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PRODUCTIVITY TRENDS IN PUBLIC
SECTOR ENTERPRISES

by

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PRODUCTIVITY TRENDS IN PUBLIC SECTOR ENTERPRISES

The first part of our study refers to the analysis of trends in growth of productivity of PSEs. For this purpose, we first give a definition of productivity, as used in the study.

Definition of Productivity:

Although productivity has been traditionally defined as ratio of outputs to inputs, there is considerable controversy on appropriate methods of counting outputs and inputs. The concept of productivity encompasses aspects relating to efficiency and effectiveness of the enterprise. Particular care therefore needs to be exercised in counting outputs. Several errors may commonly be made in this context, namely -

1. Counting outputs in a manner not related to goals
2. Counting outputs in a manner not related to inputs
3. Sub-optimisation
4. Overly simplistic measures of outputs.
5. Use of productivity indices which are not related to improvement.

In addition, there are the problems of measuring outputs whose characteristics change over time, defining and measuring capital inputs, aggregating heterogenous outputs and inputs. Thus, it is somewhat difficult to evolve one best index of productivity. However, the problem is evidently not as intriguing for a manufacturing organization as it can be for a service organization, because of difficulty in quantifying service quality. Several authors have in the past attempted enterprise level definitions of aggregate productivity index. In some way, they seek to convert the aggregate outputs and inputs into appropriate money value. In the case of PSEs, an enterprise is usually set up with specific pre-determined objectives, and the outputs are necessarily related to these. This facilitates task of quantifying productivity. For the purpose of our study, we define productivity by the following

ratio

$$PR = \frac{\text{Value of Sales}}{\text{Cost of Sales}}$$

Although consideration of sales as outputs of an enterprise is debatable, nevertheless they reflect cumulative outcome of the total management of all activities of the enterprise including marketing. Cost of sales includes cost of all inputs such as labour, materials, goods and services, capital, depreciation, energy, excise duty, and any other inputs. Taxes on profits will not be considered as input.

A characteristic feature of the growth of Public Sector, is that new enterprises go into operation, every year, fairly regularly. Thus, the number of running enterprises under Central Government with or without direct responsibility for their management was around fifty during 65-66. These have gradually increased to over hundred and fifty in 76-77. Overall productivity of an industry or the public sector as a whole, is undeniably influenced by generally low productivities of enterprise during initial years of operation. Question arises whether evaluation of productivity growth should exclude such enterprises. It needs to be recognized that ability of the public sector to generate sizeable surpluses for economy, is in the ultimate analysis, linked up with its total performance and not merely the performance of on-going units alone. In addition, in a developing economy, new enterprises becoming operational every year, whether manufacturing or service type, must be viewed as a continuing phenomenon. At least, this is a feature of the planned growth of the country's public sector. In our view, therefore, a rational analysis of public sector productivity trends should be based appropriately on all the running units during a particular year, without exclusion of new units going operational.

With the above considerations and use of productivity formula stated above, we computed the PRs for various enterprises, for the year 1965-66 to 1976-77. Thus the study period chosen is the period since inception of Bureau of Public Enterprises to the latest available annual report on the working of industrial and commercial undertakings. It is noteworthy that number of enterprises considered in our study, have steadily increased from fifty two in 65-66 to 143 in 76-77. Details of productivity ratios are given in Appendix 1. The classification of enterprises is in accordance with the current industry groups.

Model for Trend Analysis:

In analysing productivity trends in PSEs, apart from their quantitative assessment, certain hypotheses concerning productivity acquire relevance, and we thought it necessary to test for the statistical validity of these. Specifically, we propose to test

1. Whether the productivity trends within industry are significantly different from industry to industry.
2. Whether there is significant difference in productivity between industries that is not linearly explained by the trend.
3. Whether the slope of productivity trend between industries is significantly different from the common slope of productivity trends within industries.
4. Whether there is any departure from one overall trend.

In summary, the hypotheses broadly related to the two principal aspects of productivity namely (i) equality of mean productivity in different industries, (ii) equality of productivity trends in different industries. Statistical model chosen for the analysis would have to achieve the twin purpose of (i) quantitative assessment of productivity trends and (ii) test the statistical validity of hypothesis enunciated above. While analysis of variance conventionally tests hypothesis concerning equality of multiple sample means, analysis of regression, on the other hand, apart from trend assessment can test for the equality of trend slopes. The statistical technique which reconciles the features of both the above separate, widely applicable procedures, is the analysis of covariance (ANACOVA). It is therefore ideally suited to the situation stated above. Some of the important advantages offered by ANACOVA, are

- i. Increased precision in randomised experiments
- ii. Adjustment for sources of bias in observational studies
- iii. Throw light on the nature of treatment effects in randomised experiments
- iv. Study regressions in multiple classifications.

The specific model of ANACOVA which we adopted for the present study is the generalized version of model for comparison of slopes, given by

$$Y_{ij} = \mu + \phi_j + \beta_m (\bar{X}_j - \bar{X}) + \beta_a (X_{ij} - \bar{X}_j) + \delta_j (X_{ij} - \bar{X}_j) + E_{ij}$$

Y_{ij} = Productivity of industry j, enterprise i.

μ = Mean overall productivity

ϕ_j = Productivity differential pertinent to industry j, adjusted for values of X.

β_m = Slope of productivity trend between industries.

β_a = Slope of productivity trend within industries.

δ_j = Slope differential peculiar to industry j.

E_{ij} = Random, normally distributed variable with zero mean

X_{ij} = Year under consideration, for enterprise i, industry j.

The expressions for the unbiased estimates of the various parameters of the model are given in Appendix 2. The latter also provides necessary computational details and the ANACOVA table for testing hypotheses enumerated earlier. The actual values of parameter estimates obtained from the model are displayed in Exhibit 1. Reviewing the hypotheses tests performed, we draw following conclusions:

1. The within industry productivity trends do not differ significantly from industry to industry.
2. There is significant difference in productivity between industries that is not linearly explained by the productivity trend.
3. There is significant difference between the common slope of productivity within industry (β_a) and the slope of productivity trend between industries (β_m).

4. There is also a significant departure from single overall trend. The above results imply that while slopes of productivity trends within industry are not significantly different from industry to industry, the mean productivities are significantly different for different industries.

