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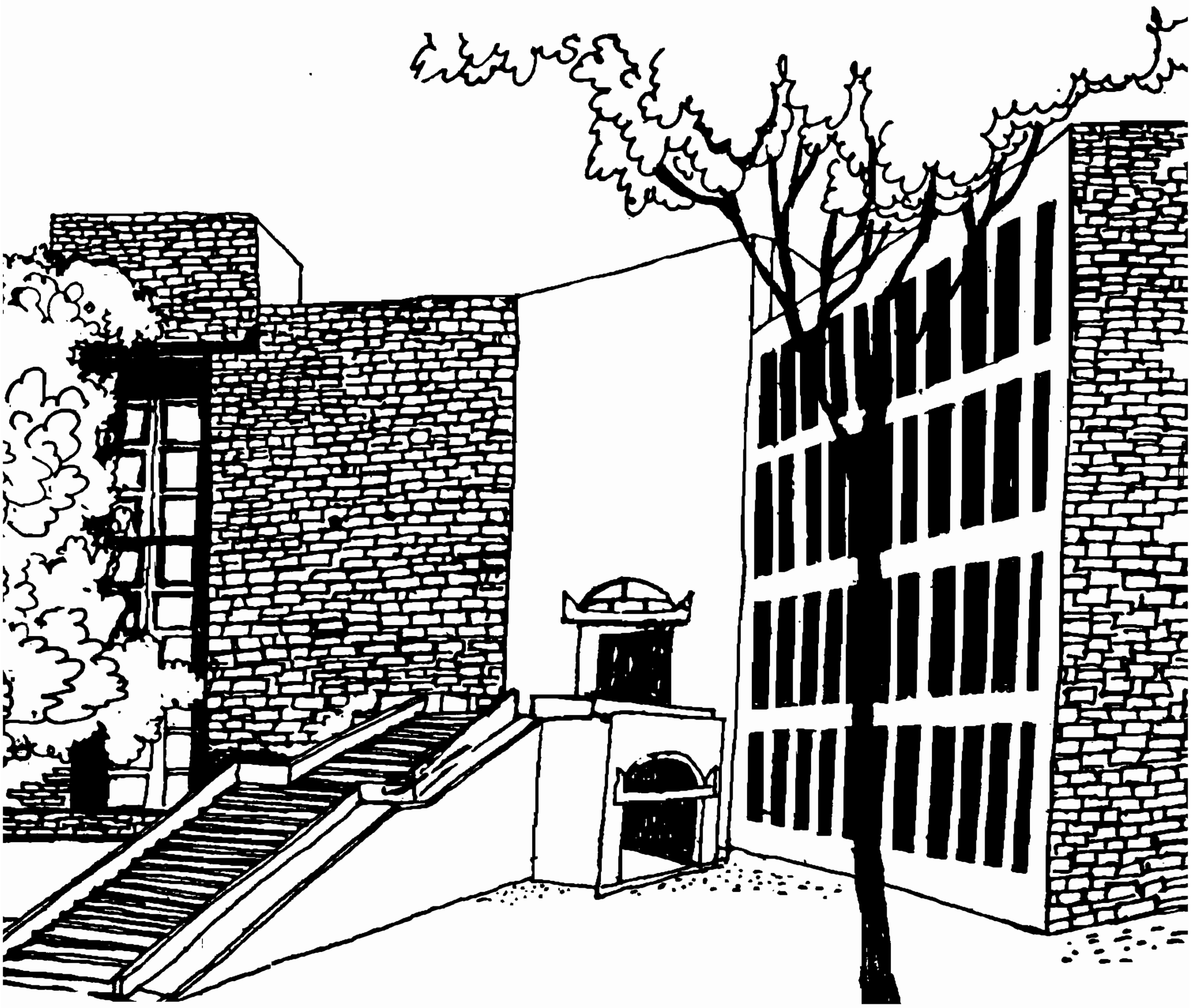

LOCALIZED AND NON-LOCALIZED COMPETITION IN THE PRESENCE OF CONSUMER LOCK-IN

By

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LOCALIZED AND NON-LOCALIZED COMPETITION IN THE PRESENCE OF CONSUMER LOCK-IN

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and
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This paper models localized competition between firms when there is consumer lock-in or loyalty. We derive the symmetric equilibrium mixed strategy price distribution under two alternative models, and compare them to symmetric equilibrium strategies under non-localized competition. Contrary to the conventional wisdom in the product differentiation literature, expected prices are lower with localized competition. The analysis questions the robustness of models of product differentiation which ignore consumer lock-in.

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1. INTRODUCTION

In this paper we model localized competition between firms under consumer lock-in or loyalty. Barring a few exceptions, the existing literature on product differentiation has by and large ignored the widely observed empirical fact that some types of consumers exhibit loyalty (or lock-in) to particular products or brands in various markets.[1] We introduce two alternative notions of localized competition in the presence of consumer lock-in. For each of these models, we derive the symmetric equilibrium prices in mixed strategies and compare them to those which arise under non-localized competition. We find that the expected prices under localized competition are lower than those under non-localized competition.

This finding is in sharp contrast to the conventional wisdom in the product differentiation literature that localized competition yields higher prices than non-localized competition. This argument has been developed in its most general form by Deneckere and Rothschild (1992) who model the demand for differentiated products starting from economic primitives: specifications of the set of possible individual preference patterns -- from which the two popular models of spatial competition on a circle [Salop (1979)] and the symmetric aggregate benefit function approach [Dixit and Stiglitz (1977)] obtain as special cases. They claim that the symmetric model is more competitive than the circle model because of the fact that in the latter model competition is localized. Our paper shows that if preferences are such that firms have a monopoly market in their locked-in or loyal customers, the localized model can actually lead to more intense price competition. This is because under localized competition any given firm is competing only with its neighbors, and hence the probability of its named price being undercut is lower than that under non-localized competition. In the absence of price-discrimination, this gives a firm an added incentive to price more aggressively to capture the price-sensitive consumers (switchers) than it would if competition over these switchers was non-localized.

Some of the relevant research concerning consumer lock-in has been on the nature of price (or, according to the marketing literature, 'promotional') competition [see, for example,

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1 Some researchers have established conditions under which rational consumers may not find it worthwhile to evaluate all products or brands available in the market. See the literature on consideration/relevant sets and switching costs, for example, Hauser and Wernerfelt (1990) and Klemperer (1987).

Shilony (1978), Varian (1980), Narasimhan (1988), Raju et al (1990) and Baye et al (1992)]. In all the aforementioned papers, consumers are segmented into two types: loyals (uninformed) and non-loyals or switchers (informed); but competition is non-localized since all firms compete for a common cohort of switchers. Our primary goal in this paper will be to set up two alternative models that localize competition over the non-loyal switchers, namely the circle model and the cluster model. We then compare the nature of price competition under these localized models with that under non-localized competition. This comparison is important for several reasons. First, loyalty and price-shopping (switching) behaviors can be rationalized as particular types of preference patterns in the context of product differentiation. Loyal consumers perceive the existing products as extremely heterogeneous and the loss in choosing a brand other than their favorite one is large; whereas price shoppers consider the brands that are in their choice-sets as homogeneous, and do not have such 'lumpy' loss functions. Given that consumers are segmented according to such perceptions regarding the existing brands, it is important to question the robustness of the standard models of product differentiation.

Second, models of localized competition capture a widely observed phenomenon that consumers often find it worthwhile to look for better deals only from neighboring stores -- stores that are located too far away may not even be relevant for their consideration. Given such segmentation of consumers, it is not apparent if firms will be more or less aggressive in competing for the price-sensitive switchers if competition were localized between neighbors. Furthermore, we are also able to show that the nature of pricing policies is markedly different under the two notions of competition. Charging a high 'regular' price and occasionally offering a significantly lower 'sale' price may be an optimal strategy under non-localized competition, but it will never be optimal under the circle model of localized competition.

