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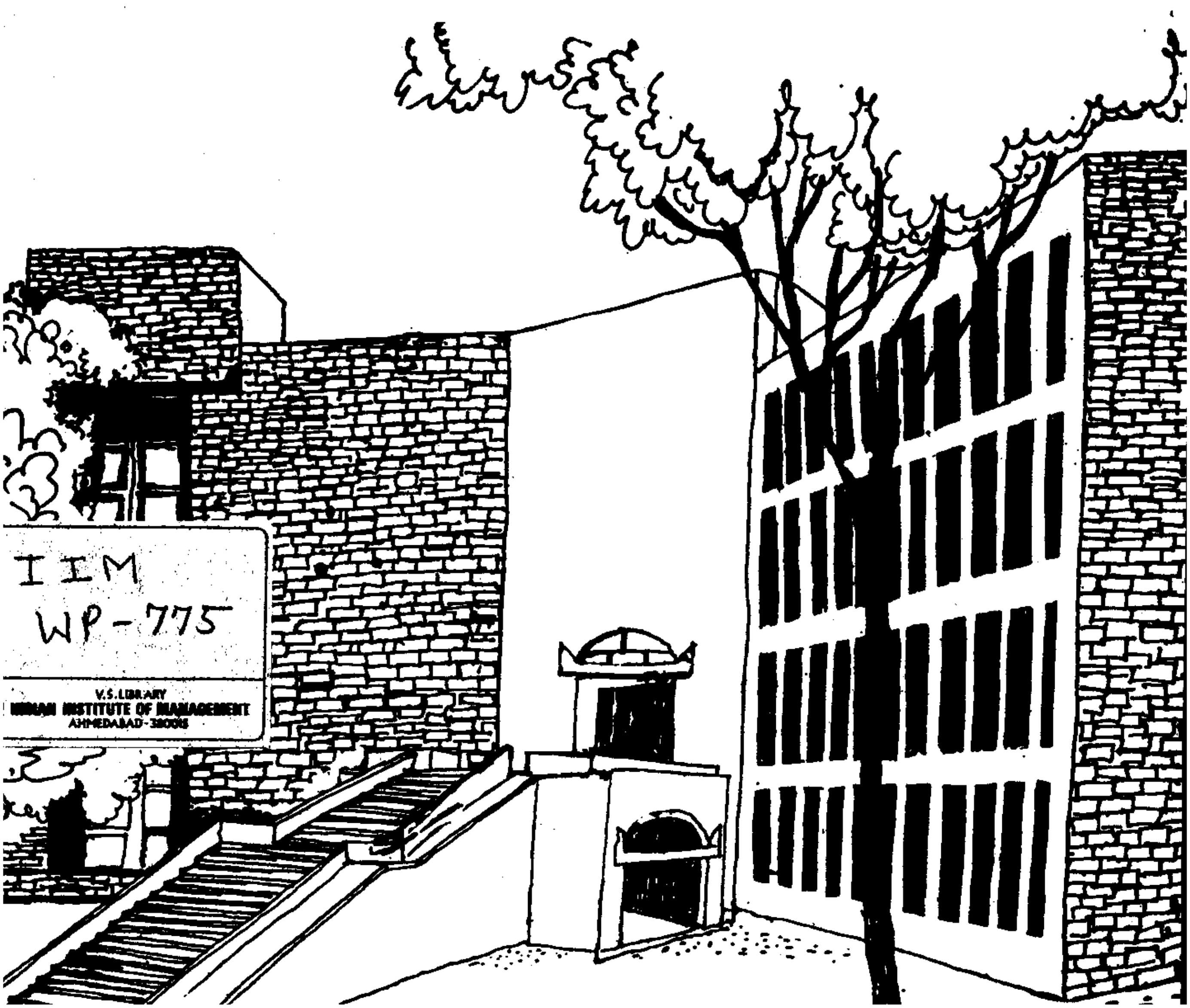

MONOTONICITY WITH RESPECT TO THE DISAGREEMENT POINT AND RISK SENSITIVITY OF A NEW SOLUTION TO NASH'S BARGAINING PROBLEM

By

Somdeb Lahiri

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INDIAN INSTITUTE OF MANAGEMENT
AHMEDABAD-380056
INDIA

ABSTRACT

We propose a solution to the bargaining problem which responds appropriately to certain changes in the disagreement point, for a fixed feasible set. If d_i increases, while for $j \neq i$, d_j remains constant, our solution recommends an increase in agent i's payoff, in agreement with intention. We also show that an increase in risk aversion is to the player's own advantage and to the disadvantage of the opponent in the two person case; to the disadvantage of all opponents in the multi-person generalization.

