MODELLING AND ANALYSIS OF LARGE SYSTEMS: SOME APPROACHES

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Modelling and Analysis of Large Systems: Some Approaches

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Abstract

The concept and nature of large scale modelling have been presented. The necessity of having a good grasp of the system has been stressed. Some of the large scale modelling efforts have been presented.

[Key Words - Large Scale Systems, Modelling, Computational Strategy, Decision Aids].

Introduction: Many real life problems[1,2] are brought forth by present day societal and environmental processes which are highly complex, "large" in dimension and stochastic in nature. The notion of "large scale" is a subjective one, in that one may ask: How large is large? Many viewpoints have been presented on this issue, one way to look at large systems as those, whose dimensions are so large that conventional techniques of modelling, analysis and optimization do not result in a reasonably good solution with commensurate computational efforts. Another way to look at large systems as composed of a number of interconnected sub-systems. Often, in such a case, the problem is de-composed for analysis and solution purposes.

Nature of Large Scale Models: The size of the problem is not only distinguishing characteristics of Large Scale models. Invariably, all large scale models are characterised by its distinctive structure. The modelling and analysis of large systems is very closely related to the exploitation of the special structure associated with the particular large system. Almost always large systems are studied in relation to a real life problem. As a result the system has also a structure similar to the decision making structure in the organisation, viz. a hierarchical structure. Sometimes the system could be laterally decomposed -- productwise, locationwise etc. Apart from the special structure of the problem, the co-efficient matrix of the problem is invariably highly sparse. This makes the classical approach to modelling/problem formulation and solution highly inefficient. The use of a matrix generator in such a case becomes very useful due to the special structral relationship. Sometimes, the problem becomes large and not necessarily complex, by multiplication of products, locations and time periods. The study of large systems almost always goes beyond usual convexity and linearity properties. One has to exploit the special nature of the large system for modelling and analysis purposes. To that extent study of each large system becomes somewhat unique.

Use of Large Scale Models: Study of large models can be used broadly for two purposes:

(a) to get the operational parameters for optimization,

(b) to decide on the policy framework.

In the first case it is essential to get the actual values of the decision variables. In case of a timetabling problem, one needs to know the day and the period when a class is to be scheduled. So also in a multistage distribution system, we need to know the amount of warehouse space to be hired at different periods, weekly/monthly/quarterly movement plan of goods etc.

In the second case, one need to analyse the implications of various policy alternatives. Here one is concerned with relative outcomes and not necessarily the actual output values. The order of magnitude with certain level of accuracy may suffice. In a location problem, one may be interested in the relative merits (demerits) of each of the locations and trade offs involved. The level of accuracy desired in respect of cost (benefit) associated with each of the locations may not be very high. Similarly in a agro-climatic regional planning model, one is not much interested in the accurate figure of acreage for various crops, actual output and such other outcomes. But one would be interested in the extent of trade-off involved for various policy alternatives in respect of cropping intensity, cropping pattern consistent with different agro-climatic zone.

The use to which the study is going to be put to, provides one of the essential input for the modelling of large system. This will help in deciding the level of aggregation/disaggregation desired, extent of details to be included in the model, decomposition structure of the model etc.

Modelling of Large Systems: Modelling of a large system will be influenced among other things,

- the use to which it is put to,
- available computing facilities,
- available softwares and solution strategies.

The modelling process itself goes through various iterations before a desirable one is obtained. Certain amount of data collection and computational effort is also quite common in between the modelling iterations. In order to have a good model of the system it is desirable to have a complete grasp of the systems being modelled. It is neither possible nor desirable to model any system in its entirety. The boundaries of the system need to be defined in line with the study objective. This will provide clarity as to which section of the reality should be modelled. Since the modelling will be related to the extent of accuracy desired, computing facilities to be used, software capability and such others; it is desirable to have these parameters firmed up as far as possible. It can be mentioned that a particular section of the reality can be modelled in several different ways, but each will have different benefits and costs and will provide different degree of insight into the reality.

Computational Strategy: Large Scale models, by definition, are large and are amenable to solution by standard computational methods. There are two broad computational strategy, which are commonly employed for large scale models.

(a) strategy aimed at improving the computations efficiency of known solution techniques taking advantage of the special nature of the problem,

(b) strategy aimed at developing essentially new solution technique.

In the first category we have the various improvements to make the simplex method more efficient. Some of these methods are "product form"[3] algorithm, which exploits the sparsness of the matrix, column generation technique and many such techniques.

In the second category the effort is more in terms of problem manipulation to find new and efficient computational methods. Here the basic nature of the problem is kept in view and manipulation is done in terms of representation in a model which is easier to solve. There is no unique way of doing this. However, good grasp of the problem being modelled, provides better opportunities in this direction. Some of the common approaches in problem manipulation are:

(a) isolating the sub system, having a familiar structure, which could be solved effciently with available algorithms,

(b) approximating linearity in a partly non linear problem,

(c) attempting meaningful decomposition, separation or partition which will aid in efficient solution.

Very rarely only one or the other of the two approaches are employed. In most cases both these approaches are used to differing degrees.

There have been certain developments with regard to efficient approaches to solving large linear programming problems, notably the algorithm developed by Karmarkar.[4] In spite of these developments, problem manipulation and exploitation of special structure for making the existing solution methods more efficient remain most prevalent for large systems.