Reviewing the actual values of parameter estimates for various industries derived from the model, it is readily noticeable that productivity trends are actually negative for several industries namely minerals and metals (other than coal), chemicals and pharmaceuticals, transportation equipment, agro-based enterprises, water transportation services, tourist services, financial services, and undertakings without direct responsibility for management. If we rate the remaining industries in the decreasing order of growth rate of productivity, the following rankings emerge 1. Section 25 - companies, 2. heavy engineering, 3. industrial development and technical consultantancy services, 4. trading and marketing services, 5. coal, 6. contracts and construction services, 7. petroleum, 8. steel, 9. small industries development, 10. medium and light engineering, 11. consumer goods, 12. air transportation services. The industry groups of section-25 companies and heavy engineering have registered, relatively speaking, highest positive growth rates while financial services have registered highest negative growth rate.

The generally low growth rates and low overall productivity levels only marginally above unity, suggest little, if any possibility of public sector contributing sizeable surpluses to the economy, from higher productivity. Another disconcerting feature is that, most of the industries with relatively higher average productivity levels are actually characterised by negative growth rates. To a certain extent, the generally low productivity levels could be attributed to the low productivity of enterprises during their initial stages of operation. From the productivity data, we removed certain productivity values which were in the nature of outliers. In all out of the 1217 original productivity values, we classified 32 as outliers and eliminated them from consideration. Though majority of outliers were low values of productivity, these also included some very high values observed in the case of some enterprises.

With outliers removed, the ANACOVA model for comparison of slopes was refitted to the remaining data. Computational details of parameter estimates and corresponding hypothesis

test are given in Appendix 2. Exhibit 2 gives the estimated values of model parameters using revised data. The new hypothesis test produced following results:

1. Productivity trends within industry are not significantly different for different industries at 0.01 level of significance.
2. There is significant difference in productivity between industries that is not linearly explained by the trend.
3. There is no significant difference in the slope of productivity trend between industries and the common slope of the trend within industry.
4. There is significant departure from single overall trend.

The tests again demonstrate that while difference in productivity trends within industry are not statistically significant for different industries, the differences in mean productivities are statistically significant. However, with the removal of outliers, the null hypothesis concerning the slope of between industry trend and the common slope of within industry trend is upheld. The study with outliers removed in a way helped to confirm the previous results concerning hypothesis tests for means and slopes of productivity trends.

In summary, the ANACOVA model resulted, among other things in the following:

1. Quantitative estimation of productivity trends for various industry groups.
2. Rejection of null hypothesis concerning mean adjusted productivities (MAP) of different industry groups.
3. Acceptance of null hypothesis concerning the within industry productivity trends (WIPT).

Consequent upon above findings, we believed it was worthwhile to identify industries which were causing rejection of the null hypothesis concerning equality of MAP. In particular, it appeared meaningful to ascertain, as indeed we suspected, whether only a few industry groups were responsible for the rejection of null hypothesis. For this purpose, we resorted to Scheffe's scheme of linear contrasts.

In this scheme, a linear contrast L, is defined, say in the following manner:

$$L = \sum_{j=1}^K C_j \mu_j \quad \text{with} \quad \sum_{j=1}^K C_j = 0$$

where K = total number of industries (in the present case) considered for the contrast.

μ_j = Mean adjusted productivity (MAP) of industry j.

An estimate of this contrast is given by

$$\hat{L} = \sum_{j=1}^K C_j [\bar{Y}_j - \hat{\beta}_m (\bar{X}_j - \bar{X})]$$

If we denote by $\hat{\sigma}^2$, the estimate of variance of deviations of productivity within industries from common trend (β_a), the estimate of variance of L can be given as

$$S_{\hat{L}}^2 = \hat{\sigma}^2 \left\{ \sum_{j=1}^K C_j^2 / n_j + \left[\sum_{j=1}^K C_j (\bar{X}_j - \bar{X}) \right]^2 / E_{XX} \right\}$$

Here E_{XX} is the quantity defined in Appendix 2 and

$$\hat{\sigma}^2 = (E_{yy} - \frac{E_{xy}^2}{E_{xx}}) / N - K - 1.$$

The terms E_{yy} and E_{xy} are defined in Appendix 2, and N is the total number of productivity observations. The 100 (1 - α) per cent confidence interval for the linear contrast L, is given by

$$\hat{L} \pm S_{\hat{L}} \sqrt{(K-1) F_{\alpha, K-1, N-K-1}}$$

Actual use of the scheme of linear contrasts involves comparison of two means at a time and check if the confidence interval for the corresponding linear contrast defined includes zero. In the case under consideration, since there were twenty industry groups, the number of linear contrasts totalled one hundred and ninety. A sample calculation to illustrate the scheme is given in Appendix 2. If the confidence interval of a contrast includes zero, we conclude that the corresponding industry means being compared are equal. This scheme was used over all the hundred and ninety comparisons. The results are summarised in Appendix 2. Some interesting inferences could be made. It was established that MAP for financial services group was not equal to the MAP for any other group. Also, MAP for consumer goods industry was not equal to two industry groups, namely petroleum and Central Government undertakings without direct responsibility for management. Thus, out of one hundred and ninety comparisons of means, significant differences were observed in the case of twenty one comparisons and these could be primarily attributed to only two industry groups viz. financial services and consumer goods. While financial services recorded the highest value of MAP, consumer goods had the lowest corresponding value. It could therefore be concluded that rejection of hypothesis concerning equality of MAP for all industry groups was mainly due to high MAP for financial services group and low MAP for consumer goods.

Exhibit 1Values of Parameters Using Analysis of Covariance Model

$\hat{\mu} = 1.0267$

$\hat{\beta}_m = 0.0950$

$\hat{\beta}_a = 0.00675$

Name of Industry Group	$\hat{\phi}_j$	$\hat{\delta}_j$	$\hat{\beta}_a + \hat{\delta}_j$	adjusted mean $\hat{\mu} + \hat{\phi}_j$
1. Steel	-0.1565	0.0013	0.00805	0.8702
2. Minerals and Metals other than coal	-0.0633	-0.0107	-0.00395	0.9634
3. Coal	-0.1721	0.015	0.02175	0.8546
4. Petroleum	0.1038	0.0026	0.00935	1.1305
5. Chemicals and Pharmaceuti- cals	0.0343	-0.0105	-0.00375	1.0610
6. Heavy Engineering	-0.1820	0.0289	0.03565	0.8447
7. Medium and Light Engg.	-0.0034	0.0001	0.00685	1.0233
8. Transportation Equipment	0.0391	-0.0138	-0.00705	1.0658
9. Consumer goods	-0.0252	-0.0007	0.00605	0.7835
10. Agrobased Enterprises	-0.0252	-0.0209	-0.01415	1.0015
11. Trading and Marketing Services	0.0392	0.0162	0.02295	1.0659
12. Transportation Services - Air	0.0556	-0.0065	0.00025	1.0823
13. Transportation Services - Water	0.1462	-0.0251	0.01836	1.1729
14. Contracts and Construction Services	0.0344	0.0118	0.01855	1.0611
15. Ind. Devpt. and Tech. Consultancy Services	-0.0666	0.0208	0.02755	0.9601
16. Devpt. of Small Industries	-0.0636	0.0012	0.00795	0.9631
17. Tourist Services	0.1229	-0.0201	-0.01335	1.1496
18. Financial Services	0.7600	-0.069	-0.06225	1.7867
19. Section 25 Companies	-0.0037	0.0292	0.03595	1.0230
20. Undertakings without direct responsibility	0.0936	-0.0120	-0.00525	1.1203

