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Goutam Dutta
Divya Pachisia

W.P. No.2014-10-01
October 2014

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Goutam Dutta
Indian Institute of Management
Ahmedabad- 380015
goutam@iimahd.ernet.in

Divya Pachisia
Former Student, Indira Gandhi Institute of Development Research
Mumbai – 400 065
pachisiadivya@gmail.com

Abstract

The National Railways of an Emerging Asian Economy (NREAE), the second largest railway network in the world, is facing growing challenges from low fare airlines. To combat these challenges, NREAE has to adopt revenue management systems where efficient forecasting plays a crucial role. In this paper, we make an attempt to compare various forecasting techniques to predict railway bookings for the final day of departure. We use NREAE data of 2005-2008 for a particular railway route, apply time series [moving average, exponential smoothing, and Auto Regressive Integrative Moving Average (ARIMA), linear regression, and revenue management techniques (additive, incremental, and multiplicative pickup] to it and compare various methods. To make an efficient forecast over a booking horizon, we employ a weighted forecasting method (a blend of time series and revenue management forecasts) and find that it is successful in producing average Mean Absolute Percentage Error (MAPE) less than 10% for all fare classes across all days of the week except one class. The advantage of the model is that it produces efficient forecasts by attaching different weights across the booking period.

Key words: Forecast Accuracy, Revenue Management, Railways, Time Series, ARIMA,

1. Introduction

A revenue management system has two key elements - the optimizer and the forecaster. The forecaster forecasts the daily passenger demand using the historical pattern of passenger arrivals. The optimizer generates optimal allocations of passengers using this forecasted demand as inputs. So, the success of the system depends crucially on accurately forecasting the passenger demand for the final day of departure.

In this paper, we consider a study on the National Railways of an Emerging Asian Economy (NREAE), the second largest railway network in the world. NREAE is the only provider of railway services in the country. It earns its revenue from the passenger and cargo segments. Passenger revenue contributes more than one third of the total revenue, and as the population increases, this segment needs special attention. NREAE is facing growing challenges from low fare airlines which promise lower travel time and better customer satisfaction. In such a scenario, NREAE needs to adopt a revenue management system which considers the dynamic characteristics of inventory and time sensitivity of demand.

There are several methods for forecasting demand. Regression is one of the easiest and most common ways to forecast passenger demand, whereas time series is one of the most conventional and popular methods of forecasting. However, in the case of airlines, hotels, and railways booking, pick up plays a crucial role on the final day forecasting. To capture the dynamics of pick up characteristics, these service industries rely on revenue management forecasting techniques like additive, incremental, and multiplicative pick up. The advantage of pick up methods is that they relate two time-indexed variables - the day of booking and the day of departure - and update the final day bookings as the day of departure approaches. In this paper, we compare these forecasting techniques in terms of forecasting accuracy measured in Mean Absolute Deviation (MAD), Mean Absolute Percentage Error (MAPE), and Root Mean Square Error (RMSE). We propose a blend of these forecasting techniques which will help NREAE to forecast final day passenger arrivals with must greater forecasting accuracy. We measure this accuracy from the early part of the booking window to the late part of the booking window.

We organize the paper as follows. In Section 2 we provide a brief review of related literature. In section 3 we present the data collection process and section 4 discusses the research methodology of our paper. Section 5 presents the data analysis part and we formulate our proposed weighted average forecasting model in section 6. Finally section 7 concludes the paper with possible extensions from our research.

2. Literature Search

A wide range of literature is available on revenue management applications in airlines, hotels, and railways. These studies focus on the optimal booking limit, pricing, overbooking, demand estimation, and other major issues of revenue management. We know that forecasting is an integral and hence a critical part of a revenue management system in a railways. According to Lee (1990), a 10% increase in forecast accuracy results in a 0.5-3% benefit in optimal revenue. Multiple regression is very common in forecasting. Sa (1987) employs multiple regression to forecast the final day bookings for an airline. Weatherford (1998) compares various forecasting techniques for airline data. Lee (1990) formulates a rigorous probability model that describes the booking process for airlines as a stochastic process with requests, reservations, and cancellations interspersed in time before the departure of a flight. He develops a full information combined model (FIC model) which uses both the advance bookings and the historical bookings to produce more efficient forecasts. He compares the censored Poisson model, the FIC model, the simple linear regression model, and the 8-week moving average model, and shows that the FIC model produces better results than the others. Wickham (1995) evaluates the performance of different forecasting methods like regression, time series, and booking pickup models (both classical and advanced) to forecast the demand for airlines. The booking pickup models in general produce superior results compared to time series and regression. In particular, an advanced pickup model produces the best results and for all models, the forecasting accuracy reduces with an increase in the forecasting period. Skwarek (1996) and Zickus (1998) discuss the detruncation method used in their forecasting model.

Skwarek (1996) compares various detruncation methods whereas Zickus (1998) captures the interactions between forecasting methods, detruncation methods, and capacity allocation algorithms.

In the field of hotel revenue management, a paper by Weatherford and Kimes (2003), use the data from Choice Hotels and Marriott Hotels to forecast arrivals. They evaluate different forecasting methods like simple exponential smoothing (SES) , double exponential smoothing (DES), moving average methods (MA), linear and logarithmic regression, additive or ‘pickup’ method, and multiplicative method. They find that pickup methods and regression give the best forecasts in case of Choice Hotels whereas exponential smoothing, pickup, and moving average method outperform the other methods for Marriot Hotel.

Talluri and Van Ryzin (2004) propose that the revenue management methods which are effectively used for forecasting in the airline and hotel industry might be effective for the cruise line industry as well. For this industry, we elucidate the work of Sun, Gauri, and Webster (2009). They use the data from a prominent North American Cruise Company and implement 24 forecasting techniques like classical pickup, advanced pickup, linear and logarithmic regression, and time series methods like MA, SES DES, ARIM. They find that classical methods are the best out of all the additive methods and multiplicative methods are the most inefficient. Among the non-pickup methods Moving Average (MA) works the best.

In the field of railway revenue management, we first look at the work of Chiang et al. (2007) that discusses a comprehensive survey of revenue management application where some application of railways are mentioned. We then consider the work of Lee and Wei (2005). They use two dynamic neural network structures to capture short-term railway passenger demand on the data from Taiwan Railway for the years 1999 and 2000. The first model is a non linear autoregressive model and the second is an extension of the first model which integrates internal recurrent so as to have a parsimonious structure. The first model produces forecasts with MAPE less than 20% in a majority of the cases. The second model performs as well as the first while keeping the model more compact. Celebi, Bolat, and Bayraktar (2009) employ both the MLP (Multi-Layer Perceptron) model as well as traditional models such as ARIMA to forecast the short-term railway passenger demand. They observe that the neural network model displays marginally more accuracy than the best ARIMA model and suggest that instead of being a substitute for ARIMA models, the neural network models should be used in conjunction with traditional statistical methods to produce robust forecasts.

Reviewing the exhaustive literature we find that there are few studies that have compared forecasting methods on railway passenger data. This is probably the first attempt (XXXXX, 2012) in South East Asia on the implementation of revenue management optimization, simulation and forecasting techniques with a large data set. XXXX did not discuss much about forecasting results. In this study, we provide the details of forecasting methods. We attempt first to implement time series, regression, and pick up methods on NREAE data and evaluate their forecast accuracy. We also propose a forecasting model which is a weighted average of time series and pick up methods and generates efficient forecasts over a booking horizon.

3. Data Collection

We collected passenger booking information for a NREAE running between a metro and a mini metro for the years 2005-2008. There are four fare classes in the train – first class air-conditioned (1^{st} AC), second class air-conditioned (2^{nd} AC), third class air-conditioned (3^{rd} AC), and Sleeper (SL). The capacity of the fare classes are 18, 138, 384, and 576 respectively. As most of the passengers travel from origin to destination, we build forecasting models for this segment. We analyze the data for 2005, 2006, 2007, and forecast final day passenger arrivals for April 2008. We build booking curves for each day of the week and for each fare class, where we plot days before departure against cumulative passenger arrivals. We have observed day wise seasonality in the booking curves, hence the rationale for day wise analysis. For pickup methods we divide the booking horizon into six parts: D_{-21} (21 days prior to departure), D_{-14} (14 days prior to departure), D_{-7} (7days prior to departure), D_{-2} (2 days prior to departure), D_{-1} (1 day prior to departure), and D_0 (day of departure).

4. Research Methodology

We apply different forecasting techniques to predict the final day passenger booking. We use 2005-2007 passenger booking information as input and forecast for April 2008 by using different forecasting techniques and compare the forecasting accuracy through MAD, MAPE, and RMSE. We categorize forecasting methods into time series, linear regression, and revenue management techniques. Time series methods include exponential smoothing, 'N' period moving average, and ARIMA process. Revenue management techniques are basically pick up methods and they are classified into three parts - additive, incremental, and multiplicative pick up. We consider linear regression as a pick up method and apply it to our data. We divide the forecasting techniques into these various sub-groups so as to determine which method is the most efficient within each sub-group. We then compare the best technique of the various sub-groups to determine which forecasting techniques suit the data at hand most efficiently.

