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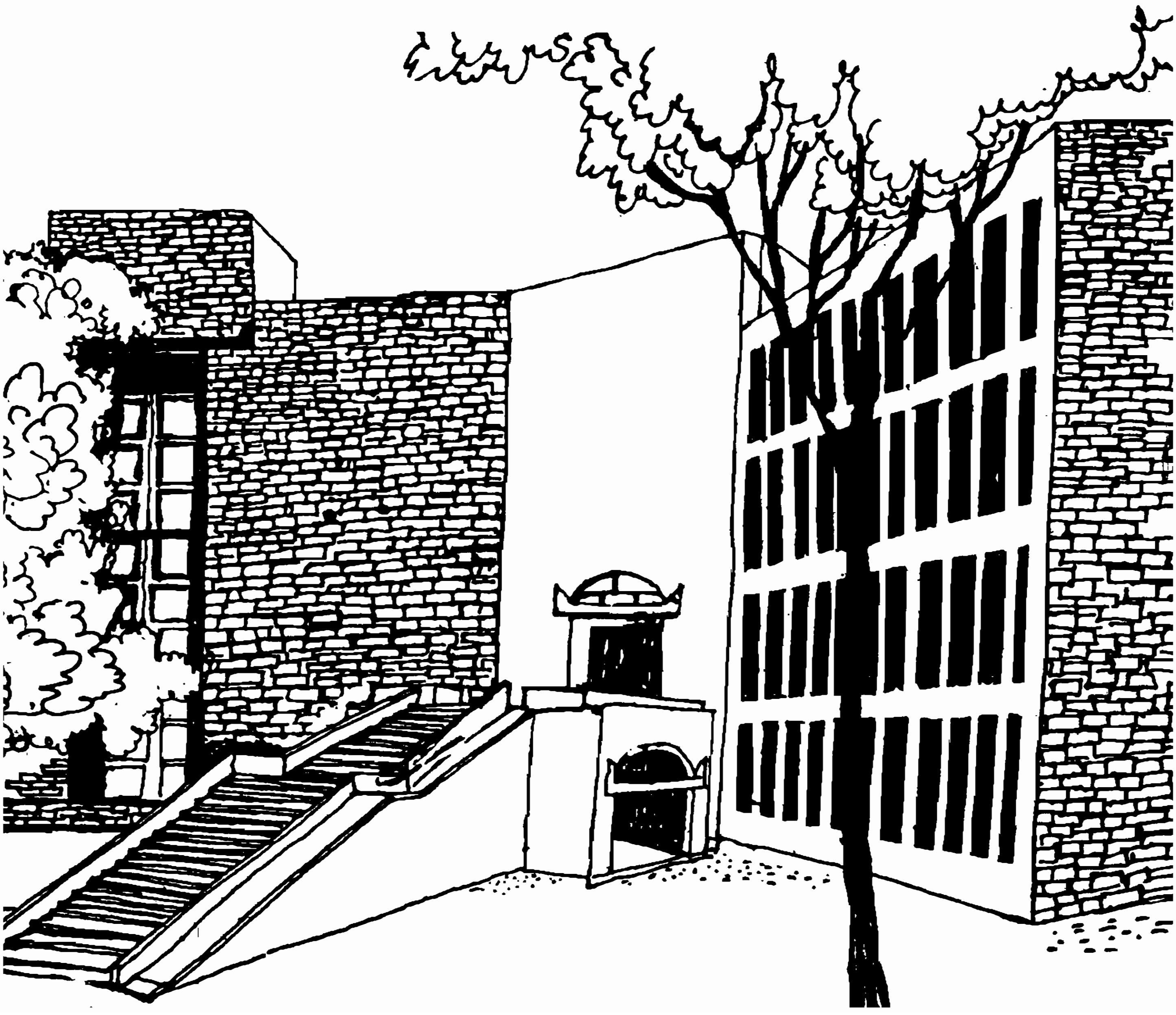

# ON THE EXISTENCE AND EFFICIENCY OF A VOTING EQUILIBRIUM FOR A PUBLIC GOOD ECONOMY

By

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W P No. 1194
June 1994

The main objective of the working paper series of the IIMA is to help faculty members to test out their research findings at the pre-publication stage.

