GROWTH OF NEW STOVES - A MODEL

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Abstract

Promotion of new wood stoves has been made into a nation-wide time-bound program. At present, setting of targets in each area proceeds somewhat arbitrarily. Generally, no account is taken of the mechanics of absorption. This paper deals with this problem. A mathematical model of the process is formulated. Under reasonable assumptions, it leads to a linear, time-invariant, discrete system. Possible growth patterns that emerge are illustrated. The model is easily built and easily solved. It can be helpful to those who plan, implement and monitor the program.

Rapid popularisation of efficient wood stoves (chulahs) has been made into a nation wide program. The goal has been converted into time-bound regional targets. In this paper a mathematical model of their possible growth pattern is formulated. Insights from the model are presented. These can help in making plans for an area, indicate what to look for in monitoring, so that besides ascertaining the extent of growth, feedback useful for further work is obtained.

Plans are usually made for 3 to 5 years at a time. Estimation of demand for the new stoves is one item that commonly presents difficulty. As a result targets are set on the basis of hunches. While some times, these may be unavoidable and even prove adequate, a degree of arbitrariness that remains associated with these is unsatisfactory. The motivation for development of model here is to suggest an alternative. It is felt that if one is able to understand the mechanics underlying the process of change, from traditional to improved ones, it may be possible to make more realistic plans.

Review of studies on introduction and acceptance of new stoves provides some clues and broad outlines of the process. It has been noted in several studies that adoption of new stoves is a slow process. Those who initially take to new stove may not always continue with it (1). On subsequent occasions some of them tend to revert to the old. The degree to which it occurs probably differs from place to place and is linked to specific designs. It is also plausible, that this may be a transitory phase. Keeping in view the above, we shall make a model of this process.

Mathematical Model

Consider a small region, may be a Block, where the program of promotion of new stoves is on.

Let,

H = total number of households in the Block, using fuel wood, assume each household uses one chulah

k = index of time (yrs), Ø,1,2 ....

X(k) = number of households having improved stove at time k

Y(k) = number of households having traditional chulah at time k

b = average life of improved stove (yrs); it will be assumed that, 1/b, fraction of stoves are retired each year

a = average life of traditional chulah (yrs), 1/a fraction to be retired each year

p = probability that those using traditional will stay traditional; given assumption (1) and (2) below, probability that traditionals will switch to improved one is (1-p).

Parameter, p, may be taken to characterise the behaviour of those using the traditional. It will be influenced by publicity about the new stoves, the fuel shortages etc. Comparative advantage of the new designs will also influence it. In particular, better the new one lower the p.

q = probability that one using improved stove will stay with improved one; accordingly probability that he will revert to traditional is (1-q).

The q can be said to characterise a given design. Intuitively, if new stoves have good attributes (saving in fuel, time, low priced and durable for instance) q will be high. A household taking to new stove will tend to stick to it. The reverse will also be true.

While p and q, will largely be independent, there may also be some relationship between them. p and q can be determined experimentally by taking a sample of both types of households. One can infer these values also from instances published in literature. Both parameters, in general, can also be expected to change with time.

Assumptions

1. The concerned region is relatively remote such that new options like LPG, biogas and even kerosene are negligible. This will leave only two options, the traditional and new stove.

2. Though the new stove come in several models with different attributes, presently these will be clubbed together.

3. Assume that the total number of households remains constant over a span of years, and so also the parameters p and q.

Change in state of the system (fig 1) can be described as follows:

