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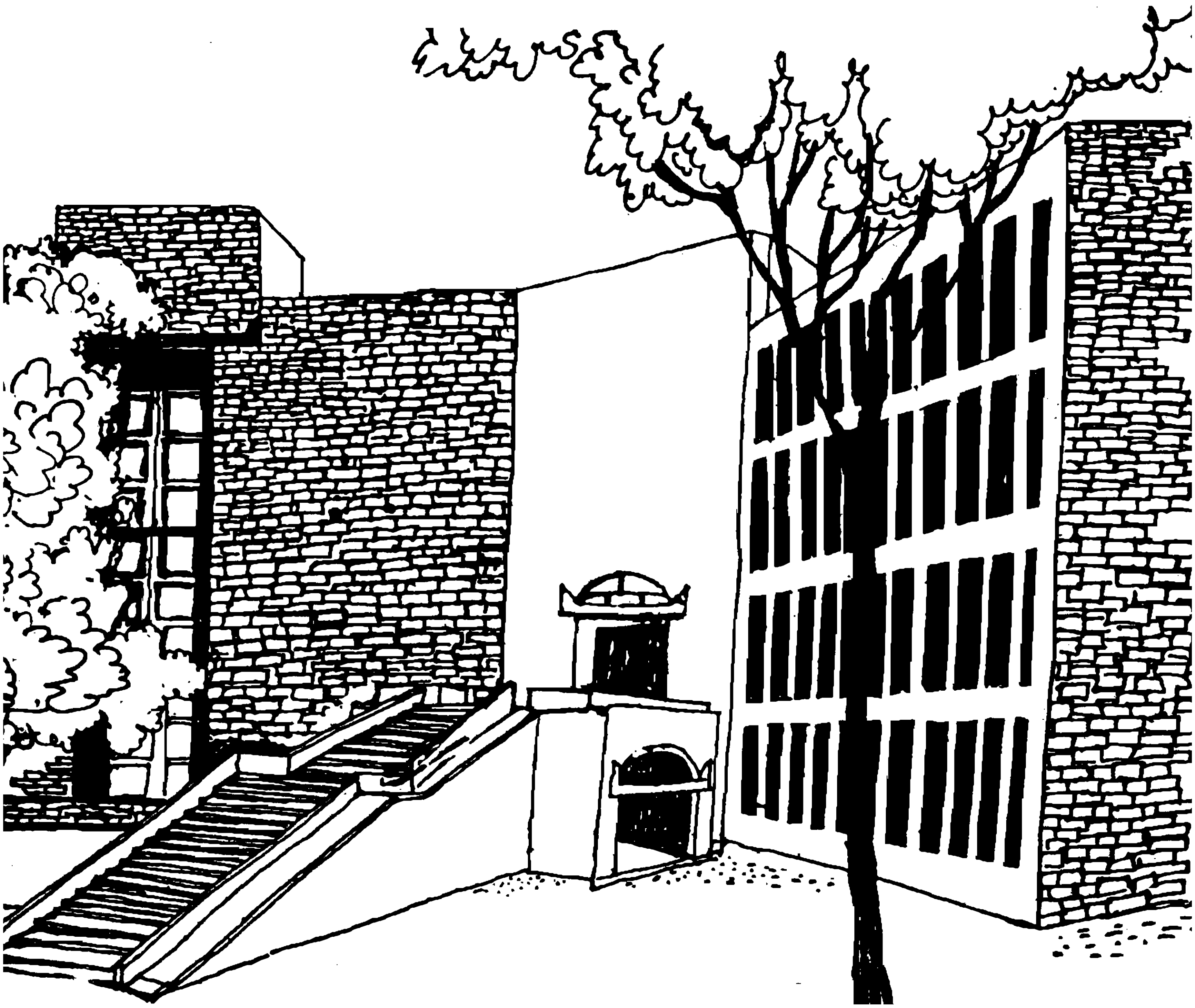

A COMPROMISE SOLUTION FOR CLAIMS PROBLEMS

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Abstract

The purpose of this paper is to axiomatically characterize a Compromise Solution for claims problems which satisfies invariance under affine utility transformations. The more well-known solutions for claims problems do not satisfy this property. This is a major deficiency of the latter class of solutions, as they fail to predict appropriate responsiveness to risk aversion by ones opponents. Our solution overcomes this deficiency.

