Information in the Term Structure – The Indian Evidence (I): Modeling the Term Structure and Information at the Short End for Future Inflation

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Abstract

This study is first in part of an on-going work on assessing the information content of the term structure in India for future inflation, future short rates and real interest rates.

In this paper, first the Indian term structure is modeled using three alternative specifications and changes in slope of the term structure at the short-end assessed for forecastability of change in inflation.

For the first time in the Indian context, two atheoretical (Nelson and Siegel, 1987 and Svensson, 1994) models are compared against empirical implications of a general equilibrium (Cox, Ingersoll and Ross, 1985) model. While Svensson is seen to offer no improvement over Nelson-Siegel, Cox-Ingersoll-Ross comes out as marginally superior to both on the criteria of mean absolute pricing and yield errors (both in-sample and out-of-sample), behaviour of the short and the long rates, stability of the parameters and behaviour of forward rates for maturities 1 – 8 years. This is encouraging because models like Nelson-Siegel and Svensson are *designed* to fit the observed yield curves, while Cox-Ingersoll-Ross is a *theoretical* model derived from intertemporal description of a competitive economy.

On the information content of the term structure, in the sample under study, change in the slope of the term structure seems to have no information for inflation changes over the horizon 1 month to 2 years. Results could be sample and/or sampling-frequency specific. Results for the long-end of the term structure (from a bigger sample) follow.

[Preliminary draft. Please do not quote.]

I. Introduction

It is aptly established that while yield curve has almost no ability to forecast future inflation changes for short horizons, at horizons greater than a year the yield curve does contain a great deal of information regarding both future paths of inflation[1] as well as future short rates[2]. To analyze the information content of the term structure of interest rates regarding future short term interest rate changes and future inflation, however, one needs a reliable term structure model.

Although currently the National Stock Exchange (NSE) publishes daily estimates of the term structure based on the Nelson and Siegel (1987; henceforth NS) specification, it is not known how well it compares with other models in the literature. This study intends to provide some evidence on comparative performance of three popular models, including NS. Further, this study also proposes to assess the information content of the estimated term structures for forecastability of future inflation.

The plan of the paper is as follows. **Section II** motivates the importance of a term structure for monetary policy analysis. **Section III** discusses the methodology of estimation of the term structures given the choice of specifications in the study. **Section IV** presents the results of the estimated term structures and discusses them on select criteria of evaluation. **Section V** presents some evidence on the forecastability of future inflation**. Section VI** concludes with a summary of results and scope for further work.

II. Motivation

Irving Fisher propounded that in a world void of uncertainty, the one period nominal interest rate is the real return plus the anticipated rate of inflation. The (expected) real rate of interest is the nominal rate less the expected rate of inflation:

$$E(r) = i - E(\pi) \qquad \textbf{[1]}$$

where $E(r)$ is the expected real interest rate; i is the nominal interest rate and $E(\pi)$ is expected inflation.

Equation **[1]** is the Fisher equation. This equation suggests that the expected interest rates changes in proportion to the changes in expected inflation or, alternatively, the (expected) real interest rates are invariant to (expected) inflation.

If the Fisher effect holds, then movements in the short-term interest rates primarily reflect fluctuations in expectations of inflation. This is also compatible with the neoclassical view of the determinants of real rate, according to which the real interest rate is unaffected by changes in money supply, and continues to be determined purely by non-monetary factors. This also explains the parallel movement of the long-term interest rate and inflation.[3]

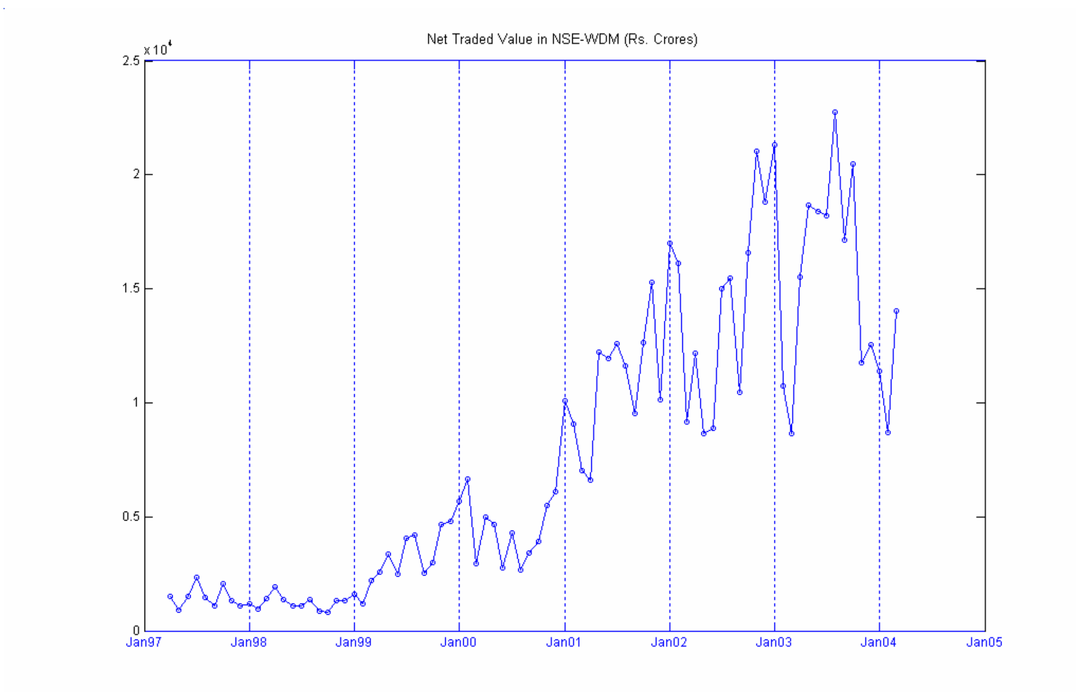
If one works under the assumptive framework of a constant long term real rate and identical inflation expectations for both the short-term and the long-term, then the changes in yield spread becomes identical with changes in the short term real rate. As Mishkin (1990a) in his survey[4] finds, yield spread[5] does contain information about future short-term interest rate changes.

[1] see Fama (1984, 1990), Mishkin (1990a, 1990b, 1991) and Jorion and Mishkin (1991) amongst others
[2] see Fama (1984), Fama and Bliss (1987) and Campbell and Shiller (1991) amongst others
[3] for empirical evidence see Fama (1975), Fama and Gibbons (1982), Mishkin (1992) among others
[4] for examples see Mishkin (1990a) and studies listed in footnote 2
[5] as defined by the difference between a long-term and a short-term rate

As regards the ability of yield curve to forecast changes in future inflation both Fama (1990) and Mishkin (1990b, 1990c) note that while yield curve has almost no ability to forecast future inflation changes for short horizons, at horizons greater than a year the yield curve does contain information regarding future inflation. This implies that at longer maturities the steepening of the yield curve is an indication of future inflationary pressures. Indeed, as Estrella and Mishkin (1995) find, that the yield spread serves as a very good leading indicator for the monetary policy stance of a central bank.

For lack of liquidity in the bond markets till the recent past (see **Figure 1** below for trading activity in the Wholesale Debt Market (WDM) segment of the NSE) there hasn't been any study on the information content of the term structure. In fact, even the term structure estimation in India is a rather nascent affair[6].

Figure 1



This study purports to evaluate three alternative term structure models to enable the assessment of information content of the term structure. The estimated term structure would be evaluated from the point of view monetary policy analysis.

While at one level this study for the first time compares the performance of atheoretical and a general equilibrium model on the Indian data, at the other, this study compares performance of alternative *parsimonious* specifications. Earlier studies by Thomas and Saple (2000) and Subramanian (2001) compare the performance of over-parameterized splines specifications with parsimonious NS and extended NS respectively, but not *amongst* competing parsimonious models.

The section that follows describes the methodology employed to estimate the term structure from the market price of the Government Securities (G-Secs) traded in the Wholesale Debt Market

[6] if one excludes the studies analyzing the stochastic dynamics of short rates (e.g. Varma, 1997 and Apte, 2001)

(WDM) segment of the NSE. The sample period is Jun, 2001 to Jun, 2003 – in total 608 trading days.

III. Methodology

Estimating a term structure, Bliss (1997) notes, requires decision on the following aspects:

1. A Pricing function
2. A functional form for the discount/rate function
3. Estimation technique

Along with the above points, one also needs well defined criteria for evaluation of the estimated term structure. This is important, because, for reasons discussed in detail in Dahlquist and Svensson (1996) and touched upon later in this section, criteria for evaluation of a term structure for the purpose of pricing derivatives/arbitrage decisions are considerably different for one for monetary policy analysis. Thus, the fourth decision aspect becomes:

4. Criteria of evaluation

3.1 The Pricing Function

In absence of arbitrage, price of a default-risk free bond can be written as:

$$P = \sum_{m=1}^{M} c_m \delta_m \qquad \textbf{[2]}$$

where M is the time to maturity of the bond, c_m is the cash flow received at time m, and δ_m is what is called the "discount function" in the term structure literature. The above equation relates the discounted cash flows from the bond in discrete time periods to the price of the bond. It is a rather straight forward matter to convert "discount function" to a "rate function" using the following equation:

$$r(m) = -\frac{\ln(\delta_m)}{m} \qquad \textbf{[3]}$$

Since, conditions for perfect markets don't exist in reality, and cash flows are received only at discrete times, in practice one needs to give a stochastic form to equation **[2]**, such as:

$$P = f[c_m, r(m)] + \varepsilon \qquad \textbf{[4]}$$

where ε is the "error" term and accounts for whatever is not captured in the function *f* about how bonds are priced. Bliss (1997) uses the term "omitted pricing factors" for

"…factors which have been omitted from the bond pricing equation which nonetheless impact the pricing of bonds[7]"

Similar pricing functions have been used by Bolder and Streliski (1999) for Canada and Darbha, Roy and Pawaskar (2003a, 2003b) and Thomas and Saple (2000) for India.

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[7] R. R. Bliss (1997), "Testing Term Structure Estimation Methods", Advances in Futures and Options Research, 9

3.2 The Discount Rate Function

The next decision in the exercise of term structure estimation involves the selection of a functional form for the discount rate function.

In the literature a number of functional forms exist to derive the zero-coupon and forward curves from observed data, with each one providing starkly different shapes for these curves. Popular examples include McCulloch (1971, 1975) and variants, NS and Svensson (1994). As suggested earlier, in this study following specifications have been used viz.

- NS
- Svensson
- Cox-Ingersoll-Ross (1985; henceforth CIR)

A glaring exclusion in the above list is the cubic splines technique (and its variants) propounded first by McCulloch (1971, 1975). The reason for its exclusion is the 'purpose' for which the term structure is to be modelled. Dahlquist and Svensson (1996) while Modeling term structure for monetary policy analysis discuss the disadvantages of using the splines technique for term structure Modeling for monetary policy analysis. For India, Thomas and Saple (2000) find that the cubic splines technique performs better (only) for in-sample data and

> *"…the performance metrics suggest that NS might be a more robust measure of the term structure for India given the current levels of liquidity…*[8]*"*

Subramanian (2001) also finds that the NS specification out-performs both cubic splines (with variable roughness penalty) and B-splines technique on the criterion of mean absolute error.

Darbha, Roy and Pawaskar (2003a, 2003b) discuss in detail the issues in estimating and pricing G-Secs traded in NSE WDM, but they provide results only for the NS specification. Darbha (2003) uses a stochastic frontier approach and finds improved error statistics when compared with other existing Indian studies. What follows is a brief discussion on models to be estimated in this study.