Time series Methods

In this section, we discuss various time series methods like moving average, exponential smoothening, and ARIMA. These are the conventional methods of forecasting and we discuss how they can be applied to our data.

Moving Average

The moving average is a straightforward and extensively used forecasting tool. It is used to smooth out the short-term variations in time series data so as to emphasize the long term trends or cycles. To forecast the final day booking for April 2008, we use the simple mean of the past N observations and the forecast so obtained is called the N-period moving average. In particular, we estimate the 10 period moving averages and the 4 period moving averages for our data set. However, we present the results of only the 4 period moving averages as it is much more efficient than the 10 period moving averages.

Exponential Smoothing

The simple moving average gives equal weight to all the N observations and ignores all the previous observations. A better method is the exponential smoothing method which discounts the past observations in a more gradual manner. It is a technique which can be applied to time series data to smooth out the data or to make forecasts. The forecasted value is a linear combination of current value and one year lagged forecasted value. The smoothing constant α, attached to the current value, varies from zero to one. We use different weights in our analysis and have found that α=0.30 produces the best results.

Auto Regressive Integrated Moving Average (ARIMA)

Before applying any time series model, it is essential to diagnose whether the underlying series is stationary or non-stationary. We determine this by conducting the Augmented Dickey-Fuller Test (ADF). If the time series model is non-stationary, we can make it stationary through successive differencing. We carry out our time series modeling in Statistical Analysis Software (SAS). The important time series models are AR (p), MA (q), ARMA (p,q) and ARIMA(p,d,q), where p and q are the orders of the AR and MA processes. ARMA (p,q) is a combination of AR and MA processes. Non-stationary processes can be represented by ARIMA (p, d, q) where 'd' denotes the differences needed to make the series stationary. In practice, the values of p, d, q are less than or equal to 2. To forecast the final day booking, we first need to identify and fit an ARIMA model by looking at the Autocorrelation Function (ACF), Partial Autocorrelation Function (PACF), and Inverse Autocorrelation Function (IACF). After estimation of the coefficients, we run the model in SAS to generate the forecasts. In our analysis, we test the models like AR(1), AR(2), MA(1), MA(2), ARMA(1,1), ARIMA(1,1,0), ARIMA(2,1,0), ARIMA(0,1,1), and ARIMA(2,1,1).

Linear Regression

Linear regression is one of the easiest ways of forecasting. We consider the data of the month of April 2005-2007 as inputs and forecast for April 2008. We have done a day wise analysis for all the fare classes. The dependent variable is the number of bookings on day zero, that is, the day of departure (denoted by D_0), and the independent variable is the cumulative booking of 1 day before departure (denoted by D_{-1}). We run the following linear regression for all days and for all fare classes.

(1) $D_0 = a_1 + b_1*D_{-1}$

Likewise, separate regressions are done for all booking period horizons and given by

(2) $D_0 = a_2 + b_2*D_{-2}$

(3) $D_0 = a_3 + b_3*D_{-7}$

(4) $D_0 = a_4 + b_4*D_{-14}$

(5) $D_0 = a_5 + b_5*D_{-21}$

We compute all the coefficients and use it to forecast for April 2008 for all days of the week and for all fare classes. We have also tried logarithmic linear regression and non linear regression but the results are poor. So we discard them from our analysis.

Revenue Management Forecasting Methods

Revenue management forecasting techniques are crucial for airlines, hotels, and railways as they capture the pick up characteristics of passenger booking during a booking horizon. Revenue management forecasting techniques are classified into three segments - additive, incremental, and multiplicative pick up. We explain all these methods with the help of a cumulative booking matrix.

We start with a booking matrix which shows daily booking during a booking period.

Table 1: Booking Matrix

Days before departure	D_0	D_{-1}	D_{-2}	D_{-3}	.	.	.	D_{-T}
Day of departure	0	1	2	3	.	.	.	T
1	b_1	a_{11}	a_{12}	a_{13}				a_{1T}
2	b_2	a_{21}	a_{22}	a_{23}	.	.	.	a_{2T}
3	b_3	a_{31}	a_{32}	a_{33}	.	.	.	a_{3T}
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	
k	b_k	a_{k1}	a_{k2}	a_{k3}	.	.	.	a_{kT}

Notations:

i: Date of departure indexed by i i= 1,2,…,k

j: Days before departure indexed by j j= 1,2,….,T

a_{ij} : Booking on i[th] date of departure from j days before departure for i= 1,2,…,k and j= 1,2,….,T

b_i : Booking on day of departure i.e. day 0 for i= 1,2,…,k

A_{ij} : Cumulative booking on i[th] date of departure from j days before departure for i= 1,2,…,k and j= 1,2,….,T

So, cumulative booking for 1[st] date of departure can be expressed as

$A_{1T} = a_{1T}$ for T days before departure

$A_{1T-1} = a_{1T-1} + A_{1T}$ for T-1 days before departure

$A_{ij} = a_{ij} + A_{ij+1}$ for i[th] date of departure from j days before departure

The cumulative booking matrix can be written as

Table 2: Cumulative Booking Matrix

Days before departure	D_0	D_{-1}	D_{-2}	D_{-3}	.	.	.	D_{-T}
Day of departure	0	1	2	3	.	.	.	T
1	A_{10}	A_{11}	A_{12}	A_{13}	.	.		A_{1T}
2	A_{20}	A_{21}	A_{22}	A_{23}	.	.	.	A_{2T}
3	A_{30}	A_{31}	A_{32}	A_{33}	.	.	.	A_{3T}
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
k	A_{k0}	A_{k1}	A_{k2}	A_{k3}	.	.	.	A_{kT}

Additive Pick up Method

We take the sum of all the columns of the Cumulative Bookings Matrix separately

Sum of column $D_0 = A_{10} + A_{20} + A_{30} + \ldots\ldots + A_{k0}$

$$= \sum_{i=1}^{k} A_{i0}$$

Sum of column $D_{-T} = A_{1T} + A_{2T} + A_{3T} + \ldots\ldots + A_{kT}$

$$= \sum_{i=1}^{k} A_{iT}$$

We generalize the column sum for D_{-j}

Sum of column $D_{-j} = \sum_{i=1}^{k} A_{ij}$

Now we calculate additive pickups for all D_{-j} s.

Additive pick up from $D_{-1} = r_1 = \dfrac{\sum_{i=1}^{k} A_{i0}}{\sum_{i=1}^{k} A_{i1}}$

Additive pick up from $D_{-T} = r_T = \dfrac{\sum_{i=1}^{k} A_{i0}}{\sum_{i=1}^{k} A_{iT}}$

We generalize the additive pick up for D_{-j}

Additive pick up from $D_{-j} = r_j = \dfrac{\sum_{i=1}^{k} A_{i0}}{\sum_{i=1}^{k} A_{ij}}$

Let us introduce two more variables

B_{ij} : Booking on i[th] date from j days before departure

F_{ij} : Forecasted final day bookings on i[th] date from j days before departure

Forecasted passenger booking from additive pick up method = F_{ij}

$= B_{ij} * r_j$

Incremental Pickup Method

We introduce the incremental pick up concept with the previous cumulative booking matrix.

We calculate incremental pickups for all D_{-j} s.

Incremental pick up from D_{-1} to $D_0 = r_1 = \dfrac{\sum_{i=1}^{k}(A_{i0} - A_{i1})}{k}$

Incremental pick up from D_{-T} to $D_{-T-1} = r_T = \dfrac{\sum_{i=1}^{k}(A_{iT-1} - A_{iT})}{k}$

Next we generalize incremental pick up for D_{-j}

Incremental pick up from D_{-j} to $D_{-j-1} = r_j = \dfrac{\sum_{i=1}^{k}(A_{ij-1} - A_{ij})}{k}$

Forecasted passenger booking from incremental pick up = F_{ij}

$$= B_{ij} + r_1 + r_2 + r_3 + \ldots\ldots + r_j$$

Multiplicative Pickup Method

Now we introduce the multiplicative pick up method with the previous cumulative booking matrix.

Multiplicative pick up from D_{-1} to $D_0 = r_1 = \dfrac{\sum_{i=1}^{k}\left(\dfrac{A_{i0} - A_{i1}}{A_{i1}}\right)}{k}$

Multiplicative pick up from D_{-T} to $D_{-T-1} = r_T = \dfrac{\sum_{i=1}^{k}\left(\dfrac{A_{iT-1} - A_{iT}}{A_{iT}}\right)}{k}$

Next we generalize multiplicative pick up for D_{-j}

Multiplicative pick up from D_{-j} to $D_{-j-1} = r_j = \dfrac{\sum_{i=1}^{k}\left(\dfrac{A_{ij-1} - A_{ij}}{A_{ij}}\right)}{k}$

Forecasted passenger booking from multiplicative pick up = F_{ij}

$$= B_{ij}\ (1+r_1)\,(1+r_2)\,(1+r_3)\ldots\ldots(1+r_j)$$

5. Data Analysis

We used the data for the years 2005-2007 to get the results from the different models and checked its accuracy against the actual observations of 2008. We first briefly compare the results of each sub group among themselves and consequently get the most effective forecasting technique.