1. *Introduction*: A *2-person bargaining problem* is a pair (S,d) of a subset S of IR^2 and of a point d S. IR^2 is the *utility space*, S is the *feasible set*, and d is the *disagreement point*. If the agents unanimously agree on a point x of S, they obtain x. Otherwise, they obtain d. This is the framework of analysis adopted for the first time by Nash (1950), which permits the selection of a unique feasible outcome as the "*solution*" to a given bargaining problem.

Given a class of 2-person bargaining problems, a *solution* is a function F associating with every (S,d) in the class a point F(S,d) S representing the compromise reached by the agents. In some contexts, F (S,d) may be interpreted as the compromise recommended to the agents by some impartial arbitrator.

In this paper as in Lahiri (1988a) we introduce a solution concept which responds appropriately to changes in d, for fixed S. A detailed investigation of this property, known as monotonicity of bargaining solutions with respect to the disagreement point for the more well known solutions e.g. those of Nash (1950), Kalai-Smorodinsky (1975), Kalai (1977), is available in Thomson (1987). Given some agent i, suppose that d_i increases while d_j remains constant for $j \neq i$. Since d_i represents agent i's fallback position, one would expect agent i's final payoff to increase (or at least not to decrease). Interesting results relating monotonicity with respect to the disagreement point to various axioms proposed by Thomson and Myerson (1980), have been established by Livne (1985).

Before we propose our solution, let us note why the monotonicity property we are interested in, is important. As noted in Owen (1982) and shown in Lahiri (1988b,1988c), the monotonicity property is particularly relevant to situation in which each agent has some control over the position of the disagreement point. Such games are variously known as variable threat games or threat bargaining games. Since the disagreement point reflects the bargaining power of the players any increase in the disagreement payoff of player i should lead to an increase in his bargaining strength and consequent diminution in the bargaining strength of player $j \neq i$. The final outcome should be consistent with the changed bargaining strength of the players.

Having proposed the solution, we then proceed to analyse the risk-sensitivity property of our new solution. Kihlstrom, Roth and Schmeidler (1981), Roth (1979), Roth and Rothbloom (1982) analyse the risk sensitivity property of the two-person Nash (1950) and Kalai-Smorodinsky (1975) solutions. Nielsen (1984) generalizes the risk sensitivity property to more than two agents bargaining problems. A recent and inspiring survey of these results are to be found in Tijs and Peters (1985). In our analysis we study the risk-sensitivity property of our proposed solution for both the two-person as well as more than 2-person case. In both cases we observe that an increase in a player's risk aversion hurts all his opponents and helps the player himself.

2. <u>The Model</u> :- We will consider a class of problems defined below:

Let $W = \{(S,d)/S \subseteq \mathbb{R}^2$, S is convex, compact, and $x \in S$ with $x > d\}$.

Here, $X = (x_1,x_2) \geqq (y_1,y_2) = y$ means $x_i \geqq y_i$, $i = 1,2$

$X = (x_1,x_2) \geq (y_1,y_2) = y$ means $x \neq y$, $x_i \geqq y_i$, $i=1,2$

$X = (x_1,x_2) > (y_1,y_2) = y$ means $x_i > y_i$, $i = 1,2$

Let $\bar{W} = \{(S,d) \in W/\text{if } x \in S \text{ and } 0 \leqq y \leqq x, \text{ then } y \in S\}$

Such games (S,d) are known as <u>comprehensive games</u>.

Let $\bar{\bar{W}} = \{(S,d) \in \bar{W}/ \exists\ u \in S \text{ with } d > u\}$.