3.2.1 NS

NS assume that the instantaneous forward rate is the solution to a second order differential equation with two equal roots. The forward rate function used by NS is:

$$f(m;b) = \beta_0 + \beta_1 \exp(-m/\tau_1) + \beta_2 \frac{m}{\tau_1} \exp(-m/\tau_1) \qquad \textbf{[5]}$$

where $b \equiv (\beta_0, \beta_1, \beta_2, \tau)$ is the vector of parameters to be estimated. The spot rate function can in turn be derived by integrating the above equation. This gives:

$$s(m;b) = \beta_0 + (\beta_1 + \beta_2) \frac{1 - \exp(-m/\tau_1)}{m/\tau_1} - \beta_2 \exp(-m/\tau_1) \qquad \textbf{[6]}$$

The spot rate function has four parameters. While β_0 and β_0 + β_1 are implied long-rate and short-rate respectively, β_2 gives the medium term component of the yield curve, and along with τ defines the shape of the curve. The possible shapes of the term structure that result as

[8] S. Thomas and V. Saple, "Estimating the Term Structure of Interest Rates in India", unpublished paper IGIDR, 2000, p.16

parameters vary can be found in NS, Svensson and Bolder and Streliski (1999) and won't be discussed here.

3.2.2 Svensson (also referred to as Extended NS)

Svensson adds a fourth term to the forward rate function given by NS, with two additional parameters, (β_3, τ_2), thereby adding to the flexibility of the shape of the term structure (possibility of a second 'hump' – or what is often referred to as an *S*-shaped curve in the literature – with β_3 and the other time decay parameter, τ_2). The corresponding functions are then given as:

$$f(m;b) = \beta_0 + \beta_1 \exp(-m/\tau_1) + \beta_2 \frac{m}{\tau_1}\exp(-m/\tau_1) + \beta_3 \frac{m}{\tau_2}\exp(-m/\tau_2) \quad \textbf{[7]}$$

$$s(m;b) = \beta_0 + (\beta_1 + \beta_2)\frac{1-\exp(-m/\tau_1)}{m/\tau_1} - \beta_2 \exp(-m/\tau_1) + \beta_3 \frac{1-\exp(-m/\tau_2)}{m/\tau_2} - \beta_3 \exp(-m/\tau_2) \quad \textbf{[8]}$$

3.2.3 Empirical Implications of the Cox-Ingersoll-Ross Model

The dynamics of the interest rate process in the CIR model is given as[9]:

$$dr_t = \kappa(\theta - r)dt + \sigma\sqrt{r}dz \quad \textbf{[9]}$$

CIR, just like other affine models, in absence of arbitrage, results in the following pricing equation:

$$P[r,t,T] = A[t,T]e^{-B[t,T]r} \quad \textbf{[10]}$$

where for $\tau = T - t$

$$A[t,T] = \{\frac{\phi_1 \exp(\phi_2 \tau)}{\phi_2[\exp(\phi_1 \tau - 1] + \phi_1]}\}^{\phi_3} \quad \textbf{[11]}$$

$$B[t,T] = \{\frac{\exp(\phi_1 \tau) - 1}{\phi_2[\exp(\phi_1 \tau) - 1] + \phi_1}\} \quad \textbf{[12]}$$

where

$$\phi_1 \equiv [(\kappa + \lambda)^2 + 2\sigma^2]^{1/2} \quad \textbf{[13]}$$

$$\phi_2 \equiv (\kappa + \lambda + \phi_1)/2 \quad \textbf{[14]}$$

$$\phi_3 \equiv 2\kappa\theta/\sigma^2 \quad \textbf{[15]}$$

[9] κ is the mean reversion coefficient, θ is the mean of the process, *r* is the instantaneous short rate, σ^2 is the scale factor for variance of *r*, and λ is the price of risk associated with *r*.

Value of a coupon bond can then be written as:

$$V[t,c,d] = \sum c_i P(r,t,d_i) \qquad \textbf{[16]}$$

where d is the vector of coupon payment dates.

Then, given the prices of the traded bonds, one can estimate the parameters ϕ_1, ϕ_2, ϕ_3 and r (though for actual dynamics it is not possible[10] to separately identify the parameters, θ, κ and λ). The long-rate and volatility of the short-rate are given as a function of the parameters ϕ_1, ϕ_2, ϕ_3 as follows:

$$r_L = (\phi_1 - \phi_2)\phi_3 \qquad \textbf{[17]}$$

$$\sigma^2 = 2(\phi_1 \phi_2 - \phi_2^2) \qquad \textbf{[18]}$$

Before moving further, it must be acknowledged that the theoretical CIR model describes the process of *real* rates, as opposed to nominal rates. However, that said, it is still attractive for Modeling nominal rates because it precludes negative interest rates.

From the point of view of monetary policy also it is intuitive because, like NS and Svensson, the model implies that the long rate $(m \to \infty)$ converges to a constant. Now, although, volatility of the yield of the longest maturity bond traded in the money market is clearly not zero, the fact that it converges to a constant makes it appealing for monetary policy purposes.

Thus, here it becomes important to state that it is not intended here to test the theoretical CIR model, as in whether the restrictions it imposes are empirically fulfilled or not. Attempt is simply to fit the data to the functional form on the lines of Brown and Dybwig (1986), Barone, Cuoco, and Zautzik (1991), and Brown and Schaefer (1994) and others, and see how well an equilibrium model compares with the atheoretical NS and Svensson functional forms.

3.3 Estimation

The optimization problem is to minimize the weighted sum of square of (price) errors

$$min\sum_i^N (\omega_i \varepsilon_i)^2 \qquad \textbf{[19]}$$

subject to non-negativity constraints imposed on the short-rate, the long rate $(m \to \infty)$ and on the τ *s;* where $\varepsilon_i = P_i - \hat{P}_i$, and

$$\omega_i = \frac{1/d_i}{\sum_j^N 1/d_j} \qquad \textbf{[20]}$$

where d_i is the Macaulay duration of the i^{th} bond.[11]

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[10] for risk-neutral dynamics with λ, the parameters of the process can be uniquely identified from equations 5.13 – 5.15

[11] This weighing scheme corrects for the heteroskedasticity problem in the error terms which occurs if the price errors are used instead of yield error. See Coleman, Fisher and Ibbotson (1995), Bliss (1997) and Bolder and Streliski (1999) for a discussion. Using duration weighted loss function is also a proxy to

The loss function above has been specified as a function of price errors. An alternative exists in taking the yield errors; and it may be argued that since for monetary policy purposes it is the yields which are more important than the prices, and hence the loss function should be specified as a function of yield errors (see Svensson, 1994). However, it is the bond *prices* that are traded in the market, and not the yields, so it makes sense to specify a loss function in terms of the variable which can be directly observed in the market. Further, the weighting scheme used – other than taking care of heteroskedasticity – also takes care of minimizing yield errors indirectly. Recall that duration is a function of first derivative of price w.r.t yield, and the weighting scheme is inverse of duration.

While studies of NS and Svensson minimize yield errors, that of Bliss (1997), Bolder and Streliski (1999), Brown and Dybwig (1986) and Brown and Schaefer (1996)[12] minimize price errors. Studies on Indian data discussed earlier have all minimized price errors.
The process of determining the parameters involves initialization of the parameters, finding pricing errors based on 'starting' values and minimization of the objective function. For the Svensson model, an additional constraint on the inequality of the two 'time-decay' parameters (the τ s) is required to identify the second 'hump' (β_3).

3.4 Criteria of Evaluation

Since the purpose of the study is to model the term structure for monetary policy analysis, the most important criterion for evaluation of the performance of the models is the robustness of parameter estimates. As Dahlquist and Svensson (1996) argue:

"The estimates in policy analysis should allow comparisons over time and across countries, with different sets of bonds and Treasury bills, and be less sensitive to missing observations and the number of bonds and bills used in the estimation.[13]"

In practice, say Dahlquist and Svensson (1996), this boils down to comparing measures of fit and convergence properties of the above models. Instead of focusing, however, on the measures of fit one checks the out of sample properties of the estimated term structure, i.e. how well the parameters of the estimated model fit the bonds that were excluded from the sample used for estimation.

The idea being that in-sample errors are less important for monetary policy than for arbitrage decisions. Expectations hypothesis tells us that forward rates can be interpreted as expectation of future interest rates, which in turn depend on real interest rates and expected inflation. Splines-based techniques, however, are in-famous for resulting in abrupt changes in implied forward rates. It is unlikely that agents would have information that would allow them to have different expectations for very long-term horizons, and that on a day-to-day basis.

In keeping with above observations, this study uses the following criteria for evaluation:

1. Objective function value
2. In-sample and Out-of-sample Mean Absolute Price Error (MAPE) and Standard of Absolute Price Error (STDAPE)

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minimize yield errors when price errors are used in the loss function. Subramanian (2001) uses a liquidity (instead of duration) weighted loss function

[12] For CIR, Brown and Dybwig (1986) assume a Gaussian distribution for errors and maximize a likelihood function w.r.t the four parameters. Brown and Schaefer (1994) specify a loss function in terms of simple squared sum of errors and minimize its value.

[13] L.E.O. Svensson and M. Dahlquist, "Estimating the Term Structure of Interest Rates for Monetary Policy Analysis", *Scandinavian Journal of Economics,* 98, 1996, p. 164

3. In-sample and Out-of-sample Mean Absolute Yield Error (MAYE) and Standard Deviation of Absolute Yield Error (STDAYE)
4. In-sample and Out-of-sample MAPE by residual term to maturity
5. Time series behaviour of the parameters (robustness of parameters) and NSE's own
6. Iterations, function count evaluations and time required for convergence
7. Time series behaviour of implied short and long rates and comparison with average of daily MIBID/MIBOR (Mumbai overnight Inter-bank Bid/Offer Rate) and NSE's own
8. Behaviour of forward rates for maturity between 1 to 8 years

While the first three in the above list would be important for evaluating *any* term structure model, the last three are more relevant from the point of view of monetary policy analysis.

On Selection of Out-of-sample Bonds and Data

It was noticed from the WDM database that around 30-60 different bonds are traded each day. To remove any biases in selection of out-of-sample bonds, 15% of bonds traded are selected each day at random to assess the out-of-sample characteristics of the estimated term structure.

Issues in estimation of the term structure for India are discussed in detail in Darbha, Roy and Pawaskar (2003b) and as far as selection of bonds and estimation strategy is concerned this study follows their approach, i.e. all bonds traded during the day are included for estimation, value weighted prices are used while calculating pricing errors, and errors are weighted by inverse of duration. Also, only bonds with $T + 0$ and $T + 1$ settlement dates have been taken for the purpose of estimation,[14] to ensure that the estimated term structure best captures the expectations on the trade date.

IV. Results and Discussion

Results are presented separately – mostly graphically – under the criteria of evaluation mentioned above. For (same) 5 days no convergence was reached for all models.

4.1 Objective function value

The loss function minimized for estimation of parameters reflects the (weighted) price errors in the units of Rupees squared. More informative, however, is the percentage error in basis points. Even though the errors have been weighted by inverse of duration, a rough idea of this can be had by taking the square root of the objective function value[15].