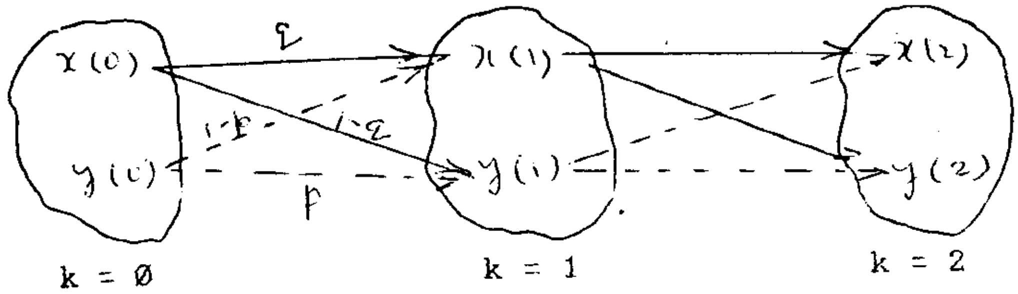

Fig 1 : State Diagram

New stoves

$$X\ (k+1) = (1 - 1/b)\ X(k) + q/b\ X(k) + (1-p)/a\ Y(k)$$

Traditional

$$Y\ (k+1) = (1-1/a)\ Y(k) + p/a\ Y(k) + (1-q)/b\ X(k)$$

rearranging

$$X\ (k+1) = (1-1/b+q/b) * X(k) + (1-p)/a * Y(k) \quad (1)$$

$$Y\ (k+1) = (1-q)/b) * X(k) + (1-1/a+p/a) * Y(k) \quad (2)$$

Equations (1) and (2) are first order difference equations. Given the assumptions, system is also linear and time-invariant. System is homogeneous. In otherwords it runs by itself and is not driven by any external force.

Note if we set p and q both equal to 1, above equations, as expected, reduce to

$$x\ (k+1) = x\ (k)$$

$$y\ (k+1) = y\ (k)$$

That is, whatever be the initial state, it will remain so for ever.

State equations can be written more compactly in matrix form,

$$X(k+1) = A\,X(k) \qquad (3)$$

where A is system matrix, X(k) state vector

$$A = \begin{bmatrix} 1-(1-q)/b & (1-p)/a \\ (1-q)/b & 1-(1-p)/a \end{bmatrix}$$

General solution of equation (3) can be written in matrix form as

$$X(k) = A^{k}\,X(\emptyset) \qquad (4)$$

or $$X(k) = \phi(k)\,X(\emptyset)$$

where $\phi(k)$ is transition matrix. Given the initial state vector, one can compute states at sucessive instants by raising the system matrix, A, to necessary powers. A less tedious way is to construct a diagonalising matrix by using eigenvalues of A and then use similarity transformation to obtain a diagonal form.

Use of z transform is yet another way to solve the state equations. The system here, being viewed as causal, one side z transform is appropriate and is given by

$$Z\left[f(k)\right] = \sum_{0}^{\infty} z^{-k} f(k) \qquad \ldots\ldots (5)$$

Transforming equation (3) yields

$$z\,x(z) - z\,x(\emptyset) = A\,x(z)$$

rearranging

$$x(z)\,[zI - A] = z\,x(\emptyset)$$

$$x(z) = (zI - A)^{-1} z\, x(\emptyset)$$

Taking inverse,

$$x(k) = z^{-1} [(zI - A)^{-1} z]\, x(\emptyset) \qquad \ldots\ldots (6)$$

Since, the present system is small one can invert the matrix (zI - A) easily. Then using partial fraction technique, one can **get** the inverse transforms, element by element. The response matrix is obtained in two convenient components given below, steady state and transient. This, when multiplied to initial state vector yields the solution.

$$P(k) = \begin{bmatrix} a12/m & a12/m \\ a21/m & a21/m \end{bmatrix} + (a11+a22-1)^{k} \begin{bmatrix} a21/m & -a12/m \\ -a21/m & a12/m \end{bmatrix} \ldots (7)$$

where m=a12+a21, aij are elements of A.

Note when k=Ø, the matrix reduces to

$$P(k) = \begin{bmatrix} 1 & \emptyset \\ \emptyset & 1 \end{bmatrix}$$

as it should.

The first matrix on RHS gives the steady state of the system. The second one gives the transient. For large values of k second matrix on RHS will tend to zero if the term in paranthesis is less than one. We will presently see that such is the case here.

Dynamics of System-Illustration

Consider a cluster of 1000 households of Sankheda block.

Initial conditions

traditional stoves Y(0) = 1000
improved stoves X(0) = 0.

