$\therefore \; \bar{r} = \lim_{K\to\infty} \bar{r}_{q_K}$.

This being true for all subsequences $\{\bar{r}_{q_K}\}_{K\in\mathbb{N}}$ we get,

$\bar{r} = \lim_{K\to\infty} \bar{r}_K$.

Q.E.D.

Denote by P the set

$\{t \in \mathbb{R}^l_{++} \,/\, \sum_{k=1}^{l} t^k = 1\}$

and denote by RP the set of egalitarian-reference trades $\{\bar{r}\,\bar{t} \in \mathbb{R}^l_{++} \,/\, \bar{t} \in P$ and $\bar{r}$ corresponds to $\bar{t}$ via Proposition 1$\}$. As a simple consequence of Proposition 3, one has RP is homeomorphic to P. Next denote by AñP the set of trades, each of them being Pareto-efficient and egalitarian equivalent to some vector in RP. By Proposition 1, 2 and 3 we have

<u>Proposition 4</u> :- Under the strict convexity assumption the correspondence that applies Pareto-efficient $\bar{t}$ in RP is a well-defined continuous function from RP to ARP.

However, note that the function of Proposition 4 is not one-to-one; one may have two distinct vectors in RP yielding the same Pareto-efficient egalitarian-equivalent trade in ARP.

<u>Proposition 5</u> :- In a two-person economy an envy free trade is egalitarian equivalent .

<u>Proof</u> :- Let x be an envy free trade for

$\therefore\ u_1(w_1+x_1) \geq u_1(w_1+w_2)$

$u_2(w_2+x_2) \geq u_2(w_2+x_1)$

Suppose $T_1 = \{t \in \mathbb{R}^1 / u_1(w_1+t) = u_1(w_1+x_1)\}$

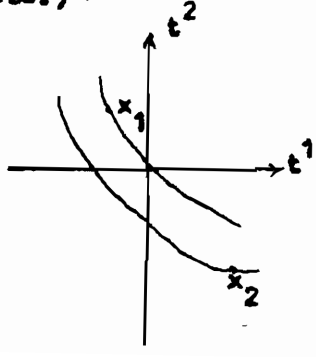
and $T_2 = \{t \in \mathbb{R}^1 / u_2(w_2+t) = u_2(w_2+x_2)\}$.

We need to show that $T_1 \cap T_2 \neq \emptyset$

first observe that $T_1 \cap \{x_2 - a / a \in \mathbb{R}^1_+ \setminus \{0\}\} = \emptyset$

and $T_2 \cap \{x_1 - a / a \in \mathbb{R}^1_+ \setminus \{0\}\} = \emptyset$. This follows by monotonicity of preferences and the fact that x is an envy free trade.

Suppose $T_1 \cap T_2 = \emptyset$. Suppose without loss of generality that $t \in T_1 \Rightarrow u_2(w_2+t) > u_2(w_2+x_2)$. (This follows from the monotonicity and convexity of preferences.)



Then we get

$$T_2 \cap \{x_1 - a/a \in \mathbb{R}^l_+ \setminus \{0\}\} \neq \emptyset$$

which is a contradiction.

Q.E.D.

The preference relation of agent i is convex if in the definition of strict convexity the "greater than or equal to" sign is substituted by "greater than".

Proposition 6 :- In a two person economy with convex preferences, if the egalitarian - reference trade lies on a positively sloped ray through the origin, the corresponding Pareto-efficient egalitarian equivalent trade is fair.

Proof :- Denote by 't' the reference point of the trade (on the ray through the origin). Suppose $x_1 = x_2 = t = 0$. Then the trade is trivially envy free. So suppose $t \in \mathbb{R}^l_{++}$.

$\therefore\ u_i(W_i+x_i) = u_i(W_i+t) \quad u_i(W_i)$ for $i = 1,2$.

Suppose towards a contradiction that

$u_1(W_1+x_2) > u_1(W_1+x_1)$. Then by convexity of preferences we get

$u_1(W_1+ \frac{x_1+x_2}{2}) > u_1(W_1+x_1)$ and by monotonicity

$u_1(W_1) \geq u_1(W_1+ \frac{x_1+x_2}{2})) > u_1(W_1+x_1)$, since $x^k_1 + x^k_2 \leq 0$ for $k = 1,\ldots,l$. This leads to a contradiction.

Similarly, $u_2(W_2+x_1) > u_2(W_2+x_2)$ leads to a contradiction.

Hence x is a fair net trade.

Q.E.D.

Let $\mathcal{E} \in \Gamma$, $\mathcal{E} = \langle W_1,\ldots,W_n; u_1,\ldots,u_n \rangle$ be such that for given $\bar{t}_i \in \mathbb{R}^l_+$, $i = 1,\ldots,n$,

$u_i(w_i+t) = u_i(w_i+s\bar{t}_i)$, $t \in \mathbb{R}^l$, $s \in \mathbb{R}$

implies $u_i(w_i+t) = s$.