Some Approaches: The approaches in case of some large scale models are discussed here.

Timetabling: The timetabling problem analysed by Tripathy[5], deals with scheduling of about 900 subjects over an academic year, in a graduate program of one academic year duration. The students are enrolled in 25 different streams (specialisation) in the graduate program. A direct mathematical programming formulation of one terms timetable, out of the three terms in the academic year, results in a binary integer linear program with about 12,000 variables and as many constraints. Here the modelling exercise is more in terms of getting relevant solution for operationalising the timetable and to a very limited extent analysing the policy issues. Due to the objective of providing output for operational decisions, the model has to be run periodically, at least a few times every term. This calls for high level of efficiency in computation, which is influenced significantly by the model representation of the problem. On the basis of the study of the system a grouping operation was carried out to reduce the size of the problem. The resultant problem formulation is presented in Annexure-I. This problem manipulation resulted in reducing the problem size considerably to the number of variables being less than 4000 and the number of constraints to less than 1500. However, the size of the problem still remains too large to be solved by conventional ILP solution methods specially in view of the fact that the model has to be run regularly. This necessitated a search for an unique solution strategy. Relaxation method based on Lagrangean multiplier with multipliers being obtained through subgradient method has been employed. Different sets of relaxed problems can be obtained by dualising different sets of constraints. Each of such relaxed problems provide different degree of ease in computation. The one finally used by dualising the constraint set (P-III) of Annexure-I. The resultant relaxed problem is presented in Annexure-II. The $_{ij}$s are the multipliers. The relaxed problem has an unimodular structure. The flow diagram of the algorithm used for solution is presented in Annexure-III.

As can be seen Branch and Brand method has also been employed here. The branching has been done on constraint sets after identifying the most infeasible constraint. The characteristics of the constraint sets, which is similar to a 'special ordered set' has been exploited in the branch and brand procedure.

This timetabling problem is characterised by its use for operatinal decisions. The knowledge of the nature of the problem has been used in "problem-manipulation" to reduce the size of the problem. A one-off algorithm has been developed for solution purposes which exploits the problem structure in obtaining a relaxed problem which is unimodular and the "special ordered set" properties has been used for branching purpose.

Location: The location problem studied by Seshadri et al.[6] deals with centralised industrial planning framework. In particular it studies the potential location for sponge iron units for a developing nation. The model has to be used essentially for evaluating various policy alternatives in order to come up with policy guidelines for the location of sponge iron units. The modelling exercise is expected to address to the following issues:

1) Which subset among an identified set of potential locations should be chosen for locating new plants. New plants are required because markets which are located at known discrete locations have known and growing demands for the product.

2) What process technology should be chosen for the new plants at each location, given that several processes are available for producing the product.

3) What should be the capacity of each of the new plants and how should this be built-up over time. The latter relates to the question of time-phasing the plants.

4) How should various types of raw materials required for manufacture of the product be allocated from their respective multiple sources to plants, both old and new.

5) How should the finished product be allocated from plants,

both old and new to the markets.

In the above, old plants refer to plants that are in operation at the beginning of the planning horizon; while new plants refer to those that are added during the planning horizon.

On the basis of the above it could be described as a Dynamic Multistage Multicommodity Process Selection Location - Production Allocation (DMMSLPA) Model.

The general formulation of the model results in the following problem presented in Annexure-IV.

The parameter with appropriate subscripts represent the following:

F - Present value of fixed cost

C - Present value of cost (as per subscripts)

D - Annual Demand

A - Annual Capacity per module

S - Annual availability

f - Allowable over capacity

r - Requirement per ton of output

M - Maximum number of locations to be decided

The model size is dependent on the number of time periods to be considered, number of potential locations, number of market centres etc. While it is desirable to have higher values for all the above parameters, the problem size becomes a hindrance. Accordingly, one has to strike balance between the size of the problem and the marginal utility of increasing the parameter dimensions. Accordingly, the nature of the policy decisions were analysed and the problem was formulated to provide maximum utility consistent with the size. Three time periods each of 5 years durations were considered, which were in line with the planning horizons. Similarly the number of potential locations, market centres etc. were decided keeping in view the nature of the problem as it existed. This resulted in a mixed integer program with over 3500 variables and nearly 1000 constraints. Even with this reduced size, experience in solving with standard MILP packages with available computing facility indicated excessive computational time. This suggested development of specialised algorithm for solution purposes. The flow chart of the algorithm is presented in Annexure-V. The algorithm is based upon partitioniting the problem into sub problems one for each period, generation of initial partly feasible solution, decomposition and branch and bound.

The modified algorithm resulted in solving the problem in 1/8th of the time initially experienced. This also helped in analysing the various policy alternatives and carrying out extensive sensitivity analysis.

Warehousing and Distribution: This study deals with the warehousing and distribution of a bulk commodity, in this case fertilizers.[7] The problem is characterised by

(i) Multistage location of warehouse due to multi-tier distribution system,

(ii) Multiproduct - multinutrient situation

(iii) Seasonality of demand

(iv) Transportation restrictions

(v) Limited warehouse space and specialised nature of warehouse hiring and fixed and variable component of warehouse rent.