Comparison of the Time Series Methods

Time series is one of the most conventional methods of forecasting. These are essentially non pick up methods since the final forecast for D_0 remains unchanged whether one is forecasting at D_{-21}, D_{-14}, D_{-7}, D_{-2} or D_{-1}. In appendix 1, we state the results obtained from exponential smoothing with α=0.3, four-period moving average, and the best ARIMA process.

As mentioned earlier, we have fitted the following ARIMA models - AR(1), AR(2), MA(1), MA(2), ARMA(1,1), ARIMA(1,1,0), ARIMA(2,1,0), ARIMA(0,1,1), and ARIMA(2,1,1) to the data at hand. The following table summarizes our findings-

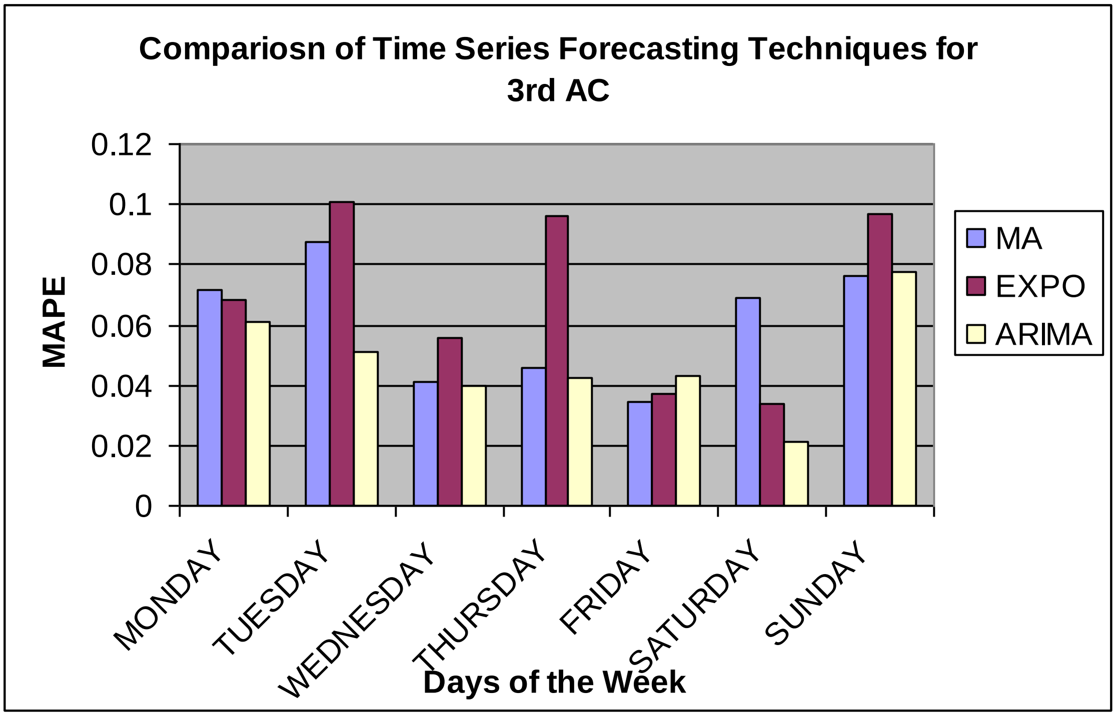
Table 3: Most Accurate ARIMA Model for the Respective Days and Fare Classes

	1st AC	2nd AC	3rd AC	Sleeper
Monday	ARIMA(2,1,0)	ARIMA(2,1,0)	ARIMA(2,1,0)	ARIMA(2,1,0)
Tuesday	ARIMA(2,1,0)	ARIMA(2,1,0)	ARIMA(2,1,0)	ARIMA(2,1,0)
Wednesday	ARIMA(2,1,1)	ARIMA(2,1,0)	ARIMA(2,1,0)	ARIMA(2,1,1)
Thursday	ARIMA(2,1,0)	ARIMA(2,1,0)	ARIMA(0,1,1)	AR(2)
Friday	ARIMA(0,1,1)	ARIMA(1,1,0)	ARIMA(1,1,0)	ARIMA(0,1,1)
Saturday	ARMA(1,1)	ARIMA(2,1,0)	ARIMA(2,1,0)	ARIMA(2,1,0)
Sunday	ARIMA(0,1,1)	ARIMA(2,1,0)	ARIMA(1,1,0)	ARIMA(0,1,1)

In most of the categories we find that ARIMA (2,1,0) performs better than the other models (as measured by MAD and MAPE).

Now, we compare the results of four period moving average, exponential smoothing, and ARIMA across days and fare classes. First we choose a particular fare class and compare the methods across all days of the week. Figure 1 shows that for 3rd AC, the MAPE of ARIMA is lowest for all days except for Friday and Sunday.

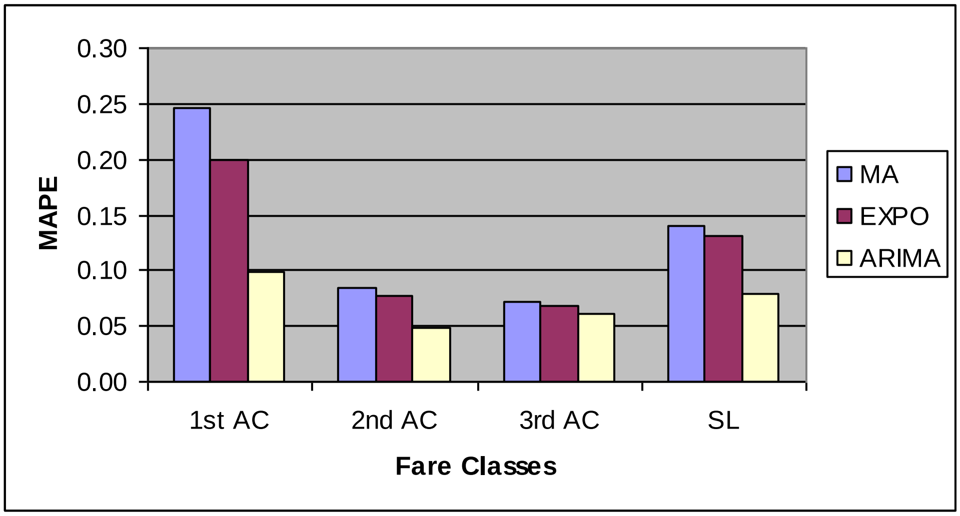
Figure 1: Comparison of Time Series Forecasting Techniques for 3rd AC



Similar analysis for other fare classes reveals that ARIMA works best for 1st AC (all days), 2nd AC (except for Tuesday and Sunday), and Sleeper class (except for Sunday). The so-called exceptions are the days when MAPE of MA(4) is lower than ARIMA. On neither of the days does Exponential Smoothing give more accurate results than the other two methods, which makes us conclude that the relative performace of this method is definitely the worst among them.

Next we compare the different methods across all the fare classes for every day in the week. Figure 2 shows that ARIMA gives the most efficient results for all fare classes on Monday.

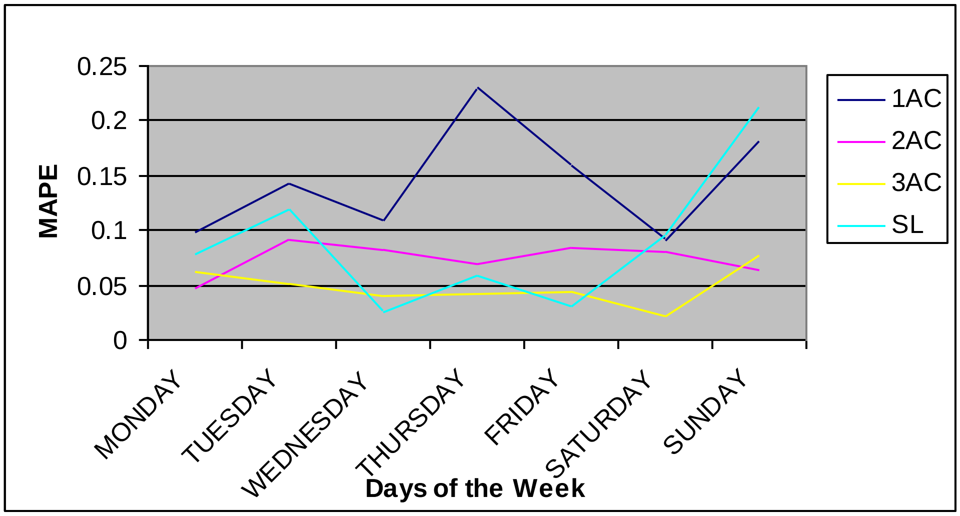
Figure 2: Comparison of Time Series Forecasting Techniques across Fare Classes



Similar analysis for other days of the week reveals that ARIMA dominates the other methods on Wednesday, Thursday, and Saturday whereas for days like Tuesday, Friday, and Sunday, MA(4) dominates ARIMA for some fare classes. It is however important to note that if we replace MAPE by MAD the above analysis will remain unchanged.