Such games (S,d) are known as <u>proper comprehensive games</u>.

Given $(S,d) \in W$, its <u>Nash</u> (1950) solution N (S,d) is the point where the product $(x_1-d_1)(x_2-d_2)$ is maximized for $x \in S$ with $x \geq d$; its <u>Kalai-Smorodinsky</u> (1975) solution outcome K(S,d) is the maximal point of S on the segment joining d to M(S,d), where for each, i, $M_i(S,d) = \{x_i/x \in S;\ x \geqq d\}$, i = 1,2 its Egalitarian (see Kalai (1977)) solution outcome E(S,d) is the maximal point x of S with $x_1-d_1 = x_2-d_2$.

We normalize the von-Neumann Morgenstern utility functions of the players so that the following blanket hypothesis holds in all our subsequent analyses:

<u>Hypothesis:</u> Let $Z(S) = (Z_1(S), Z_2(S))$ where $Z_i(S) = \min\{x_i/x \in S\}$.

Then $Z_i(S) = 0$, i = 1,2 for all $(S,d) \in \bar{\bar{W}}$.

In this paper we consider a solution F: $\bar{\bar{W}} \longrightarrow \mathbb{R}^2$ defined thus:

Let $F(S,d) = (F_1(S,d), F_2(S,d))$.

Then

(a) $$\frac{F_2(S,d)}{F_1(S,d)} = \frac{d_2}{d_1}$$

(b) $$\frac{x_2}{x_1} = \frac{d_2}{d_1}, \quad x > F(S,d) \text{ implies } x \notin S.$$

The Conditions defining the above bargaining solution are :

<u>Condition 1</u> : $F(S,d) \geqslant d$ for all $(S,d) \in \bar{\bar{W}}$

<u>Condition 2</u> : Let $a_1, a_2 \; IR_{++}$ and $(S,d), (S',d') \in \bar{\bar{W}}$ and let $d'_i = a_i d_i, \; i = 1,2, \; S' = \{ x \in IR^2 / x_i = a_i y_i, \; i = 1,2, y \in S \}$

Then $F_i(S',d') = a_i F_i(S,d), \; i = 1,2$

<u>Condition 3</u> : If $(S,d) \in \bar{\bar{W}}$ satisfies $d_1 = d_2$ and $(x_1,x_2) \in S$ if and only if $(x_2,x_1) \in S$, then $F_1(S,d) = F_2(S,d)$.

<u>Condition 4</u> :- If $x > F(S,d)$ then $x \notin S$.

<u>Condition 5</u> :- Let (S,d) and (S',d') satisfy (a) $d_1 = d'_1$, $d_2 \leqq d'_2$ and (b) $S \subseteq S'$. Then $F_2(S',d') \geqq F_2(S,d)$. If in addition $S = S'$, then $F_1(S',d') \leqq F_1(S,d)$ with $F(S',d') \neq F(S,d)$ if $(d_1,d_2) \neq (d'_1,d'_2)$.

Condition 1 stipulates individual rationality.

Condition 2 requires that the solution should be invariant with respect to positive linear utility transformation.

Condition 3 imposes symmetry

Condition 4, weak Pareto optimality, states that $x \in S$ is weakly Pareto optimal if there is no $x' \rangle S$ for which $x' \rangle x$. That is, a point is not weakly Pareto optimal if there is another point that gives more to each player.

Condition 5, is the monotonicity condition we invoke in our analysis and which has been discussed earlier.

Our basic theorem is the following:

Theorem 1 : The function $F(S,d)$ is well defined, satisfies Conditions 1 to 5 and is the only function to satisfy these conditions.

<u>Proof:</u> The function $F(S,d)$ is well defined, due to the convexity and compactness of S. That it satisfies condition 4 is apparent from its definition. Condition 1 follows from the fact, that (S,d) in definitely a comprehensive game (in fact a proper comprehensive game) together with the fact that there exists $x \in S$ such that $x > d$. These two properties imply that d belongs to the interior of S and hence Condition 1 is satisifed, due to the definition of $F(S,d)$ and the definition of a Symmetric game.