1. Introduction :- The purpose of this paper is to axiomatically characterize a Compromise Solution for claims problems which satisfies invariance under affine utility transformations. The more well-known solutions for claims problems do not satisfy this property. This is a major deficiency of the latter class of solutions, as they fail to predict appropriate responsiveness to risk aversion by ones opponents. Our solution overcomes this deficiency.

Following Richter (1982), Yu (1985), Young (1987), Lahiri (1991) and Thomson (forthcoming), we define a n-person claims problem in the following fashion :

Let $S \subseteq \mathbb{R}^n$ be compact and convex; let $PO(S) \equiv \{x \in S / \forall y \in S$ $[y \geq x \Rightarrow y=x]\}$ denote the set of Pareto-optimal points of S; and let $c \in \mathbb{R}^n$ be such that there exists $x \in S$ for which $c >> x$. Then the ordered pair (S,c) will be called a <u>claims</u> problem. Let Σ^n denote the class of claims problems as defined above. In the sequel we shall find the following property as an useful aid in our characterization theorem :

Let $m_i(S) = \min\{x_i / (x_1, x_2, \ldots, x_i, \ldots x_n) \in S\}$, $i \in \{1, \ldots, n\}$.

The point $m(S) = (m_1(S), \ldots, m_n(S))$ is called the point of minimum expectation.

We shall consider the following subclass of Σ^n:

$\Sigma^n_- = \{(S,c) \in \Sigma^n / [m(S) \leq y \leq x,\ x \in S] \Rightarrow y \in S\}$.

Σ^n_- is the class of claims problem which allow for free disposability of utility.

A <u>Compromise Solution</u> on Σ^n_- is a function $F: \Sigma^n_- \to \mathbb{R}^n$ such that $\forall (S,c) \in \Sigma^n_-$, $F(S,c) \in S$.

We shall require our Compromise Solution to satisfy the following properties.

<u>Property 1</u> :- <u>Weak Pareto Optimality</u> (WPO) : $\forall (S,c) \in \Sigma_-^n$, $y \in \mathbb{R}^n$, $y >> F(S,c) \Rightarrow y \notin S$.

<u>Property 2</u> :- <u>Independence with respect to Affine Utility Transformations</u> (IAUT) : For each $(S,c) \in \Sigma_-^n$ and each transformation $A: \mathbb{R}^n \to \mathbb{R}^n$ of the form

$A(x_1, x_2, \dots, x_n) = (a_1x_1+b_1, a_2x_2+b_2, \dots, a_nx_n+b_n)$ for all $x \in \mathbb{R}^n$,

where $b_1, b_2, \dots, b_n$ are real numbers and $a_1, a_2, \dots, a_n$ are positive real numbers, we have $F(A(S), A(c)) = A(F(S,c))$.

<u>Property 3</u> :- <u>Symmetry</u> (Sym.) : Given a one-to-one function π : $\{1,\dots,n\} \to \{1,\dots,n\}$, let $\pi(x) = (x_{\pi(1)}, x_{\pi(2)}, \dots, x_{\pi(n)}) \forall x \in \mathbb{R}^n$ and $\pi(S) = \{\pi(x) / x \in S\} \forall S \subseteq \mathbb{R}^n$.

If for all $\pi : \{1,\dots,n\} \to \{1,\dots,n\}$ which is one-to-one, $S = \pi(S)$ and $c = \pi(c)$ for $(S,c) \in \Sigma_-^n$, then $F_i(S,c) = F_j(S,c) \forall i, j \in \{1,\dots,n\}$

<u>Property 4</u> :- <u>Restricted Monotonicity</u> (R.Mon) : Given (S,c) and $(T,c) \in \Sigma_-^n$ with $m(S)=m(T)$ and $S \subseteq T$, we have $F(T,c) \geq F(S,c)$.

<u>Property 5</u> :- <u>Continuity</u> (CONT) : Let $\{(S_j,c) : j=1,2,\dots\}$ be a sequence of claims problems in Σ_-^n, be such that, in the limit as j goes to infinity, S_j converges to S (in the Hausdorff topology) where $(S,c) \in \Sigma_-^n$. Then $\lim_{j \to \infty} F(S_j,c) = F(S,c)$.