A seemingly paradoxical observation that comes from comparing the (symmetric) price equilibria in the localized and non-localized models is that an increase in the number of firms may actually relax the degree of competition. Rosenthal (1980) finds similar results, but the implicit assumptions in his model seem to be questionable. Implicit in his model is the assumption that when the number of firms increase, so does the number of locked-in customers. It is as if a new firm that comes into the market brings with it a brand of loyal customers; which in turn means that the ratio of switchers to loyal consumers per firm goes down with the addition of a new firm. We are able to establish our results without having to make such stringent assumptions. In fact we are able to show that the property of expected prices increasing with the number of firms holds for a wider class of models with consumer lock-in. This is in striking contrast to the standard result in models of monopolistic competition that entry of more firms make markets more competitive. Hence, general statements on the competitiveness of markets for differentiated products will be fraught with pitfalls if important elements of consumer preferences, namely lock-in to a particular brand or product, are not explicitly accounted for.

The rest of the paper is organized as follows: Section 2 outlines the Varian model of non-localized competition and the two models of localized competition: on a circle and in clusters. Section 3 provides the characterization of the symmetric equilibria of each of the three models described in Section 2. Section 4 compares the symmetric equilibria of the two localized models to that which would arise out of the model of non-localized competition. Finally, Section 5 provides some concluding remarks.

2. THE MODELS

The Varian Model:

First we describe the model of non-localized competition due to Varian (1980). Consider a market where $k \geq 2$ firms produce one branded product each; and where the number of consumers is fixed. Each consumer purchases one unit of the product if faced with a price less than or equal to a reservation value r, and none if faced with any price greater than r. There are two types of consumers: loyal customers and switchers -- the loyal customers are essentially locked-in to the firm they patronize, they buy only from their patron firm as long as the price is less than or equal to r. Customers loyal to firm i do not buy from any other firm $j \neq i$. Switchers, on the other hand, buy only from the stores that charge the lowest price.

All firms are assumed to have constant and identical marginal cost equal to zero. They are risk neutral, act as expected profit maximizers and behave non-cooperatively. Furthermore, firms cannot identify the types of the customers that visit their stores and hence cannot price-discriminate.

The model is symmetric in the sense that every firm has n loyal or locked-in customers and they compete for a common pool of M switchers. More importantly, competition over the switchers is non-localized since every firm can potentially sell to all the M switchers [see Figure 1(a)].

[Insert Figure 1 about here]

The Circle Model:

Consider a variant of the model presented above but with localized competition, where k identical firms (each with n loyal customers) are located along the circumference of a circle. We will call any two adjacent firms, neighboring firms. In order to maintain symmetry, we assume that switchers are distributed along the circumference of the circle such that there are m switchers between every two neighboring firms. [see Figure 2][2]. But unlike in the non-localized model, switchers buy only from the *neighboring* store that charges the lowest price.

[Insert Figure 2 about here]

We can interpret the two types of consumers as follows: the loyal consumers view all the products as extremely heterogeneous, so much so that the loss in utility from choosing any brand other than their most preferred brand is greater than r. On the other hand, the switching consumers consider the local products (i.e. the products that the neighboring firms have to offer) as homogeneous, but the other brands as being too far down in their preference ordering to make them worthwhile to purchase. In this sense inter-firm competition is localized in that firms compete for the switchers with their neighbors only -- but they cannot price discriminate between their locked-in customers and the switchers.

The Cluster Model:

The second notion of localized competition arises from splitting the set of firms in the Varian model into insulated clusters so that any firm competes only with firms within its

2 Hence we have nk locked-in customers and mk switchers for k>2.

cluster but not with firms from other clusters. The representation of this cluster model will be similar to the Varian model, except that the total number of switchers within a cluster will be M/s, where $s \in \{1,2,..., k/2\}$ is the number of clusters. Figure 1(b) gives a schematic representation of the Varian model and the cluster model for k=8 and s=2.

In the analyses that follow, firms are assumed to set prices simultaneously in all the models described.

3. THE SYMMETRIC EQUILIBRIUM

The Varian Model:

Since this game has already received extensive treatment in the literature, we shall briefly state the results. It can be shown that this game does not have any pure strategy equilibrium, but mixed strategy equilibria exist, and there is a unique symmetric equilibrium (see Baye, et al (1992)). The lower bound of the support of the symmetric equilibrium distribution of prices is given by:

$$\hat{p} = \frac{rn}{n+M} \tag{1}$$

Defining G(p) to be the c.d.f. associated with this distribution, any firm i's expected profit from charging any price $p \in [\hat{p},r]$ is:

$$E(\Pi_i)=(1-G)^{k-1}p(n+M)+[1-(1-G)^{k-1}]pn \tag{2}$$

In equilibrium all firms earn rn, i.e. $E(\Pi_i)=rn$. Hence, equating (2) to rn and solving for G(p) we obtain the k-firm symmetric equilibrium distribution in the Varian model of non-localized competition as[3]:

3 Baye, et al (1992) show that in addition to the symmetric equilibrium, there exists a continuum of asymmetric equilibria in which at least two firms randomize over $[\hat{p},r]$, with each other firm i randomizing over $[\hat{p},x_i]$, $x_i<r$, and having a mass-point at r.

$$G(p) = \begin{cases} 0 & \text{if} \quad p<\hat{p} \\ 1-\left[\frac{n}{M}\left(\frac{r}{p}-1\right)\right]^{\frac{1}{k-1}} & \text{if} \quad \hat{p}\leq p\leq r \\ 1 & \text{if} \quad p>r \end{cases} \tag{3}$$

<u>The Circle Model</u>:

Consider any firm i among the k identical firms. Let the immediate neighbors of firm i be indexed by i-1 and i+1.

Define $p^- = \min\{p_{i-1}, p_{i+1}\}$ and $p^+ = \max\{p_{i-1}, p_{i+1}\}$

For all p_i, p^-, $p^+ \leq r$, we may represent firm i's profit function as follows:

$$\Pi_i(p_i,p_{i-1},p_{i+1}) = \begin{cases} L = p_i n & \text{if} \quad p^-\leq p^+<p_i \\ T_1 = p_i(n+\frac{1}{2}m) & \text{if} \quad p^-<p^+=p_i \\ M = p_i(n+m) & \text{if} \quad p^-<p_i<p^+ \ \underline{\text{or}} \ \text{if} \ p^-=p_i=p^+ \\ T_2 = p_i(n+\frac{3}{2}m) & \text{if} \quad p_i=p^-<p^+ \\ W = p_i(n+2m) & \text{if} \quad p_i<p^-<p^+ \end{cases} \tag{4}$$

Thus if a firm sets a price which is lower than both its neighbors' prices, it sells to all the 2m switchers that consider its product in addition to its n locked-in customers. If its price is higher than one of its neighbor's but lower than the other, it sells to only m switchers. If its price is higher than both its neighbors', then it only sells to its n loyals. In case of a tie, we assume the firms share the switchers equally.[4] The payoff function in (4) is drawn in Figure 3.

[Insert Figure 3 about here]

In this normal form game firm i's strategy set is $p_i \in [0,\infty]$ and its payoff function is $\Pi_i(p_i,p_{i-1},p_{i+1})$, i=1,2,...,k. Define

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4 Our results are not dependent upon the particular sharing rule in case of a tie.

$$\underline{p} = \frac{rn}{n+2m} \tag{5}$$

i.e. $\underline{p}$ is that price at which any firm, by selling to all the 2m switchers (m on each side) and its locked-in customers makes the same profit as it would make by simply selling to its locked-in customers at the monopoly price r. See Figure 3.

We know, by the existence conditions set forth in Dasgupta and Maskin (1986), that mixed strategy equilibria exist in our game of localized competition.

Proposition 1: There exists no pure strategy equilibrium in the circle model of localized price competition.

Proof: Suppose the two neighbors of any firm i are charging prices p^{i-1} and p^{i+1}, such that $\underline{p} \leq p^{i-1} < p^{i+1} \leq r$. Then firm i's best response is to undercut firm i-1 if $\Pi_i(p^{i-1}-\varepsilon) \geq \Pi_i(p^{i+1}-\varepsilon)$ OR undercut firm i+1 if $\Pi_i(p^{i-1}-\varepsilon) \geq \Pi_i(p^{i+1}-\varepsilon)$. But these cannot be equilibrium prices since in the former case firm i-1, and in the latter case firm i+1 will be better off by revising their prices slightly below firm i's price. In the case $p^{i-1}=p^{i+1}$, firm i's best response is again to set a price slightly below its neighbors, thus contradicting an equilibrium. All firms charging the monopoly price (r) cannot be an equilibrium since any deviant is better off by undercutting. Neither can $\underline{p}$ be an equilibrium since a deviant firm is strictly better off by raising its price to r and selling only to its loyals. In fact any price below $\underline{p}$ is strictly dominated by setting r. Q.E.D.