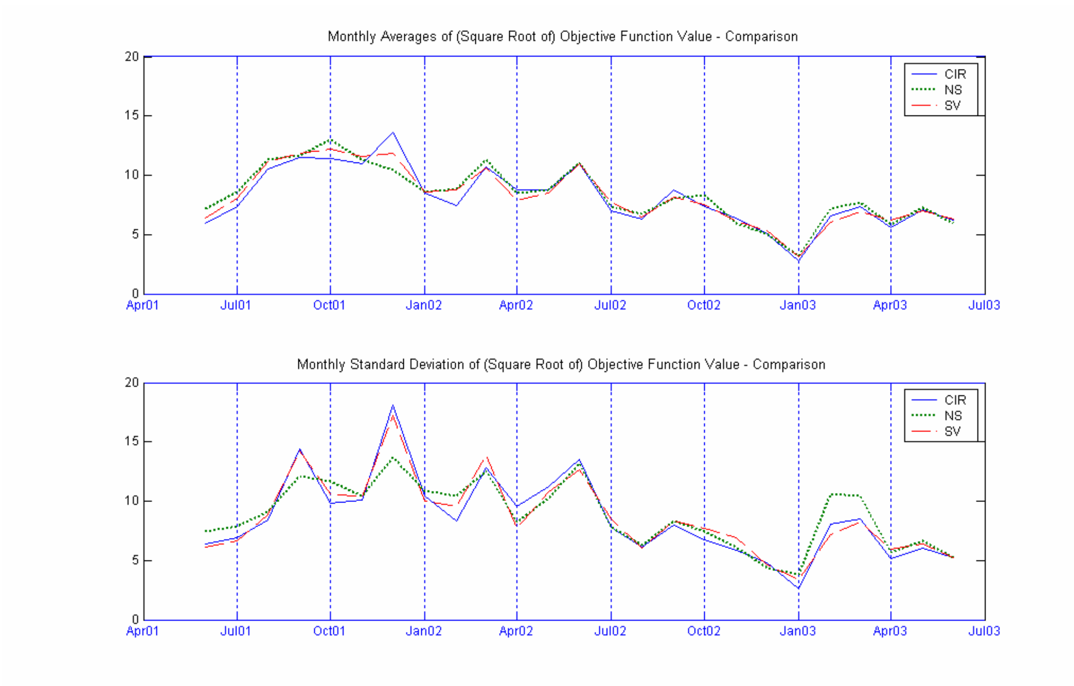
The plot below (**Figure 2***)* shows hundred times the square root of the monthly averages and standard deviations in basis points. On an average all the three models converge to similar loss function values. The following table (**Table 1***)* presents the (square root of) summary statistics of the objective function values:

Table 1
Summary statistics for 100 X (square root of the objective function)

	Min	*Max*	*Mean*	*Std. Dev.*
CIR	0.53991	38.718	8.4541	10.763
NS	0.25563	27.126	8.6647	10.174
SV	0.51061	37.902	8.5216	10.592

[14] accounting for more than 90% of the number of trades on most days; dates where none of the bonds settled on $T + 0$ or $T + 1$ dates, bonds settling on $(T + 2)^{th}$ date would also be included
[15] since taking square root is a monotonic transformation, its use creates no problems.

Figure 2



4.2 In-sample and Out-of-sample MAPE and STDAPE

The following table presents the summary statistics for in-sample MAPE in basis points.

Table 2a
Summary statistics for 100 X In-sample MAPE

	Min	*Max*	*Mean*	*Std. Dev.*
CIR	0.0007	990.09	74.13	77.05
NS	0.001	986.56	76.97	78.88
SV	0.004	1043.23	77.01	80.30

Note from the above table, that while some bonds are priced almost accurately (minimum MAPE ~ 0), the maximum pricing error for the entire sample period nears almost Rs. 10 for the three models. This lack of in-sample fit is not entirely unexpected given that models used are all parsimonious.

What is also seen is the standard deviation of the order more than that of mean, which suggests that mis-pricing is not only high but also very volatile. This tells why the specifications used in this study may not most useful in pricing contingent claims.

As to how useful these models could be to judge expectations for monetary policy purposes, other criteria for evaluation would through further light.

Summary statistics for both in-sample and out-of-sample MAPE (**Tables 2a** and **2b** respectively) suggest that CIR fits marginally better than the other two. CIR is also seen to have lesser variability. But, the differences again are only marginal, as their monthly comparisons of mean and standard deviation confirms (**Figures 3a** and **3b** respectively).

Figure 3a

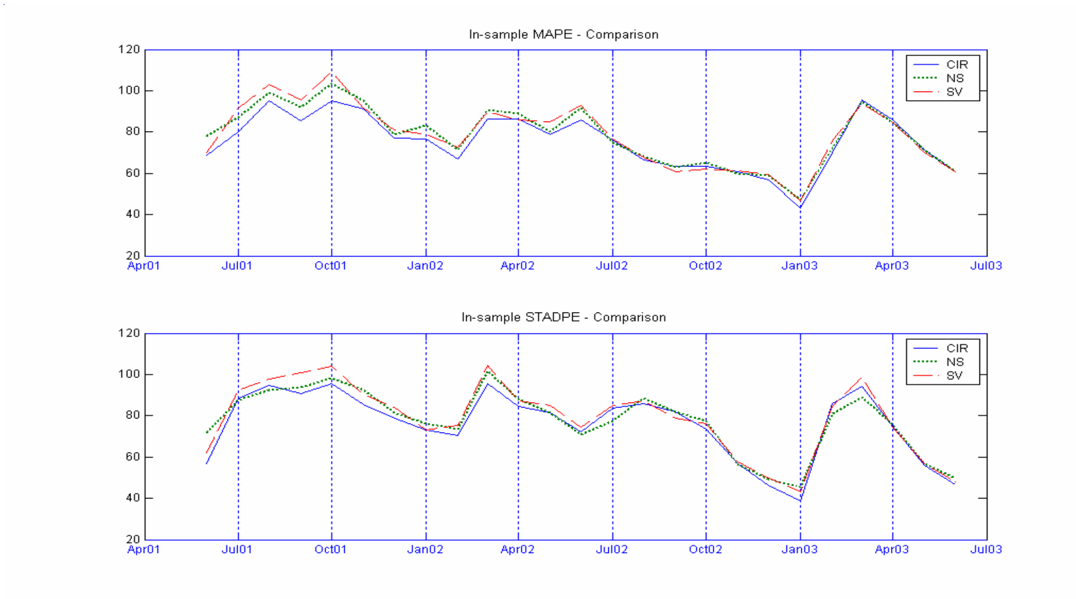


Table.2b

Summary statistics for 100 X Out-of--sample MAPE

	Min	*Max*	*Mean*	*Std. Dev.*
CIR	0.0026	1107.66	76.70	80.05
NS	0.0025	808.24	78.59	81.77
SV	0.0199	1062.81	79.25	82.00

Figure 3b

4.3 In-sample and Out-of-sample MAYE and STDAYE

Although, as argued before, price errors are used to estimate the models, it may be worthwhile to see how well the three models compare on yield to maturity errors. **Figures 4a** and **4b** respectively show results for month-by-month in-sample and out-of-sample MAYE in basis points. Clearly, while it is hard to choose between any of the three as the 'best' model, CIR has a lower mean and appears and is less variable both in-sample and out-of-sample (see **Tables 3a** and **3b** also for summary statistics).

Also, although for in-sample MAYE, SV performs slightly better than NS, pattern of overall results stays more-or-less the same, with SV coming across as the least efficient functional form, and CIR as somewhat better than the other two.

It is encouraging, however, that the mean yield error is of the order of only 12-13 basis points, a level of precision that should be acceptable for monetary policy analysis (but what is bothering is that – like for MAPE – standard deviation for yield error also has an order higher than the mean).

Figure 4a

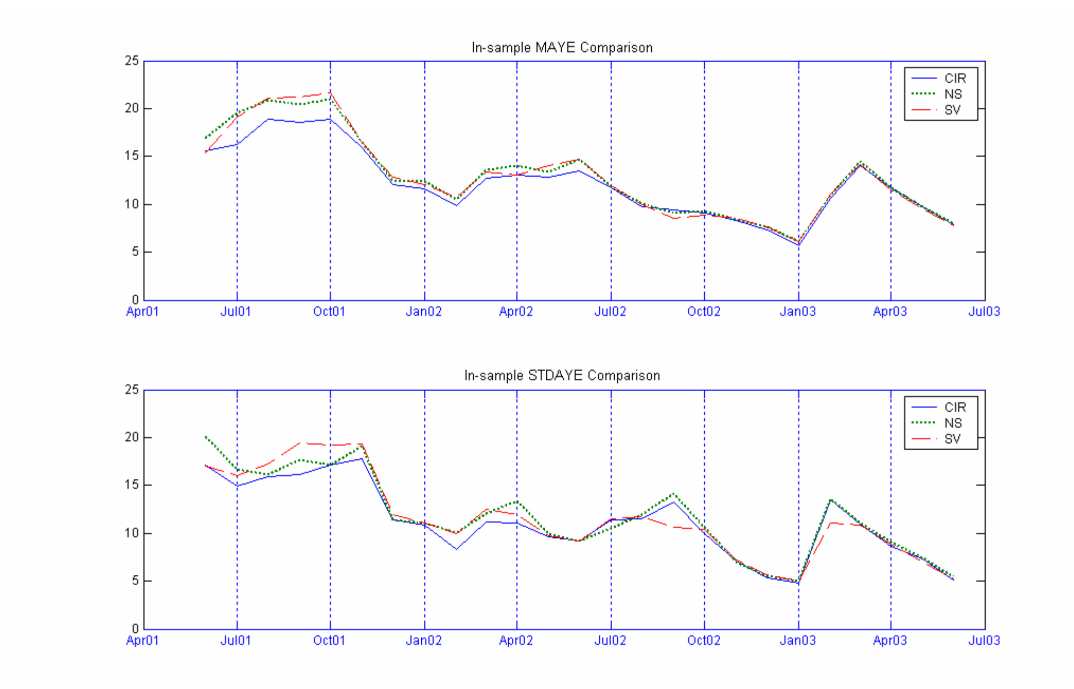
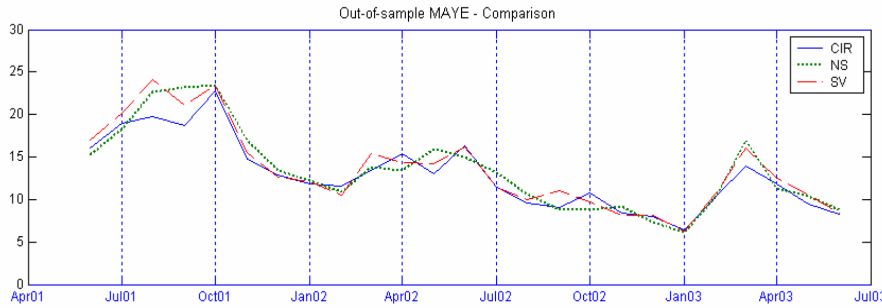


Figure 4b



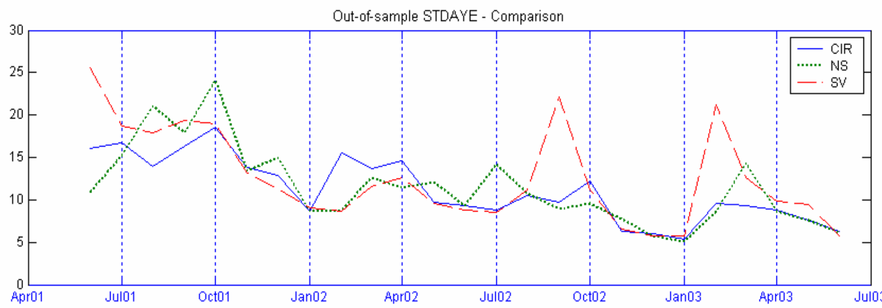


Table 3a

Summary statistics for 100 X In-sample MAYE

	Min	*Max*	*Mean*	*Std. Dev.*
CIR	0.0006	315.87	11.95	12.23
NS	0.0021	384.34	12.67	13.13
SV	0.0005	374.86	12.53	12.95

Table 3b

Summary statistics for 100 X Out-of--sample MAYE

	Min	*Max*	*Mean*	*Std. Dev.*
CIR	0.0011	165.6	12.57	12.25
NS	0.0002	244.34	13.05	12.96
SV	0.0026	277.02	13.18	14.41

4.4 In-sample and Out-of-sample MAPE by Maturity

Since prices of short-maturity bonds are relatively insensitive to yields, despite weighting by inverse of duration, there may be a tendency to over-fit the short-end. Also, there may be security-specific factors for long-maturity bonds which are not captured in the specified loss function. A reason for better fit for short-maturity bonds could also be that omitted pricing factors are less important for these bonds.

Thus, having seen that all the three models perform similarly on both MAPE/MAYE and STDAPE/STDAYE, with CIR a tad better, it would be interesting to see how the errors behave by the residual term to maturity.