$p = 0.70$ $\quad$ a = 1 year
$q = 0.50$, $\quad$ b = 3 years

Average life of traditional stove and new ones being promoted in Gujarat, was given by Gujarat Energy Development Agency (Baroda). The values of p, q are essentially arbitrary, though influenced by what can be inferred from some studies contained in (2).

Substituting the values, in eqn (3), we get

$$\begin{matrix} X(k+1) \\ Y(k+1) \end{matrix} = \begin{bmatrix} 0.83 & 0.30 \\ 0.17 & 0.70 \end{bmatrix} \begin{bmatrix} X(k) \\ Y(k) \end{bmatrix}$$

And the response matrix,

$$P(k) = \begin{bmatrix} .64 & .64 \\ .36 & .36 \end{bmatrix} + (0.53)^k \begin{bmatrix} .36 & -.64 \\ -.36 & .64 \end{bmatrix} \quad \ldots\ldots (8)$$

Fig (2a) shows the growth of new stoves (curve A) and decline of the traditional (curve A') as time advances. Number of improved stove users will increase gradually to 640. These will not register further increase. By that time the number of traditional stoves will decline from 1000 to 360, and remain there. This (640,360) is a equilibrium point of this system.

The equilibrium point is asymptotically stable. This will imply that no matter what the initial conditions are (except of course

Ø,Ø) the number of new and the traditional stoves will eventually settle at the equilibrium point. This can also be seen from the steady state component of matrix in eqn (8).

Practical implication of this feature may be of interest. Sometimes, attempt is made to saturate an area with new stoves. That is, all the traditional stoves are replaced at ones with new ones. While such efforts may be good for demonstration, it should not be expected that the area will stay saturated. It will not unless the reversion tendency (from new to traditional) is eliminated.

Let us examine the dynamics a little more closely. We have seen that given a new design (characterised by q=Ø.7Ø) only 64% of the households, can be expected to be using it on a long term basis. While it is good, one may wish to see a greater proportion of households acquire new stoves. Or may be, even desire to displace the traditionals completely and permanently. To achieve this two things will be necessary.

First, taking the new design as given, one can try to induce more of the traditionals to switch to it. If effective and wide publicity is carried out, one can expect it to manifest itself, by lowering the values of, p. That is the traditionals will become more prone to switch.

Assume that it has been possible to reduce it to Ø.35 i.e. half of the earlier. The resulting pattern is also shown in Fig 2a, curve B. It is seen now that new stove users will stablise at a higher value, 796. That is about 80% of the households will use the stove on long term basis. The traditionals of course will be the rest.

We can stretch this approach to its ultimate limit. Assume that publicity has been so effective and wide that everyone is induced to switch to new stove. The situation will imply that now p=0. The resulting graph (curve C) is shown in the figure. It shows that 85% of the households will be using it on long term basis.

This is the best that can be done with the given design. This shows that publicity alone, no matter how effective and wide, cannot achieve complete (and permanent) substitution of the traditional with the new.

Examination of curve C, also reveals another aspect that may be useful to those who monitor the progress of the program. The curve is oscillatory initially. The number of new stoves reaches a maximum of 1000 (i.e. all households) before settling to a lower steady state value. The maximum occurs one year after the start. If monitoring, is done just then it may yield a misleading picture. It is necessary therefore that monitoring program take into account the dynamics, and be so timed as not to pick the transient phase.

The solution readily yields the annual demand for the new stove under given conditions. These are shown in table 1 for all the conditions illustrated. For one situation, it is shown graphically in fig (2b). Demand is higher initially, as traditional households are switching over. Subsequently it declines asymptotically, stablising at the level equal just to the replacement requirement (214). Knowledge of demand is useful for the manufacturers/ dealers of such stoves.

Improved Designs

We shall now explore the increase that may be possible due to further improvement in design. As stated the quality of new stove is summarily characterised by the parameter, q.

If, a better design is evolved, for which say q=Ø.7, (against Ø.5Ø earlier) then the number of new stove will stabilise at a higher level 75Ø (Fig 3). Thus, the effect of increase in q is to increase the obsorption and holding capacity of the region for improved stove.