Exhibit 2Values of Parameters Using Analysis of Covariance Model

(with Outliers Removed)

$\hat{\mu} = 1.017$

$\hat{\beta}_m = 0.015$

$\hat{\beta}_a = 0.0043$

j	Name of Industry Group	$\hat{\phi}_j$	$\hat{\delta}_j$	$\hat{\beta}_a + \hat{\delta}_j$	adjusted mean $\hat{\mu} + \hat{\phi}_j$
1.	Steel	0.0725	0.0037	0.007	1.0895
2.	Minerals and Metals other than coal	0.0040	-0.0043	0.0000	1.0210
3.	Coal	-0.1631	0.0174	0.0217	0.8539
4.	Petroleum	0.1600	0.0050	0.0093	1.1770
5.	Chemicals and Pharmaceuticals	0.1710	-0.0041	0.0002	1.1880
6.	Heavy Engineering	0.1341	-0.0039	0.0004	0.8829
7.	Medium and Light Engineering	-0.0179	-0.0042	0.0001	1.0349
8.	Transportation Equipment	0.0053	-0.0114	-0.0071	1.0223
9.	Consumer Goods	-0.1912	-0.0012	0.0031	0.8258
10.	Agrobased Enterprises	0.0388	-0.0185	-0.0142	1.0558
11.	Trading and Marketing Services	0.0188	-0.0050	-0.0009	1.0358
12.	Transportation services - Air	-0.0708	-0.0041	0.0002	1.087
13.	Transportation services-Water	0.0993	-0.0227	-0.0184	1.1163
14.	Contracts and Construction Services	-0.0861	-0.0142	-0.0099	0.9309
15.	Ind. Devpt. and Tech. Consultancy Services	0.0914	0.0127	0.0170	1.1084
16.	Devpt. of Small Industries	-0.0472	0.0036	0.0079	0.9698
17.	Tourist Services	0.0387	-0.0177	-0.0134	1.0557
18.	Financial Services	0.0944	-0.0242	-0.0199	1.1114
19.	Section-25 Companies	0.0004	-0.0030	0.0013	1.0174
20.	Undertakings without direct responsibility	0.0717	-0.0111	-0.0068	1.0887

Appendix 1

Values of Productivity for Various Enterprises 1965-66 to 1976-77

Enterprise	1965-66	1966-67	1967-68	1968-69	1969-70	1970-71	1971-72	1972-73	1973-74	1974-75	1975-76	1976-77
STEEL												
Hindustan Steel	1.0086	0.9226	0.8685	0.8984	0.9722	1.0650	0.8961	0.9560	1.0068	1.0508	1.0577	1.0870
Bokaro Steel	-	-	-	-	-	-	-	0.6402	0.7749	0.8440	1.0093	1.0633
SAIL	-	-	-	-	-	-	-	1.000	1.00	1.0	1.0	1.0946
IISCO	-	-	-	-	-	-	-	-	-	-	0.9524	0.9085
MINERALS AND METALS (OTHER THAN COAL)												
Bharat Al.Co.	-	-	-	-	-	-	-	-	0.1045	0.3696	0.7486	0.747
Bharat Coal Mines	-	-	-	-	-	-	-	0.9145	0.7806	0.8194	0.7412	0.8848
Bharat Refractories	-	-	-	-	-	-	-	-	0.8571	0.8571	0.8533	0.8333
Bolani Ores	0.9709	0.9808	1.0215	1.1172	1.0556	1.0767	1.0172	1.0173	1.0182	0.8800	0.8871	0.8889
Hindustan Copper	-	-	-	-	-	-	-	1.1545	1.1720	1.0843	0.9426	1.0249
Hindustan Zinc	-	1.1873	1.0066	0.9456	0.7974	0.7946	0.9502	1.0197	1.4901	1.7199	1.5644	0.9016
Indian Rare Earths	1.2020	1.3485	1.3558	1.2044	1.1931	1.2555	1.2724	1.1483	1.2480	1.5561	1.7566	1.6926
Indian Firbricks and Insulation	-	-	-	-	-	-	-	-	-	-	0.0175	0.8693
NMDC	-	0.9501	0.8931	0.9310	0.9839	0.9033	0.8166	0.9251	1.1331	1.0827	1.032	0.9338
Pyrites, Phosphates and Chemicals	-	-	-	0.7491	0.7010	0.7475	0.5610	0.9647	0.8025	1.9022	0.9834	1.0232
Uranium Corporation of India	-	-	-	0.7168	0.9585	1.1439	1.200	1.0518	1.027	1.0243	0.9055	0.9170
COAL												
Bharat Coking Coal	-	-	-	-	-	-	-	0.9569	0.8962	-	-	0.8782
Central Coal field	-	-	-	-	-	-	-	-	-	-	-	1.0099
Coal India Ltd.	-	-	-	-	-	-	-	-	-	0.8686	0.9504	0.8095
Eastern Coal Fields	-	-	-	-	-	-	-	-	-	-	-	0.8754
Western Coal Fields	-	-	-	-	-	-	-	-	-	-	-	0.9867
National Coal Development Corporation	1.0079	0.8942	0.9473	1.0181	1.0017	0.9272	0.876	0.9588	0.8792	-	-	-
Coal Lignite Corporation	0.9987	0.6446	0.8182	0.9303	0.9431	0.6031	0.5643	0.6268	0.6241	0.7412	0.9222	1.2199
Coal Mining Authority	-	-	-	-	-	-	-	0.9001	1.054	-	-	-
PETROLEUM												
Bharat Petroleum	-	-	-	-	-	-	-	-	-	-	-	1.0109
Reliance Oil Refinery	-	-	-	-	-	-	-	-	-	-	-	1.0046
Lucan Refineries	-	-	1.0453	1.088	1.0669	1.0716	1.0453	1.0129	1.0005	0.9597	1.0218	1.0096
Hindustan Petroleum	-	-	-	-	-	-	-	-	-	1.0178	1.0071	1.0152
Hydrocarbons India	-	-	-	-	-	-	-	-	-	2.4794	2.5605	2.3079
Indian Oil Blending	-	-	-	-	-	-	-	-	-	1.2292	1.1835	1.3945
Indian Oil Corporation	1.0046	1.0275	1.0257	1.0363	1.0332	1.0224	1.0447	1.0668	1.0277	1.0273	1.0221	1.0415
Indo-Burma Petroleum	-	-	-	-	-	1.0248	1.0214	1.0223	1.0248	1.0172	1.0114	1.0064
Lubrizol India	-	-	-	1.1228	1.0984	1.1651	1.1978	1.2152	1.1947	1.3846	1.2165	1.2367
Madras Refineries	-	-	-	-	0.8784	1.1206	1.0666	1.1059	1.0496	1.0115	1.0229	1.0211
Oil and Natural Gas Commission	1.1235	1.6381	1.5702	1.5914	1.3772	1.3072	1.3251	1.1594	1.4298	1.5555	1.2943	1.2385
Indian Oil International	-	-	-	-	-	1.1667	1.200	1.2308	-	-	-	-
CHEMICALS & PHARMACEUTICALS												
Chemical Corporation of India	-	-	-	-	-	0.7922	1.0090	0.8953	0.8720	0.9744	1.0241	0.9590
Fertilizer & Chemicals (T)	0.8574	1.0429	0.9987	1.0292	0.9387	0.9428	0.8981	0.9075	0.9296	0.9477	0.8384	0.8257
Fertilizer Corporation India Ltd.	1.0118	0.9538	1.038	1.0907	1.0049	1.0072	1.0181	1.002	0.9882	0.9955	0.8768	0.8693