Figure 5 below plots MAPE by maturity, with MAPE in basis points on the ordinate and residual term to maturity on abscissa[16]. Notice that bonds with residual term to maturity less than 2 years are priced the 'best'. The huge rise in MAPE beyond that reflects the lack of trading/liquidity of bonds in the short-to-medium term segment of the bond market. Errors are higher for maturities after that too, but they stabilize reflecting that most long maturity bonds are priced with a similar order of errors.

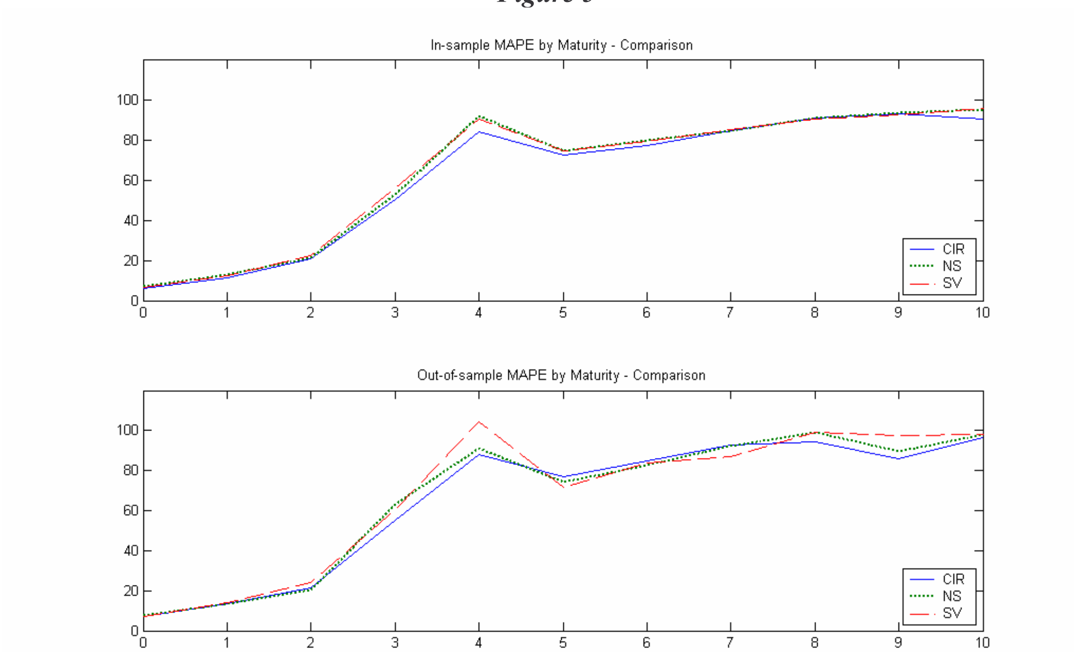
Thus, the short-end indeed is being over-fitted. This could be a useful, however, for pricing short-term interest rate derivatives, with both in-sample and out-of-sample pricing error less than 20 basis points. Although most (path-dependent) interest rate derivatives require the empirical probability *distribution* of short rates, these models could provide a starting point by looking at dynamics of the rate derived from these models.

An area of future research, and a further test of these models, would be to assess the stochastic dynamics of the short-rate derived from these models, say when compared to MIBID/MIBOR or the call rate.

For variation of errors with liquidity, as found by Darbha, Roy and Pawaskar (2003a), there was no pattern noticed across models for the sample under study. Error variation with liquidity provides no further insight, and hence they are not reported.

The next three criteria looked at – the behaviour of parameters, the short and the long rate and the forward rate for various maturities – are, as discussed earlier, more directly relevant from the point of view of monetary policy analysis.

Figure 5



[16] period 1 – 2 in abscissa refers to bonds with term to maturity greater than 1 year but less than 2 years and so on

4.5 Time series behaviour (robustness) of parameters and convergence characteristics

Table 4 below reports the summary statistics for parameters of the three models. Barring parameters ϕ_1 and ϕ_2, the coefficient of variation (CoV; standard deviation compared to the mean) for CIR parameters is the least. Also, loosely speaking, it is the parameters β_0 and β_1 of NS/SV and parameter, r of CIR which may be compared, for both relate to the short rate. CIR again comes across as slightly better than the other two. However, while convergence properties of CIR (number of iterations and function count evaluations) as reflected by CoV are better, on an average NS converges fastest. Thus, all three are computationally unproblematic, and as far as estimation is concerned converge fairly fast.

Not knowing the exact estimation strategy of NSE, although one can't directly compare results, the coefficient of variation of NSE's NS parameters is lesser than the NS model estimated in this study.

However, all the three models being highly non-linear, the problem of local versus global optima remains. Bolder and Streliski (1999) discuss this issue in detail and try to deal with it using a thorough local grid search procedure. This study uses NSE starting values for the first trading day of Jun, 2003 and from then on uses last (or next) day's values as starting point for next (or previous) day's estimation.

One of the main issues is initialization of the parameters. In this study NSE NS estimates for first trading day of June, 2003 have been used to get the starting value. For the first estimation, locally the starting values were varied and results were found to be quantitatively insensitive. From that day on, previous (or next) day's converged parameter values were taken as the starting value. For CIR, values of the implied short-rate and long-rate suggested appropriate starting values – again based on results for first trading day of June, 2003.

Table 4
Summary Statistics for Parameter Estimates and Convergence Properties

	Min	*Max*	*Mean*	*Std. Dev.*	*CoV*
			CIR		
ϕ_1	0.017	0.46	0.12	0.07	0.58
ϕ_2	0	0.41	0.07	0.07	1
ϕ_3	0.001	3.18	1.99	0.69	0.35
r	0.042	0.081	0.06	0.007	0.12
Iter	2	38	7.1	4.3	0.6
FnCount	20	260	64.9	28.02	0.43
			NS		
β_0	6.38	16.75	9.77	2.34	0.24
β_1	-10.2	-1.18	-3.85	1.8	0.47
β_2	-7.15	6.55	0.84	2.85	3.39
τ_1	1.81	19.79	8.94	5.69	0.64
Iter	1	26	4.65	4.05	0.87
FnCount	11	169	33.08	24.6	0.74
			NS_NSE		
β_0	4.62	16.32	9.68	1.72	0.18
β_1	-9.4	1.99	-3.08	1.15	0.37
β_2	-15.2	2.99	-3.97	3.92	0.99
τ_1	1.12	15.5	5.02	3.74	0.75
			SV		
β_0	6.37	20.84	9.73	2.45	0.25
β_1	-14.52	-1.15	-3.78	1.94	0.51
β_2	-12.24	0.59	-1.57	1.78	1.13
β_3	-3.34	2.9	-0.71	1.16	1.63
τ_1	1.56	9.97	3.88	1.46	0.37
τ_2	2.85	10.22	5.62	2.12	0.37
Iter	1	37	6.56	5.91	0.90
FnCount	15	312	59.7	47.8	0.81

Note: - CoV: Coefficient of Variation; Iter: Iterations; FnCount: No. of Function Count Evaluations

4.6 Time series behaviour of implied short and long rates

The evaluation criterion discussed next is the time series behaviour of implied short and long rates. If the models are correctly specified, then the short-rate derived from the models should have high correlation with the very-short (one-day) rate prevailing in the money market.

The short and the long-rate for NS and SV are given by the parameters $\beta_0 + \beta_1$ and β_0 respectively. For CIR equation **[17]** gives the long rate. **Figure 6a** shows the evolution of implied short and long rate from the three models. While the implied short-rates from the three models broadly follow the similar pattern, at the beginning of the sample long rate from CIR and SV are highly erratic.

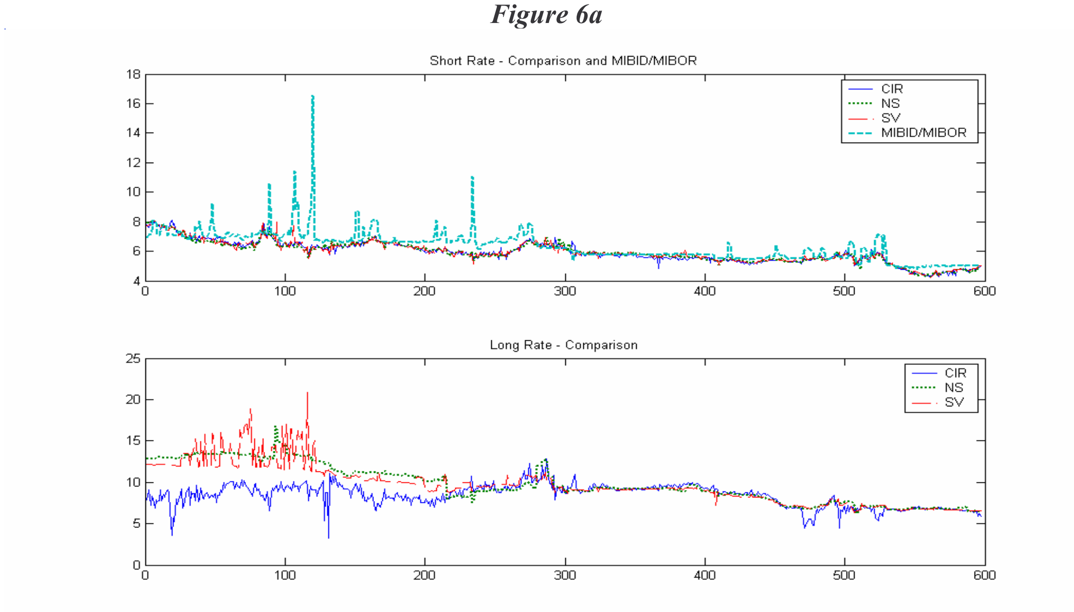
Inability of CIR to produce 'smooth' series for the long rate is not altogether surprising given that it is a nonlinear function of the parameters ϕ_1, ϕ_2, ϕ_3 (equation **[17]**). After around 200 days, CIR's estimates are closer to NS and SV estimates suggesting that the parameter values have a higher correlation than for the period prior to that[17]. Also, long rates from the three models seem to 'converge' after around 200 days, suggesting that relatively low trading activity in the market may be causing the daily long-term rate to behave erratically and also differently across models.

None of the three models, however, quite capture the high volatility ('jumps') in the MIBID/MIBOR series. This is not very surprising because the pricing model estimated in this study captures only the cash-flow and coupon effects and not short-term liquidity mis-matches – which would be the main reason for high short-term volatility of the over-night rate – and security-specific properties (see also Darbha, Roy and Pawaskar, 2003a).

Summary statistics (see **Table 5a**) also reveal that while CIR, NS and SV are virtually similar in the properties of the implied short-rate, MIBID/MIBOR is clearly more volatile with its highest coefficient of variation. Long rate from comes SV across as the most volatile (especially in the beginning of the sample period) while from CIR as the least.

NSE's own estimates of the short and the long rates are also not much different from the ones estimated in this study. **Figures 6b and 6c**, respectively, provides comparison of with NSE's estimates of short and long rates. Note the high volatility in NSE's estimates for both short and long rates.
Correlation statistics (see **Table 5b**) with the market short-rate MIBID/MIBOR are reasonable with results from all models again very similar – an indication that on most days there are a sufficient number of bonds traded with near-zero residual term to maturity allowing each model able to capture the short-rate fairly closely. It is also an indication that the yield curve at the very short-end is close to flat.

Figure 6a



[17] For further 6 days it was noticed that for CIR, lower limit for ϕ_3 (= 0.001) was reached in estimation. This would cause the long rate to become zero for those days. Hence those six days have been excluded in analysis of the short and the long rate.