In what follows, let us define $\underline{s_i}=\sup\{p:F_i(p)=0\}$ and $\overline{s_i}=\inf\{p:F_i(p)=1\}$, where $F_i(\cdot)$ is firm i's symmetric equilibrium cumulative distribution function. We can write down firm i's expected profit in a symmetric equilibrium as follows (dropping the arguments of $F(\cdot)$).

$$\begin{aligned} E(\Pi_i) &= pnF^2 + 2p(n+m)F(1-F) + p(n+2m)(1-F)^2 \\ &= p\{n + 2m[1-F]\} \end{aligned} \tag{6}$$

Because we are restricting the analysis to symmetric equilibria, we drop the subscript i from the analyses that follow.

Theorem 1: The circle model of localized competition with identical firms has a unique symmetric equilibrium in mixed strategies. In this equilibrium all firms randomize continuously over a common support, $[\underline{p},r]$ using the following distribution function:

$$F(p) = \begin{cases} 0 & \text{if} \quad p<\underline{p} \\ 1-\dfrac{n}{2m}\left(\dfrac{r}{p}-1\right) & \text{if} \quad \underline{p}\leq p\leq r \\ 1 & \text{if} \quad p>r \end{cases} \tag{7}$$

To prove the theorem we utilize the following lemma.

Lemma: $\overline{s_i} = r$ and $\underline{s}_i = \underline{p}$ $\forall$ i=1,2,...,k; and the expected equilibrium profit, $\Pi_i^* = rn$ $\forall$ i=1,2,...,k.

Proof: We prove the lemma by proceeding along the following logical claims:

(a) $\overline{s}\leq r$ and $\underline{s} \geq \underline{p}$.

(b) There are no mass-points on the interval $[\underline{p},r]$.

(c) There are no gaps in the symmetric equilibrium distribution function in the interval $[\underline{p},r]$.

(d) $\overline{s} = r$

(e) In the symmetric equilibrium, $\Pi_i^* = \Pi_j^* = rn$ $\forall$ i,j.

(f) $\underline{s} = \underline{p}$.

By setting $p_i = r$, a firm can earn rn with certainty, whereas any price greater than r will give the firm zero profit. Hence no firm will price above r implying $\overline{s_i}\leq r$ $\forall$ i=1,2,...,k. On the other hand when a firm sets any $p_i < \underline{p}$, it earns a profit which is less than $\underline{p}(n+2m)=rn$. Clearly the firm is strictly better off by charging r. Hence no firm will price below $\underline{p}$ implying $\underline{s}_i \geq \underline{p}$ $\forall$ i=1,2,...,k. This establishes claim (a).

To show that claim (b) holds we first consider the open interval $(\underline{p},r)$, and deal with the boundary cases of $p=\underline{p}$ and p=r separately. Contrary to the claim suppose one of the firms, say firm i, charges a price $p_i \in (\underline{p},r)$ with probability α. Then for any neighboring firm i+1, we can write down the profits at prices p_i-ε and p_i+ε as follows:

$$\Pi_{i+1}(p_i-\varepsilon) = (p_i-\varepsilon)\ [n+m[2 - F_i(p_i-\varepsilon) - F_{i+2}(p_i-\varepsilon)]] \tag{8}$$

and

$$\Pi_{i+1}(p_i+\varepsilon) = (p_i+\varepsilon)\ \{n + m[2 - F_i(p_i+\varepsilon) - F_{i+2}(p_i+\varepsilon)]\} \tag{9}$$

Subtracting (9) from (8) and taking limits we find:

$\lim_{\varepsilon\to 0} [\Pi_{i+1}(p_i-\varepsilon) - \Pi_{i+1}(p_i+\varepsilon)] = mp_i\alpha > 0$; which implies that any of firm i's neighbors will increase their profits by shifting mass from an ε-neighborhood above p_i to and ε-neighborhood below p_i. But this is a contradiction to the equilibrium.

On the other hand if firm i places probability mass α at $\underline{p}$, then $\Pi_{i+1}(\underline{p}) = \underline{p}(n + m(2-\alpha/2))$ and $\Pi_{i+1}(\underline{p}+\varepsilon) = (\underline{p}+\varepsilon)(n+2m-\alpha m)$, => $\lim_{\varepsilon\to 0} [\Pi_{i+1}(\underline{p}+\varepsilon) - \Pi_{i+1}(\underline{p})] = -\frac{m\underline{p}\alpha}{2} < 0$, which means that both of firm i's neighbors will shift mass from an ε-neighborhood above $\underline{p}$ to r. But it cannot be an equilibrium strategy for firm i to maintain a point mass at $\underline{p}$, given that both its neighbors do not have any density in the ε-neighborhood above $\underline{p}$.

Finally if firm i places a point-mass α at r then

$\Pi_{i+1}(r) = rn$ and $\Pi_{i+1}(r - \varepsilon)\ (n + \alpha m)$ => $\lim_{\varepsilon\to 0} [\Pi_{i+1}(r-\varepsilon) - \Pi_{i+1}(r)] = mr\alpha > 0$, which is again a contradiction.

In order to prove the claim in (c), suppose, contrary to the claim, there exists an interval $[p_l,p_h]$, such that $\underline{p}\le p_l<p_h\le r$, over which no firm randomizes, i.e. $F(p_l)=F(p_h)$. Then, using (6), we get

$\Pi_i(p_l) = p_l(n + 2m[1-F(p_l)]) < p_h(n + 2m[1-F(p_h)]) = \Pi_i(p_h)$, which is a contradiction.

Suppose claim (d) is not true and let $s = \bar{s} < r$. Then $\Pi_i(s)=s\{n+2m[1-F(s)]\}=sn$, since $F(s) = 1$. But any firm can simply sell to their locked-in customers at r and earn a profit of $rn > sn$.

The claim in (e) follows directly from the ones in (b), (c) and (d).

Finally, claim (f) is established by noting that any firm i earns the same expected profit over the entire interval of its support. Therefore by claim (e),

$$\Pi_i(\underline{s}_i) = rn \Rightarrow \underline{s}_i(n+2m) = rn \Rightarrow \underline{s}_i = \frac{rn}{n+2m} = \underline{p}. \qquad \text{Q.E.D.}$$

In order to derive the symmetric equilibrium distribution of the circle model of localized competition, we can use the expression for $E(\Pi)$ in (6) for firm i+1 and claim (e) as follows:

$E(\Pi_{i+1}) = \Pi_i^* = p\{n + 2m[1-F(p)]\} = rn.$

Solving for F(p) we obtain the symmetric equilibrium in mixed strategies as given in (7).[5]

The Cluster Model:

As noted in Section 2, the cluster model localized competition yields behavior within each cluster that is the same as the Varian model. Therefore we can easily arrive at the symmetric equilibrium c.d.f. of a local cluster by following the same steps as in deriving G(p) above. The symmetric equilibrium c.d.f. of the cluster model is:

$$H(p) = \begin{cases} 0 & \text{if} \quad p<\tilde{p} \\ 1-\left[\frac{ns}{M}\left(\frac{r}{p}-1\right)\right]^{\frac{s}{k-s}} & \text{if} \quad \tilde{p}\le p\le r \\ 1 & \text{if} \quad p>r \end{cases} \tag{10}$$

where $\tilde{p} = \frac{rn}{n+M/s}$ is the lower bound of the support of H(p). Note that k/s is the number of firms within a cluster.

4. COMPARING THE SYMMETRIC EQUILIBRIA: LOCALIZED VERSUS NON-LOCALIZED COMPETITION

Now that we have constructed two models of localized competition, on a circle and in clusters, we are in a position to compare the degree of competition in each of these models with that of the Varian model of non-localized competition. However, in order to make comparisons between the degree of 'competitiveness' in the Varian model and the circle model, we need to be specific about our notion of 'competition'. In a sense, the two models may not be directly comparable since the parameters in each model are different. We propose

5 See Banerjee (1994) for an analysis of the existence of asymmetric equilibria in the circle model of localized competition.

two alternative bases of comparison and state our results.[6] There are four parameters of interest: the number of firms (k), the total number of switchers in the non-localized model (M), the number of locked-in customers per firm (n) and the total number of switchers between each two adjacent firms in the localized model (m). When we compare the two models by holding the total market size constant, we assume that M=km. But in this case the number of switchers accessible to any firm is different. On the other hand, when we keep the number of switchers accessible to any firm constant under both models, the aggregate number of switchers in the two models differ. We examine one case at a time, i.e.,

(A1) $M = 2m < km$

(A2) $M = km > 2m$

Proposition 2: Under (A1), i.e., for equal number of switchers accessible to any given firm, the symmetric equilibrium distribution of firm i under non-localized competition (G) first-order stochastically dominates that (F) under localized competition.