Observe u_i is a well-defined continuous utility function representing the preferences of agent i.

<u>Proportion 7</u> :- Given $\bar{t} \in \mathbb{R}^l_{++}$, let $\mathcal{E} \in \Gamma$ be an economic environment satisfying the above condition with $\bar{t}_i = \bar{t}$, for $i = 1,\ldots,n$. The problem

$$\underset{i=1,\ldots,n}{\text{Min}}\; u_i(W_i+x_i) \longrightarrow \text{Max}$$

$$\text{s.t. } x = (x_1,\ldots,x_i,\ldots,x_n) \in V(\mathcal{E})$$

has as the solution the Pareto-efficient egalitarian equivalent trade defined above.

<u>Proof</u> :- Choose a feasible net trade $\bar{x} \in V(\mathcal{E})$ such that

$$\underset{i=1,\ldots,n}{\text{Min}}\; u_i(W_i+\bar{x}_i) \geqslant \underset{i=1,\ldots,n}{\text{Min}}\; u_i(W_i+x_i)$$

where $x = (x_1,\ldots,x_n) \in V(\mathcal{E})$.

Let $u_i(W_i+\bar{x}_i) = \bar{s}_i$, $i=1,\ldots,n$

$\therefore$ $u_i(W_i+\bar{x}_i) = u_i(W_i+\bar{s}_i\,\bar{t})$, $i \in \{1,\ldots,n\}$

and $\bar{s} = \underset{i=1,\ldots,n}{\text{Min}}\; \{\bar{s}_i\} \geqslant \underset{i=1,\ldots,n}{\text{min}}\; u_i(W_i+x_i)$

for every trade $x = (x_1,\ldots,x_n) \in V(\mathcal{E})$.

By monotonicity and continuity there exists a $y = (y_1,\ldots,y_n) \in V(\mathcal{E})$ such that $u_i(W_i+y_i) = \bar{s}$, $i \in \{1,\ldots,n\}$.

Suppose $y \in V(\mathcal{E})$ is not Pareto-efficient. Then there exists a trade $z \in V(\mathcal{E})$, $z = (z_1,\ldots,z_n)$ such that

$$u_i(W_i+z_i) > \bar{s} \;, \forall\, i \in \{1,\ldots,n\}$$

$\therefore \min_{i=1,\dots,n} u_i(W_i+z_i) \; \bar{\delta}$, which is a contradiction.

Hence y is Pareto-efficient.

By construction y is egalitarian equivalent and $\bar{\delta}$ satisfies the condition in Proposition 1.

<u>Q.E.D.</u>

<u>Proposition 8</u> :- If the economic environment $\mathcal{E}$ is such that the preferences of each agent $i / i \in \{1,\dots,n\}$ is convex, then there is a Pareto-efficient egalitarian equivalent trade for which the average trade and the egalitarian-reference-vector lies on the same ray through the origin.

<u>Proof</u> :- Let

$T = \left\{ \sum_{i=1}^{n} x_i / x \in V(\mathcal{E}),\ x \text{ is a Pareto-efficient trade} \right\}.$

$$\text{Let } m(x) = \sum_{i=1}^{n} x_i \Big/ \left\| \sum_{i=1}^{n} x_i \right\| \qquad \text{if} \quad \sum_{i=1}^{n} x_i \neq 0$$

$$= 0 \qquad \text{if} \quad \sum_{i=1}^{n} x_i = 0.$$

Let $\bar{T} = \{ m(x) / x \in V(\mathcal{E}),\ x \text{ is a Pareto-efficient trade} \}.$

$\hat{T}$ = convex hull of $\bar{T}$.

Since $\bar{T}$ is compact, so is $\hat{T}$. We define a correspondence $F : \hat{T} \twoheadrightarrow \hat{T}$ a fixed point of which is a ray that satisfies the conclusion of Proposition 9. For t in $\hat{T}$ we define the set of average trades corresponding to PEEE trades with reference-vector on the ray through 't'. It is obvious that the existence of a fixed point of F concludes the proof of the Proposition. We shall check the conditions of Kakutani's fixed point theorem. The set $\hat{T}$ is compact and convex and $F(t) \neq \emptyset$, $F(t) \subseteq \hat{T} \; \forall \; t \in \hat{T}$. It is equally obvious that $F(t)$ is convex set for all $t \in \hat{T}$ and that $F(t)$ is upper semicontinuous instead of continuous (as in the case of strictly convex preferences).

Q.E.D.

4. Conclusion : We have thus achieved a characterization of the egalitarian equivalent (Pareto-efficient) solution for bargaining problems in economic environments. This extends earlier results from a restricted set of problems to a larger class of problems and thereby contributes to greater generality of existing results.

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