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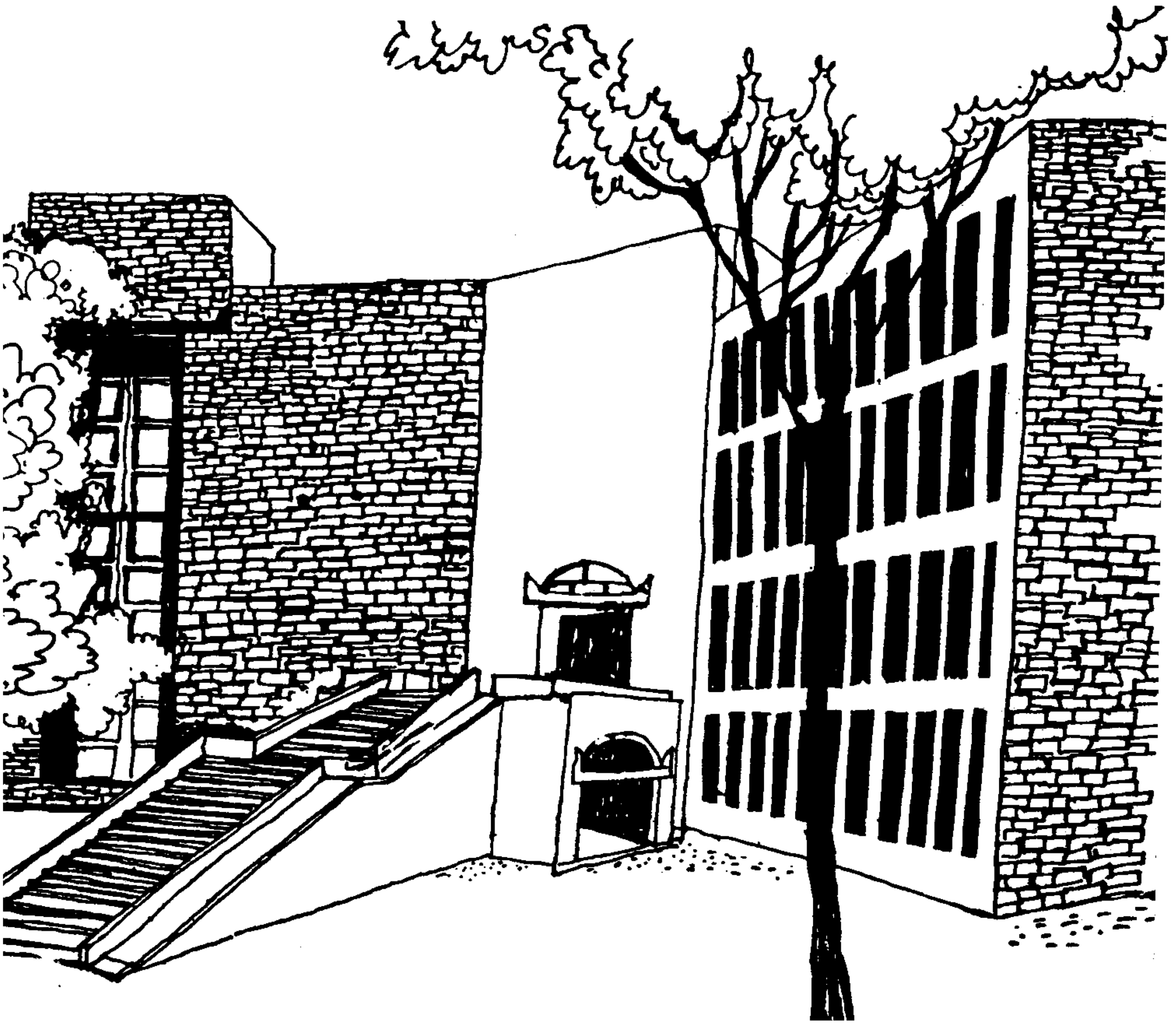

MONOTONIC SOLUTIONS TO PRODUCTION PLANNING PROBLEMS

By

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W P No. 1073
December 1992

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Abstract

A common problem in production planning for a public firm producing n distinct commodities in the same plant and facing a cost constraint, is the choice of the levels of output of the various commodities. This is a classical problem in the theory of publicly regulated firms and assumes significance in the absence of objective guidelines in determining the levels of output. It is to the study of such problems that we focus in this paper.

1. Introduction :- A common problem in production planning for a public firm producing n distinct commodities in the same plant and facing a cost constraint, is the choice of the levels of output of the various commodities. This is a classical problem in the theory of publicly regulated firms and assumes significance in the absence of objective guidelines in determining the levels of output.

The prototype model in microeconomic theory assumes that outputs are demand determined whether it be a private firm or a publicly regulated firm. If that were true without fail, then the above problem would of course never arise. However, there are instances when the market may not be able to determine output. Typically this may arise when information about demand for the outputs (both within as well as from outside the economy) are not available. This may arise when the demand functions are completely elastic; and they may also arise when cost constrains limit the outputs to levels below that which is required by the market. Such situations arise in the case of public firms more frequently than in the case of others. It is to the study of such problems that we focus in this paper.

