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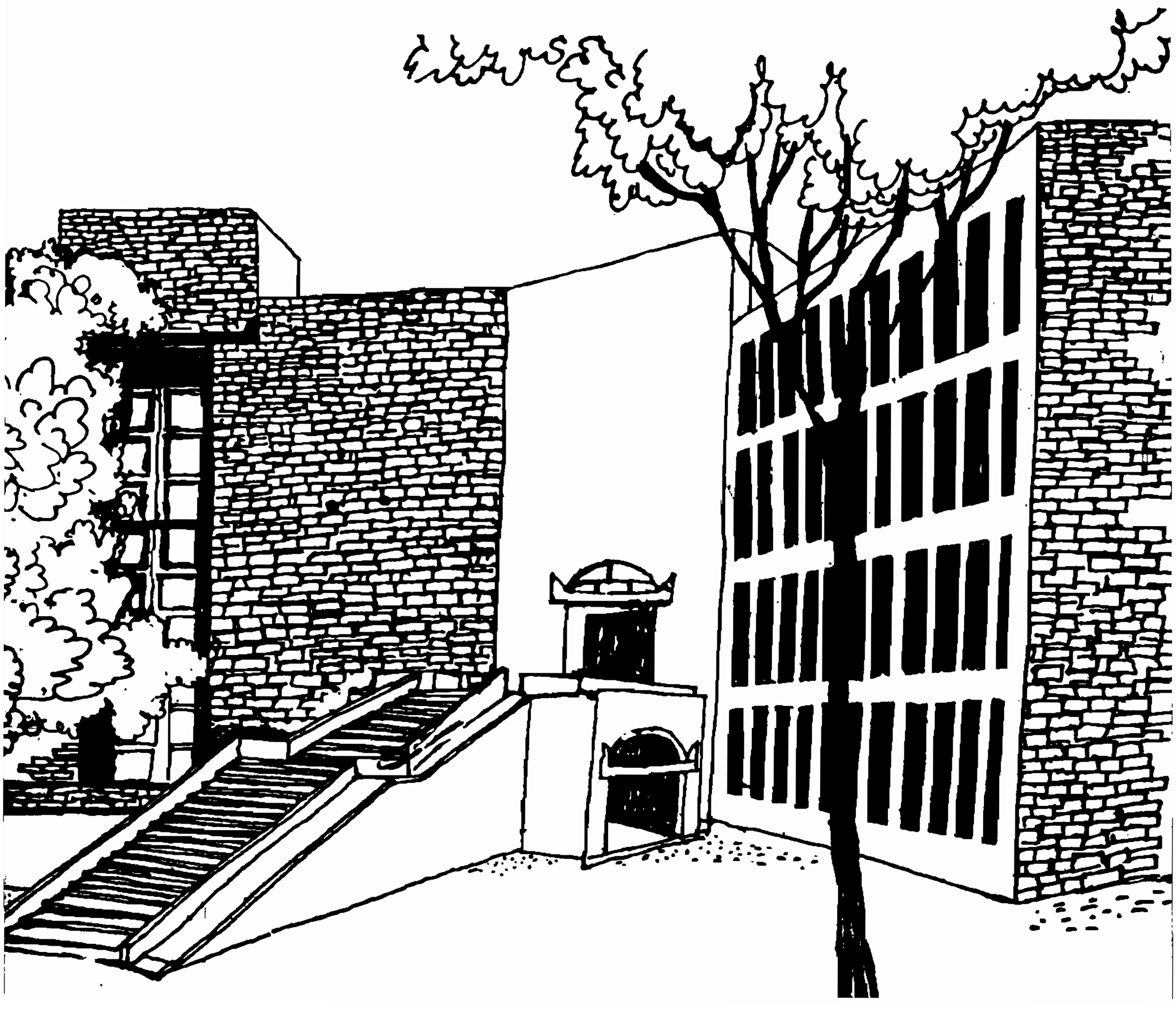

RATIONING AND THE PRIVATE PROVISION OF A PUBLIC GOOD

By

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Abstract

In this paper we study a solution concept for resource allocation in an economy with public goods and a fixed set of cost-shares for each agent. This solution is originally due to Champsaur (1979). We study the existence and some interesting properties of this solution.