Our candidate colution which the above properties are supposed to uniquely characterize is defined as follows :

$G : \Sigma_-^n \to \mathbb{R}^n$

where $G(S,c) = m(S) + \lambda(S,c)[c - m(S)]$

where $\lambda(S,c) = \max\{\lambda \geq 0 / m(S) + \lambda(S,c)[c - m(S)] \in S\}$ for all $(S,c) \in \Sigma_-^n$.

2. The Main Theorem :- We now establish the following theorem :

Theorem 1 :- The unique solution on Σ_-^n which satisfies Properties 1 to 5 is G.

Proof :- That G is well-defined and satisfies properties 1 to 5 is clear. So, let us prove that if $F : \Sigma_-^n \to \mathbb{R}^n$ satisfies properties 1 to 5 then F=G. Towards that end we define the following subdomain of Σ_-^n called $\bar{\Sigma}_-^n$.

$\bar{\Sigma}_-^n = \{(S,c) \in \Sigma_-^n / [x, y \in S, x > y \Rightarrow [\exists z \in S \text{ with } z >> y]\}$

It is easy to verify that $\forall (S,c) \in \bar{\Sigma}_-^n$, $x \in S$ is Pareto optimal if and only if it is weakly Pareto optimal.

We ~~shall now~~ establish the following lemma.

Lemma 1 :- If $F : \bar{\Sigma}^n \to \mathbb{R}^n$ satisfies properties 1 to 4, then $F = G$ on $\bar{\Sigma}_-^n$.

Proof of Lemma 1 :- By property 2 (IAUT) it is enough to consider problems $(S,c) \in \bar{\Sigma}_-^n$ such that m(S)=0 and c=(1,1,...,1).

Consider the problem (T,c) where

$T = Co.\{G(S,c), (G_1(S,c),0,\ldots,0),\ldots,(0,0,\ldots,0,G_n(S,c)),(0,0,\ldots,0)\}$

and co. denotes convex hull.

Clearly, $G_i(S,c) = G_j(S,c) \forall i, j \in \{1,\ldots,n\}$

and hence (T,c) is a symmetric claims problem with $G(S,c) \in T$ being the only symmetric and weakly Pareto optimal point.

$\therefore$ F(T,c) = G(S,c) by properties 1 (WPO) and 2(Sym).

Since $T \subseteq S$ and m(S) = m(T), we get $F(S,c) \geqq F(T,c) = G(S,c)$ by property 4 (R.Mon).

Since $S \in \bar{\Sigma}_-^n$, and G satisfies property 1, F(S,c)=G(S,c)

Proof of Theorem 1 :- Let $\{(S_j,c)/j=1,2,\ldots\}$ be a sequence of claims problems in $\bar{\Sigma}_-^n$ such that as j goes to infinity $S_j \to S$ (in the Hausdorff topology).

$\therefore F(S_j,c) = G(S_j,c) \; \forall \; j=1,2,\ldots$

$\therefore$ By property 5 (CONT)

$$\lim_{j \to \infty} F(S_j,c) = F(S,c).$$

However, $\lim_{j \to \infty} G(S_j,c) = G(S,c)$.

$\therefore F(S,c) = G(S,c)$.

Q.E.D.

Conclusion :- The purpose of this section is to note some connections of this solution with those existing in the literature.

There is some similarity between this solution and the one suggested in Lahiri (1989) for bargaining problems satisfying a certain regularity assumption. The similarity is only apparent as the context, conclusion and methods of proof are entirely different. However, one cannot completely disown ones own earlier contributions and its effects on subsequent research.

The main solution in the bargaining theory literature which resembles our compromise solution is the one due to Kalai and Smorodinsky (1975). It may be worthwhile to highlight the similarities and differences between the two.

If the claims point is less than or equal to the ideal point of Kalai-Smorodinsky, then the two problems are very similar. Otherwise, the two solutions are entirely different with regard to the method of proof.

In some senses our solution can be considered to be a generalization of the Kalai-Smorodinsky solution (calibrated appropriately to solve claims problems). That is why we must rest contended with weak Pareto optimality and invoke continuity to achieve our objectives.

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