The modelling exercise is expected to provide assistance in taking operating decisions as opposed to analysis of policy options. The various decisions involved are:

(i) Location of warehouses (At all tiers of distribution)

(ii) Space to be hired at each of the warehouses

(iii) Inventory to be carried at different periods

(iv) Distribution plan over the planning horizon, etc.

The problem discussed here is different from the classical warehousing and distribution problems due to its multistage location of warehouses and location and distribution decision in a multiperiod context.

The formulation of the problem results in a mixed 0-1 integer problem with very large size. As a first step, to make it more tactable, the problem has been decomposed into various independent problems. This decomposition has been done territorywise (which are fairly independent) keeping in view the decision making framework in the organization and the nature of distribution logistic. Direct formulation of the problem does not result in any meaningful special structure for computational purposes. Accordingly, the constraints have been restated and the variables have been re-defined to get a special structure of the problem for solution purposes. The problem structure and the structure of the subproblem has been shown in Annexure-VI & VII.

A typical territory problem has the following size:

Number of 0-1 integer variable - 120

Total number of real variables - nearly 33,000

Total number of constraints - nearly 16,000

Apart from the size of the problem, various computational difficulties like degeneracy due to large number zeros in the right hand side and precision problem were encountered. An algorithm capable of taking care of the size and the other computational problems had to be developed. The flow chart of the algorithm has been presented in Annexure-VIII.

The entire procedure of setting up the mixlp is done by specially written programs (similar to matrix generator). The algorithm includes an efficient relaxation for the lagrangean relaxation problem which was arrived at after evaluating the performance of different relaxations.

This algorithm has been implemented on a VAX-750 computer. reasonable size real life problem could be solved with about 420 minutes of C.P.U. time. VAX-750 is a relatively slow machine (0.7 MIPS), compared to VAX 8800 (20 MIPS) and CRAY-1 (60 MIPS).

Agro Climatic Modelling: In a large country the climatic conditions are different at different parts, which influences the productivity of different crops in those parts. In addition differing levels of irrigation facilities and mechanisation also necessitates provisions for varying amounts of input. The study has aimed at modelling such a situation in a large country. The model is also expected to aid in analysing various policy alternatives in order to arrive at a national policy framework for land-use pattern.

There are 15 agro climatic zones, which are further sub divided into 70 sub zones. 22 activities (different land use pattern) have been considered. Transport of both inputs and outputs have been taken into account in the model. Keeping in view the availability of data, it has been decided to maximise the net return in this case considering the production costs and transportation costs.

The formulation results in a linear programming problem with over 100,000 variable and nearly 2000 constraints. If only zones are considered instead of sub zones the problem has only about 900 variables and 400 constraints. However, in such a case the level of approximation goes up and to that extent the utility of the model becomes limited. A balance need to be struck between the computational difficulty and the utility of the model.

Accordingly the number of sub zones have been kept at 40 instead of 70 by clubbing together the near homogenous sub zones. Further the unit of transportation has been kept at the zonal level, assuming the cost of transportation within the zone to be negligible. This resulted in a problem with just about 6000 variables and about 500 constraints. In order to cut down the computational time the problem has been solved in two modules: production module and transportation module. A suitable programme has been developed to set up the problem matrix.

Transportation Logistic

The problem[9] discussed over here deals with distribution of finished steel (nearly 200 varieties) to about 50 destinations. The movement to these destinations is desired to be in rake load of about 2000 tonnes. At the originating stations, the materials are available at 7 to 10 production/loading points. The availability of the product at the production point is dependent upon the production schedule/campaign. The availability at a loading point on a particular day may be only of 2 to 3 products.

The problem has been modelled as a large mathematical programming model by many with a time horizon of one year. Most of these models indicated very high level of rake load movement. However, in reality the rake load movement was far from that indicated by the model. An analysis to the causes of such variations brought into focus the high level of aggregation in the model. This makes the model too distant from the reality. The nature of the problem suggests a time horizon of one day as compared to one year and incorporation of various other constraints apart from those related to production and demand. The mathematical programming formulation of the problem was found to be too complex. Further the model has to be used on a daily basis for taking operational decisions. The utility of mathematical programming model with this background appeared to be very limited.

The problem was redefined at this stage. The original statement of the problem was to find out the optimal level of rake load movement. The problem definition was changed to provide an aid in increasing the rake load movement. Accordingly, a micro computer based interactive decision aid has been developed to help the decision maker in increasing the rake load movement. The decision aid is based on standard data base and spreadsheet softwares. It also incorporates large number of complexities actually encountered in operation and to that extent the model is closer to reality.

Conclusion

The general nature of the large scale models and a few large scale models have been discussed here. Large scale modelling, almost always, relates to a real life problem. Each problem has its unique characteristics which calls for suitable approaches for modelling and also specific solution strategy. The standard modelling and solution approaches very rarely found to be suitable in the case of large scale systems. A good grasp of the real life problem plays a very important role in large scale modelling. An effort in understanding the real life system can provide deep insight and help developing suitable approaches in all the three phases of study: viz. (i) modelling, (ii) problem manipulation and (iii) solution strategy.

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ANNEXURE – I

TIME TABLING PROBLEM

Let
NSG = total number of subject groups,
NTG = total number of student groups,
NRG = total number of room groups,
NP = total number of periods in a week,

$x_{ij} = 1$ if subject group i (= 1, 2, ..., NSG) is scheduled in period j (= 1, 2, ..., NP),

$= 0$, otherwise.