1. Introduction :- One of the earliest investigations of private provision of a public good in a noncooperative setting is the paper by Champsaur (1979). In that paper, prices (or what may equivalently be described as cost shares) are assumed fixed. The paper tried to examine the extent to which voluntary contributions resulting from noncooperative behavior, can finance a Pareto-optimal level or an approximately Pareto optimal level of resource allocation.

This line of investigation remained almost dormant till Warr (1982) and more significantly Bergstrom, Blume and Varian (1986) came up with an analytical model, emphasizing the problem of public contributions to charity. Subsequent contributions by Andreoni and Mc Guire (1993) and Gradstein (1993) have built on the seminal work done by Bergstrom, Blume and Varian. A related investigation can be found in Lahiri (1994).

In some respects the model investigated by Champsaur (1979) and the model investigated by Bergstrom, Blume and Varian (1986) are similar. Both consider the size of individual contributions being chosen noncooperatively. Both lead to solutions which are possibly not Pareto optimal. However Champsaur (1979) considers the share of each individual in the total contribution to be fixed, unlike Bergstrom, Blume and Varian (1986). Translated into the framework of contributions to charity, Champsaur's framework would answer the following question: given an individual's share in the total contributions, how large would he/she, want the total contribution to be if one could always veto a larger size of the total contribution than what one wants. Of course the motives for the contributions are the same in both frameworks: pure altruism or a warmglow, whichever way we want to describe it.

In this paper, we investigate the solution concept due to Champsaur and study a few properties of the associated equilibria.

2. The Model :- We consider an economy consisting of a single public good which is denoted by y and a single private good, quantities of which are denoted by x. There are n agents in the economy; let $N=\{1,...,n\}$ be the set of agents. The preferences of each agent are represented by a utility function. Thus for $i \in N$, $u^i:\mathbb{R}^2_+ \to \mathbb{R}$ is the utility function of agent i. Each agent has a positive initial endowment of the private good: $w^i>0$ is the initial endowment of $i\in N$. A consumption bundle of agent i is denoted $(x^i,y)\in\mathbb{R}^2_+$. An allocation is a vector $(x^1,\ldots,x^n,y)\in\mathbb{R}^{n+1}_+$.

The production possibilities of the economy are given by a cost function $g(y)=y$ for all $y\geq 0$, which gives the amount of the private good required to produce the public good.

We make the following assumptions on the preferences:

Assumption 1 :- $\forall i\in N$, $u^i:\mathbb{R}^2_+\to\mathbb{R}$ is continuous and strictly increasing.

Assumption 2 :- $\forall i\in N$, $u^i:\mathbb{R}^2_+\to\mathbb{R}$ is quasi-concave.

Let $\Delta^{n-1}=\{z\in\mathbb{R}^n_+/z=(z^1,\ldots,z^n),\ \sum_{i\in N} z^i=1\}$.

Let $P=\{p_1,\ldots,p_l\}$ where $p_k\in\Delta^{n-1}$ for $k=1,\ldots,l$.

P is the set of fixed cost-shares at which the public good can be obtained.

Let $z^i_k\geq 0$, be the amount of the public good agent i desires at cost-shares given by the vector $p_k, k\in\{1,\ldots,l\}$.

Since each agent can veto any quantity of the public good exceeding the quantity demanded by him/her, the quantity of the public good that would be supplied at cost-shares given by p_k is simply $y_k=\min_{i\in N} z^i_k$, $k\in\{1,\ldots,l\}$. If x^i denotes the amount of the private good consumed by agent i in the above scheme,

$x^i=w^i-\sum^l_{k=1} p^i_k y_k$

Clearly, $\sum_{i\in N}x^i+\sum^l_{k=1}y^k=\sum_{i\in N}w^i-\sum^l_{k=1}y^k+\sum^l_{k=1}y^k=\sum_{i\in N}w^i$, where $y=\sum^l_{k=1}y^k$ is the total quantity of the public good supplied.

Let $Z^i=\{z\in\mathbb{R}^l_+/w^i-\sum^l_{k=1}p^i_k z_k\geq 0\}$.

Given $z^{)i(}=(z^1,\ldots,z^{i-1},z^{i+1},\ldots,z^n)\in\prod_{j\neq i}Z^j\equiv Z^{)i(}$, the problem faced by agent i is as follows:

$$\left.\begin{array}{l} u^i(x^i,\sum^l_{k=1}z^i_k)\to\max \\ \text{s.t. } x^i=w^i-\sum^l_{k=1}p^i_k z^i_k\geq 0 \\ 0\leq z^i_k\leq \min_{j\neq i} z^j_k,\ k=1,\ldots,l. \end{array}\right\} \quad (Q)^i$$

Let $\Phi^i(z^{)i(})$ be the set of solutions to Q^i.