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**Abstract**

The main purpose of this paper is to formalise the concept of a compromise function. Subsequently, given a compromise function, we define a voting equilibrium and prove the existence of such an equilibrium. We further show in our analysis that under some assumptions a voting equilibrium is Pareto optimal. We also show that voting equilibria are invariant under cost linearizing transformtions of the original economy. Finally, we close our analysis by exhibiting an intimate relationship between voting equilibria and Nash equilibria in the case of private provision of public goods.

**1. Introduction** :- The classical theory of resource allocation for the provision of a public good, concerns itself with each consumer in the economy being presented with a price for the public good (possibly different), so that no one has any incentive to make a unilateral choice distinct from the others. Given the nonexcludable nature of public consumption, differences in prices (personalized prices) are used to guarantee unanimity of choice. Such is the spirit behind the more well known solutions to problems of decentralized choice of public expenditure as in Foley (1970) or Kaneko (1977). The solution concept analysed by Foley is the Lindahl equilibrium solution. Kaneko proposed and analysed the ratio equilibrium solution.

However, the fact that public consumption must be equal for everyone, does not automatically imply that everyone will ask for the same level of public expenditure. In fact, it is quite likely that different beneficiaries of the public good, left to themselves, would ask for different levels of consumption of the good. The solution concepts mentioned above, use personalized prices to make consumers agree. However, it is quite reasonable, as insisted upon by Aczel (1987) (Section 1) that society requires a rule by which to arrive at a compromise solution for divergent proposals. To some extent, we believe, that this is precisely the method by which actual public projects are realized in practice. Society or better still a social decision maker intervenes, to arrive at a compromise solution for the public good consumption level on the basis of a list of such proposals obtained from individual consumers. As far as deciding the level of public expenditure goes, "the external arbitrator" (which for most other purposes is only a convenient fiction) plays a more active and significant role than what received theory would have us believe.

The main purpose of this paper is to formalise the concept of a compromise function. Subsequently, given a compromise function, we define a voting equilibrium and prove the existence of such an equilibrium. We also show that under some assumptions a voting equilibrium is Pareto optimal.

Ito and Kaneko (1981) discuss a theory of treating the cost of production of a public good as a proxy for the public good itself. This entails a corresponding transformation of the utility functions of the agents. Ito and Kaneko (1981) show that the set of ratio equilibrium allocations are invariant under this transformation and consequently establish the existence of ratio equilibria for a larger class of economies that what a fixed point argument would merely reveal. In this paper we carry out a similar exercise and show the invariance of voting equilibrium demands under cost-linearizing transformations. We are thus able to extend our initial existence result to a larger class of economies.

Finally, we take up the study of a model proposed by Bergstrom, Blume and Varian (1986), who investigate the private provision of public goods, as for instance in contributions to charity or contributions to family expenditure. They propose the Nash-equilibrium solution concept for analyzing voluntary gifts. We show that every Nash-equilibrium of the given economy corresponds to a voting equilibrium of a derived economy, where the preference profile remains the same, exhibiting an essential relation between the two concepts.

**2. The Model** :- Our framework of analysis draws on Lahiri (1993). Thus we assume that our economy consists of two goods: (a) a private good, which is both a numeraire, as well as an input for all production that takes place in the economy; (b) a public good, which is produced using the private good and whose final consumption level is the same for all individuals in the economy and is also equal to its level of production. We assume that there are $n \geq 2$ individuals in the economy. Let $w_i > 0$ be the initial endowment of the private good with the ith individual. The cost function for the production of the public good from the private good is $c: \mathbf{R}_+ \to \mathbf{R}_+$ satisfying (i) $c(0)=0$; (ii) c is strictly increasing; (iii) c is continuous. (Here $\mathbf{R}_+$ is the set of all non-negative real numbers.) Sometimes we shall also assume that (iv) c is a convex function.

The preferences of each individual i is represented by a function $u_i : \mathbb{R}^2_+ \to \mathbb{R}$ where if $x_i \geq 0$ denotes his consumption of the private good and $y \geq 0$ denotes society's consumption of the public good, then $u_i(x_i, y)$ denotes individual i's utility derived from the consumption bundle $(x_i, y) \in \mathbb{R}^2_+$. (Here $\mathbb{R}^2_+ \equiv \mathbb{R}_+ \times \mathbb{R}_+$). We assume that:

(v) $u_i$ is continuous

(vi) $u_i$ is strongly monotonic i.e. $(x_i, y) \geq (x'_i, y'), (x_i, y) \neq (x'_i, y') \Rightarrow u_i(x_i, y) > u_i(x'_i, y')$.