If yet another better stove is evolved, characterised by q=Ø.9, the number of new users will now stabilise at 9ØØ.

It can be demonstrated easily, and should be intutively obvious, that complete (and permanent) substitution will require that p=Ø and q=1, that is wide awareness of the need for better stove and existence of a very good design in the market.

Durability of new stoves, life, is also an aspect of quality. Naturally effect of enhancement in it has effect similar to one illustrated above. It has significant effect on absorption. Assume for instance that a model has emerged having a life of 6 years, i.e. twice as durable as the earlier one. Its effect can be readily seen by re-computing the steady state component of the response matrix (eq.7). It is

$$\begin{bmatrix} 0.78 & 0.78 \\ 0.22 & 0.22 \end{bmatrix}$$

Now 78% will be using the improved stove on a long term basis, as compared, to 64% when the life of stove was 3 years. It supports what one would expect intuitively to happen.

Refinement of Model

As stated, the motivation presently was to set down the possible mechanics of growth of improved stoves. The model will need to be refined further and ofcourse validated. The parameters (p and q) will need to be determined empirically.

Further refinement will need to admit the options also of change over to other fuels- LPG, biogas and kerosene. Accordingly more parameters will be needed.

It will also be necessary to discover the manner in which the parameters may change with time. Should these be found to change with time, the resulting equations will then become time-variant.

In the formulation of dynamic equations, equal fraction of stoves are retired each year. This will be good approximation of the situation if numbers are large and stoves of varying age uniformly present. It would lead to some errors if numbers are small. This needs to be improved upon.

Conolusions

Setting targets for replacement of traditional stoves done commonly through hunches needs to be changed. Mechanics of change needs to be understood better. Mathematical modelling can provide a means to do this.

The present effort has shown that under resonable assumptions, the process emerges as a linear, time-invariant discrete system. The parameters that characterise the model can be easily determined empirically by sampling. The model is easy to construct and easy to solve.

It enables one to generate, given a new design, the likely scenario - growth pattern - in a region. Solutions readily yield the annual demand for new stove. The nature of dynamics also has some lessons for monitoring.

The model can be useful to those who plan the program and those who implement it.

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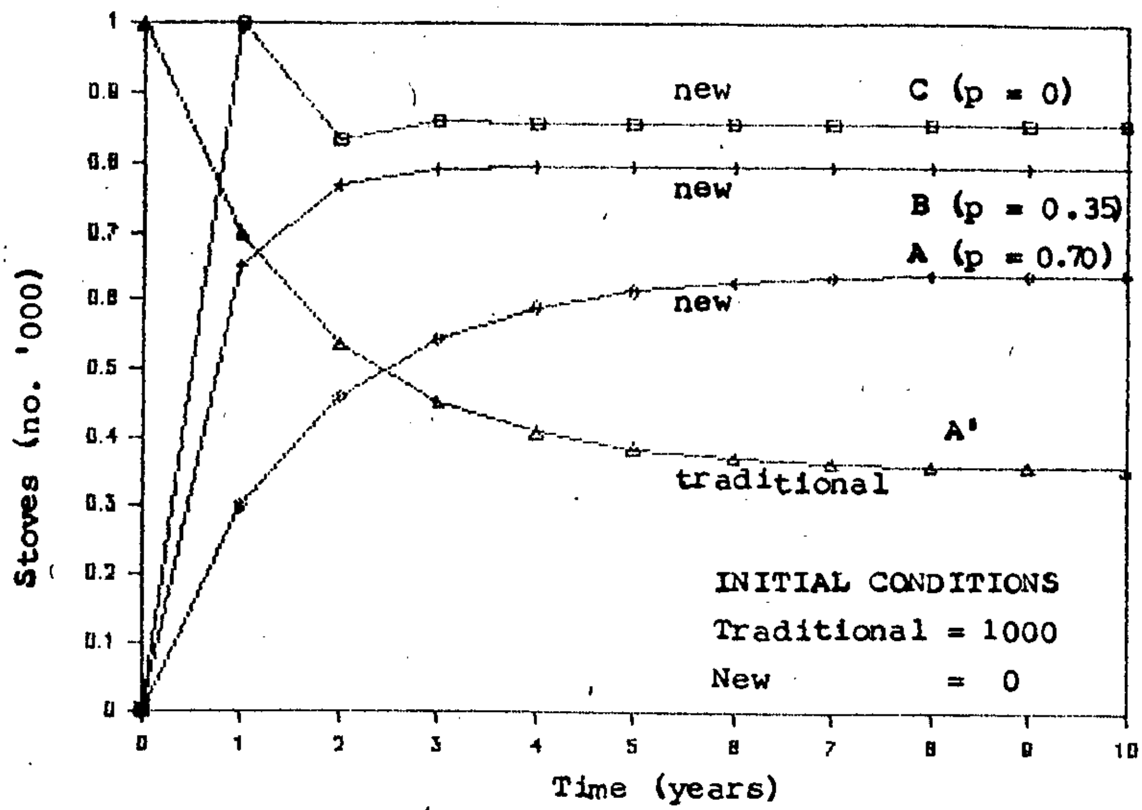