2. The Model :- Let the commodity space of the public firm be $\mathbb{R}^n_+$ and let $C:\mathbb{R}^n_+ \to \mathbb{R}_+$ be a cost function which gives for each vector $y=(y_1, \ldots, y_n)$ of the n-outputs, the cost of production expressed in terms of money, which is faced by the firm. We assume that C is continuous, quasi-convex and strictly increasing [i.e. $y, y' \in \mathbb{R}^n_+$, $y < y' \Rightarrow C(y) < C(y')$] with $C(0)=0$. This guarantees that for each $c \geq 0$, the set $F(c)=\{y \in \mathbb{R}^n_+ / C(y) \leq c\}$ is a compact, convex, comprehensive [i.e. $y \in F(c), 0 \leq y' \leq y \Rightarrow y' \in F(c)$] subset of $\mathbb{R}^n_+$. In particular the fact that C is strictly increasing, implies that $\forall c > 0, F(c)$ satisfies <u>minimal transferability</u> [i.e. $y \in F(c)$, $y_i > 0 \Rightarrow \exists\ y' \in F(c)$ with $y'_i < y_i$ and $y'_j > y_j\ \forall j \neq i$] (see Moulin (1988)). For all $c \geq 0, F(c)$ is the set of feasible output vectors for the cost function C. We shall also refer to C as a production planning problem.

Given a production <u>planning problem</u> C, we say that a solution is a function $g:\mathbb{R}_+ \to \mathbb{R}^n_+$ such that for all $c \geq 0, g(c) \in F(c)$.

We say that a solution g is efficient if for all $c \geq 0, y \in F(c), y \geq g(c)$ implies $y = g(c)$.

We say that a solution g is monotonic if $c \geq c' \Rightarrow g(c) \geq g(c')$.

Efficiency says that it should not be possible to produce more than what the solution prescribes for a given cost constraint; monotonicity says that by relaxing the constraints we could not be producing less.

Our endeavour in this paper will be to characterize, monotonic solutions to production planning problems. Towards that end we require the following definition (see Keiding and Moulin (1991)).

Definition :- A monotone path is a continuous function $\Gamma : \mathbb{R}_+ \to \mathbb{R}^n_+$ such that

(i) $\Gamma(0)=(0,\ldots,0)$

(ii) $\forall i, \Gamma_i$ is non-decreasing : $t>t' \Rightarrow \Gamma_i(t) \geq \Gamma_i(t')$

(iii) Γ is 1-1.

3.The Main Result :- In this section we present our main result.

<u>Theorem</u> :- Given a production <u>planning problem</u> C, a solution g which is efficient, satisfies monotonicity if and only if it is monotone path.

<u>Proof</u> :- The fact that a solution which is a monotone path satisfies monotonicity is obvious. We thus turn to the converse. Property (i) of the definition is obvious for `g' by feasibility. Property (ii) follows from monotonicity.

Suppose $c < c'$ and $g(c)=g(c')$

$\therefore c=C(g(c))$ by efficiency of g and continuity of C.

$c'=C(g(c'))$ by a similar argument.

$\therefore c=C(g(c))=C(g(c'))=c'$, a contradiction.

Thus g is 1-1 and property (iii) is verified.

It only remains to verify continuity.

For all $c, c' \geq 0$, $|C(g(c))-C(g(c'))| = |c-c'|$.

Let $\epsilon>0$ be given and suppose towards a contradiction g is not continuous at 'c'. Then there exists a sequence $\{c_m\}_{m\in\mathbb{N}}\subset\mathbb{R}^n_+$ such that $\lim_{m\to\infty} c_m=c$ and $\|g(c_m)-g(c)\|\geq\epsilon$ $\forall m\in\mathbb{N}$. Without loss of generality assume $\{c_m\}_{m\in\mathbb{N}}$ monotonically decreases to c (a similar argument works if $\{c_m\}_{m\in\mathbb{N}}$ monotonically increases to c). Thus $g(c_m)\geq g(c)$ $\forall m\in\mathbb{N}$ and further for some $\epsilon'>0, g(c_m)\geq g(c)+\epsilon' e$ $\forall m\in\mathbb{N}$. Since C is strictly monotonically increasing

$c_m=C(g(c_m))\geq C(g(c)+\epsilon' e)>C(g(c))=c$ $\forall m\in\mathbb{N}$.

$\therefore \lim_{m\to\infty} c_m\geq C(g(c)+\epsilon' e)>c$, contradicting $\lim_{m\to\infty} c_m=c$.

Hence the theorem.

Q.E.D.

4. Conclusion :- In this paper we have discussed and established that monotone solutions to production planning problems are in fact a monotone path. The importance of this solution and its characterization arises mainly because it prevents management in public firms from taking politically biased decisions which let the favored activities produce proportionally more than unfavored activities thereby negating balanced growth.

We could have formulated our problem in a more general setting by considering the class of problem $\{(C,c)/c\geq 0$ and $C:\mathbb{R}^n_+\to\mathbb{R}_+$ is continuous, quasi-convex and strictly increasing with $C(0)=0\}$, and defined a solution appropriately on this class. Since in our context, the cost function is assumed fixed and our hypothesis about monotonicity refers to variation in the constraint, we have pursued the formulation presented earlier. Defining $F(C,c)=\{y\in\mathbb{R}^n_+ /C(y)\leq c\}$ and requiring a solution $g(C,c)$ to belong to $F(C,c)$ we could have required efficiency and monotonicity [i.e. $F(C,c)\subseteq F(C',c')\Rightarrow g(C,c)\leq g(C',c')$] of our solution. But then the characterization of such a solution is a standard problem in axiomatic bargaining models [see Moulin (1988)].