In addition, we sometimes assume

(vii) $u_i$ is quasi-concave i.e. $\forall (x_i, y), (x'_i, y') \in \mathbb{R}^2_+ \ \forall \alpha \in [0,1]$, $u_i(\alpha x_i + (1-\alpha) x'_i, \alpha y + (1-\alpha) y') \geq \min \{u_i(x_i, y), u_i(x'_i, y')\}$.

We now define a <u>compromise function</u>. A function $G: \mathbb{R}^n_+ \to \mathbb{R}$ which is continuous and strictly increasing (i.e. $\forall y, y' \in \mathbb{R}^n_+, y \geq y', y \neq y' \Rightarrow G(y) > G(y')$) is called a <u>compromise function</u>. The interpretation of a compromise function is that corresponding to a set of proposals for public good consumption made by the individuals, G recommends a comrpomise solution by a social decision maker. (Here, $\mathbb{R}^n_+$ denotes the n-fold Cartesian product of $\mathbb{R}_+$; $\forall y, y' \in \mathbb{R}^n_+, y = (y_i)^n_{i=1}, y' = (y'_i)^n_{i=1}, [y \geq y' \Leftrightarrow y_i \geq y'_i \ \forall i = 1, \ldots n)$.

In the sequel the following notation will be found convenient: $\forall y \in \mathbb{R}^n_+, \forall i \in \{1, \ldots, n\}, y_{-i} = (y_1, \ldots, y_{i-1}, y_{i+1}, y_n)$ and $y \equiv (y_i, y_{-i})$.

A <u>voting equilibrium</u> for the above economy is an ordered pair $(y^*, t^*)$:

(i) $t^* \in \{t \in \mathbb{R}^n_+ / \sum^n_{i=1} t_i = 1\} \equiv \Delta^{n-1}$ (the n−1 dimensional simplex).

(ii) $y^*_i = \arg\max_{y_i \geq 0} u_i(w_i - t^*_i c(G(y_i, y^*_{-i})), G(y_i, y^*_{-i}))$

$w_i - t^*_i c(G(y_i, y^*_{-i})) \geq 0$

$\forall i = 1, \ldots, n$.

The close relation between a voting equilibrium and Kaneko's ratio equilibrium cannot be missed. A voting equilibrium consists of a set of personalized prices and a set of public good consumption demands, such that the corresponding compromise solution satisfies the budget equation of all individuals and no single individual can benefit by unilaterally deviating to any other public good consumption demand which along with the consumption demands of the others yields an affordable compromise solution. In the subsequent section we show that under some conditions a voting equilibrium exists.

**3. Existence of Voting Equilibrium** :- Through this section we assume that $c:\mathbb{R}_+ \to \mathbb{R}_+$ satisfies properties (i) to (iv) listed in section 2 and $\forall i=1,\ldots,n$, $u_i : \mathbb{R}_+ \to \mathbb{R}_+$ satisfies properties (vi) to (vii) listed in section 2. It is easy to show that under properties (v) to (vii), $u_i : \mathbb{R}^2_+ \to \mathbb{R}_+$ satisfies semi-strict quasi-concavity i.e. $(x_i, y), (x'_i, y') \in \mathbb{R}^2_+$, $u_i(x_i, y) > u_i(x'_i, y') \Rightarrow u_i(tx_i + (1-t)x'_i, ty+(1-t)y') > u_i(x'_i, y') \forall t \in (0,1]$. In addition we assume that the compromise solution G is linear.

Let $F=\{y \in \mathbb{R}^n_+ / c(G(y)) \leqq \sum^n_{i=1} w_i\}$. Then is a convex set. Let $F'=\{y \in \mathbb{R}^n_+ / c(G(y)) \leqq \sum^n_{i=1} w_i + 1\}$. It is easy to see that F' is a convex set. Let S be an n-dimensional symmetric cube in $\mathbb{R}^n_+$, containing F'.

The following lemma essentially due to Kaneko (1977) will be stated without proof.

