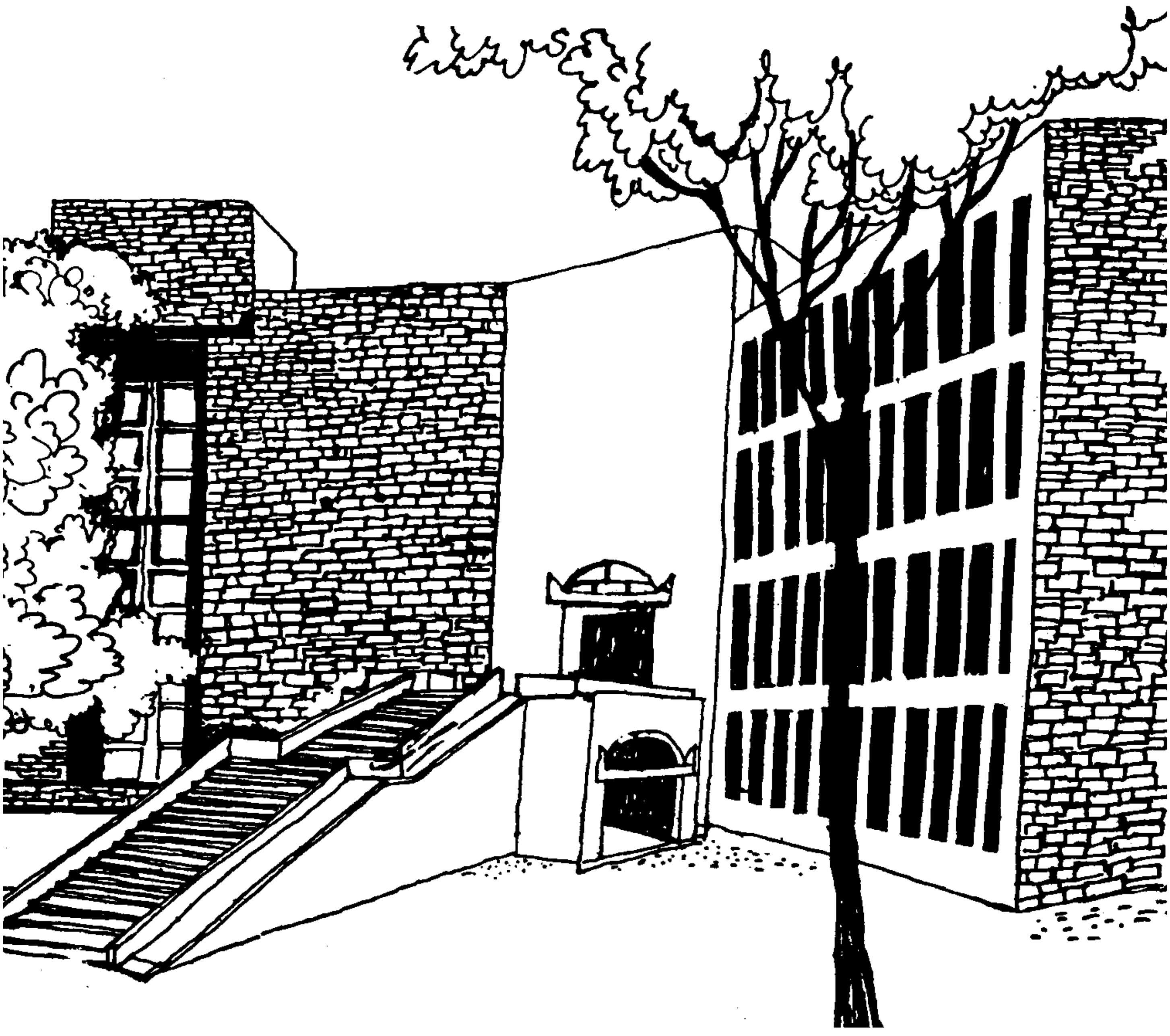
विद्याविनियोगाद्विकासः

IIM

AHMEDABAD

Working Paper



A NEGOTIATION PROCEDURE CONVERGING TO THE EGALITARIAN SOLUTION

By

Somdeb Lahiri

W P No. 981
November 1991

INDIAN INSTITUTE OF MANAGEMENT
AHMEDABAD-380 015
INDIA

A Negotiation Procedure Converging To The Egalitarian Solution

Somdeb Lahiri
Indian Institute of Management
Ahmedabad - 380015
India

November, 1991

Abstract

In this paper we propose a negotiation procedure, solutions of which converge to the egalitarian solution, in two person bargaining problems.