Figure 6b

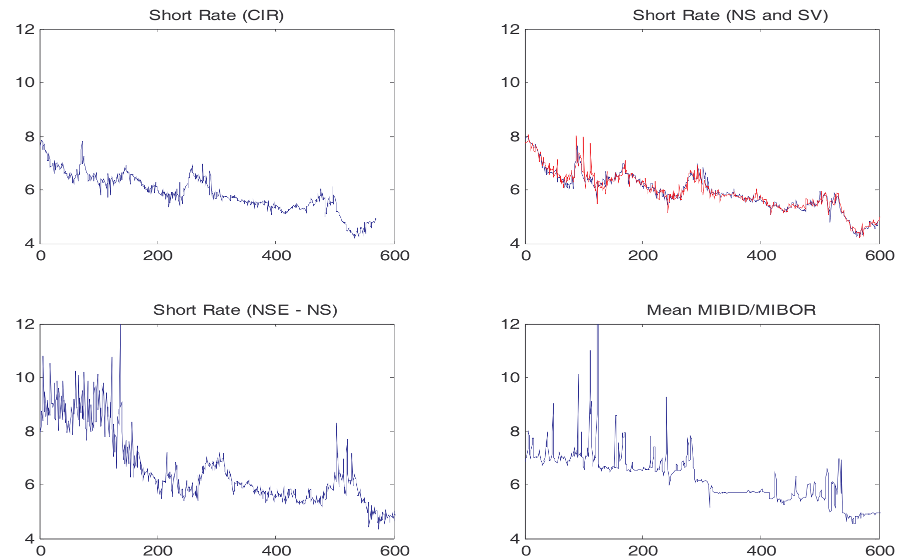


Figure 6c

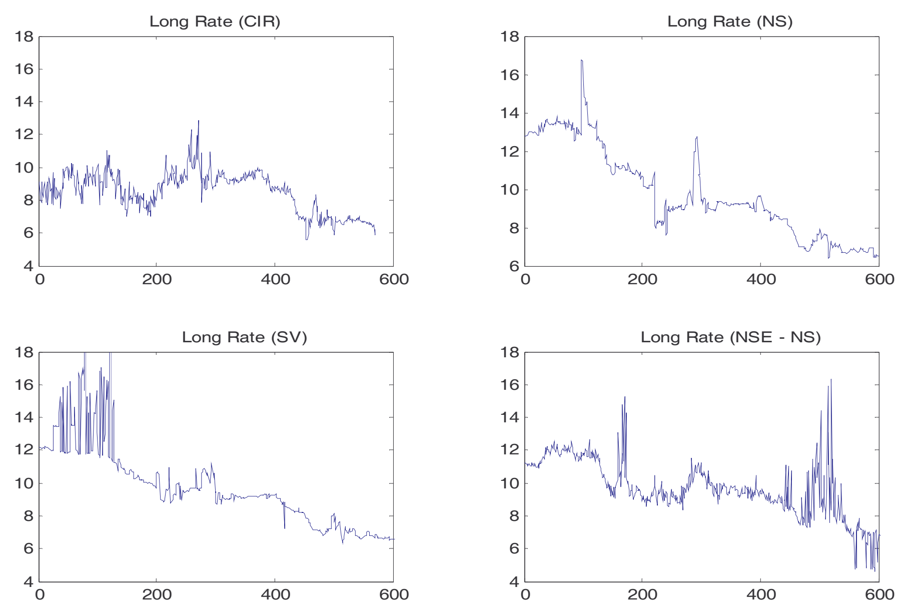


Table.5a
Implied Short and Long Rates from the three models

	Min	*Max*	*Mean*	*Std. Dev.*	*CoV*
		Short-Rate			
CIR	4.24	8.13	5.92	0.76	0.13
NS	4.26	8.00	5.92	0.73	0.12
NS_NSE	4.36	12.21	6.59	1.39	0.22
SV	4.24	8.04	5.94	0.73	0.12
MIBID/MIBOR	4.75	16.54	6.33	1.02	0.16
		Long-Rate			
CIR	3.23	12.88	8.39	1.33	0.16
NS	6.38	16.75	9.73	2.32	0.24
NS_NSE	4.62	16.32	9.68	1.72	0.18
SV	6.37	20.84	9.67	2.41	0.25

Table 5b
Correlation with MIBID/MIBOR

CIR	*NS*	*NS_NSE*	*SV*
0.72	0.69	0.65	0.71

4.7 Behaviour of forward rates for maturity between 1 to 8 years

The final criterion that is looked at is the behaviour of forward rates for maturities 1 to 8 years. The idea is that if these term structure models are going to be of any use to the central banker they should, in the least result in fairly 'smooth' series of forward rate for medium and long-term. The argument is same as offered earlier, that agents would not be expected to change expectations regarding future inflation abruptly on a day-to-day basis.

Given the parameters, forward rate for NS and SV are directly given by equations **[5]** and **[7]** respectively. For CIR a functional form for instantaneous forward rate can be derived from equation **[10]** noting that

$$f[r,t,T] = -\frac{\partial P(r,t,T)}{\partial T} = -\frac{\partial P(r,t,T)}{\partial \tau} \qquad \textbf{[21]}$$

where $\tau = T - t$

Then, using the same notation as used in the set of equations **[9]** to **[18]**, the instantaneous forward rate function for time $\tau = T - t$ at time *t* for CIR can be derived as:

$$f[r,t,T] = \phi_2\phi_3\left(\frac{\phi_1 \exp(\phi_1\tau)}{D} - 1\right) + r\left(\frac{\phi_1^2}{D^2}\right)\exp(\phi_1\tau) \qquad \textbf{[22]}$$

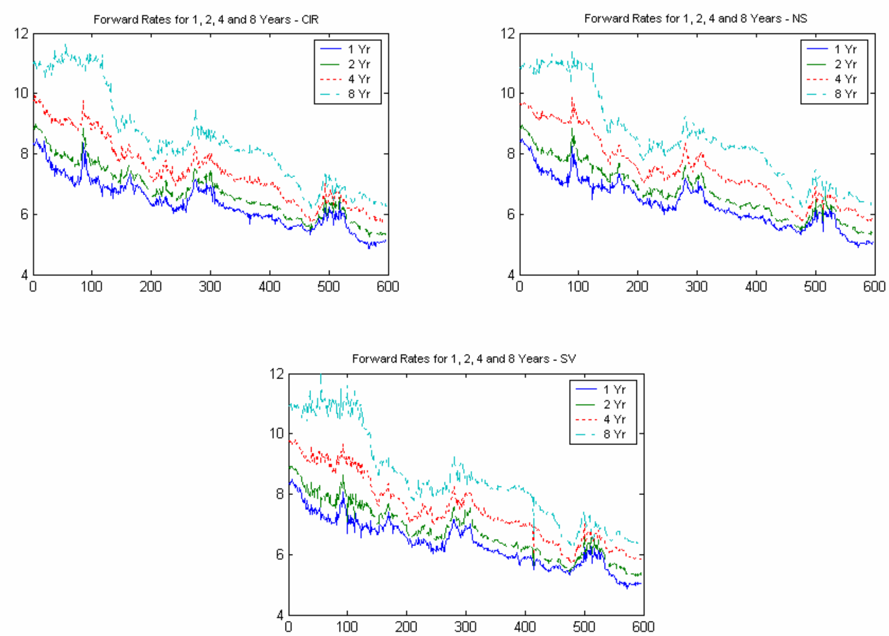
where $D = \phi_2[\exp(\phi_1\tau) - 1] + \phi_1$

Figure 7a presents results for forward rates for 1, 2, 4 and 8 years for the three models. It is apparent that not only has the term structure been upward sloping for almost the entire sample period under study, the forward rate series for all the models do not have many 'jumps'. Forward rates from SV come across as most volatile during the beginning of the sample period.

Figure 7b is the same plot as **Figure 7a** with the long-rate superimposed. The plot shows that how looking at the rate for infinity (long-rate) only may be misleading. While forward rate for maturities which matter are fairly stable, long rate at the beginning of the sample is quite erratic both for CIR and SV.

The last plot in this section, **Figure 7c**, is the monthly averages of CoV for forward rates for maturities 1, 2, 4 and 8 years. Ordinate plots 100 times the CoV. The maximum CoV for *any* month is less than 6%, which is quite good, i.e. average day-to-day variability of the forward rate for 1 – 8 years maturities is not very high, which is a good sign from the point of view of monetary policy. As on all other criteria discussed above, CIR comes across as slightly better than the other two (NS and SV being almost exact in their properties, both in mean value and standard deviation over the entire sample period).

Figure 7a



Lastly, **Table 6** below provides summary statistics on the stability of the forward rates for the four maturities for which the results are reported. The column of interest here is *CoV*. It is clear that forward rates for all the maturities are not very volatile with standard deviation of order far less than the mean ($1/5^{th}$ – $1/8^{th}$). *Min* and *Max* columns also suggest that range of forward rates for the entire sample period (more than 2 years) is not too high (between 400 – 550 basis points) across models.

Figure 7b

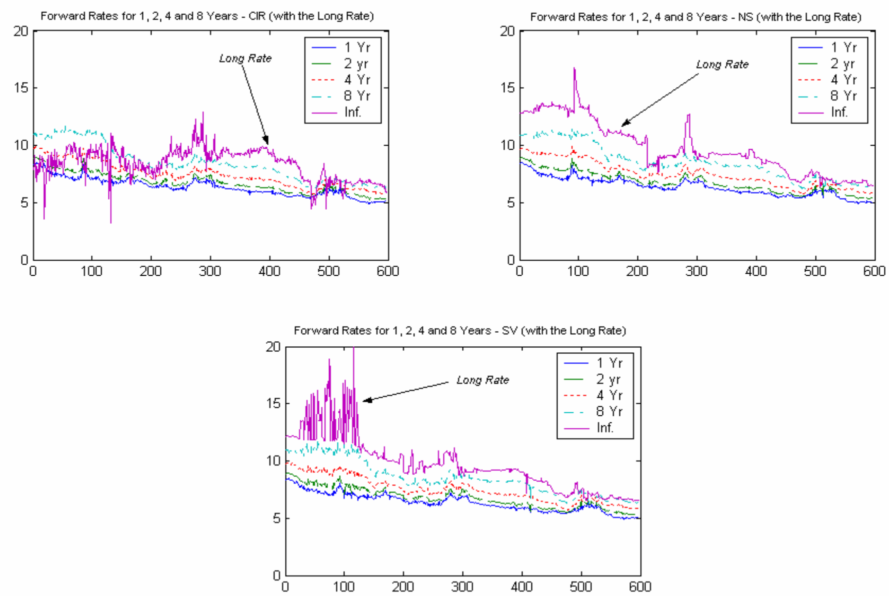


Figure 7c

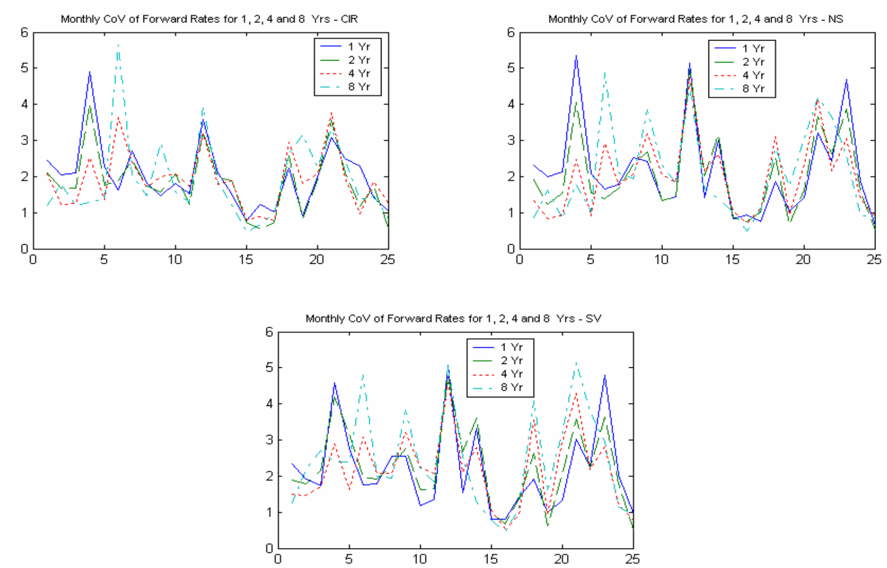


Table 6
Summary Statistics for Stability of Forward Rates for Maturities 1, 2, 4 and 8 Years

Maturity	*Min*	*Max*	*Mean*	*Std. Dev.*	*CoV*
			CIR		
1	4.86	8.51	6.36	0.79	0.12
2	5.26	8.98	6.76	0.87	0.13
4	5.70	9.95	7.45	1.09	0.15
8	6.16	11.63	8.46	1.56	0.18
			NS		
1	4.90	8.49	6.38	0.80	0.13
2	5.31	8.94	6.78	0.90	0.13
4	5.78	9.87	7.49	1.11	0.15
8	6.22	11.39	8.50	1.51	0.18
			SV		
1	4.86	8.50	6.37	0.80	0.13
2	5.30	8.93	6.78	0.91	0.13
4	5.70	9.81	7.49	1.11	0.15
8	6.24	11.97	8.49	1.51	0.18

V. Information Content of the Term Structure: Results

Taking cue from a series of works by Fama and Mishkin[18] on the information content of term structure, this study further explores for the first time the term structure for forecastability of inflation. This assumes significance because money markets in India are only now becoming increasingly efficient in the Fama (1970) sense and it would be useful to know whether the Indian central bank can use the term structure derived from the market price of G-Secs to get information about likely paths of future inflation.