Lemma 1 :- Given $y^*_{-i} \in \mathbb{R}^{n-1}_+$, $t^*_i > 0$, let $y^*_i$ solve:

$$
\begin{aligned}
& u_i(w_i - t^*_i c(G(y_i, y^*_{-i})), G(y_i, y^*_{-i})) \to \max \\
& \text{s.t. } y_i \geqq 0 \\
& \quad w_i - t^*_i c(G(y_i, y^*_{-i})) \geqq 0 \\
& \quad (y_i, y^*_{-i}) \in F'
\end{aligned}
$$

Then if $y^* \in F$, $y^*_i$ solves:

$$
\begin{aligned}
& u_i(w_i - t^*_i c(G(y_i, y^*_{-i})), G(y_i, y^*_{-i})) \to \max \\
& \text{s.t. } y_i \geqq 0 \\
& \quad w_i - t^*_i c(G(y_i, y^*_{-i})) \geqq 0.
\end{aligned}
$$

For our subsequent analysis let us define $\forall i=1,\ldots,n$, $f_i : \Delta^{n-1} \times S \twoheadrightarrow \mathbb{R}_+$ as follows: $g_i \in f_i(t,y)$ if and only if $g_i$ solves:

$$
\begin{aligned}
& u_i(w_i - t_i c(G(g_i, y_{-i})), G(g_i, y_{-i})) \to \max \\
& \text{s.t. } g_i \geqq 0 \\
& \quad w_i - t_i c(G(g_i, y_{-i})) \geqq 0,
\end{aligned}
$$

$(g_i, y_{-i}) \in F'$.

Assumption :- $\forall y \in S$, $\forall i \in \{1,\ldots,n\}$, $\exists\, g_i \geq 0$ such that $c(G(g_i, y_{-i})) \leq w_i$.

This assumption along with the ones made above guarantees that the correspondence $f_i$ is non-empty valued $\forall i=1,..,n$. It is easy to see that $f_i$ is convex valued, convex-valued and upper-semicontinuous (see Hildenbrand and Kirman (1988)).

Theorem 1 :- Under the assumptions invoked in this section, for all $t^* \in \Delta^{n-1}$, $t^* \gg 0$, there exists $y^* \in \mathbb{R}^n_+$, such that $(y^*, t^*)$ forms a voting equilibrium.

Proof :- Consider the correspondence h : S->-> $S$ defined as follows:

$$h(g) = \prod^n_{i=1} f_i(t^*, g)$$

It is easy to verify that h is well-defined, non-empty valued, convex-valued, compact-valued and upper-semicontinuous. Thus by Kakutani's fixed point theorem, there exists $y^* \in S$ such that $y^* \in h(y^*)$. By Lemma 1, $(t^*, y^*)$ is a voting equilibrium.

Q.E.D.

**4. The Pareto Optimality of a voting equilibrium with quasi-linear preferences** :- In the general case, a voting equilibrium allocation need not be Pareto Optimal, where an allocation $(x,g) \in \mathbb{R}^n_+ \times \mathbb{R}_+$ such that $\sum^n_{i=1} x_i + c(g) \leq \sum^n_{i=1} w_i$ is said to be Pareto Optimal if there does not exist $(\bar{x}, \bar{g}) \in \mathbb{R}^n_+ \times \mathbb{R}_+$ satisfying $\sum^n_{i=1} \bar{x}_i + c(\bar{g}) \leq \sum^n_{i=1} w_i$ and $u_i(x_i, g) \geq u_i(\bar{x}_i, \bar{g})\ \forall\ i=1,\ldots,n$ with at least one strict inequality. However, in one special case which is both very popular in the relevant literature as well as of considerable merit, the Pareto Optimality of voting equilibria is easy to establish.

Let $u_i(x_i, y) = x_i + v_i(y)\ \forall (x_i, y) \in \mathbb{R}^2_+$, $\forall\ i=1,\ldots,n$, where $v_i : \mathbb{R}_+ \to \mathbb{R}$ is concave, strictly increasing and differentiable for $\forall\ i=1,\ldots,n$. Let $c: \mathbb{R}_+ \to \mathbb{R}_+$ satisfy $c(0)=0$ and be convex, strictly increasing and differentiable. It is then well known that a necessary and sufficient condition for Pareto Optimality of $(x,y) \in \mathbb{R}^n_+ \times \mathbb{R}_+$ is that $\sum^n_{i=1} x_i + c(y) \leq \sum^n_{i=1} w_i$

and $\sum^{n}_{i=1} \frac{dv_i(y)}{dy} = c'(y)$.

