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A Mathematical Programming Approach with Revenue Management in Home Loan Pricing

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Abstract

We formulate a dynamic pricing model for home loans for a bank. The model optimizes the net present value of money available at the end of 15 years subject to pricing limits and cash flows. We collected the real data from a leading nationalized bank in India to develop the relationship between interest rate (price) and number of loans sanctioned (demand). We then assume different versions of the demand function (linear, exponential and rectangular hyperbolic). We also develop the relationship of default probability as a function of interest rate. In all the three cases, with the real data we demonstrate that the dynamic pricing of home loans does yield better results than the currently used static pricing. We also discuss the sensitivity of our result to change in the parameters of the demand equations.

Keywords: Revenue Management, Pricing, OR in banking, demand estimation, non-linear optimization.

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1. Introduction

Revenue management and dynamic pricing (RMDP) is a proven management science technique that helps any firm or service sector to maximize revenue, i.e., increase its top line revenue and bottom line profitability. The key to revenue management is market segmentation and charging different prices to different segments based on the willingness to pay of each segment. Revenue management, being used by several perishable services companies, is based on optimal dynamic pricing, forecasting and decision theory. It is based on maximizing revenue through pricing and is used in several services industries. This is done by charging differential pricing to different customer segments based on their willingness to pay. This trade-off needs to be made for each combination of customer segment, product, and channel and continually updated with the changes in the market conditions.

There are various approaches today which are being used for pricing of home loans. There are several publications in the risk based approach (Phillips and Rhode, 2006), competitor based pricing (Marks, 1987), and relative performance factor approach (Hoskins, 1990). However, none of these studies focuses on revenue maximization. The proposed model in this paper is a multi-period non-linear home loan pricing model based on revenue optimization. In this paper, we use concepts of ordinary least squares (OLS) statistical regression to develop a demand price relationship and use nonlinear optimization to optimally price the home loan products for n periods. Then we conduct a comparative study by using dynamic and static pricing strategies and also conduct a study on the demand price sensitivities.

This paper makes a contribution in the following areas:

- 1) This is possibly one of the initial attempts in using revenue management and dynamic pricing in the home loan application process with the demand function estimated from real data from a leading Indian nationalized bank.
- 2) We maximize the net present (future) value of all flows at the end of the home loan cycle.
- 3) We attempt to maximize the revenue using three types of demand functions, viz., linear, exponential and hyperbolic.
- 4) We consider maximizing the revenue at the end of n periods taking into consideration the default probability, which is directly proportional to the rate of interest.
- 5) We also present the sensitivity analysis with respect to the demand functions.
- 6) In all cases of demand functions, we show that dynamic pricing is better than static pricing.

This paper is organized as follows. The second section deals with the literature survey on different methods of loan pricing, the benefits of employing revenue management, differences in the application of revenue management in finance and other sectors. The third section consists of the proposed model formulation of lending home loans. The fourth section presents the multi-period optimization model with sub-sections listing the assumptions, the objective, parameters and variables, the constraints, the steps involved in developing the pricing model and the model formulation and the development of the three different demand functions, viz., linear, exponential and hyperbolic and the default probability relationship. Various experimentations of the optimization model are discussed in the fifth section. The sixth section consists of the conclusion and managerial implications. The seventh section consists of future work followed by references and appendices.

1. Review of related literature

According to a study by Phillips (2005), optimal pricing or right pricing is a challenging assignment. In case the price is too high, one may be on the reverse side of the demand curve or the sales volume may be too low. So the seller has to work hard to sell the good. In case the price is too low, then one may get the volume, but may not break even. We now discuss the different methods for estimating demand and also the methods used for pricing loans.