To see that Condition 5 is satisfied, suppose that (S,d) and (S',d') are two games satisfying $Z(S) = Z(S')$, $d_1 = d'_1$, $d_2 \leqq d'$

$$\frac{d'_2 - Z_2(S')}{d'_1 - Z_1(S')} \geqq \frac{d_2 - Z_2(S)}{d_1 - Z_1(S)}$$

Let us call the ray joining d to Z (S,d) the defining ray of d and the ray joining d' to Z(S,d') the defining ray of d'. Thus the defining ray of d' has steeper slope than the defining ray of d. Hence the defining ray of d' intersects the upper right weakly Pareto optimal frontier of S. Since S S' and the defining ray of both d and d' are positively sloped, we may conclude the following:

a) If ϕ (d ,S) is the ordinate of the point of intersection of the defining ray of d with the upper right weakly Pareto optimal frontier of S, then $\phi(d,S) = F_2(S,d)$

b) If ϕ (d',S) is the ordinate of the point of intersection of the defining ray of d' with the upper right weakly Pareto optimal frontier of S, then $\phi(d',S) \geqq \phi(d,S)$.

c) If $\phi(d',S')$ is the ordinate of the point of intersection of the defining ray of d' with the upper right Pareto optimal frontier of S', t then,

$$\phi(d',S') = F_2(S',d')$$

$$\text{and} \quad \phi(d',S') \geqq \phi(d',S)$$

$$\text{Hence} \quad F_2(S',d') \geqq F_2(S,d)$$

If in addition S = S', then $F_2(S',d') \geqq F_2(S,d)$ and the Pareto optimality of the solutions guarantee that $F_1(S',d') \leqq F_1(S,d)$. Further since $F(S',d')$ and $F(S,d)$ lie on distinct rays, $F(S',d') \neq F(S,d)$.

To see that only F(S,d) satisfies the various conditions, it is first shown to hold for games in which d = (1,1) and $F_1(S,d) = F_2(S,d)$. Extension to the full class of games follows from the requirement that the solution satisfy invariance to affine transformation (Condition 2).

Denote the true solution by $F^*(S,d)$ and let S' be the convex hull of the set of points $\{(0,1), (1,F_2(S,d)), (1,0), (0,0), (F_1(S,d), 1), F(S,d)\}$. By conditions 1,3 and 4, $F^*(S',d) = F(S,d)$. But $S' \subseteq S$ and Condition 5 implies $F_1^*(S,d) = F_1(S',d)$, $i = 1.2$. This follows because the threat point of both games being equal, the requirement on the threat point (Condition 5) easily follows. Hence $F^*(S,d) = F(S,d)$ as was required to be proved.

Q.E.D.

3. The Multi-person Generalization of the New Solution :-

An n-person bargaining game is a pair (S,d), where S is a Compact Convex Subset of IR^n, d is a point in S, and there is at least one point b in S with $d < b$.

There,

$$x = (x_1,\ldots\ldots\ldots,x_n) \geqq y = (y_1,\ldots\ldots,y_n) \text{ means } x_i \geqq y_i, \; i=1,\ldots,n$$

$$x = (x_1,\ldots\ldots,x_n) \geq y = (y_1,\ldots\ldots,y_n) \text{ means } x \geqq y \text{ but } x \neq y.$$

$$x = (x_1,\ldots\ldots,x_n) > y = (y_1,\ldots\ldots,y_n) \text{ means } x_i > y_i, \; i = 1,\ldots,n$$

The interpretation of an n-person bargaining game is the same as in the two-person case.

Let,

$$W = \{(S,d)/S \subseteq IR^n, S \text{ is convex, compact, } d \in S \text{ and } \exists\, x \in S \text{ with } x > d\}$$

$$\bar{W} = \{(S,d) \in W/\text{if } x \in S \text{ and } 0 \leq y \leq x \text{ then } y \in S\}$$

and $$\bar{\bar{W}} = \{(S,d) \in \bar{W}/ \exists\, u \in S \text{ with } d > u\}.$$

Games in $\bar{\bar{W}}$ are referred to as proper comprehensive games.

We shall assume that the von-Neumann Morgenstern utility functions of the players are normalized so as to be consistent with the following blanket hypotehsis throughout our analysis.