Proof: The terms $\frac{n}{2m}\left(\frac{r}{p}-1\right)$ in F(p) in equation (7) and $\frac{n}{M}\left(\frac{r}{p}-1\right)$ in G(p) in equation (3) are positive fractions for all $p \in [\underline{p}, r]$ and $p \in [\hat{p}, r]$ respectively. Moreover $M = 2m$ implies

$$\frac{n}{2m}\left(\frac{r}{p}-1\right) = \frac{n}{M}\left(\frac{r}{p}-1\right) \text{ and also } \hat{p} = \underline{p}.$$

But $\forall \; k > 2, \left[\frac{n}{M}\left(\frac{r}{p}-1\right)\right]^{\frac{1}{k-1}} > \frac{n}{2m}\left(\frac{r}{p}-1\right)$ which implies[7]

6 Anderson, et al (1992) also address this issue and offer a basis for comparison by an elasticity analysis of the short run equilibrium markup with respect to the number of firms. In models without any consumer lock-in, they show that prices may fall more slowly in the non-localized model as the number of firms increases. They explain this result with the observation that adding firms in the localized case means rearranging firms so that they are closer (substitutes) -- hence they tend to cut prices more. On the other hand, adding firms in the non-localized model can be viewed as adding them in the unused characteristics dimensions, which means that they retain their distances from all existing firms.

7 If $0 < a = b < 1$, then $a^{\frac{1}{k-1}} > b \; \forall \; k > 2$.

$$1-\left[\frac{n}{M}\left(\frac{r}{p}-1\right)\right]^{\frac{1}{k-1}} < 1-\frac{n}{2m}\left(\frac{r}{p}-1\right), \quad \text{i.e. } G(p) < F(p) \qquad \text{Q.E.D.}$$

Proposition 2 states that when the number of switchers that consider buying firm i's product is the same in both the models, firm i will be more aggressive in its pricing behavior if it only competes with its neighbors as opposed to competing with all firms. Since under localized competition, a firm is competing only with its neighbors, the probability of it being undercut when it names a price is lower - hence it has the leverage to be aggressive in its pricing policies to capture the switchers. On the other hand, when the firm has to compete with all the k-firms in the industry, there is a high probability that it will be undercut by any of the other k-1 firms. Since the firm cannot price-discriminate between the locked-in customers and the switchers, it pays to be more conservative in cutting prices and charge a higher price (on the average) to capture as much of the monopoly rent from the loyal customers as possible. Note that in proposition 2 the total number of switchers in the localized model (km) is higher than that (M) in the non-localized model.

Proposition 3: Under (A2), i.e., holding the size of the market constant in both models, the localized circle model will be more competitive than the non-localized Varian model if

$$\beta(.) - \beta_x(.) > n\left(\frac{km}{n}\right)^{\frac{1}{k-1}} \left[\frac{1}{2m} \ln\left(\frac{n+2m}{n}\right) - \frac{1}{n+km}\right]$$

where $\beta(.)$ is a beta-function and $\beta_x(.)$ is an incomplete beta- function, both with parameters $\left(\frac{k-2}{k-1}, \frac{k}{k-1}\right)$; and $x=\frac{n}{n+km}$.

Proof: See Appendix.

[Insert Figure 4 about here]

Proposition 3 may be explained diagramatically by noting that whenever the area T exceeds area T′ in Figure 4, the expected price under the circle model of localized competition [denoted by E(p)] will be *lower* than that under the Varian model of non-localized competition [denoted by $E^n(p)$]. A firm can sell to a *larger* pool of switchers if it is the lowest priced firm under non-localized competition (M) than if it undercuts both its neighbors under localized competition (2m). Thus the potential for larger sales (the market size effect) in the former model offers an incentive to a firm to charge a lower price than its counterpart in the localized model. However, the probability of its posted price being undercut is much higher under non-localized competition. Furthermore, in the circle model a firm can sell to *some* of the switchers (m) even if it is undercut by one of the neighbors (whereas in the Varian model a firm either sells to all or none of the switchers). Hence the higher risk involved in the competition for selling to the switching customers acts as a disincentive for a firm to charge lower prices in the non-localized model than its counterpart in the localized model (the riskiness effect). Our calculations illustrate that the latter effect dominates the market size effect, so that $E(p) < E^n(p)$ for a wide range of values for n and k.

In the appendix we show that $E(p) = \frac{rn}{2m} \ln\left(\frac{n+2m}{n}\right)$; similarly the expected price in the Varian model is: $E^n(p) = \hat{p} + \left(\frac{n}{km}\right)^{\frac{1}{k-1}} \int_{\hat{p}}^{r} \left(\frac{r}{p}-1\right)^{\frac{1}{k-1}} dp$. Unfortunately, the integrand that appears in the expression for $E^n(p)$ is a recursive one and prevents $E^n(p)$ from being expressed in a closed form. Hence the analytical condition stated in Proposition 3 does not yield a readily interpretable condition on the magnitudes of the parameters under which $E^n(p)$ will exceed $E(p)$. Beta and incomplete-beta tables can be used to verify that the condition in proposition 3 holds for a wide range of parameter values such that $n > 0$ and $k > 2$. Table 1 illustrates our own numerical approximations obtained for different values of k and n.

[Insert Table 1 about here]

Now we compare the Varian model of non-localized competition with the cluster model of localized competition described in section 2. Given the equilibrium c.d.f.s, G(p) and H(p), in (3) and (10) respectively, we immediately have the following proposition.

Proposition 4: The localized cluster model will be more competitive than the non-localized Varian model if $\left(\frac{n}{km}\right)^{\frac{1}{k-1}} I_x - \left(\frac{ns}{km}\right)^{\frac{s}{k-s}} I_y > \frac{1}{r}(\bar{p}-\hat{p})$; such that $I_x = \beta(.) - \beta_x(.)$ where $\beta(.)$ is a beta-function and $\beta_x(.)$ is an incomplete beta function both with parameters $\left(\frac{k-2}{k-1}, \frac{k}{k-1}\right)$, and $x=\frac{n}{n+km}$; $I_y = \beta'(.) - \beta'_y(.)$ where $\beta'(.)$ is a beta function and $\beta'_y(.)$ is an incomplete beta-function both with parameters $\left(\frac{k-2s}{k-s},\frac{k}{k-s}\right)$, and $y = \frac{n}{n+\frac{k}{s}m}$.

Proof: See Appendix.

[Insert figure 5 about here]

Figure 5 graphs the two equilibrium distributions G(p) and H(p). When area T exceeds area T′ the cluster model of localized competition yields a lower expected price than the Varian model of non-localized competition. Table 2 illustrates (using numerical approximation techniques) that the claim in Proposition 4 holds for a wide range of parameter values.

[Insert Table 2 about here]

Here it needs to be emphasized that the results shown in Tables 1 and 2 are the average prices posted by the firms, which are different from the average transaction prices paid by the switchers. A switcher in the Varian model will evaluate *all* the prices posted by the k firms and buy from the firm that quotes the lowest price. Hence the average transaction price paid by a switcher in the Varian model can be computed from the distribution of the lowest order-statistic among the k randomly drawn prices from G(p) in (3). On the otherhand, a switcher in the circle model evaluates only the *two* prices quoted by the neighboring stores and buys from the one with the lower price. Therefore the average price paid by the switcher in the circle model is computed using the c.d.f. of the lowest order-statistic among two firms' prices drawn from F(p) in (7). Similar argument holds for the cluster model as well. Detailed analyses are provided in the appendix.

The fact that localized competition will yield lower prices than those arising out of non-localized competition in the presence of locked-in consumers is found to be invariant to model specification. This contrasts with the Deneckere and Rothschild (1992) observation that localized competition yields higher prices as compared to those under non-localized competition. Our results cast doubt on the ability of either the standard spatial models on product differentiation (a la Salop (1979)) or the Deneckere and Rothschild (preference generated) model to explain market outcomes when there is "lumpiness", in the distribution of consumer types.