The methodology proposed by Mishkin (1990b) is now a standard in the literature. Essentially it involves estimating the following "inflation change equation" and checking for the statistical significance of the coefficient β:

$$\pi_{k,t} - \pi_{n,t} = \alpha + \beta(i_{k,t} - i_{n,t}) + \varepsilon_t \qquad \textbf{[23]}$$

where, $\pi_{k,t}$ is inflation rate from time t to $t + k$, $i_{k,t}$ is the k-period nominal interest rate at time t; i.e. assessing the information content of the term structure involves regression of the change in future k-period inflation over n period inflation on the *slope* of the term structure in the relevant range.

A value of β statistically different from zero would suggest that the change in slope of the term structure does contain information about paths of future inflation. Also, a value of β statistically different from one would indicate that term structure of *real interest rates* is not constant over time, which results directly from the Fisher equation, if it were to interpreted as an inflation forecasting equation. Mishkin (1990b) provides a detailed discussion on the interpretation of the Fisher equation as an inflation forecasting equation. The focus of this study, however, remains to assess the forecastability of inflation.

[18] See Fama (1975, 1984, and 1990), Fama and Gibbons (1982), and Mishkin (1990a, 1990b, 1990c, 1991)

Although, clearly, the estimation framework is quite simple, there is an important econometric consideration. It is that of the serial correlation in error terms arising from using overlapping forecasts when the number of periods for the interest rate and inflation are greater than the observation interval. The presence of this serial correlation would cause the standard errors to be incorrect, thereby precluding the use of OLS.

5.1 Data / Convention Used

The sample for the estimation of inflation forecasting regressions remains the same as the one for which the term structures have been estimated, i.e. Jun, 2001 to Jun, 2003.

The sampling frequency, however, is weekly. Contemporaneous end-of-week data has been used for both inflation (calculated using All-Commodities Wholesale Price Index; 1993-94 = 100) and interest rates (as estimated from all three models and NSE's own estimates as available from its website). Because the sampling frequency is weekly, monthly inflation refers to 4-weekly inflation and similarly for quarterly and so on[19].

The timing convention goes like this. One month inflation for the end of first week of Jun, 2001 is calculated from the index value of end of first week of Jun, 2001 and end of first week of July, 2001 and so on. The corresponding one month interest rate observation pertains to the interest rate for one month as at the end of the first week of Jun, 2001 from all the three estimated models. Throughout the study, all the variables have been used as annualized percentage figures. Inflation is calculated as the first difference of the natural logarithm of prices.

The following 9 inflation change regressions have been estimated:

- 3 month – 1 month
- 6 month – 1 month
- 6 month – 3 month
- 12 month – 3 month
- 12 month – 6 month
- 18 month – 6 month
- 18 month – 12 month
- 24 month – 12 month
- 24 month – 18 month

For example, in the first case above, the following regression has been estimated:

$$\pi_{3,t} - \pi_{1,t} = \alpha_3 + \beta_3 (i_{3,t} - i_{1,t}) + \varepsilon_{3\text{-}1,t} \qquad \textbf{[24]}$$

Although it was suggested in the original proposal that the regression change equations would be estimated for the very-long-end of the term structure too, the sample size, extent of overlap involved in those regressions, and concerns on tractability of study dictated the final selected intervals.

5.2 Preliminary Evidence

Before formally conducting the results, it may be useful to study the time series characteristics of the inflation/inflation change and interest rate/slope of the term structure. As we have already looked at the time series properties of estimated term structures, here we'll have a look at inflation, change in inflation, and the slope of the term structure at the relevant horizons.

[19] For 5 weeks, contemporaneous data on WPI Index and interest rate could not be obtained. The previous traded day's interest rate has been used for those weeks

The **Table 7** below presents the summary statistics for inflation for the horizons mentioned above.

Table 7
Summary Statistics for Inflation

Inflation	*Max*	*Min*	*Mean*	*Std_Dev*	*Median*	*Coeff. of Var*
π_1	18.25	-7.26	3.61	6.22	3.21	1.73
π_3	13.75	-4.83	3.67	4.36	2.96	1.19
π_6	8.37	-1.61	3.88	2.81	4.09	0.72
π_{12}	6.95	1.14	4.51	1.27	4.65	0.28
π_{18}	6.92	2.33	5.00	1.21	5.09	0.24
π_{24}	6.99	3.55	5.14	0.89	5.18	0.18

As expected the coefficient of variation for the longest horizon is the least, suggesting the oft-noticed, that high frequency inflation series are noisy. See **Figures 8a** and **8b**. Ignoring the high frequency noise, one notices that inflation has largely been within 2 to 10% throughout the sample period, and the distribution fairly symmetric as suggested by the near same-ness in the values of mean and median.

More important, however, from this study's point of view are the variations in *change* in inflation rates and interest rates.

Figure 8a
Inflation

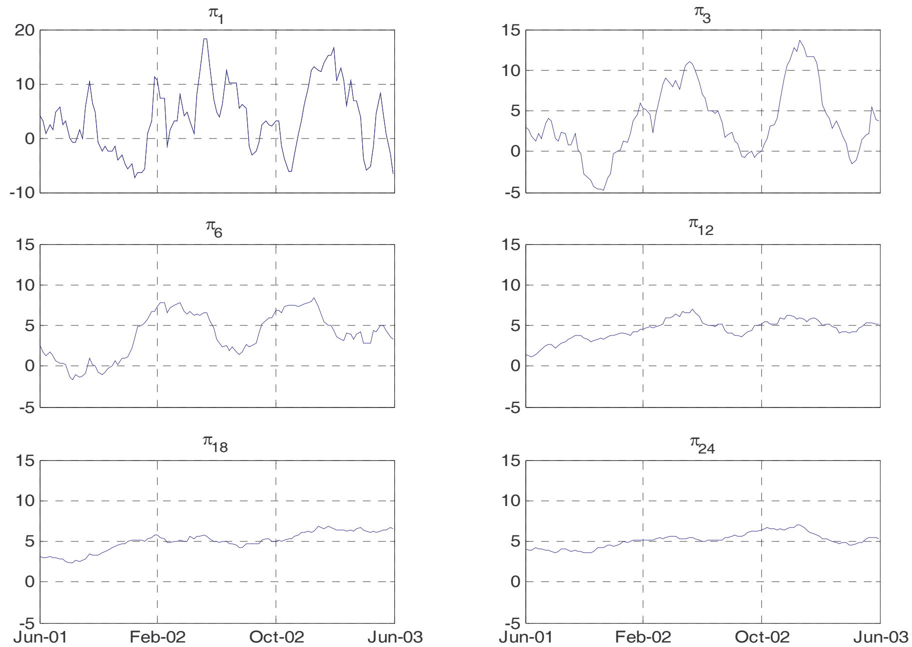


Figure 8b
Inflation

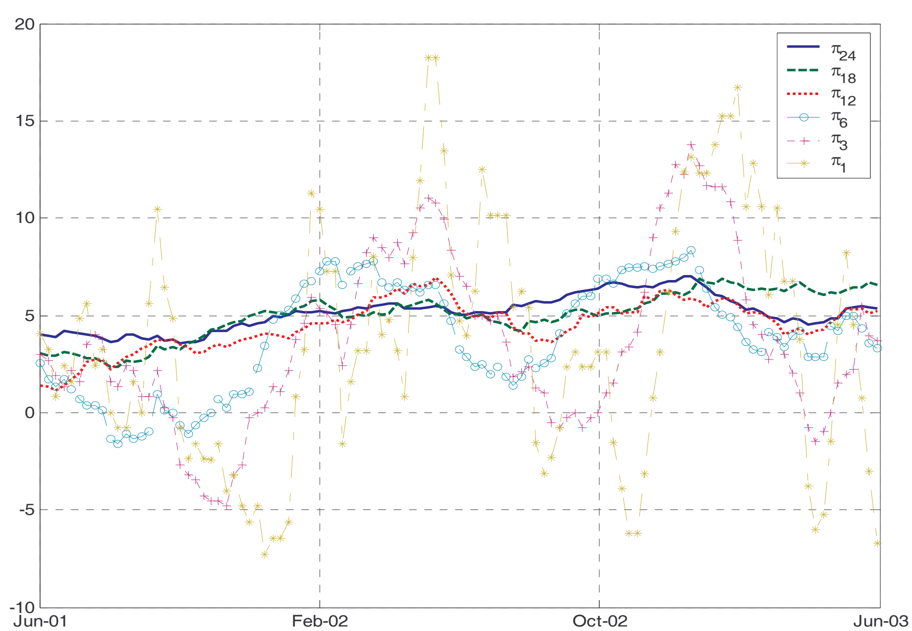


Table 8
Summary Statistics for Inflation Changes
$[\, y_{3-1} \;=\; \pi_3 - \pi_1 \,]$

$\Delta\pi$	*Max*	*Min*	*Mean*	*Std_Dev*	*Median*	*Coeff. of Var*
y_{3-1}	10.46	-8.31	0.06	4.6	-0.12	71.07
y_{6-1}	13.69	-12.31	0.27	6.11	0.13	22.45
y_{6-3}	6.87	-6.56	0.21	3.39	-0.14	16.36
y_{12-3}	8.18	-7.89	0.84	3.63	0.86	4.35
y_{12-6}	4.51	-3.19	0.63	2.05	0.64	3.27
y_{18-6}	4.66	-2.67	1.12	2.15	1.8	1.91
y_{18-12}	2.39	-1.37	0.5	0.88	0.65	1.76
y_{24-12}	2.77	-1.46	0.64	0.85	0.55	1.34
y_{24-18}	1.61	-1.9	0.14	0.9	0.19	6.68

Figure 9
Inflation Change

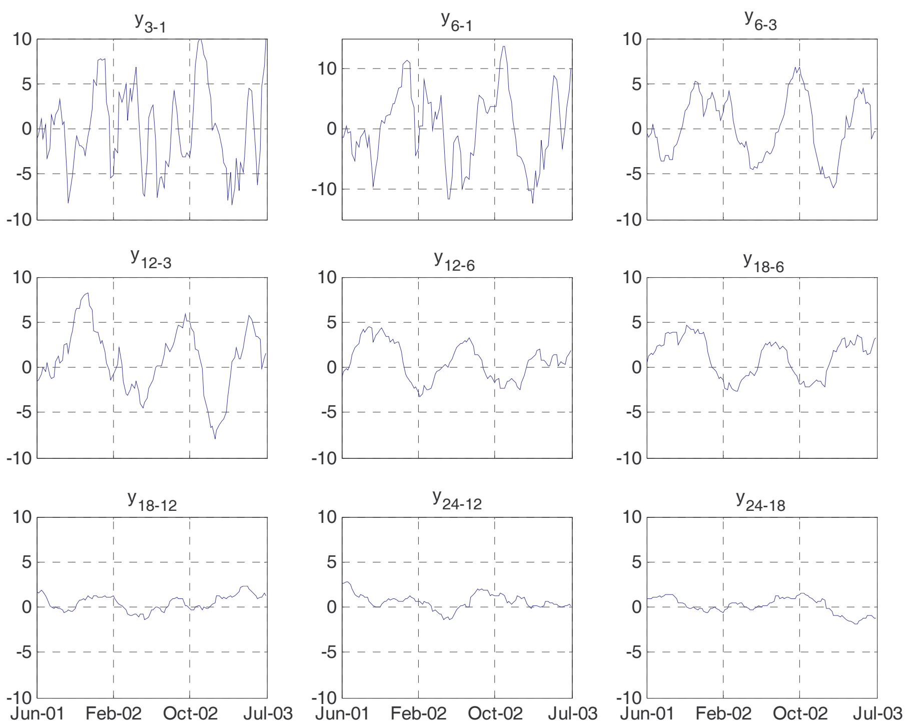


Table 9 above presents summary statistics for inflation changes for the selected horizons. The high order of standard deviation compared to the mean suggests high volatility in inflation rate changes at high frequencies. This is also borne out by high values of coefficient of variation across the horizons in **Figure 9**. Also, while at higher frequencies the level ranges between –10 to +10 %, at lower frequency, it is down to around –5 to 5% and less.