On the other hand if $(t^*, y^*)$ is a voting equilibrium for some compromise function G with $\frac{\partial G(y^*)}{\partial y_i} > 0 \ \forall = 1,\ldots,n$, then it is necessary under the above assumptions on preferences and the cost function that $t^*_i \, c'(G(y^*)) = \frac{dv_i}{dy}(G y^*)) \ \forall \ i=1,\ldots,n.$

Since $\sum^{n}_{i=1} t^*_i = 1$, we get that $\sum^{n}_{i=1} \frac{dv_i}{dy}(G(y^*)) = c'(G(y^*))$

i.e. $((w_i - t^*_i \, c(G(y^*)))^{n}_{i=1}, G(y^*))$ is a Pareto Optimal allocation. Incidentally, it is easy to verify that $(t^*, g(y^*))$ satisfies the following condition as well: $\forall \ i=1,\ldots,n, G(y^*)$ solves

$w_i - t^*_i \, c(g) + v_i(g) \rightarrow \max$

s.t. $g \geq 0$, $w_i - t^*_i \, c(g) \geq 0$,

which is the defining property of a ratio equilibrium, due to Kaneko (1977).

Thus, in case of quasi-linear concave preferences and convex cost functions, a voting equilibrium where each person asks for a different outlay of the public good, is associated to an allocation from which society as a whole has no incentive to deviate. This may be inspite of the fact, that the compromise solution agrees with nobody's demands. This explains why different individuals ask for different amounts of the public good and yet are satisfied with a compromise solution. The compromise function determines what each individual is going to ask for and not necessarily what society will provide. This is particularly true in the quasi-linear case above where $G(y^*)$ is uniquely determined, where as $y^*$ varies with the choice of G.

**5. Cost Linearization** :- In Ito and Kaneko (1981), can be found a theory of linearization of cost functions which leaves all ratio equilibria of the original economy, invariant under the transformation. Thus without loss of generality, linear cost functions can be invoked whenever we study ratio equilibrium of an economy. We will study a similar invariance property of voting equilibria in this section.

Let $\mathcal{U}=(u_i)_{i\in N}$ be a utility profile where for all $i\in N$, $u_i$ satisfies properties (v), (vi), (vii). An ordered triple $E=(\mathcal{U},c,G)$, where $\mathcal{U}$ is a utility profile, c is a cost function satisfying (i), (ii), (iii) and $\lim_{y\to\infty} c(y)=+\infty$; and G is a compromise function, is called an economy. Let $\mathcal{E}$ be the set of economies and $\tilde{\mathcal{E}}$ be the set of economies $E=(\mathcal{U},c,G)$ such that c is a convex function.

Let $F=\{f:\mathbb{R}_+ \to \mathbb{R}_+ \,/\, (a)\, f(0)=0,$ (b) f is strictly increasing, (c) f is continuous, (d) $\lim_{y\to\infty} f(y)=+\infty\}$

Given $E\in\mathcal{E}$ and $f\in F$ we define $\tilde{E}=f\circ E$ as follows:

$\tilde{E}=(\tilde{\mathcal{U}},\tilde{c},\tilde{G})$ where $\tilde{\mathcal{U}}=(\tilde{u}_i)_{i\in N}$ satisfies

$\tilde{u}_i(x_i,y)=u_i(x_i,f^{-1}(y))\ \forall (x_i,y)\in\mathbb{R}^2_+\ \forall i\in\{1,\ldots,n\}$ and

$\tilde{c}(y)=c(f^{-1}(y))\,\forall\, y\geq 0$

$\tilde{G}(y_1,\ldots,y_n)=G(f^{-1}(y_1),\ldots,f^{-1}(y_n)),\forall(y_1,\ldots,y_n)\in\mathbb{R}^n_+$

The cost-linearizing transformation T is defined as follows: $\forall E\in\mathcal{E}$, $E=(\mathcal{U},c,G)$, $T(E)=f\circ E$ where $f=c$.

It is easy to see that $T(E)\in\tilde{\mathcal{E}}\ \forall E\in\tilde{\mathcal{E}}$.

