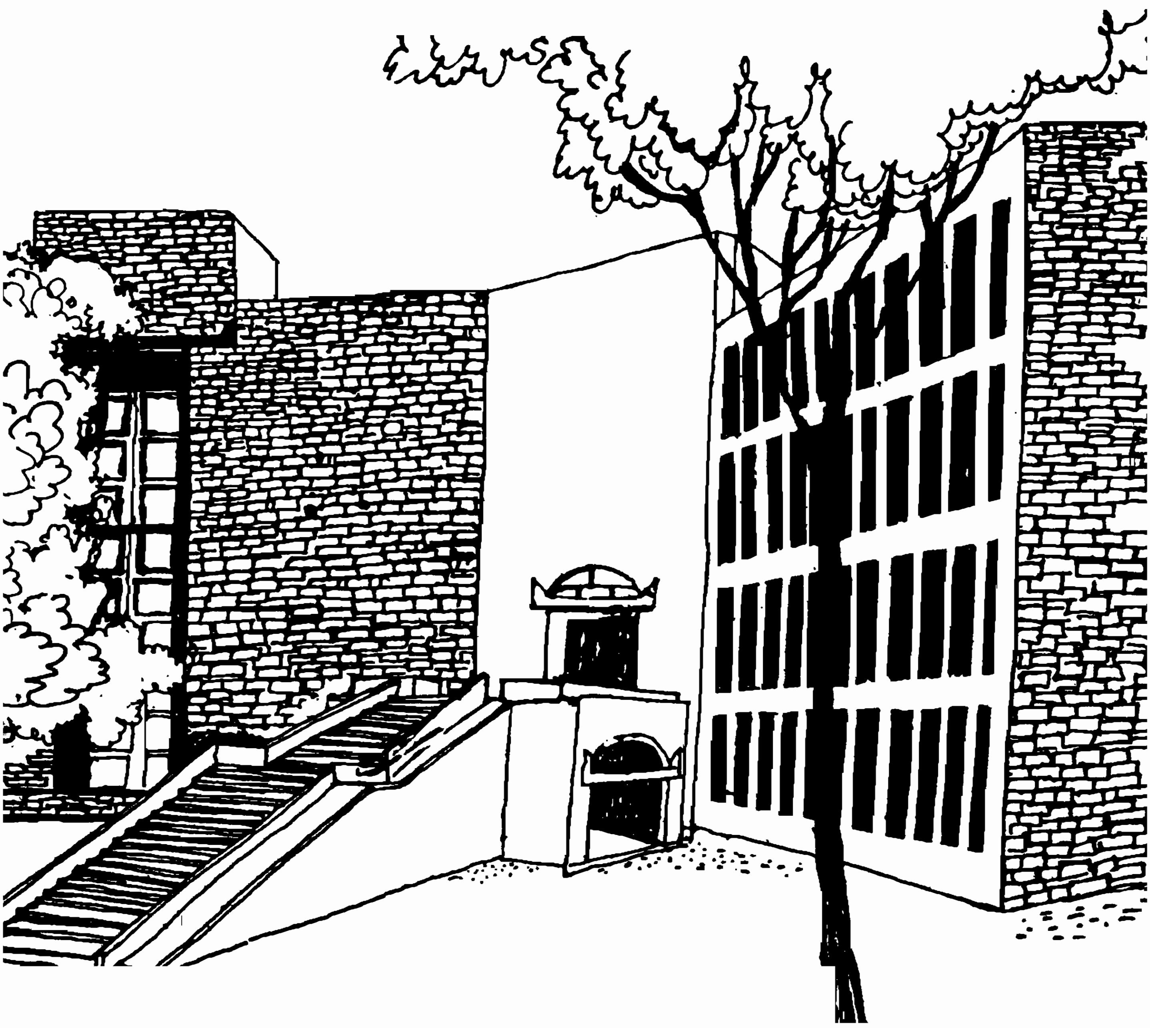
विद्याविनियोगाद्विकासः

IIM AHMEDABAD

Working Paper



IMPROVEMENT SENSITIVITY AND INVARIANCE OF SOLUTIONS TO GROUP DECISION MAKING PROBLEMS WITH CLAIMS

By

Somdeb Lahiri

W P No. 1116
June 1993

The main objective of the working paper series of the IIMA is to help faculty members to test out their research findings at the pre-publication stage.