Comparing this with the characteristics of term structure slopes across the four models (NS, CIR, SV and NS_NSE) in **Table 9** and **Figures 10a – 10d**, one notices the following difference:

1. The change in term structure slope remains (equally) volatile at both high and low frequencies (except for NS_NSE) as shown by the coefficient of variation (the last column) in **Table 9**
2. The independent variable ranges between 0 – 0.4% for most of the sample period across all horizons, suggesting that the term structure has been more or less flat in the sample (and horizons) under consideration
3. The difference between the ranges of the dependent –5 to 5% on an average) and the independent variable – coupled with a flat term structure – does not lead one to expect much information in the slope of the term structure for inflation changes.

5.3 Results

As mentioned earlier, overlapping data (sampling frequency < the frequency at which the data is collected) induces serial correlation in the error term in equation **[23]**. For example in equation **[24]**, the error $\varepsilon_{3-1,t}$ by construction would be serially correlated for (3 – 1) * 4 = 8 lags (because weekly data is used and the regressions are in the multiples of four weeks).

Figure 10a (NS)

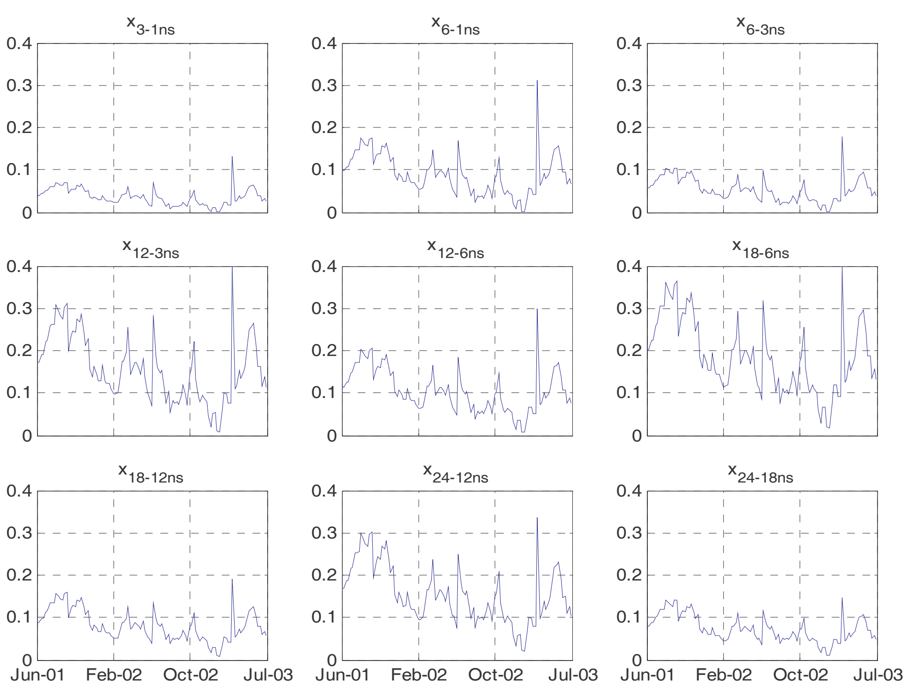


Figure 10b (CIR)

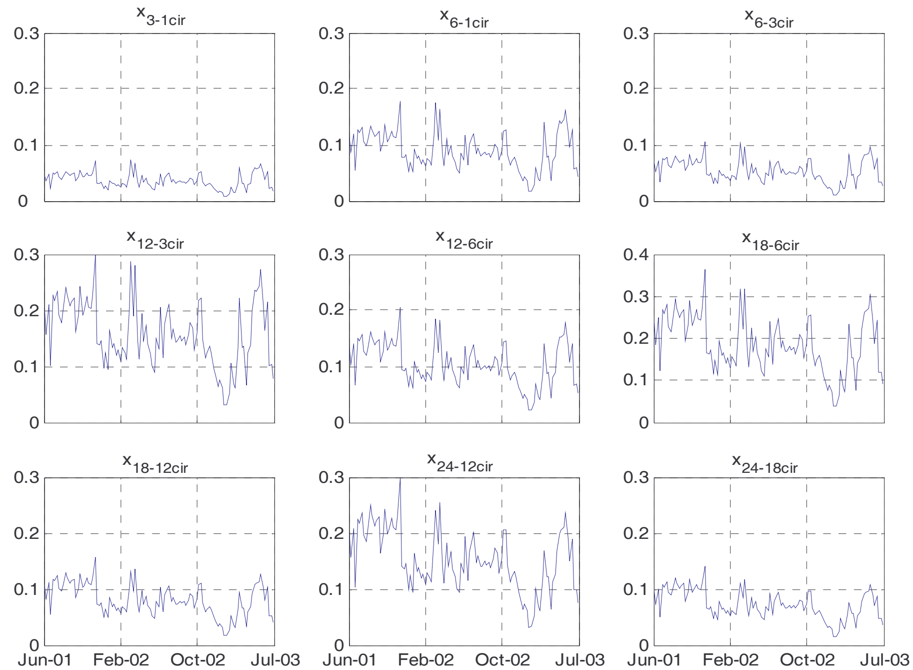


Figure 10c (NS_NSE)

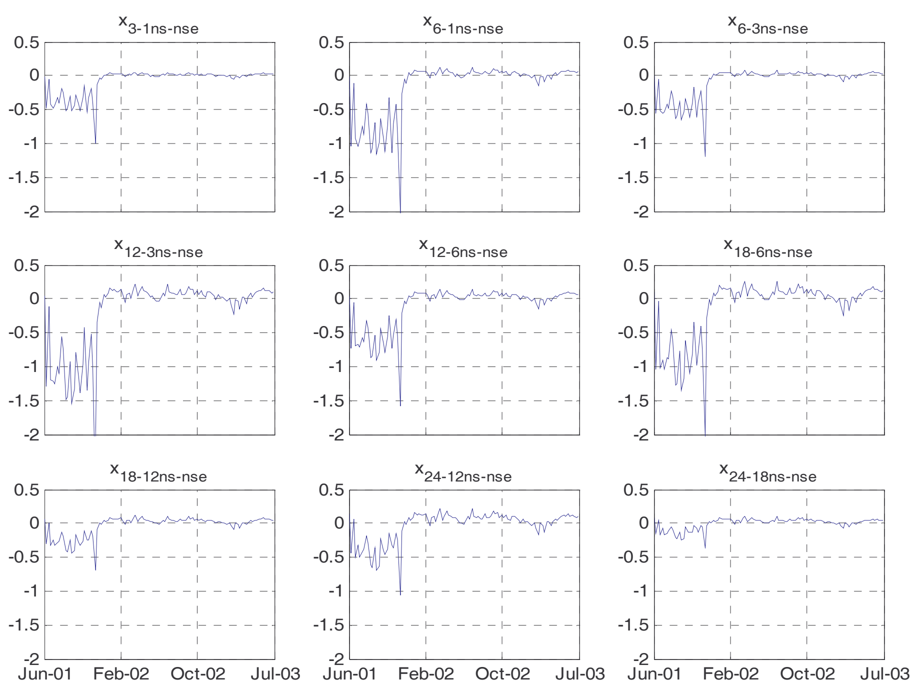


Figure 10d (SV)

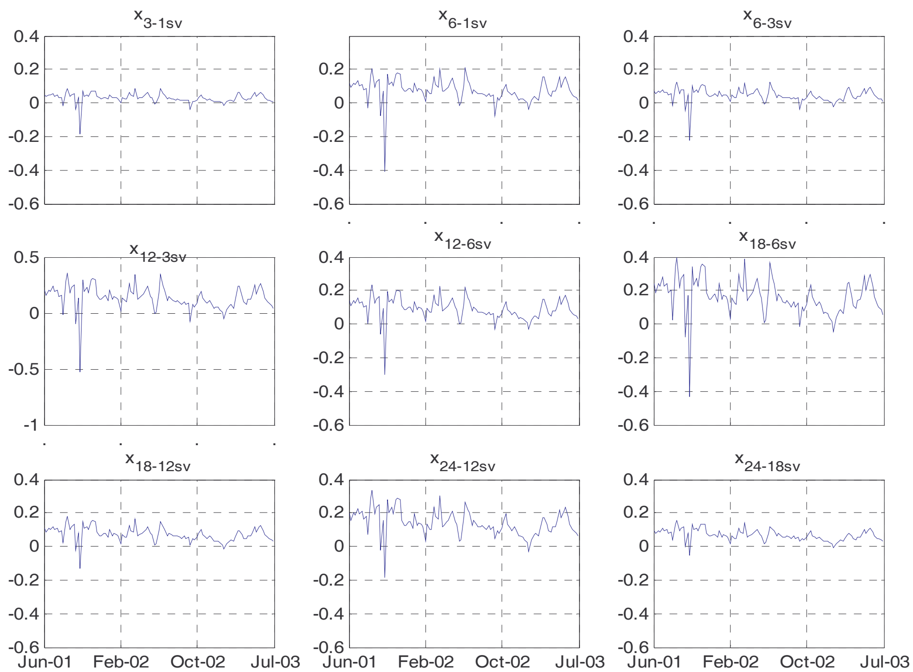


Table 9
Summary Statistics for Term Structure Slopes
$[x_{31\_ns} = i_{3\_ns} - i_{1\_ns}]$