As shown in the appendix, the expected price (average price posted by the firms) for the circle model of localized competition is: $E(p) = \frac{rn}{2m} \ln\left(\frac{r}{p}\right)$. If we interpret an increase in the number of firms as a decrease in the number of switchers per firm (recall that m=M/k) with no change in the number of locked-in customers, then it can be shown that $\frac{\partial E(p)}{\partial m} < 0$.

In all the models with consumer lock-in that we have discussed, it seems to be a pervasive phenomenon that the expected prices tend to rise as the number of firms increase, ceteris paribus[8] (see Tables 1 and 2). This is in contrast to the standard result in models of monopolistic competition that entry of more firms makes the market more competitive.

[Insert Figure 6 about here]

The next proposition describes the difference in the pattern of optimal pricing policies that would emerge out of the Varian model and the circle model.

Proposition 5: The density function of equilibrium prices in the circle model of localized competition is uni-modal, whereas Varian showed that the density function in his model of non-localized competition is bi-modal.

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8 See Rosenthal (1980) for more related to this issue.

Proof: Given the equilibrium distribution functions as in (3), we can calculate the density function of equilibrium prices in the Varian model as follows:

$$g(p) = G'(p) = \frac{A}{\left(\frac{r}{p}-1\right)^{1-\frac{1}{k-1}}} \tag{11}$$

where $A = \frac{r}{p^2(k-1)}\left(\frac{n}{km}\right)^{\frac{1}{k-1}}$. See Varian (1980) for details. Similarly, differentiating the c.d.f. given in (7), we obtain the density function of equilibrium prices in the circle model as:

$$f(p) = F'(p) = \frac{rn}{2mp^2} \tag{12}$$

The density function in (12) is monotonically declining (f'(p) < 0) along the support of the distribution of prices. Figure 6 illustrates these densities. Q.E.D.

Note that firms tend to charge the extreme prices more frequently under non-localized competition. The intuition is that if a firm decides to go after the switchers, it will have to cut prices substantially in order to undercut all the other (k-1) firms. Therefore it does not make much sense in charging any intermediate prices too often. On the other hand, under localized competition on a circle firms charge similar (low) prices with high probability and seldom indulge in any discounting. In this scenario, a firm not only competes exclusively with its two neighbors, it also can sell to half the switchers (that it has access to) by undercutting just one of its neighbors. Therefore, in equilibrium, no firm has any incentive to cut prices substantially. Thus mutually overlapping local markets generate different structural implications for optimal pricing policies than their non-localized counterparts. Frequently offering products at large discounts, which is an optimal strategy in non-localized competition, is not optimal under localized competition on the circle.[9]

9 The equilibrium price density function in the cluster model, however, has a similar shape as in the Varian model.

5. CONCLUSION

This paper presents two models of localized competition (on a circle and in clusters) between firms under consumer lock-in and derives the symmetric equilibrium prices in each of these models. Although the circle model of localized competition restricts the number of neighbors to two, it can readily be generalized to accommodate higher dimensional localization as well. For instance a similar model with 3 neighbors necessitates a sphere with interlaced hexagons.

Comparison of the symmetric equilibrium prices of the circle model to those which arise in Varian's model of non-localized competition shows, contrary to conventional wisdom, that average prices are *lower* with localized competition. Under the circle model, any given firm is competing only with its two neighbors, hence the probability of it being undercut when it names a low price is lower. Moreover, the firm can sell to half the accessible switching segment even if it is undercut by one of its two neighbors. Hence it has the leverage to be aggressive in its pricing policies to capture the price sensitive consumers. On the other hand, when the firm has to compete with all the k-firms in the industry (as in the Varian model), there is a high probability that it will be undercut by at least one of the other k-1 firms and therefore, not sell to any of the switching customers. Since the firm cannot price-discriminate between the locked-in customers (which constitute a monopoly market for the firm) and the switchers, it pays to be more conservative in cutting prices and charge a higher price (on the average) to capture as much of the monopoly rent from the loyal customers as possible.

In the case when the disadvantage of the high risk of competing with more than just the two neighbors is *partially* compensated by the attractiveness of a larger pool of switchers in the non-localized model, we derive an analytical condition on the parameters for which localized competition will yield lower expected prices than non-localized competition. Numerical estimation results show that the localized model is more competitive than the non-localized model for a wide range of parameter values. We compare the model of localized competition in clusters with the Varian model on the basis that the number of switchers *per firm* is equal in both models. Even in this case the cluster model of localized competition emerges as more competitive for a wide range of parameter values.

Furthermore, models of localized competition with overlapping markets, such as the circle model, has important implications for the pricing policies of firms. We find that the density of prices is uni-modal under localized competition, i.e. firms in general charge a (low) price with higher frequency and occasionally increase prices. Non-localized competition with sufficiently large number of firms, on the other hand, gives rise to a bimodal equilibrium price density which indicates that firms frequently charge high prices and low prices, but hardly ever charge intermediate prices.

This paper establishes that whenever firms have monopoly power over their loyal customer segment but cannot distinguish them from the non-loyal customers, the pricing policies are likely to be markedly different from those that come out of a standard spatial model of product differentiation. Hence it is important to model demand systems that accommodate possibilities of consumer lock-in before attempting generalized statements on competitiveness of markets.

APPENDIX

Proof of Proposition 3:

A standard result in probability theory states that for an arbitrary non-negative random variable X,

$$E(X)=\int_0^{\infty} [1-F(x)]\, dx$$

Therefore, denoting the expected prices in the non-localized Varian model by $E^n(p)$, we have

$$E^n(p) = \int_0^{\infty}[1-G(p)]dp = \int_0^{\hat{p}}[1-G(p)]dp + \int_{\hat{p}}^{r}[1-G(p)]dp$$
$$= \hat{p} + \int_{\hat{p}}^{r} [1-G(p)]dp = r - \int_{\hat{p}}^{r} G(p)dp$$

Using the symmetric equilibrium distribution G(p) given in (3) we can represent $E^n(p)$ in terms of the parameters as follows:

$$E^n(p)=\hat{p} + \left(\frac{n}{km}\right)^{\frac{1}{k-1}} \int_{\hat{p}}^{r}\left(\frac{r}{p}-1\right)^{\frac{1}{k-1}}dp$$

Using a variable transformation (p/r=z), we can express the definite integral in the expression above as:

$$\int_{\hat{p}}^{r}\left(\frac{r}{p}-1\right)^{\frac{1}{k-1}}dp = r\left[\int_0^1 z^{\frac{-1}{k-1}}(1-z)^{\frac{1}{k-1}}dz - \int_0^x z^{\frac{-1}{k-1}}(1-z)^{\frac{1}{k-1}}dz\right]$$

where $x = \frac{\hat{p}}{r} = \frac{n}{n+km}$. The first term within the square brackets is a beta-function $[\beta(.)]$ and the second term is an incomplete beta-function $[\beta_x(.)]$, both with parameters $\left(\frac{k-2}{k-1},\frac{k}{k-1}\right)$.

Therefore, the expected price in the non-localized Varian model is:

$$E^n(p) = \hat{p}+r\left(\frac{n}{km}\right)^{\frac{1}{k-1}} \{\beta(.)-\beta_x(.)\} \qquad \text{(B1)}$$

Using similar arguments for the localized circle model we can show that the expected price [denoted by E(p)] is:

$$E(p)=\underline{p}+\frac{n}{2m}\int_{\underline{p}}^{r}\left(\frac{r}{p}-1\right)dp$$

which on simplification yields

$$E(p) = \frac{rn}{2m}\ln\left(\frac{r}{\underline{p}}\right) \tag{B2}$$

Therefore, using (B1) and (B2), $E^n(p) > E(p)$ as:

$$\hat{p}+r\left(\frac{n}{km}\right)^{\frac{1}{k-1}}\{\beta(.)-\beta_x(.)\} > \frac{rn}{2m}\ln\left(\frac{r}{\underline{p}}\right)$$

Replacing $\hat{p}$ and $\underline{p}$ by $\frac{rn}{n+km}$ and $\frac{rn}{n+2m}$ respectively, and rearranging, we have the claim in the proposition. Q.E.D.