Then the timetabling problem is

$$\text{maximise} \sum_{i=1}^{NSG} \sum_{j=1}^{NP} c_{ij} x_{ij}, \quad \text{subject to} \qquad \text{(P)}$$

$$\sum_{j=1}^{NP} x_{ij} = NSPG_i; \qquad i = 1, 2, \ldots, NSG, \qquad \text{(P-I)}$$

$$\sum_{i \in R_k} x_{ij} \leq a_{kj}; \qquad j = 1, 2, \ldots, NP, \quad k = 1, 2, \ldots, NRG, \qquad \text{(P-II)}$$

$$\sum_{i \in T_l} x_{ij} \leq 1; \qquad j = 1, 2, \ldots, NP, \quad l = 1, 2, \ldots, NTG, \quad \text{where} \qquad \text{(P-III)}$$

c_{ij}—Objective function related to the desirability of x_{ij}, e.g. if it is desired to schedule subject 'i' to period 'j' for some reason, say due to the preference of an external teacher, c_{ij} can be given a high value. Conversely, for an undesirable combination of 'i' and 'j', c_{ij} can be given a low value (high value). Otherwise c_{ij} may be zero. When all the c_{ij} are zero, the problem is one of finding a feasible solution.

$NSPG_i$—total number of periods per week to be scheduled to subject group 'i'.

a_{kj}—number of available rooms in period 'j' of room group 'k',

R_k—subset of subject groups requiring room group 'k',

T_l—subset of subject groups attended by student group 'l',

P-I—is the constraint set related to the total number of periods required per week by the subject groups.

P-II—is the constraint set related to the availability of rooms.

P-III—is the constraint set, which takes care of the fact that a student group cannot attend more than one subject group in a particular period.

ANNEXURE - II

RELAXED TIME TABLING PROBLEM WITH UNIMODULARITY

$$\text{maximise} \quad \sum_{i=1}^{NSG} \sum_{j=1}^{NP} c_{ij} x_{ij} + \pi_{lj}\left(1 - \sum_{i \in T_l} x_{ij}\right), \quad l = 1, 2, \ldots, NTG, \quad j = 1, 2, \ldots, NP, \qquad (PR_\pi)$$

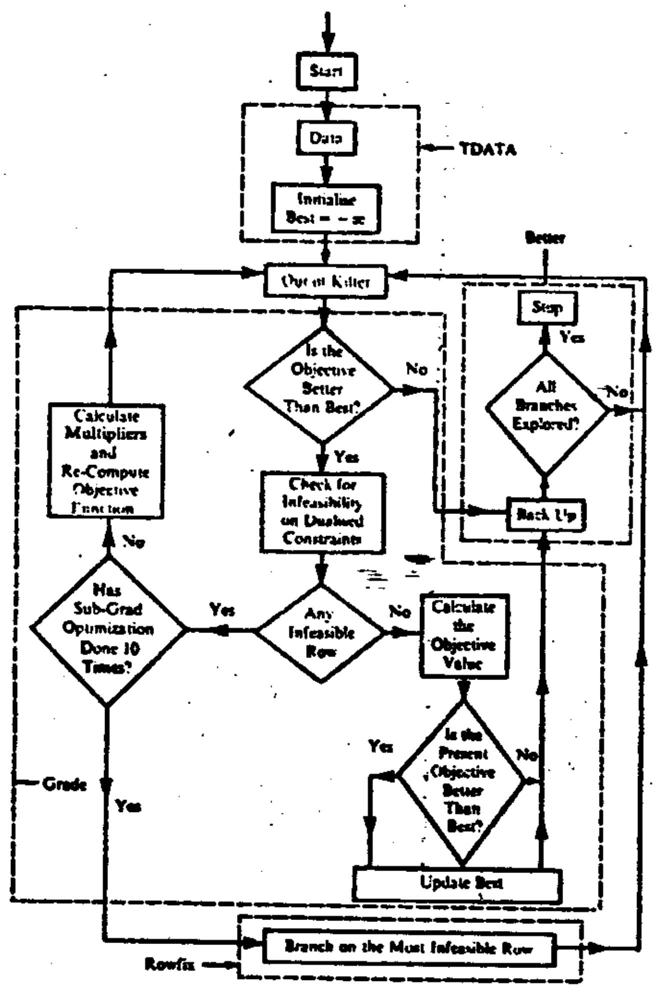
$$\text{subject to} \quad \sum_{j=1}^{NP} x_{ij} = NSPG_i \,; \quad i = 1, 2, \ldots, NSG, \qquad (PR_\pi\text{-I})$$

$$\sum_{i \in R_k} x_{ij} \leqq a_{kj} \,; \quad j = 1, 2, \ldots, NP, \quad k = 1, 2, \ldots, NRG, \qquad (PR_\pi\text{-II})$$

$$x_{ij} = 0, 1.$$

ANNEXURE - III

FLOW DIAGRAM OF THE TIME TABLING ALGORITHM



The ....Lines Enclose Various Subroutines

ANNEXURE - IV

SPONGE IRON INDUSTRY LOCATION PROBLEM

industry is formulated as follows:

$$\text{(P1) Min } Z = \sum_n \sum_k \sum_s \sum_i \sum_t F_{nksit}\, Y_{nksit}$$

$$+ \sum_k \sum_o \sum_i \sum_l C1_{koil}\; x1_{koil}$$

$$+ \sum_k \sum_p \sum_i \sum_l C2_{kpil}\; x2_{kpil}$$

$$+ \sum_k \sum_c \sum_i \sum_l C3_{kcil}\; x3_{kcil}$$

$$+ \sum_k \sum_g \sum_i \sum_l C4_{kgil}\; x4_{kgil}$$

$$+ \sum_k \sum_i \sum_j \sum_l C5_{kiji}\; x5_{kijl}$$

$$+ \sum_k \sum_m \sum_j \sum_l C6_{kmjl}\; x6_{kmjl}$$

$$- \sum_k \sum_i \sum_e \sum_l (C7_{kiel} - C8_{kiel})\; x7_{kiel}$$

s.t. 1. Demand constraints for finished product

a. Domestic

$$\sum_k \sum_i x5_{kijl} + \sum_k \sum_m x6_{kmjl} \geq D_{jl} \quad \forall\, j,l$$

b. Export

$$\sum_k \sum_i x7_{kiel} \geq D_{el} \quad \forall\; e,l$$

2. Supply constraints for finished product

a. Domestic Capacity Constraints

$$\sum_j x5_{kijl} + \sum_e x7_{kiel} \leq \sum_s A_{ks} \Big(\sum_n \big(\sum_{t \leq l} Y_{nksit}\big)\Big) \quad \forall\; k,i,l$$

b. Import supply constraints

$$\sum_j x6_{kmjl} \leq S_{kml} \quad \forall\; k,m,l$$

3. Not more than one n^{th} module for each k, s, i:

$$\sum_t y_{nksit} \leq 1 \quad \forall n, k, s, i$$

4. Precedence constraints between modules:

$$\sum_{\tau \leq t} y_{(n-1)ks i \tau} - y_{nksit} \geq 0 \; \forall n \geq 2, k, s, i$$

5. Maximum number of modules for given (k,s,i) combination:

$$\sum_n \sum_t y_{nksit} \leq 4 \quad \forall k, s, i$$

6. Not more than one technology-size type at a given location: $\sum_k \sum_s \sum_t y_{1ksit} \leq 1 \; \forall i$

7. Plants may be located in no more than M locations:

$$\sum_i \sum_k \sum_s \sum_t y_{1ksit} \leq M$$

8. Upper limit on over capacity for the country as a whole:

$$\sum_k \sum_s A_{ks} \sum_{t \leq l} \sum_i \sum_n y_{nksit} \leq (1 + f_l) \left(\sum_j D_{jl} + \sum_e D_{el}\right) \forall l$$

9. Constraints on availability of raw materials at sources.

   a) Iron ore lumps:

$$\sum_k \sum_i x1_{koil} \leq S_{ol} \quad \forall o, l$$

   b) Iron ore pelletes

$$\sum_k \sum_i x2_{kpil} \leq S_{pl} \; \forall p, l$$

   c) Coal

$$\sum_k \sum_i x3_{kcil} \leq S_{cl} \; \forall c, l$$

d) Natural gas

$$\sum_k \sum_i x4_{kgil} \leq S_{gl} \quad \forall \; g, l$$

10. Balance equations between raw material and finished product

a) Between iron ore lumps and sponge iron

$$\sum_o x1_{koil} = r1_k \left(\sum_j x5_{kijl} + \sum_e x7_{kiel}\right) \forall k, i, l$$

b) Between iron ore pellets and sponge iron

$$\sum_p x2_{kpil} = r2_k \left(\sum_j x5_{kijl} + \sum_e x7_{kiel}\right) \forall k, i, l$$

c) $$\sum_c x3_{kcil} = r3_k \left(\sum_j x5_{kijl} + \sum_e x7_{kiel}\right) \forall k, i, l$$

d) Between natural gas and sponge iron

$$\sum_g x4_{kgil} = r4_k \left(\sum_j x5_{kijl} + \sum_e x7_{kiel}\right) \forall k, i, l$$

11. Integer restrictions

$$y_{nksit} = (0, 1) \quad \forall \; n, k, s, i, t$$

12. Non-negativity restrictions

$$x1_{koil}, \; x2_{kpil}, \; x3_{kcil}, \; x4_{kgil} \geq 0 \; \forall k, o, p, c, g, i, l$$