<u>Blanket Hypothesis</u>:- Let $Z_i(S) = \min \{x_i / x \in S\}$, $i = i, \ldots, n$.
For all $(S,d) \in \bar{\bar{W}}$, $Z_i(S) = 0$, $i = 1, \ldots, n$.

The solution we consider for the n-person situation is a function $F : \bar{\bar{W}} \to IR^n$ defined thus:

$F(S,d) = (F_1,(S,d), \ldots, F_n(S,d))$ satisfies the conditions

(a) $$\frac{F_i(S,d)}{d_i} = \frac{F_j(S,d)}{d_j} \quad \forall \; i,j \in \{1, \ldots, n\}.$$

(b) $$\frac{x_i}{d_i} = \frac{x_j}{d_j} \quad \forall \; i, j \in \{1, \ldots, n\}.$$

and $x = (x_1, \ldots, x_n) > F(S,d)$ implies $x \notin S$.

The conditions defining the above bargaining solution are :

<u>Condition 1'</u>: $F(S,d) > d$ for all $(S,d) \in \bar{\bar{W}}$

<u>Condition 2'</u>: Let $a_i \in IR_{++}$ for $i = 1, \ldots, n$ and (S,d), $(S',d') \in \bar{\bar{W}}$ where $d'_i = a_i d_i$, $i = 1, \ldots, n$ and $S' = \{x \in IR^n / x_i = a_i y_i,\ i = 1, \ldots, n,\ y = (y_1, \ldots, y_n) \in S\}$

Then, $F_i(S',d') = a_i F_i(S,d)$, $i = 1, \ldots, n$.

<u>Condition 3'</u>: If $(S,d) \in \bar{\bar{W}}$ satisfies $d_1 = d_2 = \ldots = d_n$ and $x = (x_1, \ldots, x_n) \in S$ if and only if $x_\pi = (x_{\pi(1)}, \ldots, x_{\pi(n)}) \in S$ for all permutations $\pi : \{1, \ldots, n\} \to \{1, \ldots, n\}$, then $F_i(S,d) = F_j(S,d)$ $\forall \; i, j \in \{1, \ldots, n\}$.

<u>Condition 4'</u>: If $x > F(S,d)$ then $x \notin S$.

Condition 5': Let (S,d) and (S',d') satisfy

(a) $d_j = d'_j,\ j \neq i;\ d_i \leq d'_i$

(b) $S \leq S'$.

Then $F_i(S',d') \geq F_i(S,d)$.

If in addition, $S = S'$, then $F_j(S',d') \leq F_j(S,d)$ for all $j \neq i$ with $F(S',d') \neq F(S,d)$ if $d \neq d'$.

The interpretation of these conditions is the same as before. The following theorem can now be established.

Theorem 2 : The function $F : \bar{\bar{W}} \longrightarrow IR^n$ satisfying (a) and (b) is well defined, satisfies Conditions 1' to 5' and is the only function to satisfy these conditions.

Proof : Same as the proof of Theorem 1 with appropriate modification.

4. Risk Aversion :- For $(S,d) \in \bar{\bar{W}}$ and $i \in N = \{1,\ldots,n\}$, let $C_i(S,d)$ be the family of differentiable, concave and strictly increasing functions from IR_+ to IR_+ that has value 0 at 0; and for each $k_i \in C_i(S,d)$, let

$$\hat{x} \equiv K_i(x) = (x_1,\ldots,x_{i-1}, k_i(x_i), x_{i+1},\ldots,x_n) \text{ for each } x \in S$$

$$(\hat{S},\hat{d}) = K_i(S,d) = (\{K_i(x) : x \in S\},\ K_i(d)).$$

If $\hat{S}$ is convex, then $(\hat{S},\hat{d})$ is an n-person bargaining game. The assumption that $(S,d) \in \bar{\bar{W}} \subset \bar{W}$ guarantees that $\hat{S}$ is convex. In such a situation we say that the game $(\hat{S},\hat{d})$ has been derived from the game (S,d) by replacing player i, by a more risk averse player. This defintion has been motivated by earlier work done by Yaari (1969), Kihlstromm and Mirman (1974) and Roth (1979) on risk averse utility functions; the work of Nielsen (1984) in particular on risk-sensitivity property of bargaining solutions.