Proof of Proposition 4:

Since the structure of the cluster model is the same as the Varian model, Proposition 3 provides the expected price in the cluster model, $E^c(p)$, with s clusters and k/s=t firms within each cluster, as:

$$E^c(p) = \tilde{p}+r\left(\frac{n}{tm}\right)^{\frac{1}{t-1}}\{\beta'(.)-\beta'_y(.)\} \tag{B3}$$

where $\beta'(.)$ is a beta-function and $\beta'_y(.)$ is an incomplete beta-function, both with parameters $\left(\frac{k-2s}{k-s},\frac{k}{k-s}\right)$ and $y = \frac{\tilde{p}}{r} = \frac{n}{n+(k/s)m}$. Using (B1) and (B3) and rearranging proves the claim. Q.E.D.

Average Transaction Prices Paid by the Switchers:

The average price that the switchers pay will obviously be different from the average posted price derived in (B1) and (B2). Let $E_s(p)$ and $E^n{}_s(p)$ denote the average price paid by a switcher in the circle model and the Varian model respectively. In order to find $E_s(p)$ and $E^n_s(p)$, we need the distribution of the lowest order-statistic drawn from the c.d.f.s given in (7) and (3) respectively. In order to proceed further, we utilize the following theorem in statistical theory:

Let $Y_1 \leq Y_2 \leq \ldots \leq Y_n$ represent the other statistics from a c.d.f. F(.). The marginal c.d.f. of Y_α, $\alpha=1,2, \ldots, n$, is given by

$$F_{Y_\alpha}(y) = \sum_{j=\alpha}^{n} \binom{n}{j} [F(y)]^j \, [1-F(y)]^{n-j}. \text{ Therefore}$$

$$F_{Y_1}(y) = 1 - [1-F(y)]^n \tag{B4}$$

In the circle model of localized competition, only the two prices of the neighboring firms are relevant for any arbitrary switcher -- and (s)he buys from the firm that has a lower price. Therefore the marginal c.d.f. for the switchers will be the c.d.f. of the lowest order statistic among the two neighboring firms' prices drawn from the distribution given in (7). If P_1 and P_2 are two random prices drawn from the distributions F(.) of the neighboring firms, such that $P_1 \leq P_2$, then:

$$F_{P_1}(p) = 1 - [1-F(p)]^2 \Rightarrow$$

$$F_{P_1}(p) = 1 - \left[\frac{n}{2m}\left(\frac{r}{p}-1\right)\right]^2 \text{ for } p \in [\underline{p}, r] \tag{B5}$$

Hence the average price that switchers pay in the circle model is given by

$$E_s(p) = \underline{p} + \int_{\underline{p}}^{r} \left[1-F_{P_1}(p)\right] dp = r - \int_{\underline{p}}^{r} \left[F_{P_1}(p)\right] dp \tag{B6}$$

It is easy to see that $E(p) > E_s(p)$.

In contrast to the circle model, a switcher in the Varian model will evaluate all the k prices quoted by the k firms and buy from the firm that quotes the lowest price. Hence, in the non-localized competition model, we need to find the distribution of the lowest order-statistic among the k randomly drawn prices from G(p). Following the same notational conventions as in the derivation of (B5), the distribution of the lowest price in the non-localized model is:

$$G_{P_1}(p) = 1 - [1-G(p)]^k \Rightarrow$$

$$G_{P_1}(p) = 1 - \left[\frac{n}{M}\left(\frac{r}{p}-1\right)\right]^{\frac{k}{k-1}} \text{ for } p\in[\hat{p},r] \tag{B7}$$

Hence the average price that switchers pay in the non-localized model is given by:

$$E^n_s(p) = \hat{p} + \int_{\hat{p}}^{r}\left[1-G_{P_1}(p)\right]dp = r - \int_{\hat{p}}^{r}\left[G_{P_1}(p)\right]dp \tag{B8}$$

Again it is easy to show that $E^n(p) > E^n_s(p)$.

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Table 1: (a) Expected prices in the Varian model of non-localized competition, $E^n(p)$; (b) Expected prices in the circle model of localized competition, E(p).
[Calculated for r=6, M=100]

(a)

n \ k	3	4	5	6	7	8	9	10
1	0.874	1.528	2.085	2.537	2.904	3.205	3.454	3.664
10	2.400	3.080	3.558	3.906	4.169	4.374	4.538	4.673
50	4.054	4.491	4.771	4.965	5.105	5.213	5.297	5.366
100	4.714	5.017	5.206	5.334	5.427	5.497	5.552	5.596
500	5.643	5.732	5.785	5.821	5.847	5.866	5.880	5.893

(b)

n \ k	3	4	5	6	7	8	9	10
1	0.379	0.472	0.557	0.637	0.711	0.782	0.849	0.913
10	1.833	2.150	2.414	2.639	2.835	3.006	3.159	3.296
50	3.813	4.159	4.408	4.597	4.746	4.866	4.964	5.047
100	4.597	4.866	5.047	5.178	5.278	5.355	5.418	5.470
500	5.632	5.719	5.772	5.808	5.835	5.855	5.870	5.883

Table 2: **Expected prices in the Varian model of non-localized competition, $E^n(p)$ and expected prices in the cluster model of localized competition, $E^c(p)$. [Calculated for r=6, M=100, k=20]**

	s	1 $E^n(p)$	2	4	5	10
n						
1		4.728	3.952	2.903	2.528	1.438
10		5.310	4.987	4.613	4.492	4.159
50		5.678	5.596	5.529	5.512	5.470
100		5.798	5.764	5.741	5.732	5.719
500		5.947	5.945	5.943	5.942	5.941

Notice that the column under s=1 corresponds to the prices under the Varian model of non-localized competition, $E^n(p)$.

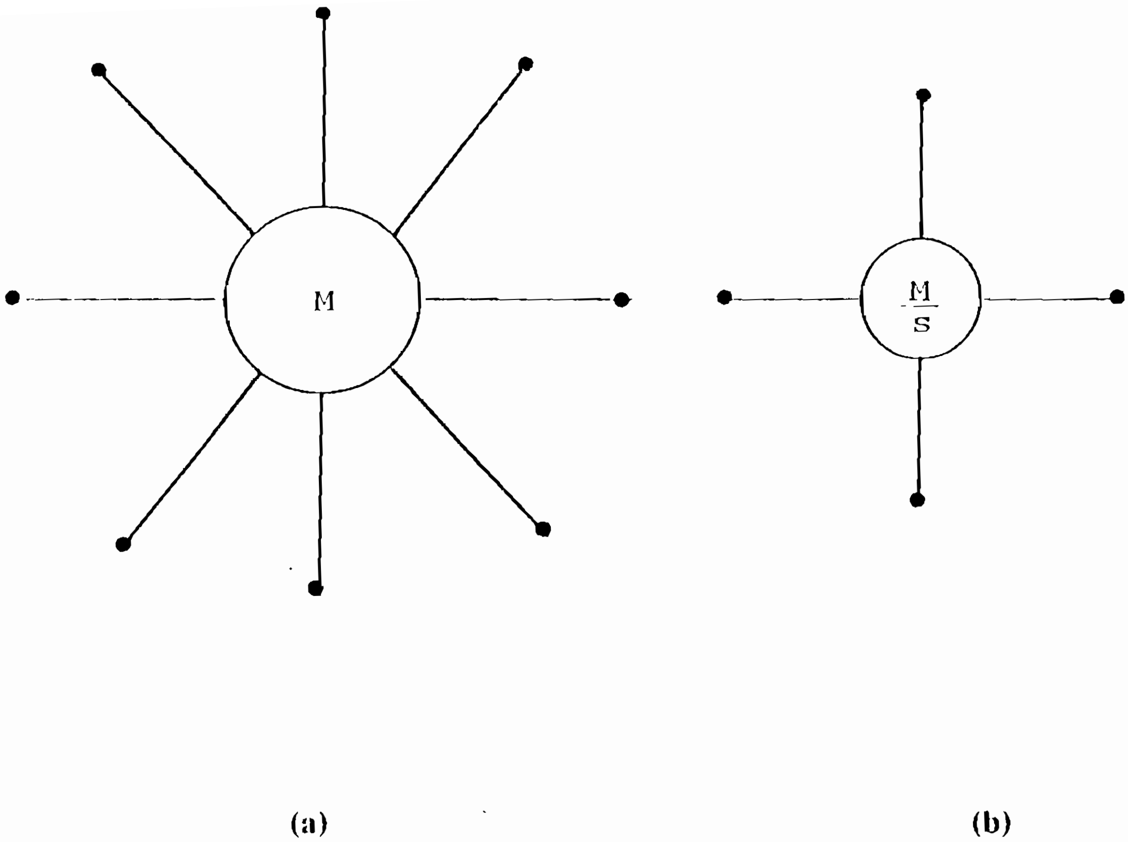

Figure 1: (a) The Varian model of non-localized competition and (b) the cluster model of localized competition.
[Drawn for k=8 and s=2]