Fig 2(a): Dynamics of Growth-Simulated

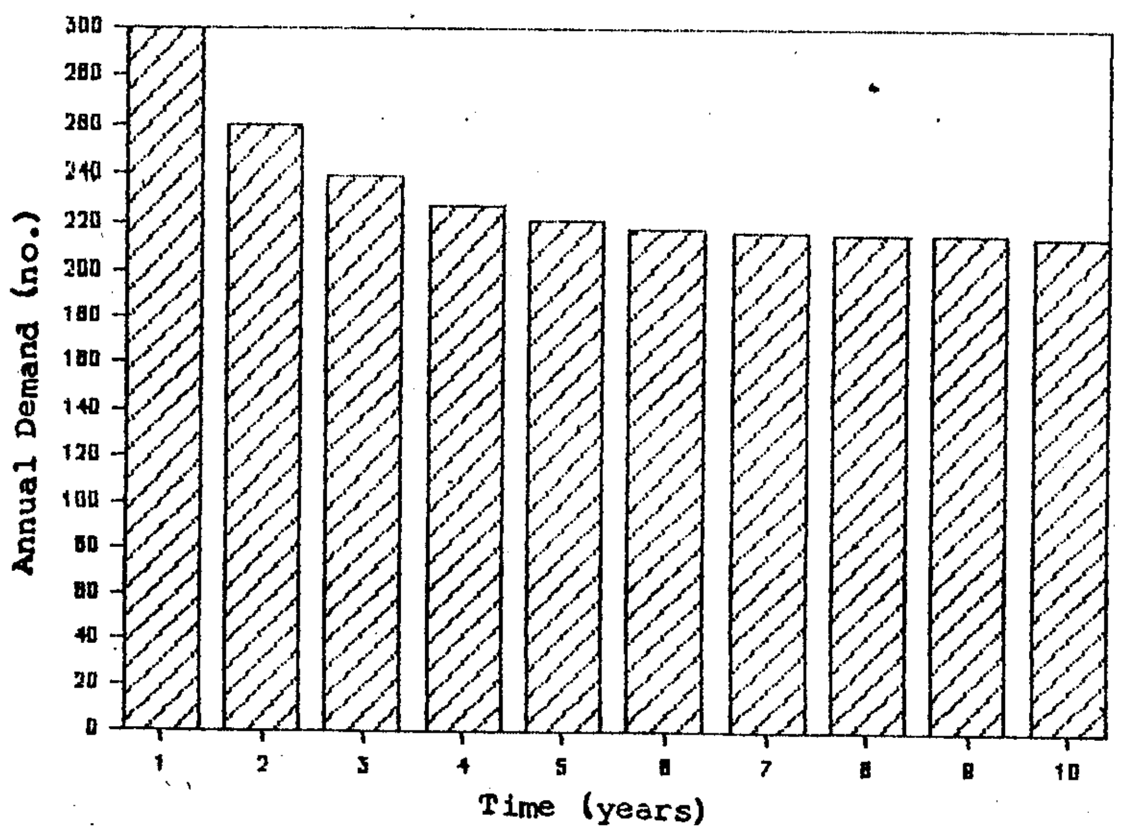

Fig 2(b): Demand Pattern-Simulated

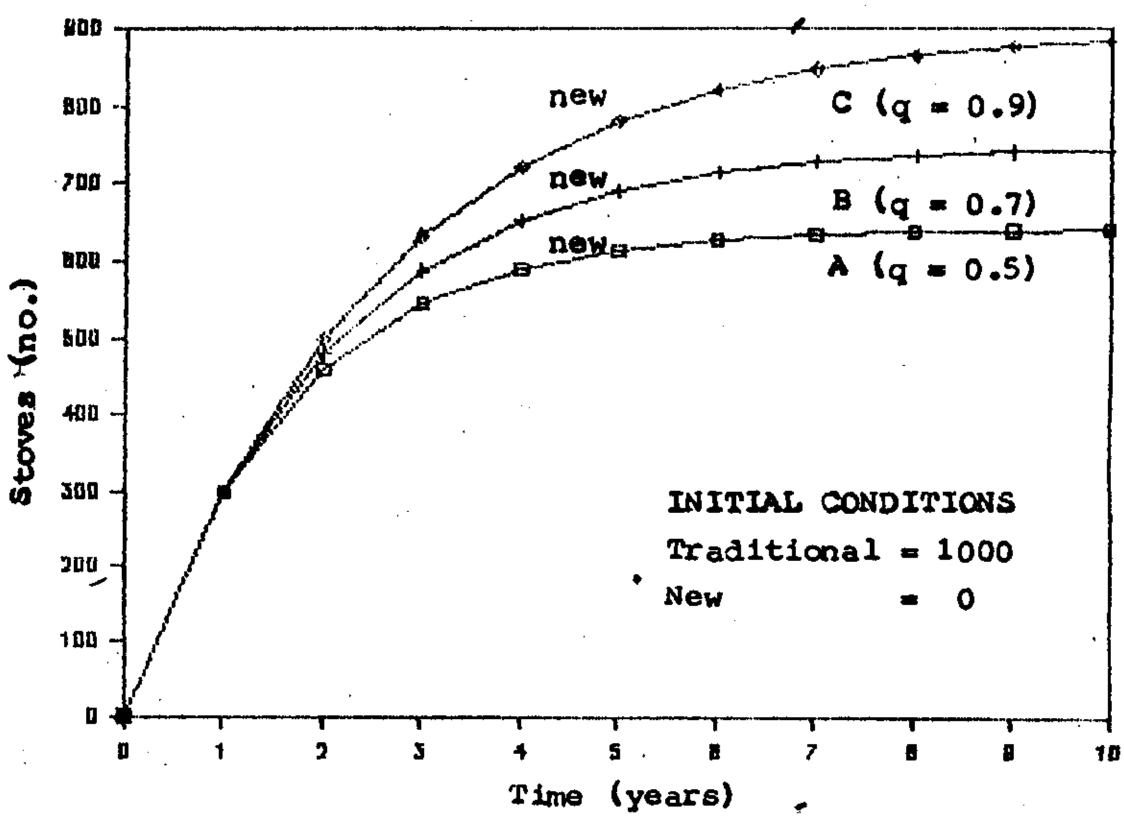

Fig 3 : Dynamics of Growth-Simulated

Table 1: Annual Demand for New Stoves- Simulated

Year	q=0.50			p=0.70		
	p			q		
	0	0.35	0.7	0.5	0.7	0.9
0	0	0	0	0	0	0
1	1000	650	300	300	300	300
2	167	336	260	260	280	300
3	306	278	239	239	268	300
4	282	268	227	227	261	300
5	286	266	221	221	256	300
6	286	265	218	218	254	300
7	286	265	216	216	252	300
8	286	265	215	215	251	300
9	286	265	215	215	251	300
10	286	265	215	215	251	300