Enterprise	1965-66	1966-67	1967-68	1968-69	1969-70	1970-71	1971-72	1972-73	1973-74	1974-75	1975-76	1976-77
Hindustan Antibiotics	1.3792	1.3342	1.2292	1.2296	1.1698	1.0853	1.040	1.0314	0.8441	0.6704	0.7639	0.9628
Hindustan Insecticides	1.3282	1.3042	1.2115	1.133	1.0326	1.0584	1.1092	1.0372	1.0892	1.1166	1.1588	1.1063
Hindustan Salts	0.7795	0.7393	0.904	0.7271	0.8814	0.9930	1.0	1.15	1.0556	1.1795	1.6667	1.1795
Hindustan Organic Chemicals	-	-	-	-	1.03	1.1212	0.9565	1.0	1.2269	1.3189	1.2148	1.2095
Indian Drugs & Pharmaceuticals	-	0.0718	0.0377	0.0979	0.3423	0.6332	0.8275	0.8830	0.9383	1.0574	1.0646	1.0628
Indian Petrochemicals Corporation	-	-	-	-	-	-	-	-	0.6496	1.415	1.6414	1.4842
Madras Fertilizers	-	-	-	-	-	-	0.8829	0.9943	1.0663	1.0754	1.0264	1.0369
Sambhar Salts	1.4907	1.9349	1.5753	1.2293	1.2835	0.9027	1.2200	1.5686	1.5769	1.5439	1.2410	0.6400
<u>CAPITAL GOODS HEAVY ENGINEERING</u>												
Bharat Heavy Electricals	-	0.0301	0.5932	0.8656	0.9967	1.0263	1.0794	1.1301	1.199	1.2152	1.1847	1.1452
Bharat Heavy Plates and Vessels	-	-	-	-	0.0214	0.1320	0.3072	0.8022	0.9292	0.8985	0.9462	0.9745
Braithwaite	-	-	-	-	-	-	-	-	-	-	0.4437	0.8316
Bridge & Roof	-	-	-	-	-	-	-	1.0038	1.0033	0.6957	0.9032	1.0103
Burn Standard	-	-	-	-	-	-	-	-	-	-	0.8500	0.8983
Heavy Engineering	-	0.3485	0.2115	0.4200	0.5012	0.6363	0.6574	0.7233	0.8351	0.9240	1.0336	1.0449
Jessop	0.9877	1.0625	1.0775	0.9351	0.8948	0.8254	0.5927	0.7975	0.8489	1.0238	1.0032	0.9994
Mining & Allied Machinery Corporation	-	-	0.1126	0.1574	0.2249	0.4195	0.6847	1.0445	1.0243	0.9920	1.0144	1.0126
Triveni Structurals	-	-	-	0.0309	0.4218	0.5633	0.7661	0.8382	0.9244	0.8514	1.0334	1.0076
Tungbhadra Steel Products	1.1119	1.5511	1.5559	1.1365	1.0501	1.0543	1.0588	1.046	1.2609	1.0350	1.0806	1.0829
Heavy Electricals	0.5240	0.6061	0.7230	0.8280	0.6832	0.8137	1.0294	-	-	-	-	-
<u>MEDIUM & LIGHT ENGINEERING</u>												
Balmer Lawrie	-	-	-	-	-	-	-	1.0272	0.8561	1.0256	1.0219	1.0344
Bharat dynamics	-	-	-	-	-	-	0.7273	1.1358	1.0786	1.1013	1.0253	1.0129
Bharat electronics	1.1871	1.2852	1.2488	1.21	1.1994	1.2556	1.2744	1.1967	1.1861	1.1318	1.1007	1.1235
Bharat Pumps & Compressors	-	-	-	-	-	-	-	-	0.5211	0.5969	0.9192	0.8239
Biccop Lawrie	-	-	-	-	-	-	-	1.014	1.0225	0.9136	0.9737	0.9203
Central Electronic	-	-	-	-	-	-	-	-	-	-	-	0.1463
Electronic Corporation of India	-	-	0.2632	0.4716	0.8431	1.0835	1.2900	1.1857	1.1893	1.100	1.0766	1.0561
Hindustan Cables	0.9566	1.0375	1.0657	1.1117	1.0251	1.0162	0.9836	1.0357	1.0851	1.0562	1.0763	1.1184
Hindustan M/c. Tools	1.1185	1.0915	0.9569	0.9735	0.9514	1.0026	1.0338	1.0072	1.0231	1.0894	1.1115	1.1010
Hindustan Teleprinters	1.1171	0.8162	1.6579	1.6762	1.8028	1.5588	1.9371	1.775	1.4191	1.1917	1.2809	1.3768
Indian Telephone Industries	1.1878	1.2087	1.2324	1.2088	1.1792	1.1880	1.2422	1.1378	1.1550	1.1573	1.1070	1.1333
Instrumentation	-	-	-	0.4481	0.757	1.3828	1.2154	1.3024	1.0313	1.0216	1.0643	1.1283
National Instruments	1.0277	0.9487	0.6612	0.3840	0.2891	0.2997	0.6625	0.747	1.0808	1.0195	1.0102	0.8412
Praga Tools	0.9835	0.8340	0.9263	0.8737	0.7262	0.8948	0.6605	0.6786	0.8607	0.9556	0.9980	1.0934
Richardson & Cruddas	-	-	-	-	-	-	-	-	0.9852	1.0123	1.0250	1.0122
M/c. Tools Corporation of India	-	-	-	-	-	0.1678	0.4894	0.6329	0.7882	-	-	-
Industrial Containers	-	-	-	-	-	-	-	1.0654	1.0843	-	-	-
Steel Containers	-	-	-	-	-	-	-	1.0800	1.0963	-	-	-
<u>TRANSPORTATION EQUIPMENT</u>												
Bharat Earth Movers	1.0634	1.0827	1.1837	1.1082	1.1498	1.2256	1.1600	1.1659	1.1201	1.1685	1.1350	1.1125
Central Inland Water Transport Corporation	-	-	0.4507	0.5705	0.5931	0.6547	0.7198	1.0045	0.6821	0.5966	0.5078	0.7352
Garden Reach Ship Builders & Engineers	1.1099	1.1094	1.1096	1.0998	1.1048	1.1174	1.0687	0.9410	0.9419	0.9313	1.0013	1.0122
Goa Shipyard	-	-	1.1927	1.1469	1.1353	1.1858	1.1573	1.1028	1.0705	1.0930	1.0726	1.1116
Hindustan Aeronautics	1.0766	1.0474	1.0328	1.0444	1.0716	1.0713	1.0552	1.0594	1.1227	1.1125	1.0895	1.0869
Hindustan Shipyard	1.0000	1.0082	1.0015	1.0114	1.0052	1.0090	1.0496	1.0949	1.0324	1.0126	1.1260	1.1072
Mazagon Dock	1.0446	1.0529	1.0948	1.1183	1.1000	1.0650	1.1437	1.0393	1.1480	1.0393	1.0728	1.1073
Scooter India	-	-	-	-	-	-	-	-	-	0.2742	0.6660	0.8502