	Max	*Min*	*Mean*	*Std_Dev*	*Median*	*Coeff. of Var*
			NS			
x31_ns	0.13	0.0002	0.036	0.020	0.034	0.554
x61_ns	0.31	0.002	0.089	0.048	0.083	0.537
x63_ns	0.18	0.002	0.054	0.028	0.049	0.526
x123_ns	0.47	0.009	0.159	0.079	0.147	0.498
x126_ns	0.29	0.007	0.105	0.051	0.099	0.485
x186_ns	0.49	0.016	0.187	0.087	0.175	0.467
x1812_ns	0.19	0.009	0.082	0.036	0.075	0.447
x2412_ns	0.34	0.019	0.155	0.068	0.142	0.435
x2418_ns	0.15	0.01	0.074	0.031	0.067	0.423
			CIR			
x31_cir	0.072	0.007	0.036	0.014	0.034	0.376
x61_cir	0.178	0.018	0.09	0.034	0.085	0.371
x63_cir	0.106	0.011	0.054	0.019	0.051	0.368
x123_cir	0.313	0.032	0.158	0.057	0.151	0.361
x126_cir	0.206	0.021	0.105	0.038	0.099	0.358
x186_cir	0.365	0.038	0.186	0.066	0.176	0.354
x1812_cir	0.158	0.017	0.081	0.028	0.077	0.351
x2412_cir	0.299	0.032	0.154	0.054	0.146	0.349
x2418_cir	0.141	0.016	0.073	0.025	0.069	0.348
			SV			
x31_sv	0.088	-0.184	0.029	0.031	0.029	1.047
x61_sv	0.209	-0.411	0.074	0.072	0.074	0.969
x63_sv	0.122	-0.227	0.045	0.041	0.045	0.919
x123_sv	0.355	-0.532	0.136	0.108	0.135	0.797
x126_sv	0.234	-0.305	0.091	0.067	0.089	0.738
x186_sv	0.412	-0.436	0.165	0.109	0.16	0.667
x1812_sv	0.178	-0.130	0.073	0.043	0.07	0.584
x2412_sv	0.336	-0.183	0.141	0.076	0.134	0.539
x2418_sv	0.158	-0.053	0.067	0.034	0.063	0.495
			NS_NSE			
x31_ns_nse	0.052	-1.00	-0.078	0.189	0.007	2.41
x61_ns_nse	0.129	-2.21	-0.167	0.419	0.018	2.469
x63_ns_nse	0.077	-1.2	-0.091	0.231	0.012	2.522
x123_ns_nse	0.225	-2.78	-0.199	0.544	0.039	2.727
x126_ns_nse	0.148	-1.57	-0.108	0.313	0.033	2.903
x186_ns_nse	0.262	-2.27	-0.141	0.461	0.065	3.272
x1812_ns_nse	0.114	-0.69	-0.033	0.148	0.031	4.502
x2412_ns_nse	0.214	-1.06	-0.034	0.231	0.062	6.805
x2418_ns_nse	0.101	-0.36	-0.001	0.084	0.032	80.76

NB: NS is Nelson Siegel; CIR is Cox-Ingersoll-Ross; NS_NSE is NSE's estimates of NS; SV is Svensson

Although valid standard errors can be computed using Hansen (1982) or Hansen-Hodrick (1980) methodology (in case of no heteroskedasticity) with appropriate corrections (e.g. Newey-West, 1987) to ensure positive-definiteness of the variance-covariance matrix, they are valid only asymptotically. The sample size in the study is, however, at best moderate. Huizinga and Mishkin (1984) note that in finite samples with overlapping data the finite sample distribution differs significantly from the asymptotic distribution. To take care of this problem critical values have been calculated using Monte Carlo simulation. Both OLS and Newey West (1987) adjusted t-stats have been reported in the results that follow.

Results for all cases for all horizons are reported in **Table 10** below. For inferences Newey-West corrected t-stats (with lag length corresponding to the induced correlation structure as explained above) have been used. OLS results have been reported only for comparison's sake. OLS standard errors would be both inconsistent and inefficient.

As could have been expected from the preliminary results, change in term structure slopes does not seem to have any information about inflation changes, except for a couple of horizons in the Nelson-Siegel case, with all t-stats less than the corresponding finite samples critical values.

The high values of β s can be explained by the difference in level (range) of the dependent and independent variables. From the results it seems that the appropriate horizons to study forecastability of inflation changes from changes in term structure slopes would be more than a year with a difference of a year or more. The **24 – 12** regressions for all cases results in much reasonable values of β (~ 1), albeit insignificant in the sample considered.

It suggests that a study considering a larger sample size with monthly frequency – maintaining a compromise between sample size and the serial correlation structure of the error terms because of the extent of overlap involved with horizons more than a year – may find more encouraging results.

Table 10
Estimates of Inflation Change Regressions

CIR					
	β	*t-stat* (OLS)	*t-critical* for $\beta = 0$ Monte Carlo (OLS)	*t-stat* (Newey West)	*t-critical* for $\beta = 0$ Monte Carlo (Newey West)
3 – 1	-26.77	-0.82	-2.67	-0.59	-2.07
6 – 1	22.58	1.29	2.83	0.84	2.42
6 – 3	56.35	3.6	3.19	1.89	2.25
12 – 3	32.21	6.1*	4.04	2.77	2.97
12 – 6	20.47	4.18*	3.47	2.32	2.54
18 – 6	10.85	3.63	3.9	2.17	3.47
18 – 12	-3.76	-1.26	-2.94	-0.93	-2.56
24 – 12	**1.95**	1.28	3.41	1.07	3.43
24 – 18	9.67	2.91	5.84	1.79	3.6

NS					
	β	*t-stat* (OLS)	*t-critical* for $\beta = 0$ Monte Carlo (OLS)	*t-stat* (Newey West)	*t-critical* for $\beta = 0$ Monte Carlo (Newey West)
3 – 1	-28.91	-1.31	-2.79	-1.13	-2.16
6 – 1	-6.67	-0.54	-2.76	-0.41	-2.41
6 – 3	9.19	0.79	3.85	0.36	2.41
12 – 3	17.0	4.2	3.47	1.84	2.71
12 – 6	21.92	6.7	4.67	**3.36****	3.10
18 – 6	12.17	5.86	4.04	**3.70****	3.21
18 – 12	-2.79	-1.19	-3.47	-0.94	-2.49
24 – 12	**0.9**	0.07	4.53	0.05	3.53
24 – 18	4.77	1.73	7.27	0.97	4.64

** - Significant at 10%

NS_NSE					
	β	*t-stat* (OLS)	*t-critical* for $\beta = 0$ Monte Carlo (OLS)	*t-stat* (Newey West)	*t-critical* for $\beta = 0$ Monte Carlo (Newey West)
3 – 1	3.98	1.71	2.26	**1.67****	1.65
6 – 1	2.24	1.61	2.61	1.29	2.13
6 – 3	0.87	0.61	3.67	0.31	2.38
12 – 3	-1.67	-2.65	-4.06	-1.45	-3.06
12 – 6	-3.69	-7.07	-3.84	**-3.92****	-2.61
18 – 6	-2.41	-6.23	-5.79	-4.41	-5.62
18 – 12	3.8	0.67	2.23	0.37	1.28
24 – 12	**-0.7**	-1.94	-6.04	-1.07	-6.49
24 – 18	-2.28	-2.24	-8.21	-1.22	-5.21

** - Significant at 10%

SV					
	β	*t-stat* (OLS)	*t-critical* for $\beta = 0$ Monte Carlo (OLS)	*t-stat* (Newey West)	*t-critical* for $\beta = 0$ Monte Carlo (Newey West)
3 – 1	13.46	0.93	2.42	1.15	1.91
6 – 1	9.47	1.16	1.92	1.58	1.64
6 – 3	7.18	0.91	2.26	0.59	2.24
12 – 3	6.49	2.04	1.59	1.03	2.03
12 – 6	5.95	2.05	1.48	1.43	1.64
18 – 6	4.43	2.39	2.15	1.75	2.93
18 – 12	0.4	0.022	2.22	0.021	2.44
24 – 12	**0.51**	0.47	2.86	0.39	3.05
24 – 18	2.47	.95	2.43	0.73	2.39

5.4 The Monte Carlo Experiment

The Monte Carlo simulation to obtain the finite sample critical values for regressions proceeded as follows.

For each regression equation, different ARMA models[20] were fitted for inflation and interest rate changes. The ARMA models were selected using top-down approach. Maximum AR and MA orders were pinned-down upon for each case using Sample Autocorrelation Function and Sample Partial Autocorrelation Function as a guide and then Akaike's Information Criterion was used for the final model selection, ensuring that the residual were white noise. The final order selected for each case is given in **Table 11** below. Also, provided are the lag-lengths used in the Newey-West t-stats for each horizon pair (3 – 1, 6 – 1 etc.) in regressions as in **[24]**.

For each horizon 10000 samples were generated using the estimated ARMA models. Beginning of the sample provided the start-up values for each experiment. Error terms were drawn from a normal distribution with standard deviation equal to the standard deviation of the error term from each ARMA model. Errors from each ARMA model were found to be conditionally homoskedastic, so no ARCH effect was introduced. Error terms from inflation change equation and interest rate change equation were assumed to be independent.

For example, for 3 month to 1 month regression, univariate ARMA models were separately fit to $y_{3-1} = \pi_3 - \pi_1$ and $x_{31} = i_3 - i_1$ for each case (i.e. NS, CIR, SV and NS_NSE). Under the null that $\beta = 0$, simulated values of ARMA values were generated and OLS and Newey West corrected t-stats recorded for each estimate of equation **[23]** for each draw of the experiment. The critical values for 10% significance level (two-tailed test) were taken to be the 5th and 95th percentile values from the finite sample distribution of t-stats.

Table 11
Order of ARMA(p, q) Models in Monte Carlo Simulation

Horizon/ARMA Order	*Lag Length for Newey West*	*y*		*x_ns*		*x_cir*		*x_sv*		*x_ns_nse*	
		p	*q*	*p*	*q*	*p*	*q*	*p*	*q*	*p*	*q*
3 – 1	*8*	6	4	4	7	4	6	5	5	6	6
6 – 1	*20*	5	7	4	6	4	5	5	5	6	6
6 – 3	*12*	4	6	4	4	4	6	5	5	5	5
12 – 3	*36*	5	4	6	6	4	5	6	5	5	5
12 – 6	*24*	5	6	6	6	4	6	5	5	5	6
18 – 6	*48*	6	4	4	6	4	6	6	5	5	5
18 – 12	*24*	5	4	4	7	4	6	5	5	5	5
24 – 12	*48*	5	4	4	4	4	6	5	5	5	6
24 – 18	*24*	4	6	4	5	4	6	4	6	5	5

VI. Conclusion

This study has been an attempt to provide evidence on the competitive performance of three parsimonious models – one of which, CIR, is a general equilibrium model.

Function from derived from the theoretical CIR model comes across as best in pricing G-Secs in the Indian market as compared to NS and Extended NS, on all the criteria discussed in the study. Results, however, are only marginally superior to NS, and there is very little to choose between the two. Though from the point of arbitrage decisions results only confirm what other Indian studies have found – that MAPE and MAYE are far too high and volatile to have much faith in using these for pricing derivatives.

For monetary policy purposes, however, the results are encouraging, in that all the models result in smooth series for forward rates and a high correlation with average of MIBID/MIBOR. Further,

[20] Unit Root tests revealed no unit roots in both inflation changes and term structure slopes for all horizons

the MAYE is of the order of 12 – 13 basis points only for all the models, which should be considered acceptable given the across the board liquidity in the bond market during the sample under study (mid 2001 – mid 2003). Also, the fact that order of errors for both in-sample and out-of-sample is very similar is an indication that results are robust to the selection of bonds used to estimate the term structure. This is encouraging from the point of view of monetary policy analysis.

It would be interesting to see, with increased trading and more efficient operation of NSE WDM over the years, how these models compare in a sample period which includes data till date.

If computational resources permit, rigour can be added to this study by doing a local grid search on the lines of Bolder and Streliski (1999) of Bank of Canada, which deals with estimation issues quite comprehensively. It is not expected that results would improve drastically from what has been reported in this study, as day-to-day term structures are not expected to vary by too much.

Extensions to this study would include checking for the predictability of future-short rates and inflation from change in slope across the term structure and investigating the dynamics of the short-rate derived from these models. Since CIR comes across as the best model, albeit marginally, it would also be interesting to see how well the dynamics of the short-rate captured by CIR compares with MIBID/MIBOR and call rate.

In the sample (and horizons) under study the term structure does not seem to contain any information for changes in inflation. However, the findings here indicate that a study conducted over a longer sample using lower frequency data at the longer-end may provide more encouraging results. This is part of an on-going work.

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