Tobie and Houthakker (1950) discuss the effects of rationing on demand elasticities. Kent (1980) analyzes the importance of credit rationing in the home mortgage market by giving statistical evidence on how the aggregate demand for home mortgage funds and aggregate proportion of owner occupiers is affected by loan maturity. According to credit rationing, mortgages are rationed on the basis of default risk which is empirically tested by Duca and Rosenthal (1991) using time series data. Further, Rosenthal, Duca and Gabriel (1991), apply cross-sectional data to test whether default risk-induced credit constraints affect the demand for owner-occupied housing. Chakravarty and Scott (1999) show that the relationship between individual households and a potential lender significantly lowers the probability of being credit-rationed and further determine the interest rates of consumer loans by examining the role of the relationships. In case of consumer lending, Phillips and Raffard (2011) explain differential price-sensitivity between lenders who will default and who will not default, using price-driven adverse selection. This method also explains credit rationing in commercial credit markets. However, the demand for home loans depends on credit rationing by the banks, in this paper we estimate the demand from the past data as a function of interest rate.

According to a study by Phillips and Rhode (2006), risk based pricing is considered to be one of the best methods, where customers are separated by willingness to pay. According to this study, it would be profitable to identify higher-risk customers and to charge them higher rates to offset the possible higher losses. However, customer price sensitivity, which is one of the key elements required to maximize financial return, cannot be incorporated using risk based pricing.

Marks (1987), discusses the pricing done to match a competitor's price which may result in a loss of revenue due to redundant under-pricing. Hoskins (1988) outlines an approach on how to price loans to increase profits by determining the expected return on assets (ROA). The spread over costs needed to provide the required ROA is determined. Three major categories of loans – real estate, instalment, and commercial – plus investments are analyzed for their contribution toward ROA. Operating expenses decrease as the loan size increases. Hoskins (1990) mention a tool developed by the Federal Reserve that helps bankers to accurately assess the cost of money and gives a guide for pricing long term loans. One approach to improving loan pricing is the relative performance factor (RPF). RPF is a relative measure which allows a bank to rank commercial loans based on their profitability, thus making it easier to find out which loans need attention. It is composed of capital, risk, operating expenses and cost of funds. The prime plus one half method of pricing mentioned by Owens et al. (1991) is another easy method but it may not necessarily maximize revenue. A publication by Ulrich (1980), states that a bank can price its loans based on the profitability of a customer's relationship with it. It is a cost accounting approach in which the profitability of a commercial customer's relationship with a bank can be measured, including loans, deposits, and other services. Finn et al. (1992) discuss compensable pricing which is the customers' favourite as they are allowed to save money by doing something that is of value to the bank. Seshadri et al. (1999) present a methodology that assists the process of asset-liability selection in a stochastic interest rate environment. Hannagan (2002), in his paper, has cited that banks successfully use a loan-pricing approach for commercial lending report that they benefit in several ways either directly or indirectly to enhance earnings. A good loan-pricing model is able to bring the bank's lending costs, risks, funding, and loan accounting considerations to the individual loan level in a reasonable and consistent manner. A recent study by Bent et al. (2017) study the various lending techniques and loan pricing models for foreign and domestic banks and explore the difference between them.

There are several studies on commercial loan pricing and the various approaches that are being used for pricing of loans like the risk based approach, competitor based pricing, and relative performance factor approach. However, there is no focus on revenue maximization. Although there are several applications of revenue management in other sectors, we did not find a lot of literature that discusses the application

of revenue management principles in home loan pricing and demonstrates its potential impact. In this paper we focus on home loan pricing which in turn has an impact on revenue maximization for banks. This paper makes the first attempt in using dynamic pricing of interest rates for home loans using real data from the Indian banking sector and also studying the impact on the home loan process using static pricing and dynamic pricing of interest rates.

2. Proposed Home Loan Pricing Model

The bank makes a quote available for different home loan amounts and periods. The customer either chooses to apply for the loan after seeing the quote or does not apply for the loan resulting in a lost quote. The applications received by the bank are either accepted or declined depending on the risk characteristics of the customer. If a customer is more risk prone but within the risk limit of the bank, it then quotes a higher rate of interest and if the customer agrees to it then it results in the sanctioning of the loan, else if the customer does not agree to the new rate of interest, we say it is a failed sale. The following figure 1 depicts the home loan process.