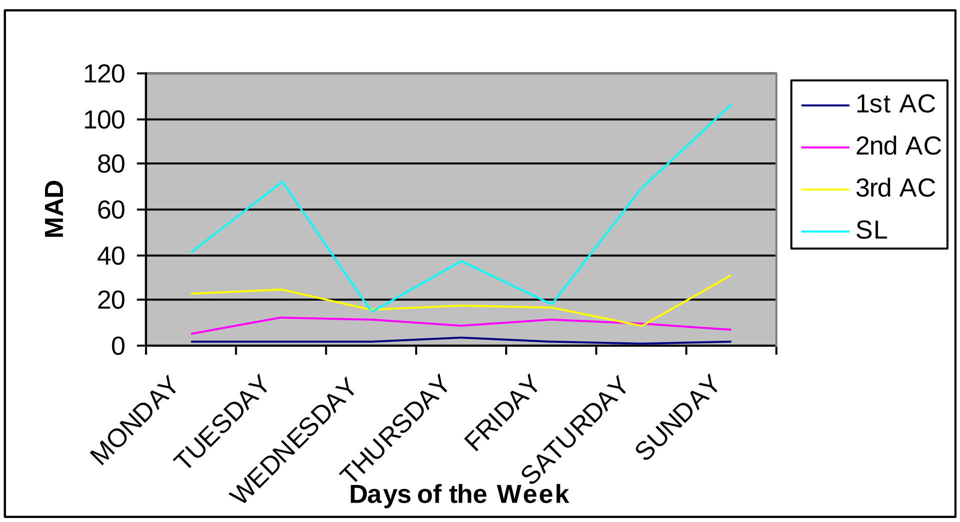
From the preceding discussion, we may conclude that the ARIMA technique is the best forecasting method among the time series methods. Figure 3 shows the performance of ARIMA for the various fare classes across the week. It can be readily observed that ARIMA produces the smallest MAPE for 3rd AC, followed by Sleeper class and 2nd AC whereas that of 1st AC is relatively high. This may lead us to conclude that the performance of ARIMA for 1st AC is significantly worse than for the other fare classes. However, this can be explained as being on account of the variation in the capacity of each fare class.

Figure 3: Comparison of ARIMA Technique for Different Fare Classes by MAPE



1st AC has the lowest seats(18), followed by 2nd AC (138 seats), 3rd AC (384 seats), and Sleeper class (576 seats). If we compare the different fare classes by the MAD obtained from ARIMA we find that the opposite result holds true. MAD is the smallest for 1st AC, followed by 2nd AC, 3rd AC, and Sleeper class. The following figure illustrates this-

Figure 4: Comarison of ARIMA Technique for Different Fare Classes by MAD



Analysis of Linear Regression

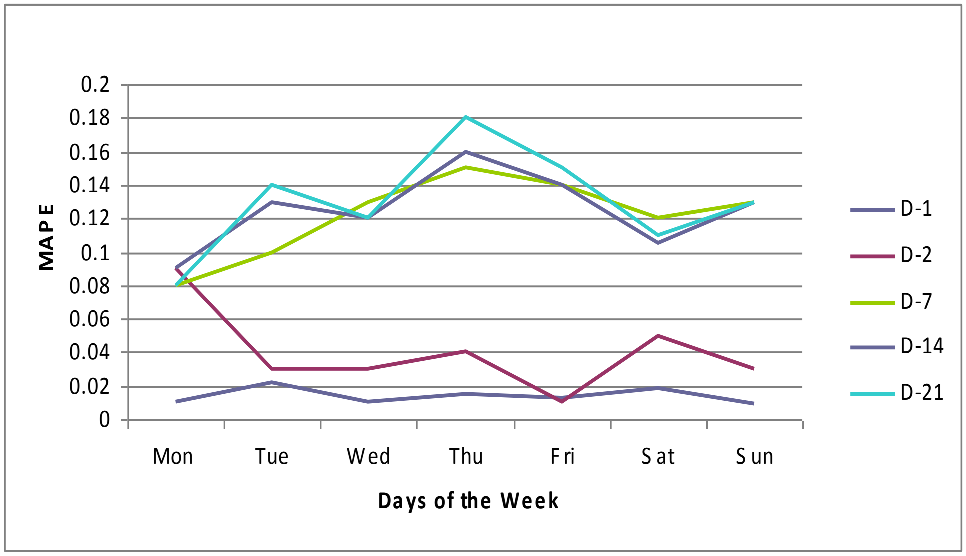
We carry out linear regression separately for each fare class with D_0 as the dependent variable and D_{-1}, D_{-2}, D_{-7}, D_{-14}, D_{-21} as the independent variables. We summarize the MAD, MAPE, and RMSE for 3rd AC in the following table-

Table 4: Forecasting Accuracy of Linear Regression for 3rd AC for all Days of the Week

Day before Departure		Mon	Tue	Wed	Thu	Fri	Sat	Sun
D_{-1}	MAD	5.53	9.07	6.32	6.8	5.22	7.99	3.66
	MAPE	0.01	0.02	0.01	0.02	0.01	0.02	0.01
	RMSE	9.58	13	8.01	10.2	5.51	9.16	4.41
D_{-2}	MAD	38.3	11.7	13.1	18.8	5.91	22.3	14.5
	MAPE	0.099	0.03	0.03	0.04	0.01	0.05	0.03
	RMSE	40.3	14.8	16.2	34.4	6.77	25.2	5.69
D_{-7}	MAD	33.51	44.7	51.8	67.3	59.9	51.5	53.4
	MAPE	0.08	0.1	0.13	0.15	0.14	0.12	0.13
	RMSE	118.7	45.1	54.4	68.3	61.4	7.24	54.7
D_{-14}	MAD	36.67	55.7	47.7	73.7	59.8	42.7	52.8
	MAPE	0.09	0.13	0.12	0.16	0.14	0.11	0.13
	RMSE	84.27	62.7	52.3	78.4	60.7	53.2	57.4
D_{-21}	MAD	32.85	57	47	82.4	62.9	45.8	53.2
	MAPE	0.08	0.14	0.12	0.18	0.15	0.11	0.13
	RMSE	65.4	65.9	57.8	90.5	63.7	51.9	57.1

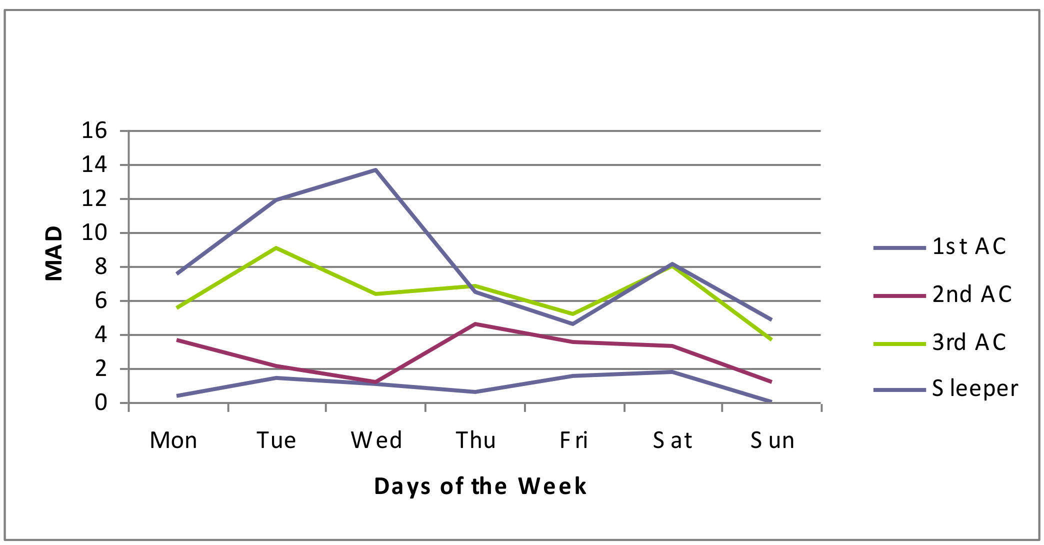
As expected, the forecasting accuracy for D_{-1} is the most with MAPE lying between 1%-2% followed by D_{-2} (1%-9%) whereas that for D_{-7}, D_{-14} and D_{-21} is between 8%-18%. As the departure date comes closer, the forecasting accuracy improves. Figure 5 captures this result.