1. Introduction :- The simple analytical freamework of dividing a fixed sum of money between two risk averse agents has been discussed in Roth (1979). It may briefly be described as follows:

There are two agents indexed $i=1,2$ whose preference for money is given by utility functions $u_i : \mathbb{R}_+ \to \mathbb{R}$. Agent i, possesses initial wealth $w_i \geq 0$. A fixed sum of money Q, has to be divided betweeen the two agents. We assume that each u_i is increasing (i.e. more money is preferred to less money), concave (i.e. risk-averse) and continuously differentiable. We assume $u_i' > 0$, $i=1,2$.

A <u>feasible</u> <u>allocation</u> is a split (c_1, c_2) such that $c_1+c_2 \leq Q$, $c_1 \geq 0$, $c_2 \geq 0$. In Lahiri (1991), we adopted this framework to obtain a negotiation procedure, solutions of which converge to the Nash bargaining solution.

The <u>egalitarian</u> <u>solution</u> to the above problem is a feasible allocation (c_1^*, c_2^*) such that

(i) $c_1^* + c_2^* = Q$

(ii) $u_1(c_1^* = u_2(c_2^*)$.

Under our given hypotheses, there exists a unique egalitarian solution.

The purpose of this paper is to suggest a negotiation (or adjustment) procedure, solutions of which converge to the egalitarian solution.

2. The Negotiation Procedure and Main Result :- We consider the following negotiation procedure :

$$\left.\begin{aligned}
\frac{dc_1}{dt} &= u_2(Q-c_1(t))-u_1(c_1(t)) \\
c_2(t) &= Q-c_1(t), \\
t &\geqq 0
\end{aligned}\right\} \qquad (1)$$

It is clear that the unique critical point (or equilibrium) of (1) is (c_1^*, c_2^*). Our purpose in this paper is to show that if $(c_1(t), c_2(t))_{t \geqq 0}$ be any solution of (1) then $\lim_{t \to \infty} (c_1(t), c_2(t)) = (c_1^*, c_2^*)$.

<u>Theorem 1</u> :- Let $(c_1(t), c_2(t))_{t \geqq 0}$ be any solution of (1). Then $\lim_{t \to \infty} (c_1(t), c_2(t)) = (c_1^*, c_2^*)$.

<u>Proof</u> :- To prove this theorem we construct the function

$V(c_1) = [u_1(c_1)-u_2(Q-c_1]^2, \; 0 \leqq c_1 \leqq Q.$

$V(c_1^*) = 0$ and $V(c_1) \neq 0$ if $c_1 \neq c_1^*$

Further,

$$\frac{dV(c_1(t))}{dt} = 2[u_1(c_1)-u_2(Q-c_1)][u_1'(c_1)+u_2'(Q-c_1][u_2(Q-c_1)-u_1(c_1)]$$

$$= -2[u_1(c_1)-u_2(Q-c_1)]^2[u_1'(c_1+u_2'(Q-c_1)] < 0$$

whenever $c_1(t) \neq c^*$ and $c_1(t)$ solves (1).

Further $c_1(t) \in [0, Q]$ (a compact set) for all $t \geqq 0$. Hence, by the theorem on global asymptotic stability in Varian (1981), $\lim_{t \to \infty} c_1(t) = c_2^*$ and $\lim_{t \to \infty} c_2^*(t) = c_2^*$.

Q.E.D.

<u>Conclusion</u> :- In this paper we have defined a procedure, solutions of which converge to the egalitarian solution, in a division problem. In Maschler, Owen and Peleg (1988), we find a procedure, solutions of which converge to the Nash set for bargaining problems. Their procedure is independent of the underlying set of physical alternatives, and thus lacks the economic implications that arises out of the specific context, which is under investigation.

References :-

1. S. Lahiri (1991) : "A Procedure to the Nash Bargaining Solution", mimeo.

2. M. Maschler, G. Owen, B. Peleg (1988) : "Paths Leading to the Nash Set", in A.E. Roth (ed.) "The Shapley Value : Essays in honor of Lloyd S. Shapley", Cambridge University Press, Cambridge.

3. A.E. Roth (1979) : "Axiomatic Models of Bargaining", Springer-Verlag (Lecture Notes in Economic and Mathematical Systems).

4. H. Varian (1981) : "Dynamical Systems", Ch. 3 in K.J. Arrow and M.D. Intrilligator (ed.) "Handbook of Mathematical Economics", Vol. 1, North-Holland Publishing Company, Amsterdam.