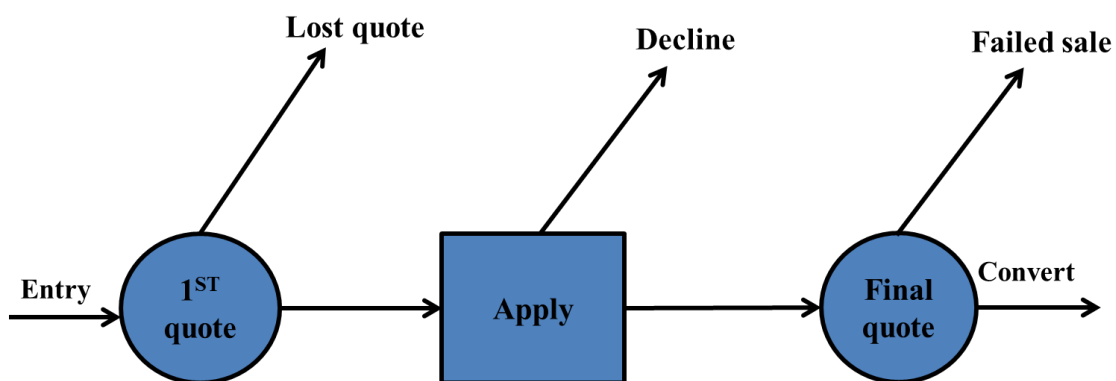


Figure 1: Home loan process (Phillips, Pricing and Revenue Optimization 2005)

Let's assume that in the n^{th} period, x_n people are willing to take the home loan being offered at the first quote. There is a p_1 probability that they would not apply. Out of the total number of $(1-p_1)*x_n$ people who apply for the loan, the probability that they would be declined is p_2 . Thus, now the number of applications accepted is $(1-p_1)*(1-p_2)*x_n$. When the bank makes the final quote of the rate of interest, the probability that the customer won't accept and it would result in a failed sale is p_3 . Finally, the number of loans sold by the bank would be $(1-p_1)*(1-p_2)*(1-p_3)*x_n$, when the demand for the loan was x_n . The process is as shown in figure 2 and 3.

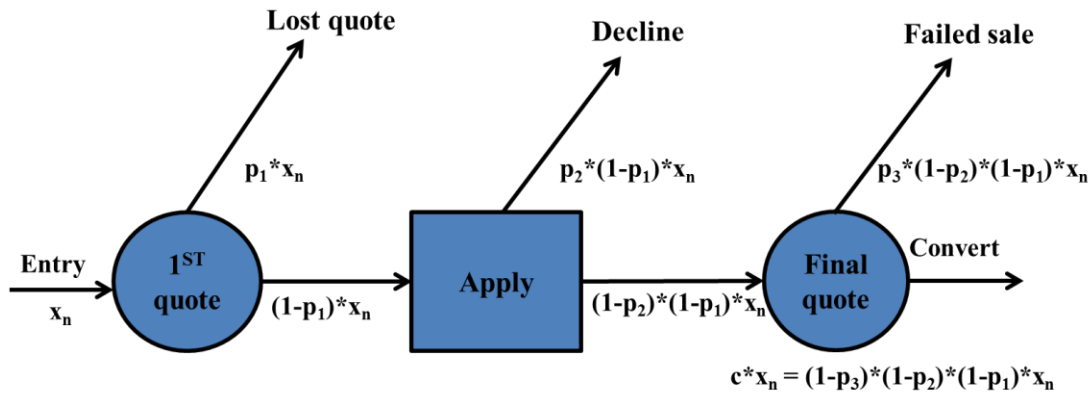


Figure 2: Home loan process depicting probabilities of decline at each stage



Figure 3: Home loan process as a black box

3. Multi Period Optimization Model

After surveying the existing literature on the application of revenue management in the financial sector, we propose a model using revenue management for home loan pricing with dynamic pricing model. This model is based on maximizing the expected revenue of the bank providing the loan. Here we consider one particular market segment only.

4.1. Assumptions:

Following are the assumptions made:

- (i) We assume a linear relationship of interest rate versus the number of people taking the loan as can be extended to other demand functions (linear demand function with negative slope).
- (ii) We also assume a linear relationship of interest rate versus default probability, such that, the default probability increases as there is an increase in the interest rate.