Figure 5: Daywise Comparison of MAPE for 3rd AC



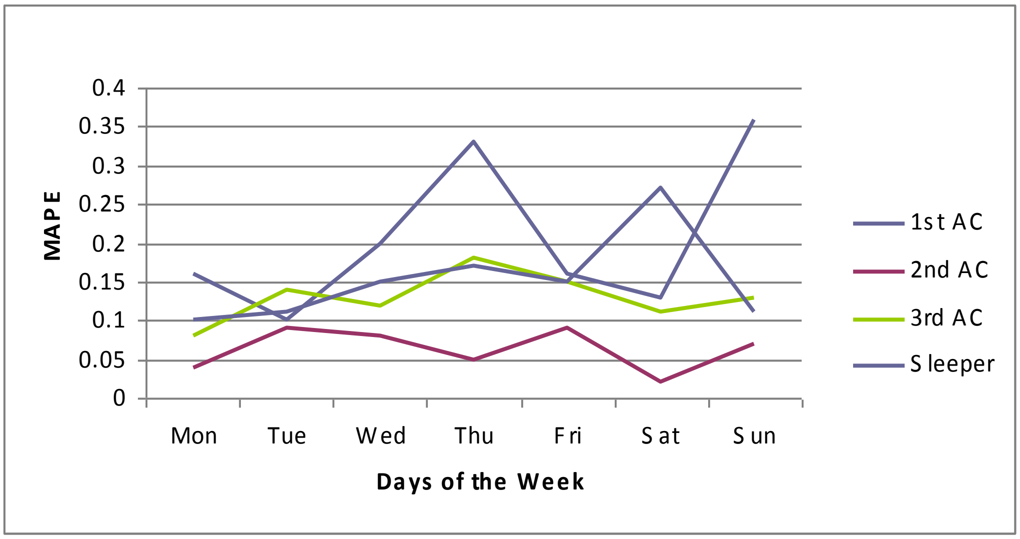
For the other fare classes as well a similar trend has been noted. The precise results have been given in appendix 2. The Sleeper class follows a trend similar to 3rd AC. However it is essential to note that for 1st AC, the MAPE is on the average much higher for D_{-1},D_{-2} *vis-a-vis* the other classes. This can again be explained due to the variation in the capacity of the different classes. But if we take MAD as the yardstick, it is seen that it is the lowest for 1st AC, followed by 2nd AC, 3rd AC and Sleeper class (Figure 6).

Figure 6: Comparison of MAD for all the Fare Classes on D_{-1}



We will now consider the forecasting accuracy on specific days before departure of all the fare classes. As noted earlier, the MAPE for D_{-1} and D_{-2} have been found to be very low indicating high forecasting accuracy. For D_{-7}, MAPE is found to lie in the range of 10%-30% for 1st AC and Sleeper. In 3rd AC, it lies between 8%-15% and for 2nd AC it varies between 5%-9%. For both D_{-14} and D_{-21}, the MAPE is quite high (lying between 10%-36%) for all the classes with the exception of 2nd AC (below 10%). The following graph gives the MAPE for the different fare classes on D_{-21}.

Figure 7: Forecasting Accuracy on D_{-21}



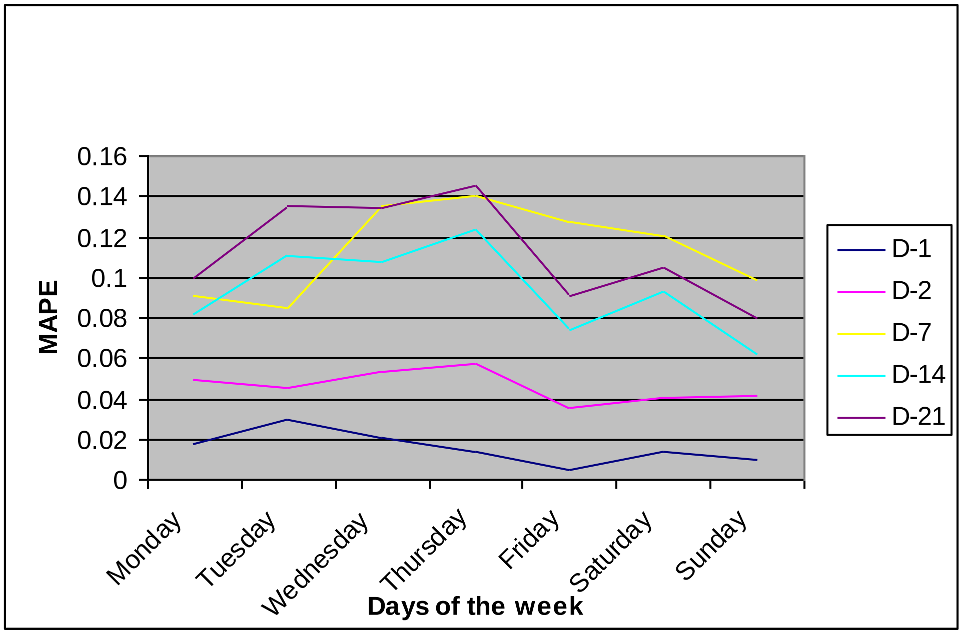
Thus, from the analysis of the linear regression, we may conclude that although forecasting accuracy is high for D_{-1} and D_{-2}, we need to implement other forecasting techniques to improve the forecasting accuracy of D_{-7}, $D\text{-}_{14}$, and D_{-21}.

Comparison of Pick up Methods

Revenue management forecasting techniques are different from time series methods as they capture the pick up characteristics of booking curves. We build booking curves for 2005, 2006, and 2007 to analyze the pick up pattern. We forecast for April 2008 final day passenger arrivals using additive, incremental, and multiplicative pick up methods. In the case of NREAE, it appears that incremental pick up is superior to the other two methods in terms of MAD and MAPE. So we focus on the incremental pick up method and carry on our analysis.

We analyze incremental pick up results for a specific fare class across all days of the week over a booking horizon. In figure 8, we show the variation of MAPE for 3rd AC over a booking horizon and for all days of the week.

Figure 8: Daywise Analysis of Incremental Pick up Method for 3rd AC

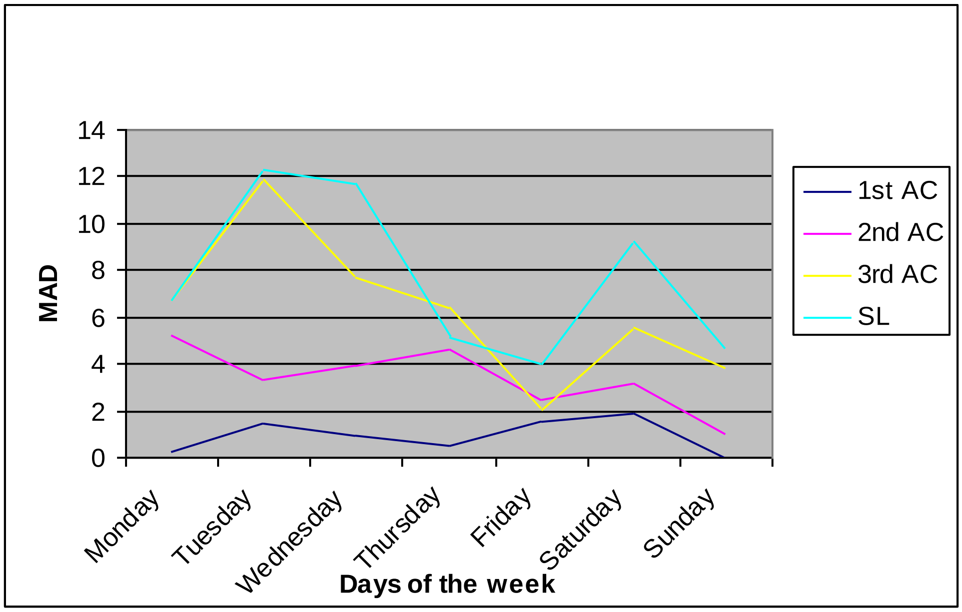


As the departure date arrives, the forecasting accuracy increases, which is quite obvious. But in the middle of the week, D_{-14} forecasts play better than D_{-7}. Friday pick up performs better than other days except D_{-7} forecasts. Overall, the forecast accuracy from 21 days prior to departure is below 15%, irrespective of all days.

Next we carry out our analysis on the basis of MAD for all fare classes and for all days of the week. Here we consider only the D_{-1} forecasts. In figure 9 we show the variation of MAD.

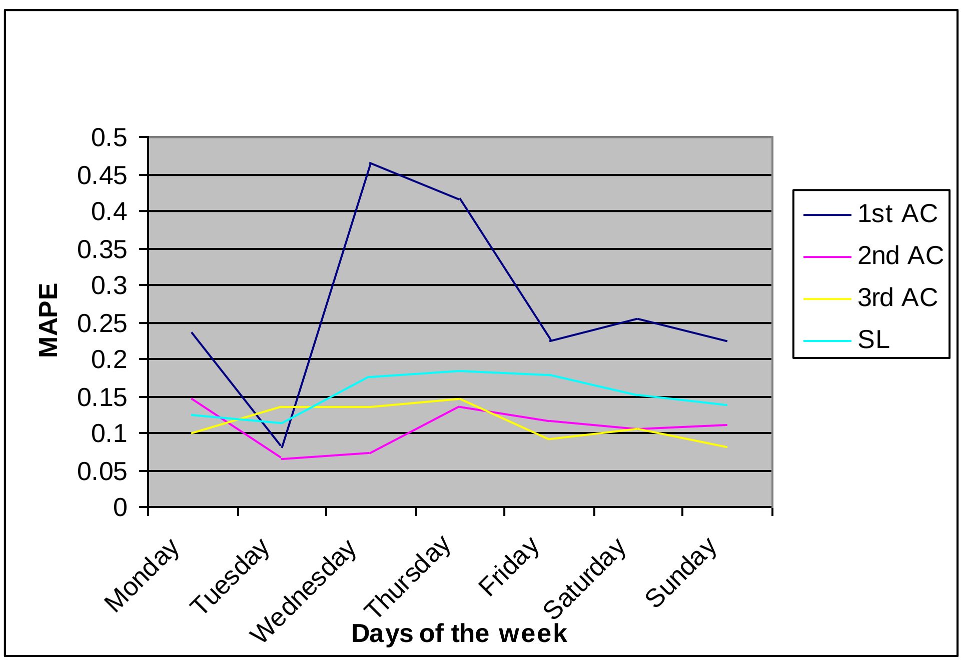
As the capacity of the fare class increases, MAD also increases; only for Thursday Sleeper is the MAD lower than 3rd AC. For Tuesday, the MAD is more than 10 for 3rd AC and SL. The variation is not as high for the other days of the week.