5. <u>The Situation With Two Players</u> : Suppose there are just two players and suppose without loss of generality that player 2 becomes more risk averse or is replaced by a more risk-avers player. By methods similar to the ones used in Theorem 3.3 of Jansen and Tijs (1983), we can show that our solution F satisfying (a) and (b) is continuous with respect to the Hansdorff metric topology on W. In the following theorem we shall show that the new outcome is preferred to the old outcome by the new player 2 (or, for that matter, by the old player 2). Player 1 stands to lose in the process and it is in this sense that player 2 is at an advantage.

<u>Theorem 3</u> : Suppose that $(\hat{S},\hat{d})$ is a game derived from (S,d) by making player 2 more risk-averse. Let $Y = F(S,d)$ and let x S be chosen such that $\hat{x} = F(\hat{S},\hat{d})$. Then $x_1 < y_1$.

<u>Proof of Theorem 3</u> : Since F is invariant under positive affine transformation, it can be assumed that $d = 1$ and $k(1) = \hat{d}_2 = 1 = \hat{d}_1$. Since $\hat{x} = F(\hat{S},\hat{d})$ and $y = F(S,d)$, it follows that

$$\phi(y_1) = y_1$$

where $u_1 \mapsto \phi(u_1)$ is the nonincreasing, concave function for which $\{(u_1, \phi(u_1)) : u_1 \in [\underline{u}_1, \bar{u}_1]\}$ is the set of all weakly Pareto-optimal outcomes of S. We can without loss of generality assume that ϕ is a differentiable since differentiable weakly Pareto-optimal boundaries are dense in the class of continuous weakly

Pareto - optimal boundaries and F is continuous in the Hausdorff metric - topology.

Clearly $\hat{x}$ satisfies the condition

$$k(\phi(x_1)) = x_1$$

where $\hat{x}_1 = x_1$, $\hat{x}_2 = k(\phi(x_1)) = k(x_2)$ and $\hat{x} = (\hat{x}_1, \hat{x}_2)$.

Let,

$$A(u_1) = k(\phi(u_1)) - u_1$$

Clearly, $A(x_1) = 0$.

Also,

$$A(y_1) = k(\phi(y_1)) - y_1$$

Now, the concavity and nonnegativeness of k implies

$$\frac{k(a)}{k(b)} - \frac{a}{b} \gtreqless 0$$

according as $\frac{k(a)}{a} - \frac{k(b)}{b} \gtreqless 0$

according as $a \lesseqgtr b$.

Since $\phi(y_1) > 1$, $k(\phi(y_1)) < \phi(y_1)$ as $k(1) = 1$

$\therefore A(y_1) < 0.$

Further,

$$A'(u_1) = k'(\phi(u_1)).\ \phi'(u_1) - 1 < 0$$

which is easily observed since $\phi'(.) \leq 0.$

Thus, $A(x_1) = 0$, $A(y_1) < 0$ and $A'(u_1) < 0$ implies $x_1 < y_1$ as was required to be proved.

Q.E.D.

In the case where $n = 2$, i.e., the player who becomes more risk averse has only one opponent, it follows that this opponent does prefer the old outcome.

6. <u>The Situation With More Than Two Players:</u>

With more than two players an analogous result can be obtained. Nielsen (1984) obtains risk-sensitivity results for the Nash and the Kalai-Smorodinsky solution with more than two players. As before we show that if player i becomes more risk averse or is replaced by a more risk averse player, then the older outcome is not preferred to the new one by any of the opponents and both the old player i and the new player i prefers the old outcome. The old and/or the new player i can only be indifferent between the outcomes if all players are indifferent between them.

<u>Theorem 4 :</u> Let $(S,d) \in \overline{\overline{W}}$. Then $(\hat{S},\hat{d}) \in W$. Let $y = F(S,d)$, and let $x \in S$ be such that $\hat{x} = F(\hat{S},\hat{d})$. Then $x_j \leq y_j$ for $j \neq i$ and $x_i \geq y_i$. If $x_i = y_i$, then $x = y$.