INDIAN INSTITUTE OF MANAGEMENT
AHMEDABAD-380 015
INDIA

Abstract

In this paper we have developed a very general framework for studying group decision making problems with claims which subsumes the set of problems studied in axiomatic models of bargaining. Subsequently we proceed to establish the improvement sensitivity of some solutions defined on a smaller domain.

We also extend an earlier framework for studying multicriteria group decision making problems to incorporate claims as an additional parameter for reference. We define the concept of an equilibrated state for such problems and show that a solution which chooses only equilibrated state is of necessity invariant under monotone increasing transformations of the value functions of the respective agents.

1. Introduction :- In a recent paper, Peters (1992) has defined the concept of improvement sensitivity for group decision making problems. The concept of improvement sensitivity is based on the notion of a faster improving value function. This latter concept is the non-stochastic analogue of the risk-aversion criteria which pervades the analysis of choice under risk and uncertainty. Peters (1992) proceeds to show that the class of symmetric and non-symmetric Nash bargaining solutions (see Nash (1950), Harsanyi-Selten (1972)) satisfies improvement sensitivity.

Group decision theory has recently been extended to a similar theory with a claims or a target point in a series of papers by Chun and Peters (1991), Chun and Thomson (1992), Bossert (1992a), Bossert (1992b), Lahiri (1993a), Lahiri (1993b). The main difference between the two theories is the presence of an additional vector of parameters which influences the solution outcome to the given problem. It may be worthwhile to extend the idea of improvement sensitivity to such domains - and that is the precise rationale of this paper. In another paper by Abad and Lahiri (1993), we discuss the essential similarity of an important sub-class of these problems to the problem of choosing an output vector in a regulated firm.

In this paper we also establish that on a sub-domain of the set of problems on which improvement sensitivity can be actually defined, a solution satisfying Pareto optimality, scale-covariance and restricted monotonicity actually implies improvement sensitivity.

2. The Framework :- An <u>n-person group decision making problem with claims</u> Ω has the form $\Omega=\langle A,\bar{a},u^1,\ldots,u^n v\rangle$ where

(i) A is a nonempty set of alternatives

(ii) $\bar{a}\in A$ is a status-quo alternative

(iii) for each $i=1,\ldots,n$, $u^i : A\to\mathbb{R}$ is a value function

(iv) $v=(v_1,\ldots,v_n)\in\mathbb{R}^n$, $v_i > u_i(\bar{a})\ \forall\ i=1,\ldots,n$.

We denote by $\mathcal{G}$ the family of all <u>admissible</u> group decision making problems with claims.

A solution on $\mathscr{B}$ is a non-empty valued correspondence

$\Psi : \mathscr{B} \to \bigcup_{A:\Gamma=\langle A,\bar{a},u^1,\ldots,u^n,v\rangle\in\mathscr{B}} A$ such that for each $\Omega\in\mathscr{B}$, $\Omega=\langle A,\bar{a},u^1,\ldots,u^n,v\rangle$, $\Psi(\Omega) \subseteq A$.

A solution Ψ on $\mathscr{B}$ is said to be efficient or Pareto-optimal if $\forall\Omega\in\mathscr{B}$, $\Omega=\langle A,\bar{a},u^1,\ldots,u^n,v\rangle$, $a\in\Psi(\Omega)$, implies 'a' is a Pareto optimal element of A i.e. $b\in A$, $u^i(b)\geq u^i(a)\ \forall i=1,\ldots,n \Rightarrow u^i(b)=u^i(a)\ \forall i=1,\ldots,n$.

A solution Ψ on $\mathscr{B}$ is said to be individually rational if $\forall\Omega\in\mathscr{B}$, $\Omega=\langle A,\bar{a},u^1,\ldots,u^n,v\rangle$, $a\in\Psi(\Omega) \Rightarrow u^i(a)\geq u^i(\bar{a})\ \forall i=1,\ldots,n$.

A solution Ψ on $\mathscr{B}$ is said to be a bargaining solution if $\forall\Omega\in\mathscr{B}$, $\Omega=\langle A,\bar{a},u^1,\ldots,u^n,v\rangle$ $a\in\Psi(\Omega)$, $b\in A$, $[u^i(a)=u^i(b)\ \forall i=1,\ldots,n \Leftrightarrow b\in\Psi(\Omega)]$.