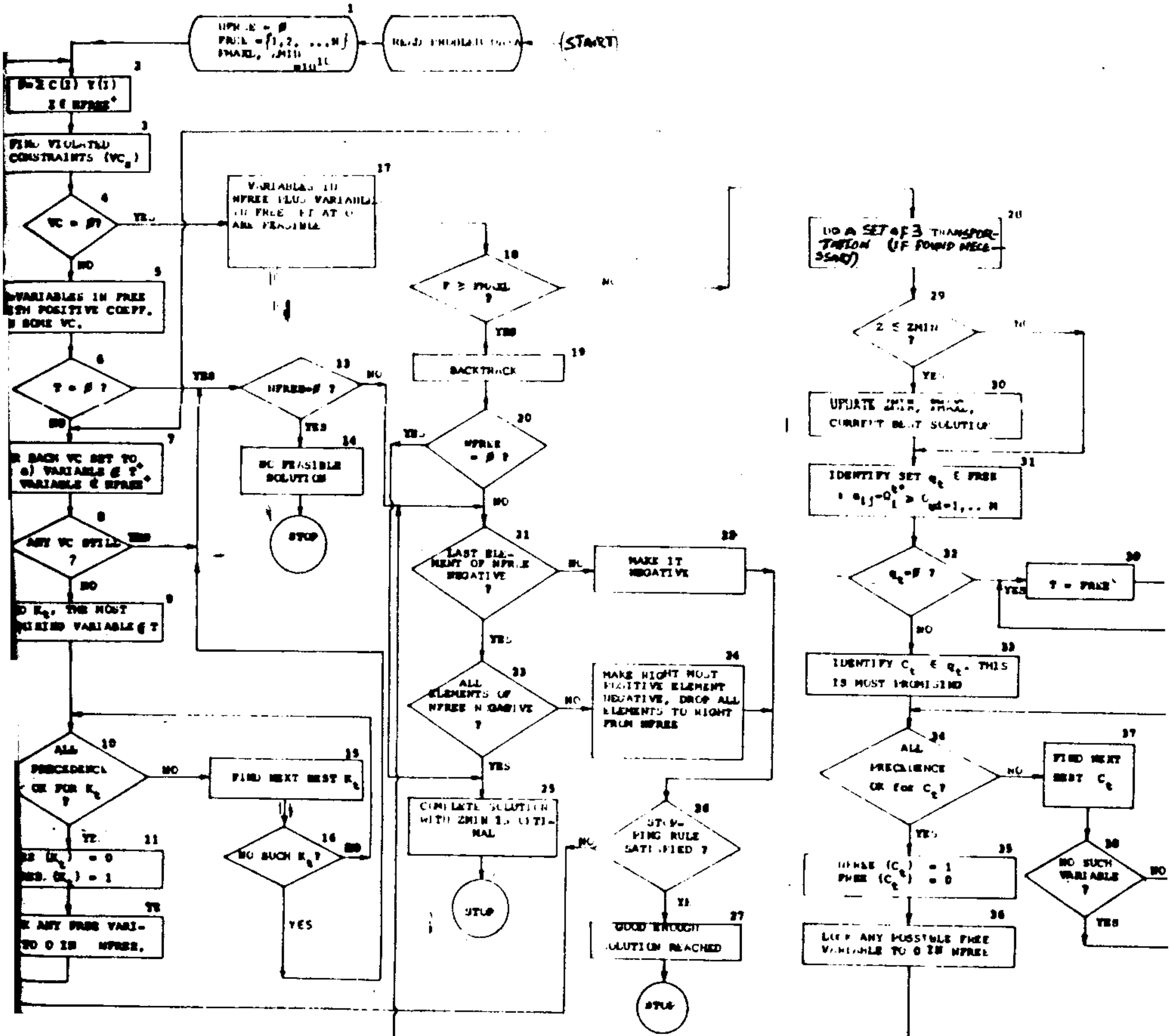
$$x5_{kijl}, \; x6_{kmjl}, \; x7_{kiel} \geq 0 \; \forall k, i, m, j, e, l$$

Subscripts

n	module number
k	index for technology
s	index for size of module
i	potential plant location index
t	set up time period
j	domestic market for sponge iron
l	production time period
m	source of import of sponge iron
e	destination of export of sponge iron
o	index for lump iron ore mine
p	index for pelletised iron ore source
c	index for coal mine
g	index for gas source