- (iii) We assume that the bank sanctions a loan of Indian Rupees (INR) 2.5 million for a period of n months. The typical value of the period is $n = 180$ months, i.e., 15 years and we maximize the expected revenue of the bank after 30 years. The bank has a total amount of INR 5 billion to lend home loans.
- (iv) The amount not paid by defaulters is accounted for as loss, assuming the bank has not mortgaged the house.
- (v) The default probability depends only on the rate of interest of that month and is independent of the individual customer or the number of instalments that he is paying.
- (vi) The internal discount rate is a constant.
- (vii) The instalment to be paid in each period is a constant.

The cash flow each month is due to:

- (i) The amount remaining after the transactions of last month.
- (ii) The instalments paid by customers that month taking into account the defaulters.
- (iii) The amount sanctioned to customers that month in the form of home loans.

4.2. Steps for Model Building

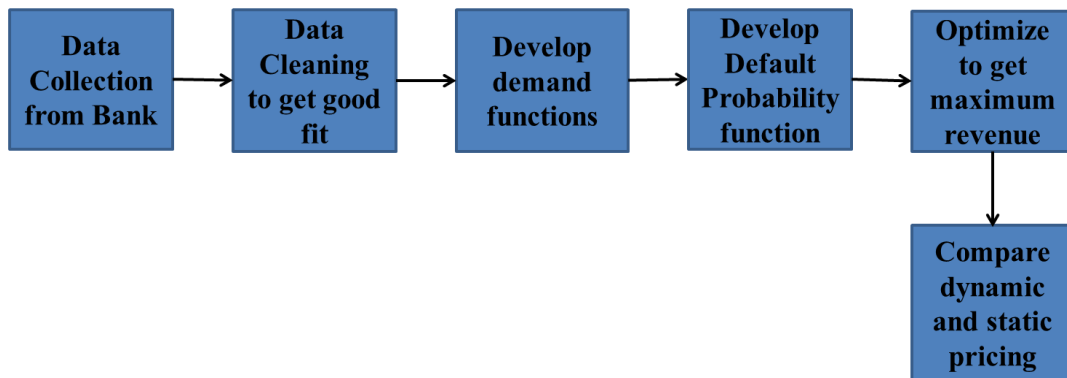


Figure 4: Block diagram representing the steps to model the problem

- (i) We consider the assumptions as above.
- (ii) We then evaluate different options through a sensitivity analysis of the optimization model.

- (iii) We use AMPL (Fourer, 1990) with Minos (AMPL version 12) solver for obtaining the optimized solution to this model which is a multi-variable model with a quadratic objective function and one quadratic constraint.
- (iv) We then compare the revenue we obtained through the dynamic and the static pricing models.

4.3. Parameters and Variables

Parameters:

i = time period indexed by i such that $i = 1$ to i_{\max} ($i_{\max} = 360$)

n = number of periods (here, $n = 180$ months for 15 years home loan) indexed by I ($n > 0$)

α = overall internal discount rate (assumed constant in a 15 year period $n = 180$) ($\alpha \geq 0$)

K = amount of money available at the beginning of $2n$ months ($2n = 360$) ($K \geq 0$)

P = monthly instalment (assumed to be the same for each month period) paid to the bank by each customer

Q_i = amount of money available to the bank at the end of the i^{th} period (revenue) ($Q_i \geq 0$ for all i)

L = total home loan amount (assumed to be INR 2.5 million) sanctioned to one customer ($L \geq 0$)

p_1 = probability of lost quote

p_2 = probability of declining the application by bank

p_3 = probability of failed sale

$c = (1-p_1)*(1-p_2)*(1-p_3)$ such that $0 \leq c \leq 1$

a_i = demand equation parameter for the time period I ($a_i \geq 0$ for all i)

b_i = demand equation parameter for the time period I ($b_i \geq 0$ for all i)

e = default probability equation parameter ($e \geq 0$)

d = default probability equation parameter ($d \geq 0$)

p_D = default probability such that $0 \leq p_D \leq 1$ for all I

Variables:

x_i = demand or number of people applying for home loans in the beginning of the i^{th} period

Decision Variable:

I_i = rate of interest in the i^{th} period (will vary in each period)

4.4. Model Formulation

The cash flow is as shown in figure 5.