Enterprises	1965-66	1966-67	1967-68	1968-69	1969-70	1970-71	1971-72	1972-73	1973-74	1974-75	1975-76	1976-77
CONSUMER GOODS												
Bharat Ophthalmic Glass	-	-	-	-	-	-	0	0.1048	0.2740	0.3869	0.7678	0.5738
Hindustan Latex	-	-	-	-	0.1195	1.0512	1.1702	1.1649	0.8866	0.7889	1.0980	1.1657
Hindustan Photo Films	-	-	0.3709	0.5227	0.6460	0.5501	0.6220	0.6360	0.7395	0.8929	1.0072	1.0480
Mandya National Paper Mills	-	-	-	-	-	-	-	-	1.0853	1.2219	1.0699	0.8578
Modern Bakeries	-	-	0.6881	0.9975	0.9617	1.0216	1.1021	1.0931	1.1005	1.0905	1.0145	1.0330
Rehabilitation Industries Corporation	1.1538	0.6931	0.6730	0.5542	0.7160	0.5432	0.5828	0.5260	0.2682	0.3421	0.4338	0.4796
Tannery & Footwear Corpn. of India	-	-	-	-	0.4828	0.7298	0.7537	0.7211	0.8641	0.8205	0.9589	0.6985
National Newprints & Paper Mills	1.2111	1.1193	1.1569	1.0689	0.8340	0.8888	1.0159	1.0036	1.1259	1.0732	1.0939	1.0674
AGRO BASED ENTERPRISES												
Banana and Fruit Devpt. Corpn.	-	-	-	-	-	-	-	0.8333	1.000	1.4000	0.6226	0.5625
National Seeds Corporation	-	1.0801	1.0746	0.9075	1.0271	1.0005	1.0	1.1402	1.1744	1.2058	1.2058	1.0425
State Farms Corpn.	-	-	-	-	1.5840	1.0188	1.0246	1.0270	1.3986	1.5166	0.9456	0.7864
SERVICE ENTERPRISES												
Cashew Corpn. of India	-	-	-	-	-	1.0682	1.0843	1.0964	1.1275	1.1365	1.1354	0.9688
Central Fisheries Corpn.	-	0.8194	0.7354	0.8023	0.7938	0.6460	0.4800	0.7168	1.0105	0.8971	1.0541	0.7758
Cotton Corpn. of India	-	-	-	-	-	-	1.0109	1.0038	1.0447	1.0342	1.0539	1.0540
Central Warehouse Corpn.	0.9328	1.0119	1.0976	1.2977	1.6375	1.3086	1.2238	1.3459	1.2978	1.2793	1.2876	1.3277
Elect. Trade & Tech. Development Corpn.	-	-	-	-	-	-	-	-	-	1.0983	1.1000	1.0261
Food Corpn. of India	1.0008	1.0123	1.0110	1.0020	1.0037	1.0023	1.0011	1.0005	1.0007	1.0014	1.0015	1.0011
HMT International	-	-	-	-	-	-	-	-	-	-	1.1881	1.2178
Handicrafts & Handloom Export Corporation	-	0.9080	0.7392	0.9896	1.0003	1.0083	0.9665	1.0136	1.0109	1.0091	1.0225	1.0246
Indian Motion Pictures Export Corporation	-	-	1.4255	1.6892	1.5181	1.5886	1.0	0.9167	0.3939	0.5385	0.5385	1.0
Jute Corpn. of India	-	-	-	-	-	-	-	1.04	1.0177	1.0029	1.0173	1.0165
Metal Scrap Trading Corpn.	-	-	-	-	-	-	-	1.0	1.3333	4.25	4.200	4.750
Mica Trading Corpn.	-	-	-	-	-	-	-	-	-	1.0137	1.0014	1.0117
Minerals & Metals Trading Corporation	1.0182	1.1157	1.0167	0.9952	1.0139	1.0343	1.0620	1.0455	1.0815	1.0626	1.0439	1.0618
Projects & Equipment Corporation	-	-	-	-	-	-	1.0210	1.0239	1.0112	1.0130	1.0301	1.0588
State Chemicals and Pharmaceuticals Corpn.	-	-	-	-	-	-	-	-	-	-	-	1.0529
SPIL International	-	-	-	-	-	-	-	-	-	2.375	1.0237	1.0506
State Trading Corporation	1.0734	1.0462	1.0564	1.0775	1.0814	1.0299	1.0567	1.0339	1.0291	1.0238	1.0149	1.0318
Tea Trading Corpn.	-	-	-	-	-	-	-	0	0.1842	1.0787	1.0752	1.0365
TRANSPORTATION SERVICES & AIR												
Air India	1.0166	1.1008	1.0735	1.0626	1.0332	1.0113	0.9783	0.9903	1.0025	0.9359	1.0358	1.0749
Air India Chaiters	-	-	-	-	-	-	0.9955	1.0008	0.9970	0.9971	1.0057	0.9957
Indian Airlines	1.0066	0.9029	0.9923	1.0442	1.0538	0.9025	0.9229	1.0001	0.9841	1.0111	1.0798	1.2024
International Air Ports of India	-	-	-	-	-	-	-	1.8633	1.5539	1.4064	1.5518	1.3138
TRANSPORTATION SERVICES WATER												
Moghul Lines Ltd.	-	1.1826	1.2331	1.1067	1.1161	1.1440	1.0302	1.0269	1.1277	1.0435	0.8901	0.8862
Shipping Corporation of India	1.1240	1.2122	1.2269	1.1463	1.1259	1.1289	1.1271	1.1164	1.1374	1.2175	1.0512	1.0169
CONTRACTS & CONSTRUCTION SERVICES												
Hindustan Housing Factory	1.1352	1.0532	1.0911	1.1032	1.1366	1.1026	1.1340	1.0547	1.0046	0.7740	0.8741	0.8542
Hindustan Steel Works	1.3695	1.2271	1.0172	1.0117	1.0112	1.0175	1.0270	1.0277	1.0260	1.0218	1.0205	1.0209
Indian Road Const. Corporation	-	-	-	-	-	-	-	-	-	-	-	1.2220
National Building Const. Corporation	0.9454	0.9579	0.9701	0.9024	0.9200	0.9529	1.0254	1.0233	1.0193	1.0160	1.0721	1.0288