Associated with each bargaining solution Ψ on $\mathscr{B}$ is a function $\phi_\Psi : \mathscr{B}\to\mathbb{R}^n$ defined as follows :

$\phi_\Psi(\Omega)=(u^1(a),\ldots,u^n(a))$, $a\in\Psi(\Omega)$

It is easy to check that ϕ_Ψ is well defined. Such a function is called an n person bargaining solution on $\mathscr{B}$.

Let $\Omega\in\mathscr{B}$ and $S_\Omega=\{x\in\mathbb{R}^n / u^i(a)\geq x_i\geq u^i(\bar{a})\forall i=1,\ldots,n$ for some $a\in A\}$

where $\Omega=\langle A,\bar{a},u^1,\ldots,u^n,v\rangle$ and $d_\Omega=(u^1(\bar{a}),\ldots,u^n(\bar{a}))$.

Associated with each function $\phi:\mathscr{B}\to\mathbb{R}^n$ such that $\forall\Omega\in\mathscr{B}$, $\Omega=\langle A,\bar{a},u^1,\ldots,u^n,v\rangle$, $\phi(\Omega)\in S_\Omega$, there exists a bargaining solution Ψ on $\mathscr{B}$ defined as follows :

$\Psi(\Omega)=\{a\in A/(u^1(a),\ldots,u^n(a))=\phi(\Omega)\}$

where $\Omega=\langle A,\bar{a},u^1,\ldots,u^n,v\rangle\in\mathscr{B}$. Such a Ψ is of necessity individually rational .

If $\phi:\mathscr{B}\to\mathbb{R}^n$ is an n-person bargaining solution, such that for each $\Psi\in\mathscr{B}$, ϕ depends only on (S_Ω, d_Ω, v), then ϕ is called a welfarist n-person bargaining solution .

In axiomatic bargaining (see Moulin (1988) for a recent treatment), we study the following class of admissible n-person group-decision making problems with claims , denoted $\mathscr{B}^n$:

$\mathscr{B}^n=\{\Omega=\langle A,\bar{a},u^1,\ldots,u^n,v\rangle/\Omega$ satisfies properties (i), (ii), (iii) and (iv) mentioned below)

Property (i) :- A is a non-empty compact, convex subset of $\mathbb{R}^n_+$ satisfying minimal transferability i.e. $a \in A$, $a_i > 0 \Rightarrow \exists b \in A$ with $0 \leq b_i < a_i$, $b_j > a_j \forall j \neq i$.

Property (ii) :- $\bar{a}=0$

Property (iii) :- $u^i(a)=\pi^i(a) \forall a \in A$, where $\pi^i : \mathbb{R}^n \to \mathbb{R}$ is the projection onto the ith coordinate (see Spivak (1965)).

Property (iv) :- $v \geqq 0$.

Such problems are also known as multi-attribute choice problems.

In the subsequent analysis we will study a somewhat larger class of problems denoted $\mathcal{B}^0_l$

$\Omega=\langle A,\bar{a},u^1,\ldots,u^n,v\rangle \in \mathcal{B}^0_l$ where $l \in \mathbb{N}$ if and only if

(i) A is a non-empty compact, convex subset of $\mathbb{R}^l$

(ii) $\bar{a} \in A$

(iii) for each $i=1,2,\ldots,n$, $u^i : A \to \mathbb{R}$ is a concave, continuous function

(iv) $v \geqq (u^1(\bar{a}),\ldots,u^n(\bar{a}))$

The following function $I:\mathcal{B}^0_l \to \mathbb{R}^n$ called the ideal (or utopia) point function will prove useful in isolating an essential subdomain for our analysis :

$$I_i(\Omega)=\max\{u_i(a)/a \in A,\ u_i(a) \geq u_i(\bar{a})\}\ i=1,\ldots,n,.$$

Using the ideal point function $I:\mathcal{B}^0_l \to \mathbb{R}^n$ we can consider the following domain :

$$\bar{\mathcal{B}}^0_l=\{\Omega \in \mathcal{B}^0_l / \Omega=\langle A,\bar{a},u^1,\ldots,u^n,v\rangle, v \geqq I(\Omega) \geqq d_\Omega\}$$

Before we close this section, let us note that there is a natural embedding of problems studied in axiomatic models of bargaining into the set $\mathcal{B}^0_l$. If we restrict $\bar{a}$ to be equal to o and $A \subseteq \mathbb{R}^l_+$ in the definition of $\mathcal{B}^0_l$, we get a natural one-to-one correspondence between the two classes of problems.