Definition :- An equilibrium with rationing is an n-tuple $\bar{z}=(\bar{z}^i)\in\prod_{i\in N}Z^i$, such that $\forall i\in N$, $\bar{z}^i\in\Phi^i(\bar{z}^{)i(})$.

Writing the constraint $0\leq z^i_k\leq\min_{j\neq i} z^j_k, k=1,\ldots,l$ in Q^i explicitly as $0\leq z^i_k\leq z^j_k, k=1,\ldots,l; j\in N, j\neq i$, we see that under our Assumptions 1 and 2, the correspondence $z \to\!\!\to \prod_{i\in N}\Phi^i(z^{)i(})$ :: $\prod_{i\in N}Z^i\to\!\!\to\prod_{i\in N}Z^i$, is nonempty valued, compact valued, convex valued and upper-hemicontinuous. Thus by appealing to Kakutani's Fixed Point Theorem there exists $\overset{*}{z}\in\prod_{i\in N}Z^i$ which is an equilibrium with rationing.

Theorem 1 :- Under Assumptions 1 and 2, there exists an equilibrium with rationing $\overset{*}{z}\in\prod_{i\in N}Z^i$. Further $\forall i,j\in N$, $\overset{*}{z}^i=\overset{*}{z}^j$.

Proof :- Since $\overset{*}{z}^i_k \leqq \min_{j\neq i} \overset{*}{z}^j_k$ $\forall k=1,\ldots,l$ and $i\in N$, we must have $\overset{*}{z}^i_k=\overset{*}{z}^j_k \forall k=1,\ldots,l$, $\forall i$, $j\in N$.

Q.E.D.

Fixed Price Equilibria With Rationing :- The case where $P=\{p\}, p\in\Delta^{n-1}$, is the situation where there is a single vector of cost-shares. This corresponds to the situation studied by Lahiri (1993) for an economy solely with private goods. However the solution concepts are different. Interesting possibilities now open up with the following assumption:

Assumption 3 :- $\forall i\in N$, $u^i:\mathbb{R}^2_+\to\mathbb{R}$ is strictly quasi-concave

Lemma 1 :- Under assumptions 1, 2 and 3, the following problem has a unique solution:

$$u^i(w^i-p^iz,z)\to\max$$

$$\text{s.t. } z\geqq 0, w^i-p^iz\geqq 0.$$

The unique solution is denoted by $f^i(w^i)$.

Proof :- Immediate.

The set of equilibria with rationing, now has an interesting characterization:

Theorem 2 :- Under assumptions 1, 2 and 3, $\overset{*}{z}\geqq 0$ is an equilibria with rationing if and only if $\overset{*}{z}=\min\{f^i(w^i),\overset{*}{z}\}\forall i\in N$.

Proof :- Suppose $\overset{*}{z}=\min\{f^i(w^i),\overset{*}{z}\}$. If $\overset{*}{z}<f^i(w^i)$ and $z'<\overset{*}{z}$ solves the problem for agent i, then we have $u^i(w^i-p^if^i(w^i),f^i(w^i))\geqq u^i(w^i-p^iz',z')>u^i(w^i-p^iz,z)$ which along with $z'<z<f^i(w^i)$ contradicts the quasi-concavity of u^i. Hence $\overset{*}{z}$ solves the problem for each i. On the other hand $f^i(w^i)<\overset{*}{z}$ contradicts $\overset{*}{z}=\min\{f^i(w^i),\overset{*}{z}\}$. Thus $\overset{*}{z}=\min\{f^i(w^i),\overset{*}{z}\}\forall i\in N$ is an equilibria with rationing.

Conversely suppose $\overset{*}{z}$ is an equilibria with rationing and $f^i(w^i)<\overset{*}{z}$ for some $i\in N$. Thus $f^i(w^i)$ solves agent i's constrained optimization problem i.e. $z^i=f^i(w^i)<\overset{*}{z}$ is agent i's best response to z. This contradicts Theorem 1, where it is required that $z^i=\overset{*}{z}$.