Enterprises	1965-66	1966-67	1967-68	1968-69	1969-70	1970-71	1971-72	1972-73	1973-74	1974-75	1975-76	1976-77
National Projects Const. Corporation	1.0169	1.0309	0.9711	0.8824	0.8311	0.7586	0.8532	1.0084	1.0328	1.0503	1.0106	1.0
Central Road Transport Corporation	0.4259	0.8191	0.7912	0.7097	0.8187	0.7831	0.734	0.5682	0.6279	0.7368	-	-
Mineral Exploration Corporation	-	-	-	-	-	-	-	0.6579	0.8571	0.8132	0.8198	1.2220
<u>INDUSTRIAL DEVPT. TECH. CONSULT. SERVICES</u>												
Engineering India Ltd.	0.2552	0.6691	1.4463	1.2831	1.3772	1.3333	1.1789	1.1144	1.1918	1.2210	1.1691	1.2417
Engg. Projects India	-	-	-	-	-	0.0114	0	1.0147	1.0087	1.0065	1.0056	1.0064
Metallurgy Engg. Consultants	-	-	-	-	-	-	-	-	0.7941	1.1121	1.1107	1.1090
National Ind. Devpt. Corporation	1.0529	1.0706	1.0078	1.0299	1.1747	1.1337	1.1376	1.1495	1.0685	1.0943	1.1414	1.0472
Rail India Tech & Economic Services	-	-	-	-	-	-	-	-	-	0	1.0952	1.1935
Water & Power Devtp. Consult. Services	-	-	-	-	0.1667	0.05	1.4	1.0	1.3636	1.2	1.5306	1.2371
<u>DEVPT. OF SMALL INDUSTRIES</u>												
National Small Ind. Corpn.	0.8920	0.9574	0.9285	0.8669	0.9852	0.9473	0.9688	1.0535	0.9682	0.9200	1.0028	0.9482
Delhi State Ind. Devt. Corporation	-	-	-	-	-	-	-	1.0164	1.0283	1.0026	1.0032	1.0145
<u>TOURIST SERVICES</u>												
Hotl. Corpn. of India	1.2785	1.1816	1.3028	1.0862	-	-	-	-	0.8963	0.9455	0.7575	0.9886
Hotel Janpath	-	0.9355	0.9760	0.8784	-	-	-	-	-	-	-	-
Indian Tourism Devt. Corporation	-	1.0880	1.0172	1.0339	1.0466	1.0751	1.0372	1.0559	1.0520	1.0642	1.0562	1.0808
<u>FINANCIAL SERVICES</u>												
Film Finance Corpn.	-	-	1.00	1.02	1.0332	1.0392	1.1667	1.1429	0.5455	1.0278	1.2278	1.0303
Housing & Urban Devt. Const. Ltd.	-	-	-	-	-	-	1.8461	1.7400	1.5814	1.4295	0.5357	0.6099
Rural Elect. Corpn.	-	-	-	-	1.5510	4.7063	5.1944	3.7460	3.8010	2.9356	1.9757	1.6571
<u>SECTION 25 COMPANIES</u>												
Indian Dairy Corpn.	-	-	-	-	-	1.0289	0.9529	1.0970	1.0395	1.0404	1.0778	1.1289
National Research Devt. Corporation	0.5576	0.7925	0.7938	0.9650	0.9133	1.2251	1.1923	1.1515	1.0690	1.0882	1.1053	1.1
<u>UNDERTAKING WITHOUT DIRECT RESP. FOR MOT</u>												
Andrew Yule & Coy	-	-	-	-	-	-	-	-	-	-	1.0718	1.0892
British India Corpn.	1.0236	0.9992	0.8815	0.9652	1.0705	1.0972	1.1353	1.0661	1.0794	1.0038	1.0777	1.0407
Damodar Valley Corpn.	0.9693	1.0374	0.9789	0.9713	0.9163	0.9175	0.9618	0.9456	0.8303	0.9319	1.1237	1.2944
Indian Explosive Ltd.	1.3563	1.3739	1.3624	1.2292	1.2504	1.0014	1.1795	1.0791	1.0791	1.1590	1.1411	1.1624
M/c Ry. Mfg. Corpn.	1.1038	0.911	0.9295	0.9519	1.0321	1.0511	0.7553	0.6900	1.0062	1.0604	1.0468	1.0416
Manganese Ore (India)	1.1833	1.3533	1.2146	1.0158	1.0118	1.0368	1.0029	0.9083	0.8611	1.016	1.1372	1.2384
Oil India	1.2125	1.2018	1.5676	1.7425	1.5999	1.6436	1.6505	1.714	1.7491	1.5676	1.3200	1.4588
Sikkim Mining Corpn.	-	-	0.2527	0.8209	1.2222	0.4526	0.4167	0.8333	1.2727	0.8947	0.6667	1.0
Sindhu Resettlement Corporation	-	-	1.0205	0.900	1.2679	1.4571	0.8182	1.000	1.000	1.000	1.0385	1.0417
Singareni Collieries	1.0395	0.9861	0.9331	0.8793	0.8446	0.9435	1.0309	1.0252	1.0030	1.0730	1.1235	0.9412
Vishveshwaraya Iron & Steel	-	-	-	-	-	-	0.9162	1.0021	1.0071	1.0483	1.0659	0.9326
Wagons India	-	-	-	-	-	-	-	-	-	-	1.0	1.0
Lube India	-	-	-	-	0.0015	1.1619	1.3655	1.1451	-	-	-	-

Appendix 2

In this appendix, we give computational details of ANACOVA model used for comparison of slopes and the scheme of linear contrasts.