3. A recapitulation of the improving faster than criterion :- Let A be a non-empty set of alternatives and $u:A \to \mathbb{R}$, $v:A \to \mathbb{R}$ be given (value) functions. We call v improving faster than u, denoted vIFu, if $\forall$ a,b,c,d$\in$A with $u(a) \geq u(b) \geq u(c) \geq u(d)$ we have :

$\Rightarrow u(c)-u(d)\geq u(a)-u(b) \Rightarrow v(c)-v(d)\geq v(a)-v(b)$

and $u(c)-u(d)>u(a)-u(b) \Rightarrow v(c)-v(d)>v(a)-v(b)$

This is the definition as in Peters (1992).

<u>Theorem 1</u> (Peters (1992)) : Let $u:A\to\mathbb{R}$ and $v:A\to\mathbb{R}$ be given with $u(A)$ an interval. The following two statements are equivalent :

i) $vIFu$

ii) There exists a strictly increasing concave function $k:V\to\mathbb{R}$ with $v(a)=k\circ u(a)\ \forall a\in A$ where $V=[\min_{a\in A}\{u(a)\},+\infty)$.

<u>Proof</u> :- See Peters (1992). The function k in the proof of Peters (1992) can be extended as desired.

It is easy to see that for the domain $\mathcal{D}^0{}_1$, given $\Omega\in\mathcal{D}^0{}_1, \Omega=\langle A,\bar{a},u^1,\ldots,u^n,v\rangle$, $u^i(A)$ is an interval for each $i=1,\ldots,n$.

Given $\Omega\in\mathcal{D}^0{}_1, \Omega=\langle A,\bar{a},u^1,\ldots,u^n,v\rangle$, let $k^i(\Omega)$ denote the problem $\langle A,\bar{a},u^1,\ldots,u^{i-1},k\circ u^i,u^{i+1},\ldots,u^n, v_1,\ldots,v_{i-1},k(v_i), v_{i+1},\ldots,v_n\rangle$ where $k:V_i\to\mathbb{R}$ is a strictly increasing concave function and $V_i=[\min_{a\in A} u^i(a),+\infty)$. Here player i of Ω has been replaced by a player with a faster improving value function $k\circ u^i$. It is easy to check as in Wakker (1989) that $k^i(\Omega)\in\mathcal{D}^0{}_1$ if $\Omega\in\mathcal{D}^0{}_1$ and $k^i(\Omega)\in\bar{\mathcal{D}}^0{}_1$ if $\Omega\in\bar{\mathcal{D}}^0{}_1$.

Let Ψ be a bargaining solution on $\mathcal{D}^0{}_1(\bar{\mathcal{D}}^0{}_1)$. We shall call Ψ <u>improvement sensitive</u> (IS) if for all $i=1,2,\ldots,n$, and $\Omega\in\mathcal{D}^0{}_1(\bar{\mathcal{D}}^0{}_1)$, we have

$u^i(a)\geq u^i(b)\ \forall a\in\Psi(\Omega),\ b\in\Psi(k^i(\Omega))$ and consequently

$k\circ u^i(a)\geq k\circ u^i(b)\ \forall a\in\Psi(\Omega),\ b\in\Psi(k^i(\Omega))$

where $\Omega=\langle A,\bar{a},u^1,\ldots,u^i,\ldots,u^n,v\rangle$. Here k^i is as above.

4. A preliminary result on $\mathcal{D}^0{}_1$: Let Ψ be a solution on $\mathcal{D}$. Consider the following property :

<u>Property 1 :- Ψ satisfies Scale Covariance</u> : If $\alpha\in\mathbb{R}^n{}_{++}$, $\beta\in\mathbb{R}^n$, $\Omega\in\mathcal{D}$, $\Omega=\langle A,\bar{a},u^1,\ldots,u^n,v\rangle$, $\Omega'=\langle A,a,\alpha_1 u^1+\beta_1,\ldots,\alpha_n u^n+\beta_n,\alpha_1 v_1+\beta_1,\ldots,\alpha_n v_n+\beta_n\rangle\in\mathcal{D}$, then $\Psi(\Omega)=\Psi(\Omega')$.