The cost linearizing transformation transforms an economy in such a way that cost of producing the public good acts as a proxy for the public good. Thus the transformed economy has a cost function which is linear with unit slope.

The following proposition is easy to verify:

Proposition 1 :- $E'=foE$ for some $f\in F$ if and only if $T(E)=T(E')$.

Proof :- Let $E'=foE$

Necessity :- $T(E')=(\tilde{u}',\tilde{c}',\tilde{G}')$

Then $\tilde{c}'(y)=y\forall y\geq 0$

$$\tilde{u}'_i\ (x_i,y)=u'_i(x_i,c'^{-1}\ (y))\forall(x_i,y)\in\mathbb{R}^2_+,\forall i\in N$$

$$=u_i(x_i,f^{-1}oc'^{-1}(y))\forall(x_i,y)\in\mathbb{R}^2_+,\forall i\in N$$

since $E'=foE$

$$=u_i(x_i,(c'of)^{-1}(y))\forall(x_i,y)\in\mathbb{R}^2_+,\forall i\in N$$

$$=u_i(x_i,c^{-1}(y))\forall(x_i,y)\in\mathbb{R}^2_+,\forall i\in N$$

since $c=c'of$

$$=u_i(x_i,y)\forall(x_i,y)\in\mathbb{R}^2_+,\forall i\in N.$$

$$\tilde{G}'(y_1,\ldots,y_n)=G'(c'^{-1}(y_1),\ldots,c'^{-1}(y_n))$$

$$=G(f^{-1}oc'^{-1}\ (y_1),\ldots,f^{-1}oc'^{-1}\ (y_n))$$

$$=G((c'of)^{-1}\ (y_1),\ldots,(c'of)^{-1}(y_n))$$

$$=G(c^{-1}(y_1),\ldots,c^{-1}(y_n))$$

$$=\tilde{G}(y_1,\ldots,y_n)\ \ \forall(y_1,\ldots,y_n)\in\mathbb{R}^n_+.$$

Since the cost function of both $T(E')$ and $T(E)$ are the same, we have $T(E)=T(E')$.

Sufficiency :- Suppose

$$\tilde{u}'_i(x_i,y)=\tilde{u}_i(x_i,y)\forall(x_i,y)\in\mathbb{R}^2_+,\ \ \forall i\in N$$

$$\tilde{G}'(y_1,\ldots,y_n)=\tilde{G}(y_1,\ldots,y_n)\forall(y_1,\ldots,y_n)\in\mathbb{R}^n_+$$

then,

$$u'_i(x_i,c'^{-1}(y))=u_i(x_i,c^{-1}(y))\forall(x_i,y)\in\mathbb{R}^2_+,\forall i\in N$$

$$G'(c'^{-1}(y_1),\ldots,c'^{-1}(y_n))=G(c^{-1}(y_1),\ldots,c^{-1}(y_n))\forall(y_1,\ldots,y_n)\in\mathbb{R}^n_+$$

i.e.

$$u'_i(x_i,y)=u_i(x_i,f^{-1}(y))\forall(x_i,y)\in\mathbb{R}^2_+,\forall i\in N$$

$$G'(y_1,\ldots,y_n)=G(f^{-1}(y_1),\ldots,f^{-1}(y_n))\forall(y_1,\ldots,y_n)\in\mathbb{R}^n_+$$

where $f=c^{-1}oc'\in F$

Thus $E'=foE$.

Q.E.D.

We define an equivalence relation $\sim$ on $\mathcal{E}$ by $E\sim E'$ if and only if $E'=foE$ for some $f\in F$.

We denote the equivalence class containing $E$ by $\mathcal{E}(E)$.

We call $\mu(.)$ a solution function if $\mu(E)\subseteq\{(x,y)\in\mathbb{R}^n_+\times\mathbb{R}^n_+/$

$\Sigma^n_{i=1} x_i + c(G(y)) \leq \Sigma^n_{i=1} w_i\} \forall E=(u,c,G) \in$ .

Let $\mu_v(E) = \{(\underline{x},\underline{y}) \in \mathbf{R}^n_+ x \mathbf{R}^n_+ / \exists t \in \Delta^{n-1}$ with $(\underline{y},t)$ a voting equilibrium E and $\underline{x}_i = w_i - t_i c(G(\underline{y})) \forall i \in N\}$.

The following proposition is also easy to establish.

