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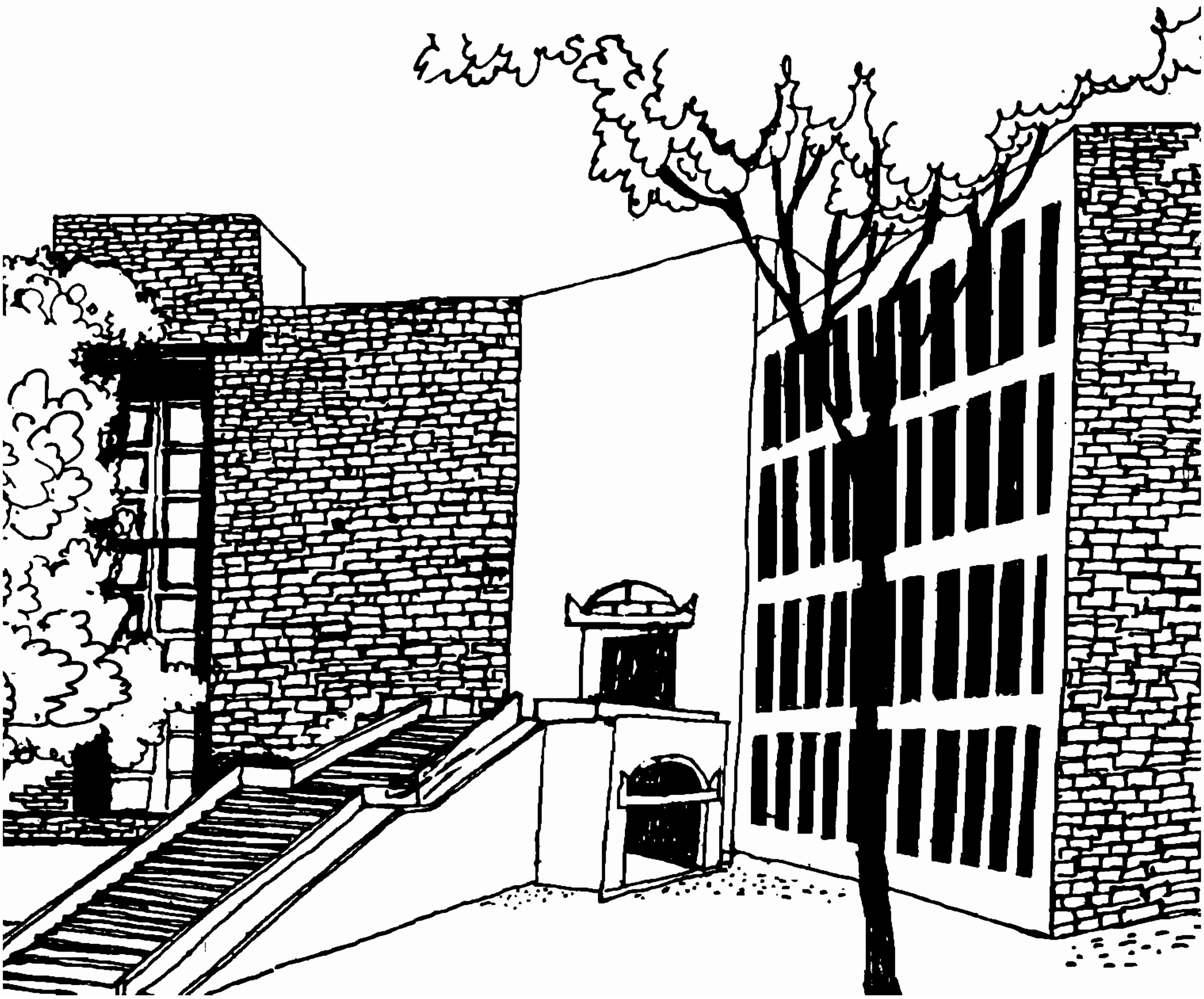

REVEALED PREFERENCE UNDER RATIONING

By

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W P No. 1111
June 1993

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Abstract

In this paper we extend the weak and strong axioms of revealed preference to markets with rationing and establish that if the observed demand behavior in such markets satisfy the strong axiom of revealed preference, then it is representable by a utility function.

1. Introduction :- In Polterovich (1993) and Lahiri (1993) can be found a theory of rationing or dual pricing which goes as follows: there is a market for commodities with fix-prices and quantity constraints and a market for the same commodities with flexible prices and without any quantity constraints. Consumer's choose their consumption bundles subject to their budget constraints, while adhering to the rules prevailing in each market. At an equilibrium, all markets clear.