APPENDIX - V

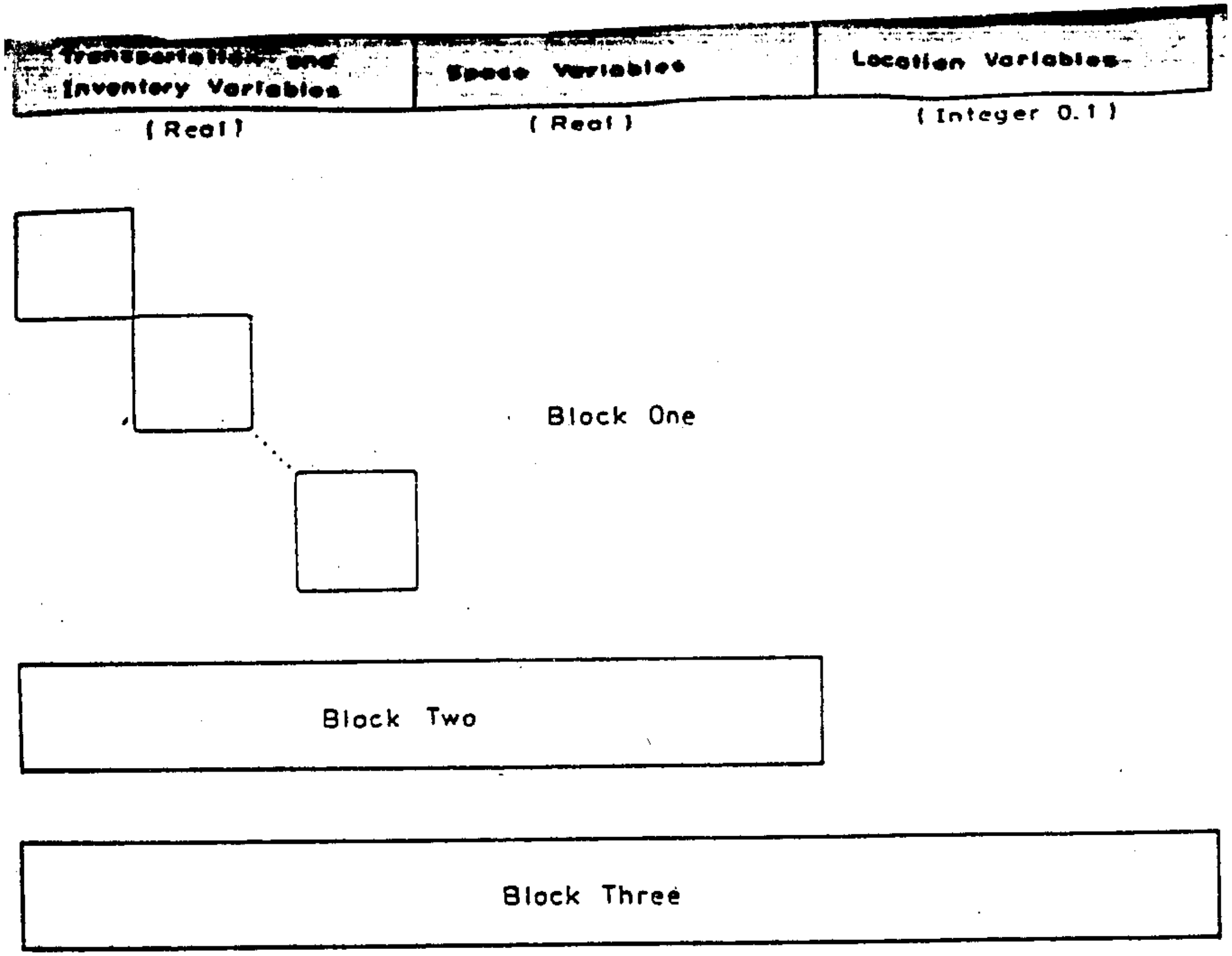
FLOW CHART OF THE LOCATION ALGORITHM



here t refers to the t th iteration

ANNEXURE - VI

STRUCTURE OF DISTRIBUTION AND WAREHOUSING PROBLEM

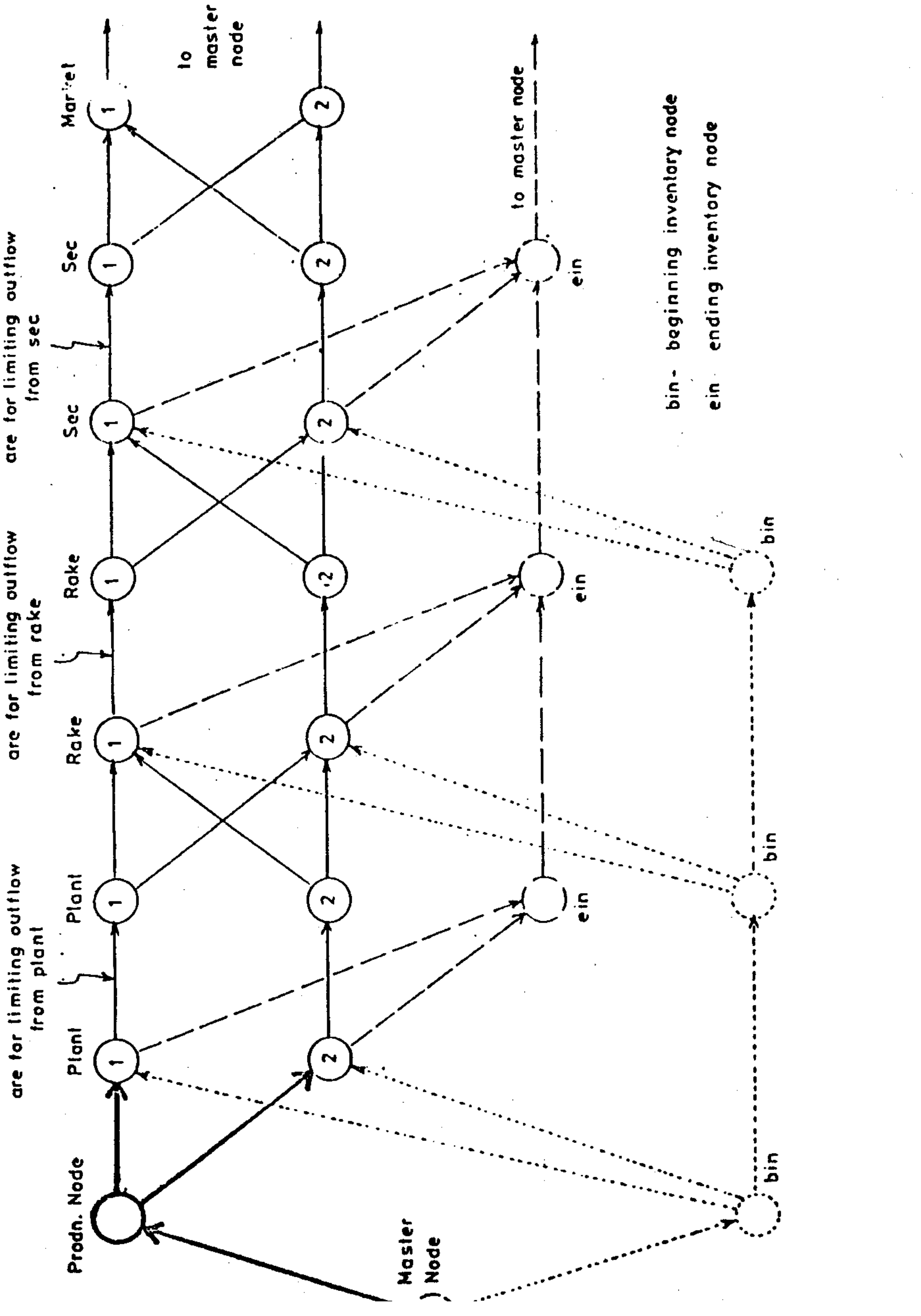


Block One	Out of killer subproblems
Block Two	All real constraints that link outkil and space variables