The ANACOVA model used is

$$Y_{ij} = \mu + \phi_j + \beta_m(\bar{X}_j - \bar{X}) + \beta_a(X_{ij} - \bar{X}_j) + \delta_j(X_{ij} - \bar{X}_j) + \epsilon_{ij}$$

The unbiased estimates of the various parameters are

$$\begin{aligned}\hat{\mu} &= \frac{\sum_i \sum_j Y_{ij}}{n} = \bar{Y} \\ \hat{\phi}_j &= \bar{Y}_j - \hat{\mu} - \hat{\beta}_m(\bar{X}_j - \bar{X}) \\ \hat{\beta}_m &= \frac{C_{XY}}{C_{XX}} \\ \hat{\beta}_a &= \frac{E_{XY}}{E_{XX}} \\ \hat{\delta}_j &= \frac{E_{XYj}}{E_{XXj}} - \frac{E_{XY}}{E_{XX}}\end{aligned}$$

The quantities C_{xy} , C_{xx} , E_{xy} , E_{xx} , E_{xyj} and E_{xxj} are defined in Exhibit 1. The actual values for the model are shown in Exhibit 2. Exhibit 3 is the ANACOVA table for hypothesis testing.

Exhibit 1Sums of Squares, Cross Products for One Way ANACOVA Different slopes

Source of variation	Sums of Squares for Y	Sums of Cross products for X and Y	Sums of squares for X
1. Differences between columns	$C_{YY} = \sum_j \frac{C_{yj}^2}{n_j} - K_y$	$C_{xy} = \sum_j \frac{C_{xj} C_{yj}}{n_j} - K_{xy}$	$C_{xx} = \sum_j \frac{C_{xj}^2}{n_j} - K_x$
2. Differences within column j	$E_{yyj} = \sum_i Y_{ij}^2 - K_{yj}$	$E_{xyj} = \sum_i X_{ij} Y_{ij} - K_{xyj}$	$E_{xxj} = \sum_i X_{ij}^2 - K_{xj}$
3. Sum of differences within columns	$E_{YY} = \sum_j E_{yyj}$	$E_{xy} = \sum_j E_{xyj}$	$E_{xx} = \sum_j E_{xxj}$
4. Total	$S_{YY} = \sum_i \sum_j Y_{ij}^2 - K_y$	$S_{xy} = \sum_{ij} X_{ij} Y_{ij} - K_{xy}$	$S_{xx} = \sum_{ij} X_{ij}^2 - K_x$

where $K_x = (\sum_{ij} X_{ij})^2 / N$, $K_y = (\sum_{ij} Y_{ij})^2 / N$

$K_{xj} = (\sum_i X_{ij})^2 / n_j$, $K_{yj} = (\sum_i Y_{ij})^2 / n_j$

$K_{xy} = (\sum_{ij} X_{ij}) (\sum_{ij} Y_{ij}) / N$

$K_{xyj} = (\sum_i X_{ij}) (\sum_i Y_{ij}) / n_j$

$C_{xj} = \sum_i X_{ij}$ $C_{yj} = \sum_i Y_{ij}$

Exhibit 2

j	E_{yyj}	E_{xyj}	E_{xxj}	E_{xyj}^2/E_{xxj}	K_{yj}	K_{xyj}	K_{xj}
1.	0.2483	1.9983	249.6250	0.0160	22.0376	192.6074	1683.375
2.	7.3825	-3.3612	845.5596	0.0134	84.5676	663.2293	5201.4404
3.	2.4158	8.6066	395.8750	0.1871	23.3123	203.1397	1770.125
4.	6.8379	6.4181	687.9445	0.0599	101.1905	685.2222	4640.0555
5.	7.4947	-4.5696	1207.6575	0.0173	115.7196	802.2197	5561.3425
6.	4.7613	33.7650	946.4045	1.2046	61.2859	538.5548	4732.5955
7.	10.1822	10.0892	1465.2648	0.0695	141.8169	1031.3733	7500.7352
8.	2.8077	-6.7178	949.2290	0.0475	85.3386	581.0165	3955.771
9.	6.2410	3.9237	653.8429	0.0235	45.9338	438.2425	4181.1571
10.	1.3923	-2.7532	194.6250	0.0389	27.2608	207.8253	1584.3750
11.	41.1568	30.2824	1321.7143	0.6938	156.2982	1115.9877	7968.2857
12.	1.4557	0.1048	392.7429	0.0000	41.4733	286.2898	1976.2571
13.	0.1852	-4.6719	254.4348	0.0858	28.1074	171.3478	1044.6652
14.	5.7753	14.1825	763.3594	0.2635	54.1113	394.4675	2875.6406
15.	6.9190	13.1641	477.2174	0.3631	46.0102	362.0401	2848.7826
16.	0.0401	1.5595	196.2353	0.0000	16.0221	124.2639	963.7647
17.	0.3043	-3.8544	288.3636	0.0515	23.6996	143.2315	865.6364
18.	38.6447	-9.7913	157.3334	0.6093	79.1816	363.2758	1666.6666
19.	0.4672	7.1350	198.6316	0.2563	19.6434	143.3673	1046.3684
20.	8.4467	-7.4193	1405.5883	0.0392	136.7823	885.5671	5733.4117

$$\begin{aligned}
 C_{YY} &= 26.9995 & C_{xy} &= 26.4586 & C_{xx} &= 278.385 & E_{xy}^2/E_{xx} &= 0.5946 \\
 K_Y &= 1282.7935 & K_{xy} &= 9306.8106 & K_X &= 67521.959 & C_{xy}^2/C_{xx} &= 2.5147 \\
 E_{YY} &= 153.1587 & E_{xy} &= 83.0905 & E_{xx} &= 13051.646 & S_{xy}^2/S_{xx} &= 0.9844 \\
 S_{YY} &= 180.1582 & S_{xy} &= 114.5491 & S_{xx} &= 13330.031 & &
 \end{aligned}$$