This property will turn out to be significant in most of the subsequent analysis.

Now let Ψ be a bargaining solution on $\mathcal{B}'$. Consider the following property :

<u>Property 2</u> :- <u>Ψ satisfies Nash's Independence of Irrelevant Alternatives Assumption</u> :- Let Ω, $\Omega' \in \mathcal{B}'$, with $d_{\Omega} = d_{\Omega'}$. $S_{\Omega'} \subseteq S_{\Omega}$ and $u[\Psi(\Omega)] \in S_{\Omega'}$. Then $u'[\Psi(\Omega')] = u[\Psi(\Omega)]$, where $\Omega = \langle A, a, u^1, \ldots, u^n, v \rangle$, $\Omega' = \langle A', \bar{a}', u^{1'}, \ldots, u^{n'}, v' \rangle$, $u = (u^1, \ldots, u^n)$ and $u' = (u^{1'}, \ldots, u^{n'})$.

The following theorem a variant of which exists in Peters (1992) can now be established.

<u>Theorem 2</u> :- Let Ψ be a bargaining solution on $\mathcal{B}^0{}_1$ satisfying individual rationality, Scale Covariance (Property 1) and Nash's Independence of Irrelevant Alternatives Assumption (Property 2). Then Ψ is improvement sensitive.

<u>Proof</u> :- Let Ω, $k^i(\Omega) \in \mathcal{B}^0{}_1$ and be as in the definition of (IS). Suppose $u^i(\Psi(k^i(\Omega))) > u^i(\Psi(\Omega))$. We will derive a contradiction. In view of property 1,we may assume $d_{\Omega} = 0$ and $k(0)=0$. Since k is strictly increasing and $u^i(\Psi(k^i(\Omega))) > u^i(\Psi(\Omega)) \geq 0$, we may further assume in view of property 1, that $k(u^i(\Psi(k^i(\Omega)))) = u^i(\Psi(k^i(\Omega)))$. Let $A' = \{a \in A : u^i(a) \leq u^i(\Psi(k^i(\Omega)))\}$, and let $\Omega' = \langle A', a, u^1, \ldots, u^n, v \rangle$. Then, $S_{\Omega'} \subseteq S_{\Omega}$ and $u(\Psi(\Omega)) \in S_{\Omega'}$ implies by property 2, that $u(\Psi(\Omega')) = u(\Psi(\Omega))$. On the other hand the concavity of k implies $k(t) \geq t \;\forall\; 0 \leq t \leq u^i(\Psi(k^i(\Omega)))$. Therefore $S_{\Omega'} \subseteq S_{k^i(\Omega)}$. Further $u'(\Psi(k^i(\Omega))) \in S_{\Omega'}$ where $u'^j = u^j \;\forall\; j \neq i$, $u'^i = k o u^i$. Thus by property 2, $u(\Psi(\Omega')) = u'(\Psi(k^i(\Omega)))$ and hence $u^i(\Psi(k^i(\Omega))) = u^i(\Psi(\Omega))$ which is a contradiction. This establishes the theorem.

Q.E.D.

The proof of the above theorem has been borrowed almost verbatim from Peters (1992), to show that the same proof works in establishing a result which is stronger than what has been cited in the original document. Further, we feel that the appropriate definition of Nash's Independence of Irrelevant Alternatives assumption is, as we have stated it. Our restatement is particularly crucial in the second application of the property in the above proof. The statement of the assumption in Peters (1992) does not apply when we consider $S_{k^i(\Omega)}$.

Often times, in group decision theory, solutions are characterized by appealing to the following property 3 where Ψ on $\mathcal{B}'$ is a solution.

Property 3 :- Ψ satisfies anonymity : For any permutation σ on $\{1,\ldots,n\}$ and any $\Omega=\langle A,\bar{a},u^1,\ldots,u^n,v\rangle$, if $\Omega^\sigma=\langle A,\bar{a},u^{\sigma(1)},\ldots,u^{\sigma(n)},\sigma(v)\rangle$ then $\Psi(\Omega^\sigma)=\Psi(\Omega)$ where for any $x\in\mathbb{R}^n$, $\sigma(x)$ is the vector with $\sigma(x)_i=x_{\sigma(i)}$.