Clearly consumer choice in such markets is determined by the quantity constraints on the rationed market, the fixed prices prevailing on the rationed market and the other price vector prevailing on the flexible price market. It is easily seen that this consumer choice theory is a generalization of the consumer choice theory with flexible prices on the one hand and the consumer choice theory with fixed prices and quantity constraints as in Dreze (1975) on the other. The theory of consumer choice in such situations is the subject of study in Neary and Roberts (1980), Howard (1977).

For flexible price markets there is the theory of revealed preference in the tradition of Samuelson (1938,1950), Houthakker (1950), Richter (1966), Hurwicz and Richter (1966,1971) and Sondermann (1982). The theory focuses on a set of sufficient conditions which guarantee the existence of a utility maximizing consumer generating observed demand behavior.

In this paper we will in line with the work of Sondermann (1982), extend the conventional analysis to situations discussed in Polterovich (1993) and Lahiri (1993).

2. The Model :- Let $\mathbb{R}^n$ denote n-dimensional Euclidean space. Let X be a non-empty subset of $\mathbb{R}^n$, denoting a fixed consumption set. We assume that X is of the form $X_1 + X_2$ where X_i is a non-empty subset of $\mathbb{R}^n$ for i=1,2. $\mathcal{B}$ will be a family of non-empty budgets $B(p,\psi,L,m)=\{x \in X: \exists x_1 \in X_1, x_2 \in X_2, x_1 \leq L, p.x_1 + \psi.x_2 \leq m\}$ defined on some (arbitrary) subset $P_1 \times P_2 \times Q \times M \subset \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}$. Here p denotes the price vector on the fixed price market, ψ denotes the quantity constraints on the rationed (fixed price) market, L denotes the quantity constraints on the rationed (fixed price) market and m is income. Let h be a demand correspondence on $\mathcal{B}$ i.e. $h: \mathcal{B} \rightarrow\rightarrow \mathbb{R}^n$ such that $\forall (p, \mathcal{q}, L, m) \in P_1 \times P_2 \times Q \times M$, $h(p, \mathcal{q}, L, m) \subseteq B(p, \mathcal{q}, L, m)$. The range of h, is the set $\mathcal{R}(h) = \bigcup_{B \in \mathcal{B}} h(B)$. We say that h is representable if there exists a real-valued utility function u on X such that, for all $B \in \mathcal{B}$, $h(B) = \{x \in B : u(x) \geq u(y) \forall y \in B\}$.

If for some budget $B \in \mathcal{B}$, x is chosen, although a different consumption plan y could have been chosen [i.e. $x \in h(B)$ and $x \neq y \in B$], then we say that x is revealed preferred to y and write xSy. This defines a binary relation S on X which depends on h.

We postulate the following two axioms :

Weak Axiom of Revealed Preference : S is asymmetric; that is, xSy implies not ySx.

Strong Axiom of Revealed Preference : S is acyclic; that is, $x^1 S x^2 S \ldots S x^n$ implies not $x^n S x^1$.

Let H denote the transitive hull of S; that is, xHy if $x S u_1 S \ldots S u_n S y$ for some finite (possibly empty) sequence $u_1, \ldots, u_n$ in X. Then the Strong Axiom is equivalent to : H is irreflexive.

The above is a faithfull reproduction of the model as in Sondermann (1982).

Before we proceed we make the following observation :

$\forall \lambda > 0$, $\lambda \in \mathbb{R}$, $\forall (p, \mathcal{q}, L, m) \in P_1 \times P_2 \times Q \times M$ if $(\lambda p, \lambda \mathcal{q}, L, \lambda m) \in P_1 \times P_2 \times L \times m$, then $B(p, \mathcal{q}, L, m) = B(\lambda p, \lambda \mathcal{q}, L, \lambda m)$. The verification of this observation is a routine exercise.

We now make the following assumption which is crucial to what follows :

Assumption 1 :- Given any linear transformation $A: \mathbb{R}^n \rightarrow \mathbb{R}^n$, and any $(p, \mathcal{q}, L, m) \in P_1 \times P_2 \times Q \times M$, there exists a $\lambda \in \mathbb{R}$, $\lambda > 0$ such that $(\lambda A(p), \lambda A(\mathcal{q}), L, \lambda m) \in P_1 \times P_2 \times Q \times M$.

This assumption is used in proving the following proposition.

