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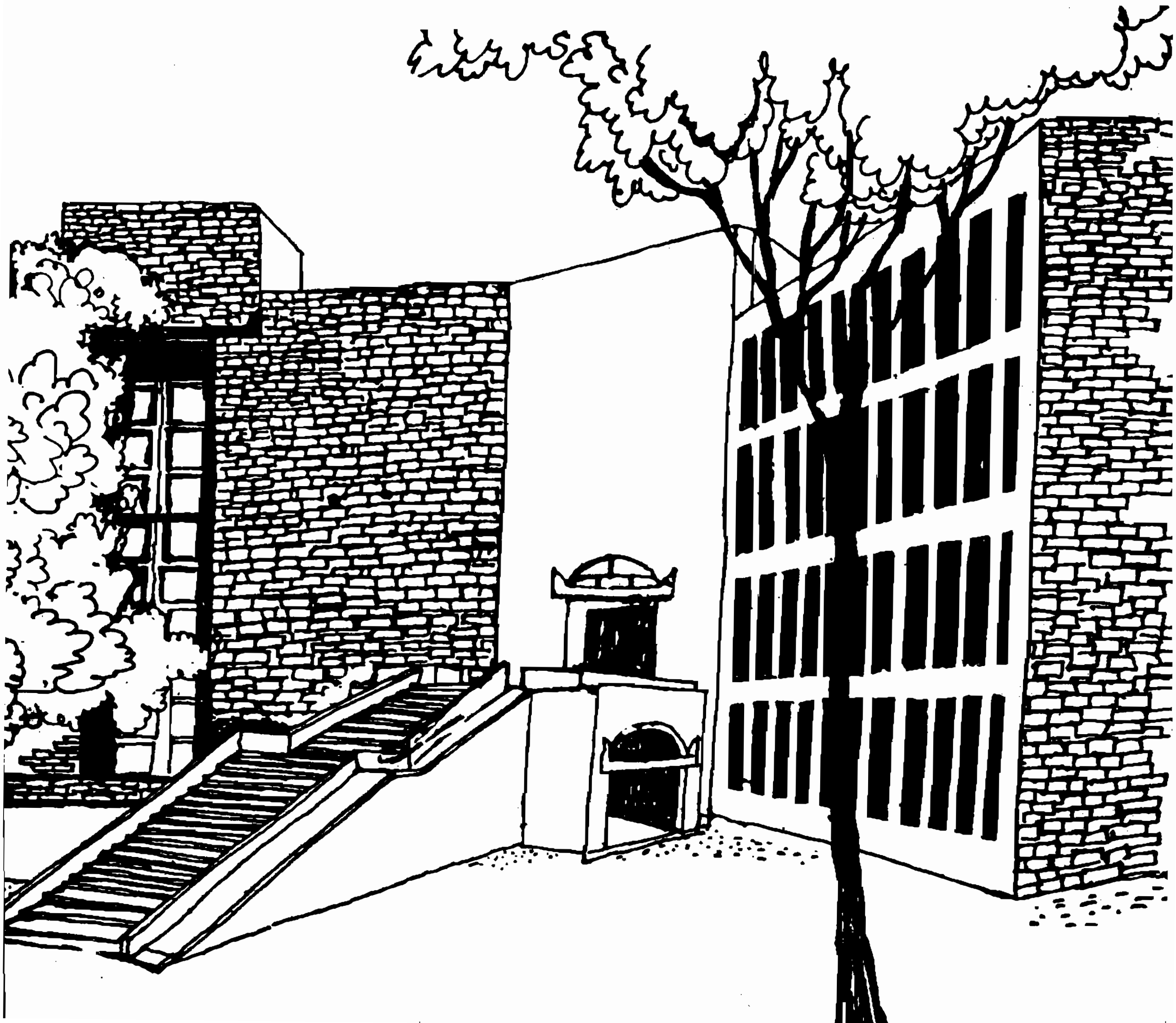

THE NON-MANIPULABILITY OF THE UTILITARIAN SOLUTION

By

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W P No.1062
October, 1992

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Abstract

In this paper we model threat bargaining problems as fixed threat bargaining games (with the threat point at the origin) and show that the utilitarian solution to threat bargaining problems is not manipulable.

1. Introduction :- A threat-bargaining game as defined for instance in Owen (1982), has been shown to reveal certain undesirable properties (as for instance in Lahiri (1988), Lahiri (1989) and some desirable properties (as for instance in Lahiri (1990), Lahiri (1991). The desirability of solutions to threat bargaining games was related to its non-manipulability as a solution concept. With perfect information and suitable domains the more well known solutions to bargaining problem e.g. Nash (1952), Kalai-Smorodinsky (1975) were shown to be manipulable. With incomplete information, these very solutions were shown to be non-manipulable, when the beliefs of the agents were suitably chosen.

The importance of threat-bargaining games lies in its ability to model many endowment manipulation games in pure exchange economies as discussed for instance in Postelwaite (1979), Yi (1991), although the solutions considered in these two papers are different from ours. Other applications of threat bargaining games can also be conceived.

It is in light of these considerations that we propose to show in this paper that the utilitarian solution to bargaining problems is non-manipulable.