Figure 9: Comparison of MAD for all the fare classes on D_{-1}



We also carry out the same analysis for D_{-21} with MAPE. In figure 10, we show the variation of MAPE for all fare classes and for all days of the week.

Figure 10: Forecasting Accuracy on D_{-21}



It is evident that for 1st AC, D_{-21} forecasts are not convincing as per MAPE, and for 2nd AC and 3rd AC, MAPE is below 15% for all days. But for Sleeper, it lies in a range of 15-20% in the middle of the week.

Comparison of the three methods

We now compare the best technique of each of the three methods discussed above - ARIMA, linear regression, and incremental pick up. From the analysis above, it is evident that both linear regression and incremental pick up methods emerge as being superior to the ARIMA technique to serve as a good method to forecast the final day booking from D_{-1} and D_{-2}. Thus either of the two methods can be used to make forecasts. But the ARIMA technique works better for D_{-7}, D_{-14}, and $D_{-21.}$ Since ARIMA is a non-pickup method, its forecast for D_{-1} is equivalent to its forecast for D_{-21}. The following graphs for 3rd AC illustrate our argument.

Figure 11: Comparison of Different Methods for Forecasting Accuracy on D_{-1}

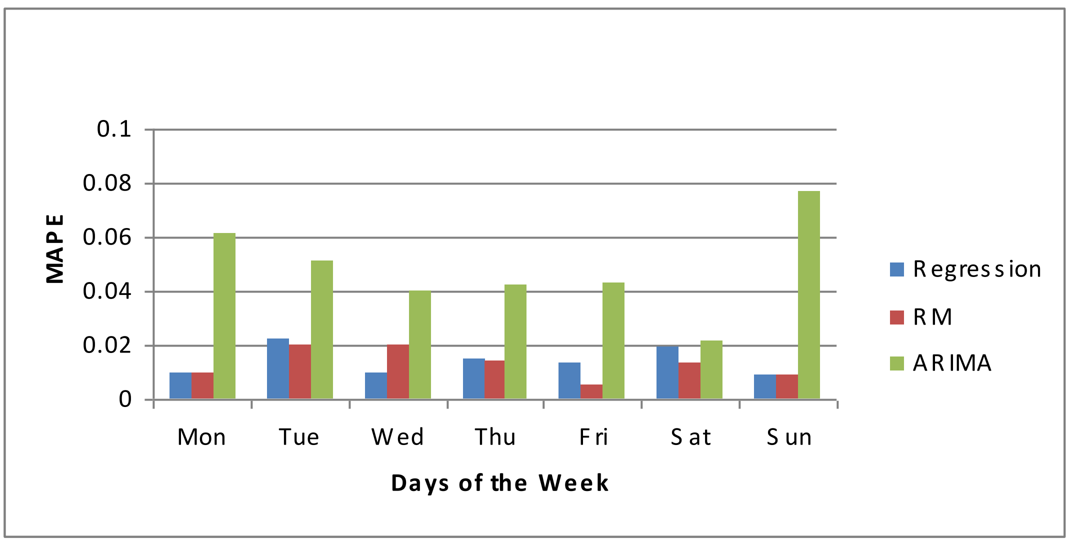
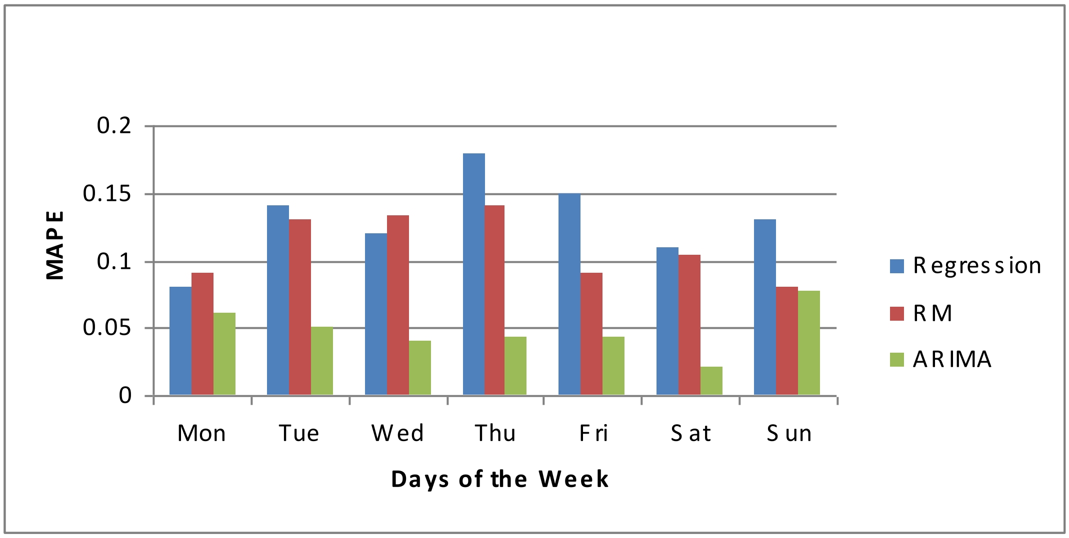


Figure 12: Comparison of Different Methods for Forecasting Accuracy on D_{-21}



It is inconvenient to switch models to forecast for different time horizons. Hence our objective is to look for a forecasting model which will keep the MAPE below 10% over the booking horizon.

6. A Weighted Average of Revenue Management and Time Series Technique

Description of the Model

We start by defining a set of variables

A_{ijf} = Cumulative booking on ith date from j days before departure for fare class f.

T_{if} = Time series forecast for final day booking on ith date of departure for fare class f

F_{ijf} = Fractional build for ith date from j days before departure for fare class f

Hence,

$$F_{ijf} = \frac{\sum_{i=1}^{k} A_{ijf}}{\sum_{i=1}^{k} A_{i0f}}$$

where notations have their usual significance (refer to cumulative booking matrix in section 4).

$$RM_{ijf} = \frac{A_{ijf}}{F_{ijf}}$$

= Forecast from the revenue management technique on i^{th} date from j days before departure for fare class f

FB_{ijf} = Booking adjusted end point forecast for i^{th} date from j days before departure for fare class f

We now consider a blend of revenue management and time series forecast, which is given by

$$FB_{ijf} = (1 - w_{jf}) * T_{if} + w_{jf} * RM_{ijf}$$

We have thus formulated the problem in terms of TF (which is non-pickup in nature) and RM (which considers pick up booking data). We shall refer to this method as the Weighted Forecast Method (WF method). Here w_{jf} corresponds to the weight given to the f^{th} fare class for j days before departure. *A priori*, it is expected that to make accurate forecasts from D_{-1} and D_{-2} it would be pragmatic to place more weight on RM_{ijf} whereas if the objective is to predict from D_{-7}, D_{-14} and D_{-21} we should choose a smaller value for 'w' (implying placing more weight on the time series forecast). For the time series component in the above formulation, we have used the forecasts from the ARIMA technique.

Comparison of WF method with Linear Regression and Incremental Pick up

As before, we have continued with our day wise analysis and implemented this model for all the fare classes. For D_{-1} and D_{-2} we use weights ranging from 0.7-0.9 and for D_{-7}, D_{-14}, and D_{-21} we use weights ranging from 0.05-0.3. We check the accuracy of the forecasts through MAPE. We take the average of the weights that produce the lowest MAPE across all days of the week over a booking horizon and give it in the following table.