<u>Proof of Theorem 4 :</u> Clearly $(\hat{S},\hat{d}) \in \overline{\overline{W}}$. To see that $\hat{S}$ is convex, let $p,\ q \in S$, $t \in [0,1]$, and put $v = tp + (1-t)\ q$. Since k is concave, $tk(p_i) + (1-t)k\ (q_i) \leq k(v_i)$, so that $\hat{d} \leq t\hat{p} + (1-t)\hat{q} \leq \hat{v}$. But since (S,d) has disposable utility, this implies that $t\hat{p} + (1-t)\ \hat{q} \in \hat{S}$. It can be assumed that $d = (1,1)$, $k(1) = 1$, and $k(0) = 0$, because F is invariant under positive linear transformations. Then $Z(\hat{S}) = Z(S)$. Since k is concave, $\frac{k(t)}{t}$ is a decreasing function of t. As before, let

$x_1 = \phi(x_2,\ldots\ldots,x_n)$, and let us assume that player 1 is replaced by a more risk averse player. Let ϕ be the Pareto-optimal surface of S, and as in the proof of Theorem 3 (for the same reasons as before) we may assume that ϕ is a differentiable function.

Clearly,

$$\phi(y_2,\ldots\ldots,y_n) = y_2 = \ldots\ldots = y_n.$$

Similarly,

$$k(\phi(x_2,\ldots\ldots,x_n)) = x_2 = \ldots\ldots = x_n.$$

Let,

$$A(u_2,\ldots\ldots,u_n) = k(\phi(u_2,\ldots\ldots,u_n)) - u_2.$$

$$A(x_2,\ldots\ldots,x_n) = 0 \text{ and}$$

$$A(y_2,\ldots\ldots,y_n) = k(\phi(y_2,\ldots\ldots,y_n)) - y_2$$

$$\frac{k(\phi(y_2,\ldots\ldots,y_n))}{\phi(y_2,\ldots\ldots,y_m)} < 1 = \frac{k(1)}{1}$$

$$\therefore\ k(\phi(y_2,\ldots,y_n)) < \phi(y_2,\ldots,y_n).$$

$$\therefore\ A(y_2,\ldots\ldots,y_n) = k(\phi(y_2,\ldots,y_n) - y_2 < 0$$

Further,

$$dA(u_2,\ldots\ldots,u_n) = k'(\phi(u_2,\ldots,u_n))\, d\phi(u_2,\ldots\ldots,u_n) - 1$$

Since ϕ is the equation of the Pareto optimal surface, $\frac{\partial \phi}{\partial u_2} < 0 \quad \forall i \in \{2,\ldots,n\}$.

$$\therefore\ \frac{\partial A}{\partial x_i}(u_2,\ldots\ldots,u_n) < 0 \quad \forall\ i \in \{2,\ldots\ldots,n\}.$$

$A(y_2,\ldots\ldots,y_n) < 0$, $A(x_2,\ldots\ldots,x_n) = 0$ and $\frac{\partial A}{\partial u_j}(u_2,\ldots\ldots,u_n) < 0$ implies

$x_j \leq y_j$ for some $j \in \{2,\ldots\ldots,n\}$.

But $x_2 = \ldots\ldots = x_n$ and

$y_2 = \ldots\ldots = y_n$

implies,

$$x_j \leq y_j \quad \forall\ j \in \{2,\ldots\ldots,n\}.$$

$\therefore\ X_1 = \phi(x_2,\ldots\ldots,x_n) \geq \phi\ (y_2,\ldots\ldots,y_n) = y_1$, since ϕ is the equation of the Pareto optimal surface.

Now by going to the limit for the general case, we get $X_1 \geq y_1$ as was required to be proved.

Q.E.D.

FOOT NOTE

This paper is a revised version of two earlier working papers, Nos. 724 & 769 of the Indian Institute of Management, Ahmedabad.

The idea behind the second half of this paper grew out of an informal discussion with Jim Jordan. However, I hold myself solely responsible for whatever lapses that remain.

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