Hence $f^i(w^i)\geqq\overset{*}{z}$ $\forall i\in N$, proving the theorem.

Q.E.D.

Notice the similarity of the expression for equilibrium with rationing to the one suggested by Bergstrom, Blume and Varian (1986), in computing their equilibria.

Corollary to Theorem 2 :- Under assumptions 1, 2 and 3, $0\leq\overset{*}{z}\leq\min_{i\in N}\{f^i(w^i)\}\Longleftrightarrow\overset{*}{z}$ is an equilibrium with rationing.

4. Conclusion :- If the public good is a normal good for all agents and if the social planner desires to implement $\overset{*}{z}=\min_{i\in N}\{f^i(w^i)\}$, then one way in which the allocation of the public good can be increased is to increase the income of the agent who desires the public good least. If in addition all agents have identical preferences, giving rise to $f^i=f\,\forall i\in N$, an increase in the public good is brought about by increasing the income of the poorest individual. Thus, in this situation growth can lead to an expansion of public good consumption if and only if the poorest individual benefits from it.

We shall finally try to relate this work, with earlier work reported in Lahiri (1994). There we had defined the concept of a compromise function $G:\mathbb{R}^n_+\to\mathbb{R}_+$ which was assumed to continuous and strictly increasing in each of its arguments. For the present analysis if we retain continuity but relax the requirement of strict increasingness and require that for $y,y'\in\mathbb{R}^n_+$, (a) $y>y'\Rightarrow G(y)\geq G(y')$, and (b) $y\gg y'\Rightarrow G(y)>G(y')$, then our above analysis can be easily accommodated in the earlier framework, where G was used to aggregate the possibly different demands for the public good (made by different individuals) into a single quantity which society was prepared to provide. In that framework, we had defined an ordered pair $(\overset{*}{y},\overset{*}{p})\in\mathbb{R}^n_+\times\Delta^{n-1}$ to be a voting equilibrium if

$\forall i\in N$, $\overset{*}{y}{}^i$ solved:

$$u^i(w^i-\overset{*}{p}{}^iG(\overset{*}{y}{}^{)i(},y^i),\ G(\overset{*}{y}{}^{)i(},y^i))\to\max$$

$$\text{s.t. } y^i\geq 0,\ w^i-\overset{*}{p}{}^iG(\overset{*}{y}{}^{)i(},y^i)\geq 0$$

Suppose $\overset{*}{p}\in\Delta^{n-1}$ was fixed. We could thus define a voting equilibrium with rationing to be a vector $\overset{*}{y}\in\mathbb{R}^n_+$ such that $\forall i\in N$, $\overset{*}{y}{}^i$ solves

$$u^i(w^i-\overset{*}{p}{}^iG(\overset{*}{y}{}^{)i(},y^i),G(\overset{*}{y}{}^{)i(},y^i))\to\max$$

$$\text{s.t. } y^i\geq 0,\ w^i-\overset{*}{p}{}^iG(\overset{*}{y}{}^{)i(},y^i)\geq 0.$$

It is easy to see that if $\overset{*}{y}$ is a <u>voting equilibrium with rationing</u> for fixed cost-shares given by $\overset{*}{p}\in\Delta^{n-1}$, then $(\overset{*}{y},\overset{*}{p})$ is a <u>voting equilibrium</u> and conversely.

Now, if $G(y^1,...,y^n)=\min_i y^i$, then a <u>voting equilibrium with rationing</u> for fixed cost-shares given by $\overset{*}{p}\in\Delta^{n-1}$, is obtained from an equilibrium with rationing for fixed cost-shares given by $\overset{*}{p}\in\Delta^{n-1}$. In Lahiri (1994), we show that a voting equilibrium exists whenever, G is linear and strictly increasing. Here we obtain that a voting equilibrium obtains when G is as specified above.

Conversely, if $G(y^1,...,y^n)=\min_i y^i$ and $\overset{*}{y}$ is voting equilibrium with rationing for fixed cost-shares given by $\overset{*}{p}\in\Delta^{n-1}$, then the vector $G(\overset{*}{y}).e$ (where e is the vector in $\mathbb{R}^n$ with all coordinates unity) is an equilibrium with rationing for fixed cost shares given by $\overset{*}{p}$.

Thus, there is an intimate relationship between the concept of a voting equilibrium and equilibrium with rationing for a single fixed cost share vector.

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