2. The Model :- Following Moulin (1988), we develop the following framework of analysis :

Let a society consist of N (≥ 2) agents, where N belongs to the set of positive integers. The utility space of the agents is the non-negative orthant of Euclidean N-dimensional space denoted $\mathbf{R}^N_+$. Let Σ be a collection of subsets of $\mathbf{R}^n_+$. An element $S \in \Sigma$ is called a <u>bargaining problem</u> indicating a collection of utility possibilities for the agents from which society is required to make a choice. A <u>solution</u> is a function $F: \Sigma \to \mathbf{R}^N_+$ such that $\forall S \in \Sigma$, $F(S) \in S$.

In the sequel it will be found convenient to impose the following restrictions on Σ.

(i) S is strictly convex $\forall S \in \Sigma$

(ii) S is compact $\forall S \in \Sigma$

(iii) S is comprehensive $\forall S \in \Sigma$, i.e. $\forall S \in \Sigma$, $x \in S$, $0 \leq y \leq x$

$\Rightarrow y \in S$.

(iv) $\forall S \in \Sigma$, $\exists x \in S$ with $x >> 0$

To each problem $S \in \Sigma$, we can naturally associate the following N-tuple :

Define $M_i(S) = \max\{x_i / \exists (x_{-i}, x_i) \in S\}$, $i = 1, \ldots, N$ where $x_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_N) \in \mathbb{R}^{N-1}_+$ and $(x_{-i}, x_i) = (x_1, \ldots, x_{i-1}, x_i, x_{i+1}, \ldots, x_N) \in \mathbb{R}^N_+$.

Denote, $M(S) = (M_i(S))^N_{i=1}$. Clearly M is a function from Σ to $\mathbb{R}^N_+$.

Let u_i be the ith co-ordinate vector in $\mathbb{R}^N_+$. For each $S \in \Sigma$ and for each $\alpha_i \in [0, M_i(S)]$, define,

$S_i(\alpha_i) = (S - \alpha_i u_i) \cap \mathbb{R}^N_+$.

Clearly $S_i(\alpha_i) \in \Sigma$ for each $S \in \Sigma$, $\alpha_i \in [o, M_i(S)]$ and $i \in \{1, \ldots, N\}$.

Given $S \in \Sigma$, define a function

$P_S : \pi^N_{i=1} [0, M_i(S)] \to \mathbb{R}^N$ as follows :

$$P_S(\alpha_1, \ldots, \alpha_N) = F(\cap^N_{i=1} S_i(\alpha_i)) + (\alpha_1, \ldots, \alpha_N) \quad \text{if } \cap^N_{i=1} S_i(\alpha_i) \neq \emptyset \qquad (1)$$
$$= 0 \quad \text{if } \cap^N_{i=1} S_i(\alpha_i) = \emptyset.$$

The collection $\{P_S : S \in \Sigma\}$ is called a threat bargaining game.

Significant to our discussion is the concept of <u>manipulability</u> which is defined as follows :

A threat bargaining game $\{P_S : S \in \Sigma\}$ is said to be <u>manipulable</u> if there exists $S \in \Sigma$, $i \in \{1, \ldots, N\}$ and $\alpha_i \in (0, M_i(S))$ such that

$P^i_S(0, \ldots, 0, \alpha_i, 0, \ldots, 0) > P^i_S(0)$

where,

$P_S \equiv (P^1_S, \ldots, P^N_S)$.

3. The Main Result :- Let $\lambda_i > 0$, $i = 1, \ldots, N$ be given.

Denote $\lambda = (\lambda_i)^N_{i=1} >> 0$. Define,

$F_\lambda : \Sigma \to \mathbb{R}^N_+$ as $F_\lambda(S) = \arg\max_{x \in S} (\Sigma^N_{i=1} \lambda_i x_i)$.

It is easy to see that F_λ is well defined and is a bonafide bargaining solution.

Let $\{P_S : S \in \Sigma\}$ be the corresponding threat bargaining game.

Theorem :- $\{P_S : S \in \Sigma\}$ is nonmanipulable.

Proof :- Let $S \in \Sigma$. $i \in \{1,\ldots,N\}$ and $\alpha_i \in (0, M_i(S))$.

Then,

$F_\lambda(S(\alpha_i)) = F_\lambda(S) - \alpha_i u_i$.

since $\Sigma^N_{j=1} \lambda_j F^j_\lambda(S) \geq \Sigma^N_{j=1} \lambda_j x_j \forall (x_1,\ldots,x_N) \in S$

$\Leftrightarrow \Sigma_{j \neq i} \lambda_j F^j_\lambda(S) + \lambda_i [F^i_\lambda(S) - \alpha_i] \geq \Sigma_{j \neq i} \lambda_j x_j + \lambda_i (x_i - \alpha_i)$

$\forall (x_1, \ldots, x_N) \in S$

$\Rightarrow \Sigma_{j \neq i} \lambda_j F^j_\lambda(S) + \lambda_i [F^i_\lambda(S) - \alpha_i] \geq \Sigma^N_{j=1} \lambda_i x_i \forall (x_1, \ldots, x_N) \in S(\alpha_i)$

$\therefore P^i_S(0,\ldots,0,\alpha_i,0,\ldots,0) = P^i_S(0) \ \forall \alpha_i \in [0, M_i(S)]$.

i.e. $\{P_S : S \in \Sigma\}$ is non-manipulable.

Q.E.D.

Note : Here $F_\lambda(S) = F^j(S))^N_{j=1} \forall S \in \Sigma$.

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