Where as both the Nash (1950) as well as the family of non-symmetric Nash solutions due to Harsanyi and Selten (1972) satisfy properties 1 and 2, if we require in addition that solution Ψ satisfy property 3, then we automatically eliminate the latter class of solutions.

5. Restricted Monotonicity and a result on $\bar{\mathcal{B}}'^{*}_{l}$:- A desirable property for bargaining solutions to group decision making problems with claims to satisfy is the following :

Let Ψ be a bargaining solution on $\mathcal{B}'$.

Property 4 :- Ψ satisfies restricted monotonicity : Let $\Omega=\langle A,\bar{a},u^1,\ldots,u^n,v\rangle$, $\Omega'=\langle A',\bar{a}',u^{1'},\ldots,u^{n'},v\rangle\in\mathcal{B}'$ with $S_\Omega\supseteq S_{\Omega'}$, $d_\Omega=d_{\Omega'}$. Then, $u(\Psi(\Omega))\geq u(\Psi(\Omega'))$.

In Lahiri (1993b) we establish that the class of solutions due to Yu (1973) satisfy property 4. However this class of solutions do not satisfy improvement sensitivity. A solution which does satisfy improvement sensitivity on $\bar{\mathcal{B}}^0_l$ is the one that seeks a Pareto optimal outcome on the straight line connecting the claims point to the vector of status quo utilities. This solution has been given independent characterizations by Chun and Thomson (1992) and Lahiri (1993a). The significant reason for this dissimilarity of behavior is that in addition to property 4, the latter solution satisfies property 1, whereas the former group of solutions do not.

Let $\mathcal{Q}\subseteq\bar{\mathcal{B}}^0_l$ satisfy, the following property :

$\Omega\in\mathcal{Q}\Rightarrow k^i(\Omega)\in\mathcal{Q}\ \forall i=1,\ldots,n$, where $k^i(\Omega)$ is as defined in section 3.

<u>Theorem 3</u> :- Let Ψ be a bargaining solution on $\mathcal{Q}$ satisfying Pareto optimality, individual rationality, properties 1 and 4. Then Ψ satisfies improvement sensitivity.

<u>Proof</u> :- Let $\Omega=\langle A,\bar{a},u^1,\ldots,u^n,v\rangle\in\mathcal{Q}$ and suppose $k^i(\Omega)$ is as in the definition of IS. Then by property 1, we may assume $d_\Omega=0$, $k(0)=0$, $k(v_i)=v_i$. Now since k is strictly increasing and concave, $k(t)\geqq t$ $\forall 0\leqq t\leqq v_i$. Since $I_i(\Omega)\leqq v_i$ and $I_i(k^i(\Omega))\leqq k(v_i)$, we get $S_\Omega\subseteq S_{k^i(\Omega)}$. Thus by property 4, $u_j(\Psi(k^i(\Omega)))\geqq u_j(\Psi(\Omega))$ $\forall$ $j\neq i$ and $k o u_i(\Psi(k^i(\Omega)))\geqq u_i(\Psi(\Omega))$. However, $u(\Psi(k^i(\Omega)))\in S_\Omega$ and Ψ satisfies Pareto optimality. Hence $u_j(\Psi(k^i(\Omega)))\geqq u_j(\Psi(\Omega))$ $\forall$ $j\neq i$ implies $u_i(\Psi(k^i(\Omega)))\leqq u_i(\Psi(\Omega))$. This proves the theorem.

Q.E.D.

Admittedly it would be desirable to obtain a similar result on $\mathcal{B}^0_I$. However, such a result does not appear to be true. Property 4 has been defined, discussed and used in characterizing a solution in Lahiri (1993a).

6. Multicriteria group decision making problems with claims :- In Polterovich (1990) and Lahiri (1993c) can be found a model of multicriteria group decision making problem. In this section we propose to extend the model within our framework to group decision making problems with claims and establish an immediate invariance property of solutions to such problems.