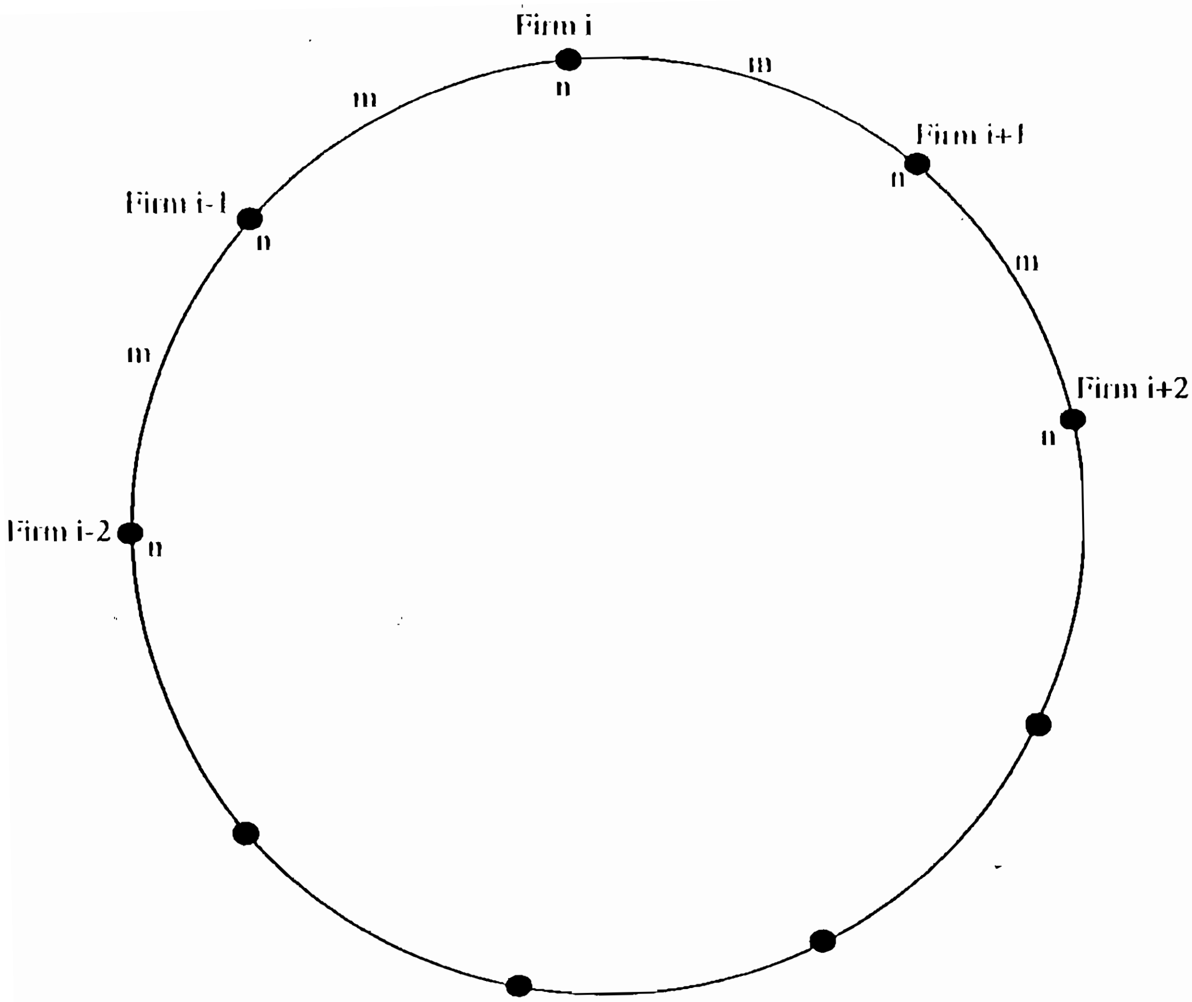

Figure 2: **The circle model of localized competition with n locked-in consumers per firm and m switchers between every two adjacent firms.**

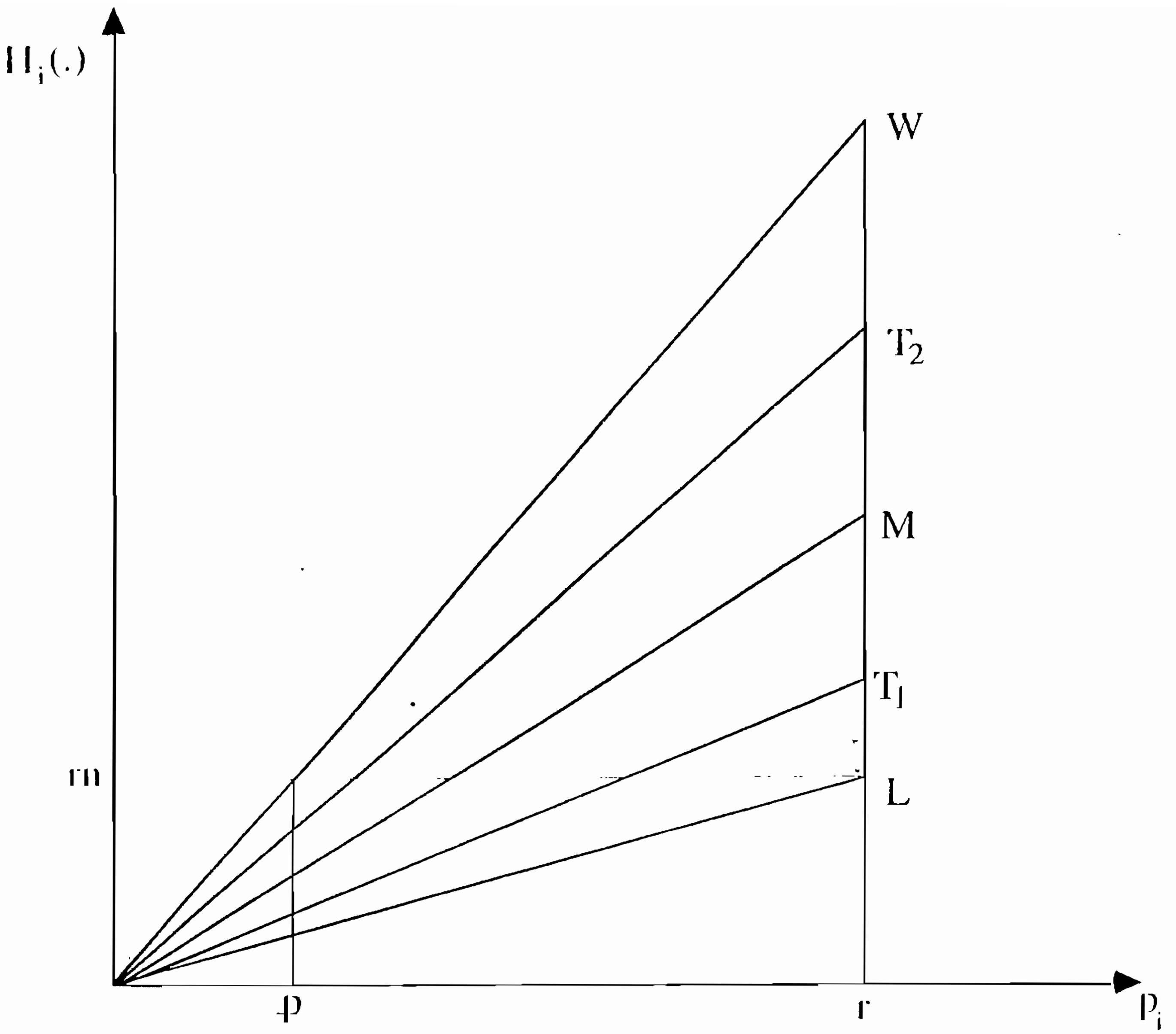

Figure 3: The profit functions of firm i in the circle model of localized competition.