Table 5: The Respective Weights for D_{-1}, D_{-2}, D_{-7}, D_{-14} and D_{-21}

Days before Departure	Weight
D_{-1}	0.8
D_{-2}	0.8
D_{-7}	0.2
D_{-14}	0.2
D_{-21}	0.1

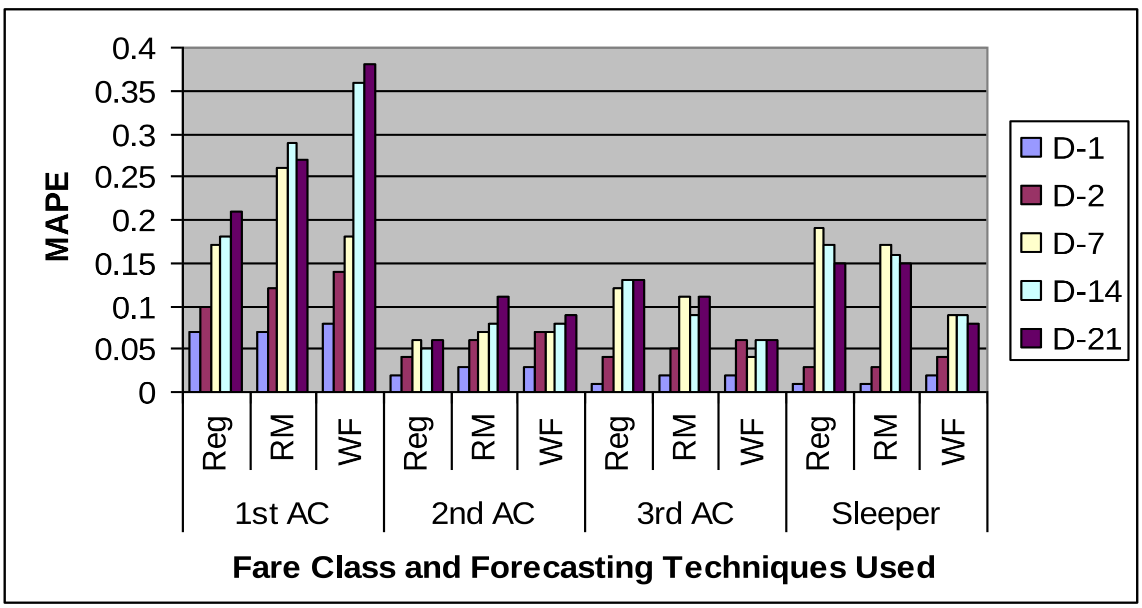
Now we compare the MAPE obtained from the linear regression, incremental pick up method, and the WF method for different fare classes. First we obtain the MAPE for every fare class day wise for each of D_{-1}, D_{-2} and so on. To facilitate easy comparison we took the average MAPE (i.e. averaged over Sunday, Monday to Saturday) for each of the methods. Table 6 explains the average MAPE obtained for each fare class.

Table 6: The Average MAPE from Different Forecasting Methods

Fare Class	1st AC			2nd AC		
Days before Dep.\Method	Reg	RM	WF	Reg	RM	WF
D_{-1}	0.07	0.07	0.08	0.02	0.03	0.03
D_{-2}	0.10	0.12	0.14	0.04	0.06	0.07
D_{-7}	0.17	0.26	0.18	0.06	0.07	0.07
D_{-14}	0.18	0.29	0.36	0.05	0.08	0.08
D_{-21}	0.21	0.27	0.38	0.06	0.11	0.09
Fare Class	3rd AC			Sleeper		
Days before Dep.\Method	Reg	RM	WF	Reg	RM	WF
D_{-1}	0.01	0.02	0.02	0.01	0.01	0.02
D_{-2}	0.04	0.05	0.06	0.03	0.03	0.04
D_{-7}	0.12	0.11	0.04	0.19	0.17	0.09
D_{-14}	0.13	0.09	0.06	0.17	0.16	0.09
D_{-21}	0.13	0.11	0.06	0.15	0.15	0.08

We can readily observe that for D_{-1} and D_{-2}, both regression and revenue management (incremental pick up) give marginally better results than WF, with MAPE varying between 1%-3% and 3%-6% respectively. But if we look at D_{-7}, D_{-14} and D_{-21} we find that for 2nd AC, 3rd AC, and Sleeper class, the average MAPE from the WF method is lower than 10%. It may be argued that regression serves as a better method for forecasting booking in the 1st AC and 2nd AC as indicated by the lower MAPE. In light of the fact that 1st AC has very low capacity (18 seats only), errors in forecasting are bound to be high so we shall not include it actively in our debate over which forecasting model is superior. Barring 1st AC, we can say that for the other three classes, for each day before departure (D_{-1}, D_{-2} to D_{-21}), we find the WF method to be better than the other methods as it consistently produces MAPE below 10%. Figure 13 summarizes the results.

Figure 13: Average MAPE for all the Fare Classes using Different Methods



Hence, we can conclude by saying that if our objective is to predict only for D_{-1} or D_{-2}, then regression or incremental pick up - both are equally suitable. But when we have to forecast early in the booking window, the WF method emerges as superior for all the fare classes. In reality, a good forecasting model updates its forecasts regularly and smoothens its final day booking as the day of departure approaches. So it is convenient and practical to use one single forecasting model for the entire booking period without compromising on the forecasting accuracy. The advantage of the WF method is that it consistently produces MAPE below 10% for any day of the week and for any fare class by attaching suitable weights.

7. Conclusion and Extensions

In our paper, we find that both incremental pickup and linear regression are efficient for short term forecasts whereas ARIMA gives efficient long term forecasts. Our objective is to devise a forecasting model such that the level of error in forecasting (MAPE) does not exceed 10% for any period of booking horizon. So we make an attempt to build a blend of the revenue management and ARIMA forecasts to arrive at a weighted forecast. The advantage of this model is that one can use a single model to make both short term and long term forecasts simply by varying weights. If the objective is to make short term forecasts one should attach greater weights to the revenue management forecast and if one wants to make a long term forecast he should attach greater weights to the time series forecast. This technique is successful in producing MAPE less than 10% over a booking horizon.

Of course one can advocate the use of regression for 2^{nd} AC. However, for practical reasons, a good forecaster would prefer to apply a single model for the entire data set rather than applying a whole range of models. Neither time series nor incremental pick up alone can serve the purpose of making good forecasts. A blend of these two methods, that is, the traditional method (time series) and a pick up method (revenue management) serves as the best technique.

Based on the result of the paper, we can successfully apply the weighted average forecast method for railway booking. This can be extended to other fields as well, such as airlines, hotels, cruise line bookings, and so on. Moreover in the railways itself it can be applied to other routes and other trains as well.

References:

1. Celebi D., Bolat B, and Bayraktar D (2009), Light Rail Passenger Demand Forecasting by Artificial Neural Networks, International Conference on Computers and Industrial Engineering, Issue date 6-9th July,2009, pp. 239-243.

2. Chiang, W-C., Chen, J.C.H. and Xu, X. (2007) 'An overview of research on revenue management: current issues and future research', *International Journal of Revenue Management,* Vol. 1, No. 1, pp.97–128.

3. Dutta G. and Ghosh P. (2012) A passenger revenue management system (RMS) for a National Railway in an Emerging Asian Economy, Journal of Revenue and Pricing Management (2012) **11,** 487–499. Advanced Online publications , doi:10.1057/rpm.2012.10; published online 6 April 2012

4.Dutta G. Ghosh P and Kaul Wanchoo A. , A Logarithmic Goal Programming Approach to Develop the Utility Function for a Railway Travel, accepted for publication in International Journal of Revenue Management

5. Lee, A.O. (1990), Airline reservations forecasting: Probabilistic and statistical models of the booking process. PhD Dissertation, Massachusetts Institute of Technology.

6. Sa, J. (1987), Reservation forecasting in airline yield management. Master's thesis, Massachusetts Institute of Technology.

7. Skwarek, D.K. (1996), Competitive impacts of yield management system components: forecasting and sell-up models. Master's thesis, Massachusetts Institute of Technology.

8. Sun, X., Gauri, D.K. and Webster, S. (2010) Forecasting for cruise line revenue management. Journal of Revenue & Pricing Management, advance online publication, 29 January 2010, doi:10.1057/rpm.2009.55.

9. Talluri, K. and van Ryzin, J. (2004), The Theory and Practice of Revenue Management. Boston, MA: Kluwer Academic.

10. Tsai, T-H., and Lee C-K., Wei C H (2005), Design of Dynamic Neural Networks to Forecast Short-Term Railway Passenger Demand Journal of the Eastern Asia Society for Transportation Studies, Vol. 6, pp. 1651 – 1666.

11. Weatherford, L.R. (1998), Forecasting issues in revenue management. In Presentation at Spring INFORMS Conference, Montreal, Canada.

12. Weatherford, L.R. and Kimes, S.E. (2003), A comparison of forecasting methods for hotel revenue management. International Journal of Forecasting 19(3): 401–415.

13. Wickham, R.R. (1995), Evaluation of forecasting techniques for short-term demand of air transportation. Master's thesis, Massachusetts Institute of Technology.

14. Zickus, J.S. (1998), Forecasting for airline network revenue management: Revenue and competitive impacts. Master's thesis, Massachusetts Institute of Technology.