Exhibit 3ANACOVA Table for Hypothesis Tests

Source of variation	Component of total sum of squares	d.f.	MS
Deviation from regressions within groups	$Q_e = E_{YY} - \sum_j \frac{E_{xyj}^2}{E_{xxj}}$ $= 149.1185$	$N-2K$ $= 1177$	$S_e^2 = Q_e / (N-2K)$ $= 0.1267$
Differences between regressions within groups	$Q_g = \sum_j \frac{E_{xyj}^2}{E_{xxj}} - \frac{E_{xy}^2}{E_{xx}}$ $= 3.4456$	$K-1$ $= 19$	$S_g^2 = Q_g / (K-1)$ $= 0.1813$
Deviations within groups from common regression β_a	$Q_a = E_e + Q_g$ $= 152.5641$	$N-K-1$ $= 1196$	$S_a^2 = Q_a / (N-K-1)$ $= 0.1276$
Deviations between groups from linear regression β_m	$Q_g = C_{YY} - C_{xy}^2 / C_{xx}$ $= 24.4848$	$K-2$ $= 18$	$S_g^2 = Q_g / (K-2)$ $= 1.3602$
Differences between β_a and β_m	$Q_{m-a} = \frac{C_{xy}^2}{C_{xx}} + \frac{E_{xy}^2}{E_{xx}} - \frac{S_{xy}^2}{S_{xx}}$ $= 2.1249$	1	$S_{m-a}^2 = Q_{m-a}$ $= 2.1249$
Common overall regression β_o	$Q_o = S_{xy}^2 / S_{xx}$ $= 0.9844$	1	$S_o^2 = Q_o$ $= 0.9844$
Total	$S_{YY} = 180.1582$	$N-1=1216$	

Hypothesis Tests:

1. Hypothesis: Regression Slopes within industry do not vary from industry to industry i.e. $\delta_j = 0$

$$\frac{S_{\delta}^2}{S_e} = 1.4309$$

$$F_{0.05} (V_1 = 19, V_2 = 1177) = 1.57 > 1.4309$$

Hence the null hypothesis is valid.

2. Hypothesis: There is no difference in productivity between industries that is not (linearly) explained by the regression i.e. $\phi_j = 0$

$$\frac{S_{\phi}^2}{S_a^2} = 10.6599$$

$$F_{0.05} [V_1 = 18, V_2 = 1196] \approx 1.6 < 10.6599$$

The null hypothesis is rejected.

3. Hypothesis: The common slope of regression of productivity within industry is the same as the slope of regression of productivity, between industries i.e. $\beta_a - \beta_m = 0$

$$S_{m-a}^2 / S_a^2 = 16.6528$$

$$F_{0.05} (V_1 = 1, V_2 = 1177) = 3.84 < 16.6528$$

We reject the null hypothesis.

4. There is no departure from one overall regression

$$\frac{(Q_{\delta} + Q_{\theta} + Q_{m-a}) / 38}{s_e^2} = 6.2425$$

$$F_{0.05} (V_1 = 38, V_2 = 1177) \approx 1.45 < 6.2425$$

The null hypothesis is rejected.

Exhibit 4ANACOVA Table for Hypothesis Tests (Outliers Removed)

Source of Variation	Component of total sum of squares	d.f.	MS
Deviations from regressions within groups.	$Q_e = 63.7989$	1145	$s_e^2 = 0.0557$
Differences between regressions within groups	$Q = 1.8038$	19	$s_\delta^2 = 0.0949$
Deviations within groups from common regression β_a	$Q_a = 65.6027$	1164	$s_a^2 = 0.0564$
Deviations between groups from linear regression β_m	$Q_0 = 9.5421$	18	$s_\emptyset^2 = 0.5301$
Differences between β_a and β_m	$Q_{m-a} = 0.0284$	1	$s_{m-a}^2 = 0.0284$
Common overall regression β_o	$Q_o = 0.2615$	1	$s_o^2 = 0.2615$

Hypothesis Tests (with Outliers removed):

1. Hypothesis : $\delta_j = 0$

$$s_\delta^2 / s_e^2 = 1.7038$$

$$F_{0.01}(19, 1145) \approx 2 > 1.7038$$

The null hypothesis is not rejected at 0.01 significance level.

2. Hypothesis : $\emptyset_j = 0$

$$s_\emptyset^2 / s_a^2 = 9.4$$

$$F_{0.05}(18, 1164) = 1.7 < 9.4$$

The null hypothesis is rejected.

3. Hypothesis: $\beta_m - \beta_a = 0$

$$\frac{s_{m-a}^2}{s_a^2} = 0.5$$

$$F_{0.05}(1, 1145) = 3.84 > 0.5$$

The null hypothesis is accepted.

4. There is no departure from single overall regression

$$\frac{(Q_6 + Q_7 + Q_{m-a})/38}{s_e^2} = 5.3739$$

$$F_{0.05}(38, 1145) = 1.45 < 5.3739$$

The null hypothesis is rejected.

Sample Calculation for the Method of Linear Contrasts:

Suppose our interest is to test if $\mu_1 - \mu_3 = 0$. μ_1 and μ_3 are the adjusted means of groups 1 and 3, respectively.

Null Hypothesis: $\mu_1 - \mu_3 = 0$

Define $[C_1, C_2, \dots, C_{20}] = [1, 0, -1, 0, \dots, 0]$

$$\therefore \hat{L} = \mu_1 - \mu_3 = 0.0156$$

$$\hat{\sigma}^2 = 0.1276$$

$$\therefore \hat{s}_L = \sqrt{0.1276 [0.0729 + 0.0001]} = 0.0964$$

$$F_{\alpha, K-1, N-K-1} = F_{0.05, 19, 1197} = 1.59$$

$$\hat{s}_L \sqrt{(K-1) F_{0.05, 19, 1197}} = 0.0964 \sqrt{19 \times 1.59} = 0.5$$

The confidence interval of the contrast is

$$-0.5146 \leq \mu_1 - \mu_3 \leq 0.5406$$

Since this interval contains zero, we conclude that μ_1 and μ_3 are not significantly different.

Exhibit 5 Summary of Results With Scheme of Linear
Contrasts

1. $\mu_{18} \neq \mu_j$ ($j = 1, 2, \dots, 17, 19, 20$)
2. $\mu_9 \neq \mu_{4, 20}$
3. All Other means are equal.