<u>Proposition 2</u> :- $\mu_v(T(E)) = T_E o\ \mu_v(E) \forall E \in$ .

<u>Proof</u> :- Immediate.

Thus the set of voting equilibria for an economy is invariant under a cost-linearizing transformation. Given that $\mu_v(E) \neq \phi \forall E \in \mathcal{E}_c$ (as established in Theorem 1), we can now conclude from Propositions 1 and 2 that

$$\mu_v(E') \neq \phi \forall E' \in \bigcup_{T(E) \in \mathcal{E}_c} \xi(E)$$

Hence we have an existence result for economies with possible nonconvexities in the cost-function if their linearized form corresponds to an economy in $\mathcal{E}_c$. Clearly, this class is larger than the class considered in Theorem 1.

## 6. Relationship Between A Nash Equilibrium and a Voting Equilibrium:-

In a seminal paper, Bergstrom, Blume and Varian (1986), outlined a theory of private provision of public goods. Specifically, they were concerned with endogenising private contributions to charity as the Nash equilibrium solution outcome of a contribution game. They considered without loss of generality a cost-function which was linear with unit slope. In this section our purpose is to show that a Nash equilibrium solution outcome is always a voting equilibrium for a suitable economy. Thus we relate two morphologically dissimilar solution concepts, by resorting to an analytical device.

Assume $c(y) = y \forall \geq 0$.

Given a utility profile $u = (u_i)_{i \in N}$, an ordered n-tuple $(y^*_1, \ldots, y^*_n) \in \mathbf{R}^n_+$ is a Nash-equilibrium if $\forall i \in N$, $y^*_i$ solves

$u_i(w_i - y_i, y_i + \Sigma_{j \neq i} y^*_j) \rightarrow \max$

s.t. $0 \leq y_i < w_i$

Theorem 2 :- Let $y^*=(y^*_1,\ldots,y^*_n)$ be a Nash equilibrium for a utility profile $\mathbf{U}=(u_i)_{i\in N}$ and income profile $(w_i)_{i\in N}$. Then, $(y^*,\frac{e}{n})$ is a voting equilibrium for an economy with utility profile $\mathbf{U}=(u_i)_{i\in N}$, compromise function G, cost function c', and income profile $(w_i+\sum_{j\neq i} y^*_j)_{i\in N}$ where $G(y_1,\ldots,y_n)=\sum^n_{i=1} y_i$, $c'(g)=ng\forall g\geq 0$. Here e is the vector in $\mathbb{R}^n$ with all coordinates equal to unity.

Proof :- Immediate.

A few things ought to be noted about Theorem 2. First, there is a striking similarity between the revised incomes of the agents and what the Groves-Clarkes mechanism recommends for truthful revelation of preferences (i.e. each agent is made a contribution of what everyone else commits). However, given the nature of our investigation this observation needs to be dismissed as a mere coincidence, at this stage of our knowledge. What is however significant, is that the Nash equilibrium concept for private provision of public goods can be tailored to conform to a voting equilibrium for a suitable economy.

Second, the cost function in Theorem 2, bears a striking resemblance to the cost function used by Gradstein (1993), in establishing that in the presence of rent-seeking (or other directly unproductive) activities, a Nash equilibrium (i.e., private provision of the public good) will perform better than public provision of the public good. It is probably appropriate at this juncture, to mention, that in the kind of environments studied by Gradstein (1993), a voting equilibrium will coincide with a socially optimal level of the public good. Given that a voting equilibrium is the outcome of a meaningful decentralized procedure, it is necessary to attach caveats to Gradstein's interpretation of his results before accepting them lock-stock and barrell.

**7. Conclusion** :- In this paper we have essentially established the existence of a voting equilibrium in a simple mixed economy. The linearity of the compromise function, which was required in the proof of existence of a voting equilibrium, does not severely restrict our environment, since we still have available all functions which takes the weighted average of the individual bids as society's choice of public good consumption level. The assumptions on preferences and cost functions are natural ones in the literature.

The discussion of the efficiency of a voting equilibrium allocation with quasi-linear preferences, also implied that the role of the compromise function was largely to allow for the possibility of divergent claims and not necessarily in choosing a different allocation from the ones suggested by the conventional solution concepts. Compromise functions may thus explain what has always been observed in a democratic society : Conflicting claims and yet unanimity in realized choices.

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