A multicriteria group decision making problem with claims Γ is of the form $(\langle A,\bar{a},u^1,\ldots,u^n,v\rangle,g)$ where $\langle A,\bar{a},u^1,\ldots,u^n,v\rangle\in\mathcal{B}'$ (i.e. is a group decision making problem with claims and $g:A\to\mathbb{R}^m$, for some $m\in\mathbb{N}$. Let M denote the class of all multicriteria decision making problems with claims. In the above 'g' is called the criteria function.

Given $\Gamma\in M$, $\Gamma=(A,\bar{a},u^1,\ldots,u^n,v\rangle,g)$, an element $a\in A$ is called an <u>equilibrated state</u> if

(i) a is Pareto optimal for $\langle A,\bar{a},u^1,\ldots,u^n v\rangle$

(ii) $g(a)\leqq 0$.

Let $\Psi(\Gamma)=\{a\in A/a$ is an equilibrated state for $\Gamma\}$ where, $\Gamma=(\langle A,\bar{a},u^1,\ldots,u^n,v\rangle,g)$.

Theorem 4 :- Let $k^i : u^i(A) \to \mathbb{R}$ be any strictly increasing function for $i=1,\dots,n$. $\Gamma' = (\langle A, \bar{a}, k^1 o u^1, \dots, k^n o u^n, w\rangle, g)$ where $\langle A, \bar{a}, k^1 o u^1, \dots, k^n o u^n, w\rangle \in \mathcal{D}'$. Then $\Psi(\Gamma) = \Psi(\Gamma')$.

Proof :- It is easy to see that Ψ is independent of the claims point. Since the set of Pareto optimal alternatives are invariant with respect to strictly increasing transformations of the utility functions and since g remains unaltered, the result is immediate.

Q.E.D.

Conclusion :- In this paper we have developed a very general framework for studying group decision making problems with claims which subsumes the set of problems studied in axiomatic models of bargaining. Subsequently we proceed to establish the improvement sensitivity of some solutions defined on a smaller domain.

We also extend an earlier framework for studying multicriteria group decision making problems to incorporate claims as an additional parameter for reference. We define the concept of an equilibrated state for such problems and show that a solution which chooses only equilibrated state is of necessity invariant under monotone increasing transformations of the value functions of the respective agents.

References :-

1. P. Abad and S. Lahiri (1993) : "Group Decision Theory and Production Planning Problems", Indian institute of Management, Working Paper No. 1106.

2. W. Bossert (1992a) : "Monotonic Solutions for Bargaining Problems With Claims", Economics Letters 39, 359-399.

3. W. Bossert (1992b) : "An alternative solution to bargaining problems with claims", Mathematical Social Sciences (forthcoming).

4. Y. Chun and H. Peters (1991) : "The lexicographic equal-loss solution", Mathematical Social Sciences 22, 151-161.

5. Y. Chun and W. Thomson (1992) : "Bargaining problems with claims", Mathematical Social Sciences, forthcoming.

6. J.C. Harsanyi and R. Selten (1972) : "A Generalized Nash Solution for Two-Person Bargaining Games with Incomplete Information", Management Science 18, 80-106.

7. S. Lahiri (1993a) : "A Compromise Solution for Claims Problems", Forthcoming in European Journal of Operations Research.

8. S. Lahiri (1993b) : "Monotonicity of Solutions With Respect to the Claims Point", mimeo.

9. S. Lahiri (1993c) : "Existence of Equilibrated States In Multicriteria Decision Making Problems", Indian Institute of Management, Ahmedabad, Working Paper No. 1101.

10. H. Moulin (1988) : "Axioms of Cooperative Decision Making", Cambridge University Press.

11. J.F. Nash (1950) : "The Bargaining Problem", Econometrica 28, 155-162.

12. H. Peters (1992) : "A Criterion for Comparing Strength of Preference With An Application To Bargaining", Operations Research, Vol. 40, No. 5, 1016-1022.

13. V.M. Polterovich (1990) : "Equilibrated states and optimal allocations of resources under rigid prices", Journal of Mathematical Economics 19, 255-268.

14. M. Spivak (1965) : "Calculus on Manifolds", Addison-Wesley Publishing Company.

15. P.P.Wakker (1989) : "Additive Representations of Preferences : A New Foundation of Decision Analysis", Kluwer Academic Publishers, Dordrecht, The Netherlands.

16. P.L. Yu (1973) : "A class of solutions for group decision problems", Management Science 19, 936-947.