Proposition 1 :- Let $(p, \mathcal{q}, L, m)$ and $(p, \mathcal{q}, L', m) \in P_1 \times P_2 \times Q \times M$. Then there exists a linear transformation $A: \mathbb{R}^n \rightarrow \mathbb{R}^n$ and $\lambda \in \mathbb{R}$, $\lambda > 0$, such that $B(p, q, L', m) = B(A(p), A(q), L, \lambda m)$ where $(A(p), A(q), L, \lambda m) \in P_1 \times P_2 \times Q \times M$.

Proof :- The transition from (p, q, L, m) to (p, q, L', m) transforms the budget set $B(p, q, L, m)$ to $B(p, q, L', m)$ by a linear transformation $A': \mathbb{R}^n \to \mathbb{R}^n$. Given A' and (p, q, L', m), by Assumption 1, there exists $\lambda > 0$ such that $(\lambda A'(p), \lambda A(q), L, \lambda m) \in P_1 \times P_2 \times Q \times M$. Let $A = \lambda A'$. Then $B(A(p), A(q), L, \lambda m) = B(p, q, L', m)$.

Q.E.D.

Hence we can assume that the quantity constraints are fixed at $\hat{L}$ and any observed variation in the quantity constraints are accounted for by suitable variations in the other parameters as proposition 1 suggests.

The Main Theorem :- We now prove the main theorem of our analysis, which turns out to be a minor adaptation of a similar result in Sondermann (1982).

Theorem 1 :- If $\mathcal{R}(h)$ has the following "connectedness" property : for all $x, y \in \mathcal{R}(h)$, if xSy, then $tx + (1-t)y \in \mathcal{R}(h)$ for some $t \in (0,1)$ (in particular, if $\mathcal{R}(h)$ is convex), then the Strong Axiom implies h is representable.

Proof :- (We shall provide one for completeness inspite of its essential similarity with the proof in Sondermann (1982)).

Since S is asymmetric, $h(B)$ is always a singleton. Hence h is representable, if S has a utility representation u; that is, xSy implies $u(x) > u(y)$.

The topology of $\mathbb{R}^n$ has a countable base of open sets, say $\{\theta m\}_{m \in \mathbb{N}}$. For $x \in \mathcal{R}(h)$, define $N(x) = \{m \in \mathbb{N} : x \in \theta m$ or wHx for some $w \in \theta m \cap X\}$ and $u(x) = \Sigma_{m \in N(x)} 2^{-m}$. For $x \in X \setminus \mathcal{R}(h)$ set $u(x) = -1$. Let xSy. Clearly, by transitivity of H, $N(x) \subseteq N(y)$; hence $u(x) \geq u(y)$. If $y \notin \mathcal{R}(h)$, then $u(x) \geq 0 > u(y) = -1$. Otherwise, by assumption there exists a $z = tx + (1-t)y \in \mathcal{R}(h)$ for some $t \in (0,1)$. Let $x = x_1 + x_2 = h(p, q, \hat{L}, m)$ and $z = z_1 + z_2 = h(\bar{p}, \bar{q}, \hat{L}, \bar{m})$. xSy means, there exists $y_1 \in X_1, y_2 = X_2$, $y = y_1 + y_2$ and $p.y_1 + q.y_2 \leq m$, where $y_1 \leq \hat{L}$ and $x \neq y$. Hence $p.z_1 + q.z_2 \leq m$ and $x \neq z$, i.e. xSz. Consequently by the weak axiom $\bar{p}.x_1 + \bar{p}.x_2 > \bar{m}$, which in turn implies $\bar{p}y_1 + \bar{q}\, y_2 < \bar{m}$. Thus there exists some neighborhood $\mathcal{O}m$ of y such that for all $w\in\mathcal{O}m\cap X$, $\exists w_1\in X_1, w_2\in X_2$, $w_1 \leq \hat{L}$, $w_1+w_2=w$ and $\bar{p}w_1+\bar{q}w_2<\bar{m}$ and $w\neq z$, that is, xSzSw. By the Strong Axiom, not wHx. Hence $m\in N(y)\setminus N(x)$, which proves $u(x)>u(y)$.

Q.E.D.

4. Conclusion :- In the introduction to this paper we have claimed that this consumer choice theory can be treated as a generalization of the received theory of consumer choice without quantity constraints. In the concluding section of this paper we propose to show just that.

Let $X=\mathbb{R}^n_+$, $Q=\mathbb{R}^n_+$, $P_1=\mathbb{R}^n_{++}$, $P_2=\mathbb{R}^n_{++}$, $M=\mathbb{R}_{++}$. Setting $\hat{L}=0$, we get the pure flexible price situation. Thus our revealed preference axioms can be considered to be an appropriate generalization of the standard one already existing in the literature, to economies in which there is dual pricing.

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