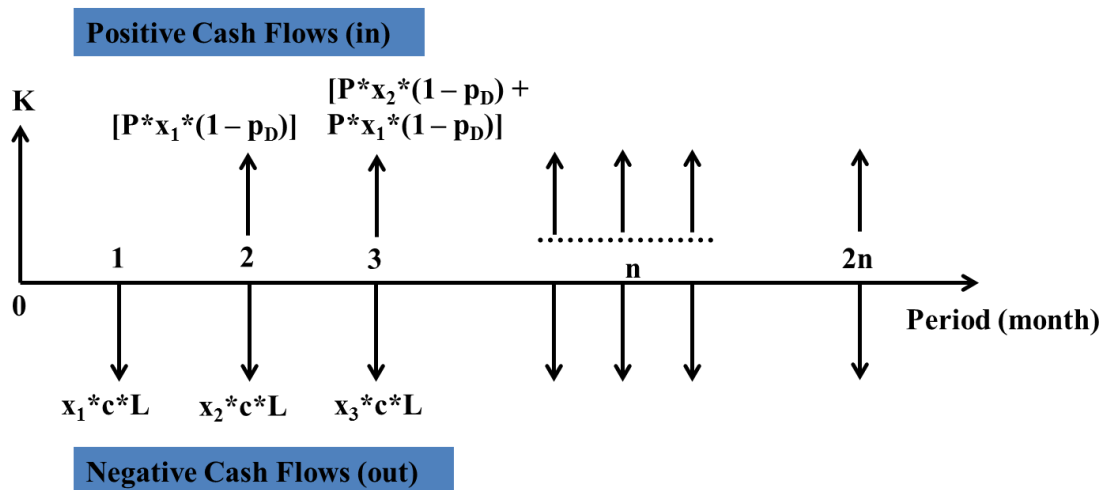


Figure 5: Cash flow diagram for 2n period

Transactions for the first month:

The amount available at the beginning of the 1st month = K

The amount sanctioned in the 1st month = $x_1 * c * L$

Thus, the amount available at the end of the 1st month =

$$Q_1 = K - x_1 * c * L \quad \text{----- (1)}$$

Transactions for the second month:

The amount available at the beginning of the 2nd month = Q_1

The amount sanctioned in the 2nd month = $x_2 * c * L$

The amount recovered via instalments = $P * x_1 * (1 - p_D)$

Thus, the amount available at the end of the 2nd month =

$$Q_2 = Q_1 - x_2 * c * L + P * x_1 * (1 - p_D) \text{ ----- (2)}$$

Transactions for the third month:

The amount available at the beginning of the 3rd month = Q_2

The amount sanctioned in the 3rd month = $x_3 * c * L$

The amount recovered via instalments = $P * x_2 * (1 - p_D) + P * x_1 * (1 - p_D)$

Thus, the amount available at the end of 3rd month =

$$Q_3 = Q_2 - x_3 * c * L + P * x_1 * (1 - p_D) + P * x_2 * (1 - p_D) \text{ ----- (3)}$$

Thus, generalizing, we get

$$Q_n = Q_{n-1} - x_n * c * L + P * (1 - p_D) * (\sum_{i=1}^{(n-1)} x_i) \text{ ----- (4)}$$

Transactions for the (n+1)th month:

The amount available at the beginning of (n+1)th month = Q_n

The amount sanctioned in the (n+1)th month = $x_{n+1} * c * L$

The amount recovered via instalments = $P * (1 - p_D) * \sum_{i=1}^n x_i$

Thus, the amount available at the end of (n+1)th month =

$$Q_{n+1} = Q_n - x_{n+1} * c * L + P * (1 - p_D) * \sum_{i=1}^n x_i \text{ ----- (5)}$$

Transactions for the (n+2)th month:

The amount available at the beginning of (n+2)th month = Q_{n+1}

The amount sanctioned in the (n+2)th month = $x_{n+2} * c * L$

The amount recovered via instalments = $P * (1 - p_D) * \sum_{i=2}^{(n+1)} x_i$

Thus, the amount available at the end of (n+2)th month =

$$Q_{n+2} = Q_{n+1} - x_{n+2} * c * L + P * (1 - p_D) * \sum_{i=2}^{(n+1)} x_i \quad \text{----- (6)}$$

Thus, the transactions for n months are as follows:

$$Q_1 = K - x_1 cL$$

$$Q_2 = Q_1 - x_2 cL + Px_1(1 - p_D) = K - cL \sum_{i=1}^2 x_i + Px_1(1 - p_D)$$

$$Q_3 = Q_2 - x_3 cL + Px_1(1 - p_D) + Px_2(1 - p_D) = K - cL \sum_{i=1}^3 x_i + P(1 - p_D) \sum_{i=1}^2 (3 - i)x_i$$

proceeding the same way

|

|

$$Q_n = Q_{n-1} - x_n cL + P(1 - p_D) \sum_{i=1}^{(n-1)} x_i$$

$$\text{i.e. } Q_n = K - cL \sum_{i=1}^n x_i + P(1 - p_D) \sum_{i=1}^n (n - i)x_i \text{ for all } n = 1..n$$

$$Q_{n+1} = Q_n + P(1 - p_D) \sum_{i=1}^n x_i = K - cL \sum_{i=1}^n x_i + P(1 - p_D) \sum_{i=1}^n (n + 1 - i)x_i$$

$$Q_{n+2} = Q_{n+1} + P(1 - p_D) \sum_{i=2}^n x_i = K - cL \sum_{i=1}^n x_i + P(1 - p_D) \{nx_1 + \sum_{i=2}^n (n + 2 - i)x_i\}$$

$$Q_{n+3} = Q_{n+2} + P(1 - p_D) \sum_{i=3}^n x_i = K - cL \sum_{i=1}^n x_i + P(1 - p_D) \{n \sum_{i=1}^2 x_i + \sum_{i=3}^n (n + 3 - i)x_i\}$$

|

|

$$Q_{2n} = Q_{2n-1} + P(1 - p_D)x_n = K - cL \sum_{i=1}^n x_i + P(1 - p_D) \sum_{i=1}^n n x_i$$

To calculate P, we use NPV = 0

$$NPV = L = P * \sum_{i=1}^n \left[\frac{1}{\left(1 + \frac{\alpha}{100n}\right)^i} \right]$$

The present value of Q_n is the objective function that has to be maximized. But as we have assumed the discount rate to be constant, we simply maximize the expression of Q_n

$$Q_n = K - cL \sum_{i=1}^n x_i + P(1 - p_D) \sum_{i=1}^n (n - i)x_i \text{ for all } n = 1..n$$

$$\text{i.e. } Q_n = K - cL \sum_{i=1}^n x_i + \left(\frac{L}{\sum_{i=1}^n \left[\frac{1}{\left(1 + \frac{\alpha}{100n}\right)^i} \right]} \right) (1 - p_D) \sum_{i=1}^n (n - i)x_i \text{ for all } n = 1..n \quad \text{----- (7)}$$

Linear relation between demand and rate of interest

In the expression of Q_n , x_n and p_D depend on I_n . If we assume a linear relation for both, then,

$$x_i = a_i - b_i * I_i \text{ and } p_D = e + d * I_i$$

The objective function in this case is to maximize

$$Q_n = K - cL \sum_{i=1}^n (a_i - b_i * I_i) + \left(\frac{L}{\sum_{i=1}^n \left[\frac{1}{\left(1 + \frac{\alpha}{100n}\right)^i} \right]} \right) (1 - (e + d * I_i)) \sum_{i=1}^n (n - i) (a_i - b_i * I_i)$$

----- (8)