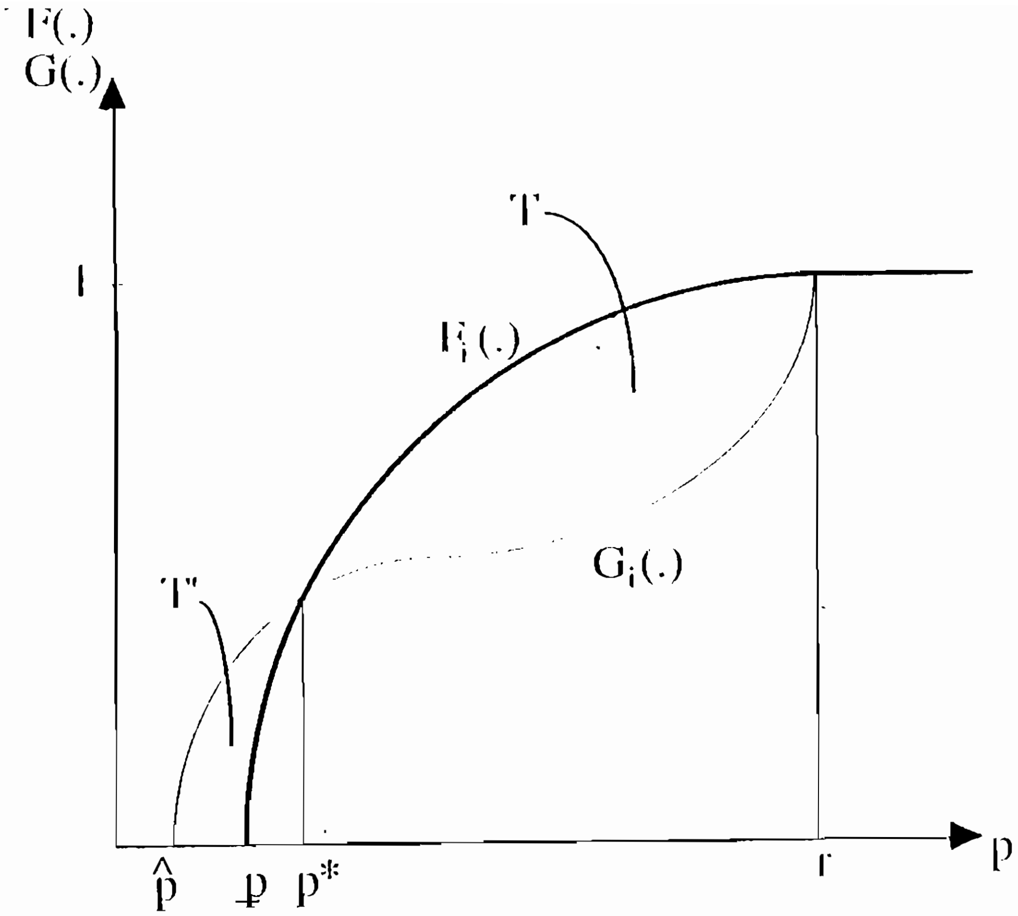

Figure 4: **Equilibrium distribution functions under A2 (i.e. when M = km < 2m).**

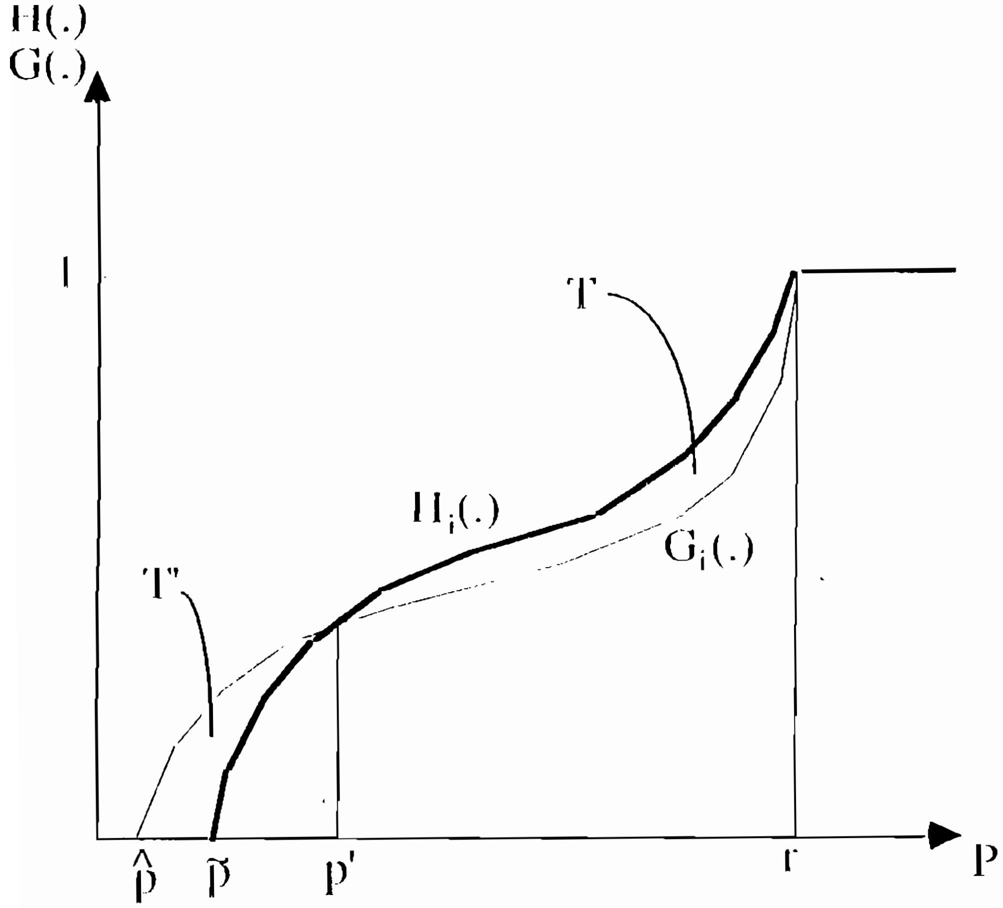

Figure 5: Equilibrium distributions in the Varian model and the cluster model.

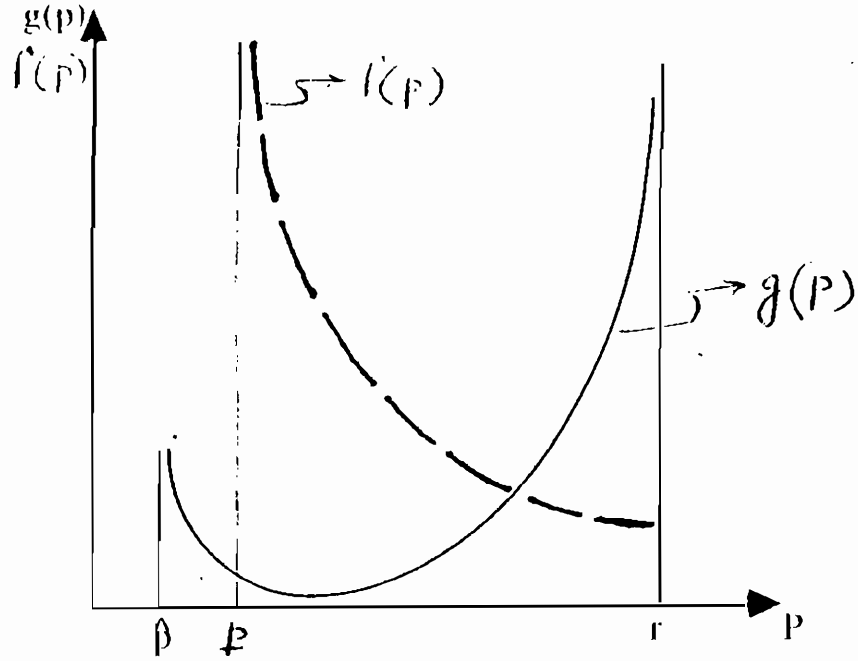

Figure 6: Equilibrium density functions in the Varian model and the circle model.