Block Three	These constraints link location variables and real variables

ANNEXURE - VII

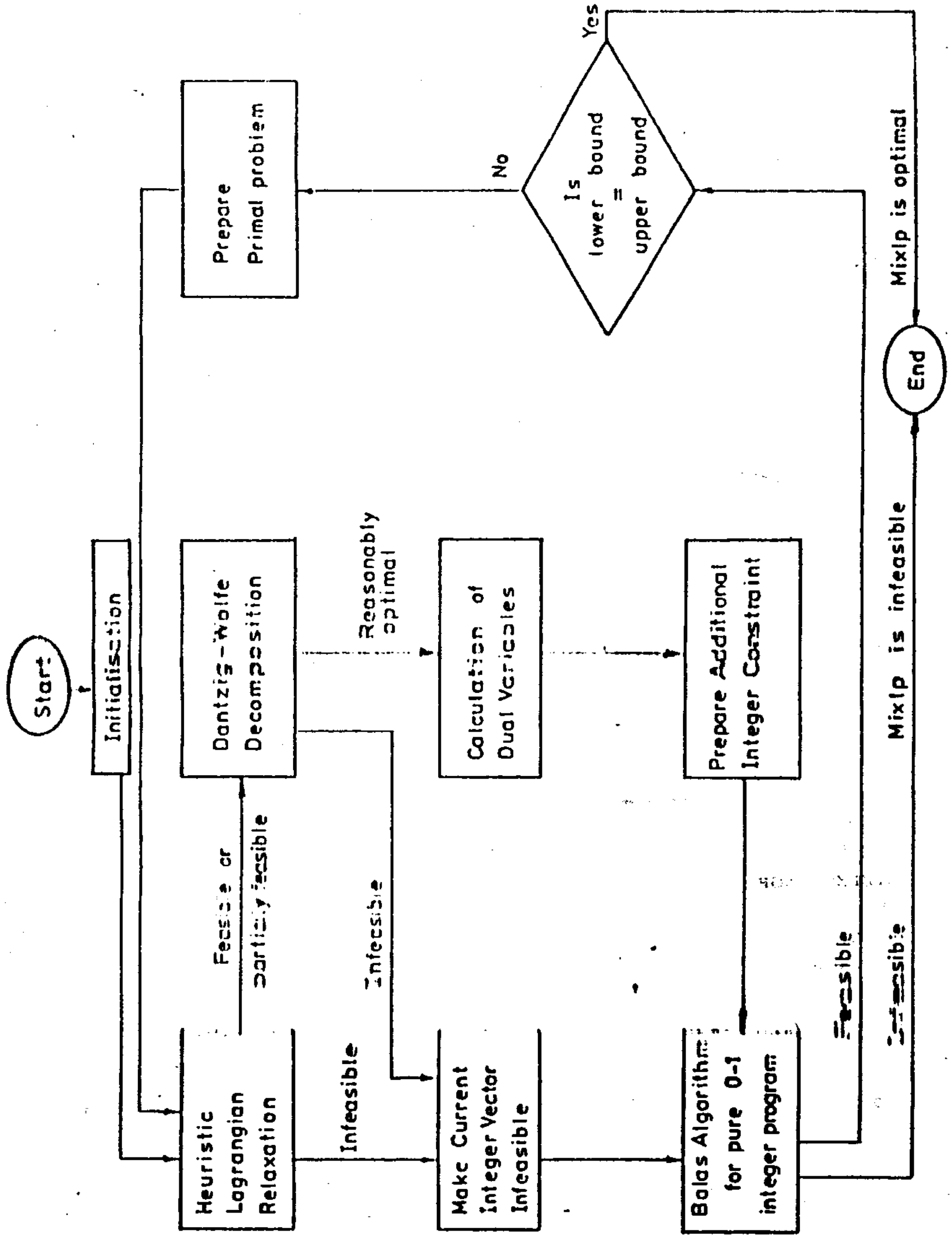
OUT OF KILTER SUBPROBLEM CONFIGURATION - WAREHOUSING AND DISTRIBUTION PROBLEM

(for a particular product and period)



ANNEXURE - VIII

FLOW CHART OF ALGORITHM FOR WAREHOUSING AND DISTRIBUTION PROBLEM



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