Appendix 1: Results for Time Series Methods - 4 period Moving Average, Exponential Smoothing and ARIMA

	1st AC Fare Class																				
	Monday			Tuesday			Wednesday			Thursday			Friday			Saturday			Sunday		
	MA	EXPO	ARIMA	MA	EXPO	ARIMA	MA	EXPO	ARIMA	MA	EXPO	ARIMA	MA	EXPO	ARIMA	MA	EXPO	ARIMA	MA	EXPO	ARIMA
MAD	3.25	2.62	1.50	2.75	2.83	1.80	2.00	1.88	1.60	5.31	4.95	3.75	1.38	1.58	1.50	2.13	2.12	1.25	3.38	2.12	2.00
MAPE	0.25	0.20	0.10	0.22	0.22	0.14	0.14	0.13	0.11	0.42	0.38	0.23	0.14	0.16	0.16	0.14	0.14	0.09	0.30	0.19	0.18
RMSE	3.97	3.40	2.12	3.03	3.22	0.16	2.72	2.53	0.13	5.75	5.33	4.33	2.19	2.26	2.55	2.63	2.61	1.32	3.61	2.45	2.24
	2nd AC Fare Class																				
	Monday			Tuesday			Wednesday			Thursday			Friday			Saturday			Sunday		
	MA	EXPO	ARIMA	MA	EXPO	ARIMA	MA	EXPO	ARIMA	MA	EXPO	ARIMA	MA	EXPO	ARIMA	MA	EXPO	ARIMA	MA	EXPO	ARIMA
MAD	10.06	9.33	5.25	8.95	9.59	12.00	13.20	14.01	11.40	11.69	10.14	8.50	10.11	12.57	11.25	11.25	9.68	9.75	4.75	5.98	7.50
MAPE	0.08	0.08	0.05	0.07	0.07	0.09	0.10	0.10	0.08	0.10	0.08	0.07	0.08	0.09	0.08	0.09	0.08	0.08	0.04	0.05	0.06
RMSE	13.80	11.93	8.26	2.99	13.84	0.11	3.63	16.08	0.09	12.59	11.41	9.67	0.29	14.06	12.64	13.94	10.86	11.15	5.49	7.10	8.15
	3rd AC Fare Class																				
	Monday			Tuesday			Wednesday			Thursday			Friday			Saturday			Sunday		
	MA	EXPO	ARIMA	MA	EXPO	ARIMA	MA	EXPO	ARIMA	MA	EXPO	ARIMA	MA	EXPO	ARIMA	MA	EXPO	ARIMA	MA	EXPO	ARIMA
MAD	27.69	27.37	23.00	39.65	46.05	24.80	15.60	21.77	15.80	17.72	41.95	18.00	14.25	15.63	17.00	27.81	13.82	8.50	31.19	39.74	31.00
MAPE	0.07	0.07	0.06	0.09	0.10	0.05	0.04	0.06	0.04	0.05	0.10	0.04	0.03	0.04	0.04	0.07	0.03	0.02	0.08	0.10	0.08
RMSE	29.15	33.84	30.37	6.30	64.69	0.09	3.95	26.92	0.05	0.21	48.18	19.91	18.43	25.72	19.89	37.94	14.35	10.12	36.93	46.87	34.47
	Sleeper Fare Class																				
	Monday			Tuesday			Wednesday			Thursday			Friday			Saturday			Sunday		
	MA	EXPO	ARIMA	MA	EXPO	ARIMA	MA	EXPO	ARIMA	MA	EXPO	ARIMA	MA	EXPO	ARIMA	MA	EXPO	ARIMA	MA	EXPO	ARIMA
MAD	70.13	68.13	41.50	103.50	108.66	72.40	53.15	49.02	15.00	94.00	85.23	37.00	42.81	46.02	18.50	109.06	78.61	69.75	31.19	39.74	106.50
MAPE	0.14	0.13	0.08	0.17	0.17	0.12	0.09	0.08	0.03	0.15	0.13	0.06	0.07	0.08	0.03	0.19	0.13	0.10	0.08	0.10	0.21
RMSE	82.86	77.48	46.27	10.17	151.38	0.15	7.29	64.10	0.03	105.83	99.73	45.02	48.22	52.78	19.27	150.57	105.08	99.65	36.93	46.87	109.49

Appendix 2: Table showing the Linear Regression Results for all the fare classes for all days of the week

		1st AC Fare Class							2nd AC Fare Class						
		Mon	Tue	Wed	Thu	Fri	Sat	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Sun
D-1	MAD	0.31	1.42	1.04	0.64	1.48	1.709	0	3.68	2.09	1.17	4.58	3.55	3.25	1.14
	MAPE	0.02	0.105	0.08	0.043	0.12	0.11	0	0.03	0.016	0.01	0.03	0.02	0.02	0.009
	RMSE	0.45	3.0116	2.51396	2.60768	2.28473	2.82666	1.91311	0.17321	3.45299	3.697296	2.54951	2.13542	2.851315	2.19089
D-2	MAD	0.78	1.399	1.95	0.317	1.814	1.84	0.98	6.55	2.76	2.22	5.9	6.24	9.51	9.37
	MAPE	0.06	0.113	0.148	0.018	0.16	0.12	0.08	0.05	0.02	0.017	0.04	0.04	0.07	0.07
	RMSE	1.05	0.1483	0.1	0.12247	0.11402	0.13784	0.09487	0.22361	0.14153	0.158114	0.1	0.08367	0.104881	0.09487
D-7	MAD	1.75	1.54	1.78	3.57	2.57	1.42	1.97	4.61	12.54	10.6	7.99	6.69	7.08	10.13
	MAPE	0.14	0.12	0.144	0.26	0.244	0.09	0.16	0.04	0.09	0.07	0.06	0.04	0.05	0.08
	RMSE	2.51	3.6045	2.23354	3.70603	3.16511	1.57994	2.43337	0.2	3.79615	4.475709	2.7513	2.46131	3.079214	2.22571
D-14	MAD	2.86	0.832	2.12	4.64	2.54	1.67	1.71	7.44	8.83	9.3	5.79	8.87	2.34	8.44
	MAPE	0.23	0.06	0.163	0.345	0.22	0.1	0.15	0.06	0.06	0.067	0.05	0.066	0.019	0.06
	RMSE	3.17	0.9878	2.4996	4.74087	2.82538	1.97785	2.31054	0.24495	10.7856	11.16338	9.57352	9.18222	3.308881	10.3432
D-21	MAD	2.1	1.34	2.72	4.66	1.71	2.21	4.24	5.3	12.03	11.99	6.47	12.32	2.7	8.6
	MAPE	0.16	0.1	0.2	0.33	0.16	0.13	0.36	0.04	0.09	0.08	0.05	0.09	0.02	0.07
	RMSE	2.19	3.4264	3.62353	4.3359	2.43105	4.7244	3.8092	0.2	4.78435	4.780167	4.53762	3.65377	4.1833	3.10805
		3rd AC Fare Class							Sleeper Fare Class						
		Mon	Tue	Wed	Thu	Fri	Sat	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Sun
D-1	MAD		9.07	6.32	6.80	5.22	7.99	3.66	7.55	11.9232	13.67	6.5	4.56	8.13	4.8
	MAPE	0.01	0.02	0.01	0.02	0.01	0.02	0.01	0.01	0.02003	0.025	0.01	0.007	0.011	0.009
	RMSE	9.58	12.99	8.01	10.85	5.51	9.16	4.42	9.58541	14.4107	20.03197	7.56968	6.05805	9.481561	4.95379
D-2	MAD	38.30	11.74	13.13	18.80	5.91	22.32	14.51	15.71	22.89	22.85	20.59	13.35	17.5	9.66
	MAPE	0.10	0.03	0.03	0.04	0.01	0.05	0.03	0.03	0.03	0.04	0.03	0.02	0.02	0.019
	RMSE	40.30	14.76	16.16	34.36	6.77	25.23	18.00	20.6034	26.8393	27.37864	22.698	13.8214	21.34713	10.6442
D-7	MAD	33.51	44.68	51.84	67.26	59.90	51.48	53.43	118	84.98	120.9	115.1	101.5	189.8	69.59
	MAPE	0.08	0.10	0.13	0.15	0.14	0.12	0.13	0.21	0.14	0.2	0.18	0.16	0.29	0.13
	RMSE	118.70	45.14	54.37	68.32	61.37	52.41	54.65	118.775	87.4352	123.7607	124.98	105.809	192.8785	74.0783
D-14	MAD	36.67	55.69	47.70	73.73	59.84	42.69	52.78	83.05	77.89	106.77	107.9	98.8	190.21	65.82
	MAPE	0.09	0.13	0.12	0.16	0.14	0.11	0.13	0.15	0.12	0.18	0.17	0.16	0.3	0.128
	RMSE	84.27	62.72	52.30	78.42	60.69	53.17	57.42	84.271	85.71	108.9034	115.511	103.183	196.9555	69.2423
D-21	MAD	32.85	56.99	46.96	82.39	62.85	45.77	53.19	61.39	73.72	94.41	111.09	93.53	174.81	61.13
	MAPE	0.08	0.14	0.12	0.18	0.15	0.11	0.13	0.1	0.11	0.15	0.17	0.15	0.27	0.11
	RMSE	65.4	65.928	57.7812	90.4529	63.7342	51.8758	57.1069	65.4079	90.3391	96.96804	116.241	97.5639	181.8006	65.4098