Where, $L = 2.5$ million

Exponential relation between demand and rate of interest

In the expression of Q_n , x_n and p_D depend on I_n . If we assume an exponential relation for demand and a linear relation for default probability, then,

$$x_i = a_i * \exp(-b_i * I_i) \text{ and } p_D = e + d * I_i$$

The objective function in this case is to maximize

$$Q_n = K - cL \sum_{i=1}^n (a_i * \exp(-b_i * I_i)) + \left(\frac{L}{\sum_{i=1}^n \left[\frac{1}{\left(1 + \frac{\alpha}{100n}\right)^i} \right]} \right) (1 - (e + d * I_i)) \sum_{i=1}^n (n - i) (a_i * \exp(-b_i * I_i))$$

----- (9)

Where, $L = 2.5$ million

Hyperbolic relation between demand and rate of interest

In the expression of Q_n , x_n and p_D depend on I_n . If we assume a hyperbolic relation for demand and a linear relation for default probability, then,

$$x_i = a_i / I_i \text{ and } p_D = e + d * I_i$$

The objective function in this case is to maximize

$$Q_n = K - cL \sum_{i=1}^n (a_i/I_i) + \left(\frac{L}{\sum_{i=1}^n \left[\frac{1}{\left(1 + \frac{\alpha}{100n}\right)^i} \right]} \right) (1 - (e + d * I_i)) \sum_{i=1}^n (n - i)(a_i/I_i) \text{ ----- (10)}$$

Where, L = 2.5 million

Constraints:

The rate of interest at each month I_i will have upper and lower bounds, i.e.,

$$I_{\min} \leq I_i \leq I_{\max}, \text{ where } I_{\min} = 8, I_{\max} = 12, \text{ for all } i = 1 \dots n$$

Developing the Demand Price and default probability equations

We have collected real data from a leading nationalized bank from one of the leading states in India. The data consisted of the demand for home loans spread over interest rates ranging from 4% to 15%. We had discussions with the bank officials and cleaned the data. As the bank was operating currently with an interest rate of 8 to 15%, we develop the relationship of interest rate ranging from 8% to 15%. Based on these, we developed three different demand curves: linear, exponential and rectangular hyperbola.

The data used to estimate the linear and exponential demand function is as given in Table 1:

Table 1: Data for estimation of demand function	
Price	Demand
0.0800	11535
0.0900	22099
0.0915	15340
0.0925	21348
0.0935	29213
0.0955	9699
0.1000	12691
0.1100	7461
0.1150	5939
0.1200	3846
0.1250	3068
0.1300	457

0.1350	212
0.1400	162
0.1450	27
0.1500	22

4.5. Linear Demand Function:

For the linear fit, we assume $x = a - bI$ where x is the demand and I is the interest rate. After running the regression, we get $x = 47455 - 339853 * I$ with the regression coefficient $R^2 = 0.6855$.

The graph of the linear fit equation for the data in table 1 is as shown in figure 6:

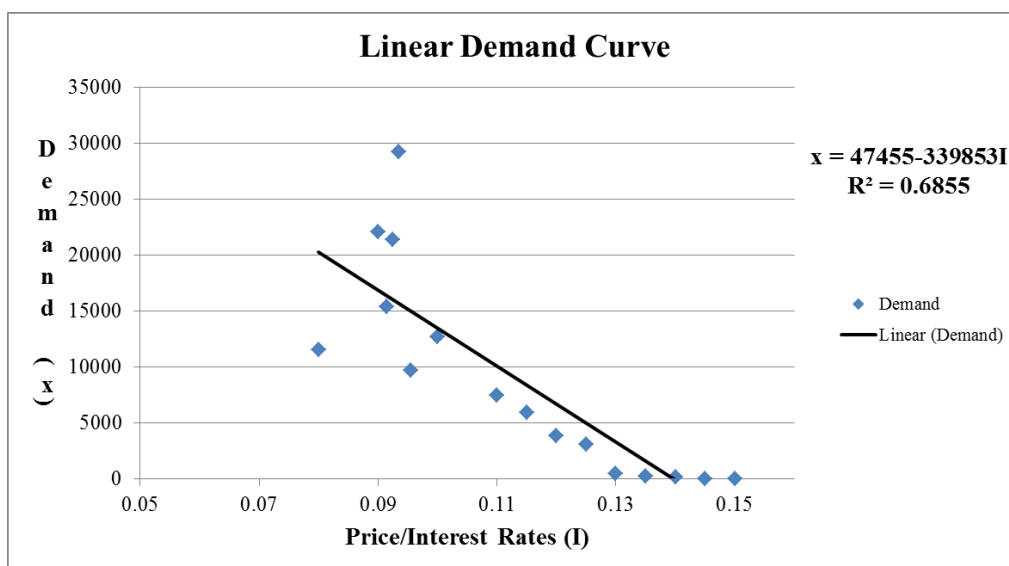


Figure 6: Graph of Linear Demand function with R^2 value

4.6. Exponential Demand Function:

For the exponential fit, we assume $x = a * e^{-bI}$, where x is the demand and I is the interest rate. After running the regression, we get $x = 2 * 10^8 e^{-100.51I}$ with the regression coefficient $R^2 = 0.8654$.

The graph of the exponential fit for the data in table 1 is as shown in figure 7:

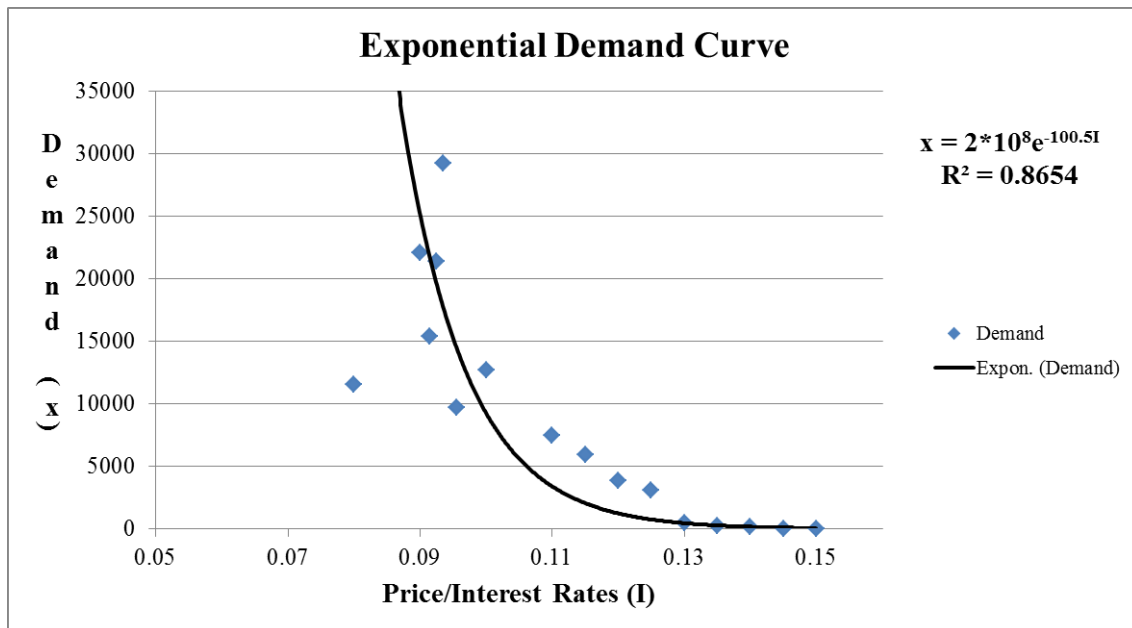


Figure 7: Graph of Exponential Demand function with R^2 value

4.7. Hyperbolic Demand Function:

For the hyperbolic fit, we considered the function $x = a / I^\alpha$, where x is the demand, I is the interest rate with α as a parameter. Taking Log on both sides we get the equation $\text{Log } x = \text{Log } a - \alpha \text{Log } I$. For $\alpha = 1$, we run an optimization to find the optimal value of 'a' that minimizes the difference between x and $\text{Log } x$, giving the hyperbolic equation to be $x = 270.2035 / I^1$.

The graph of the hyperbolic fit for the data in table 1 is as shown in figure 8:

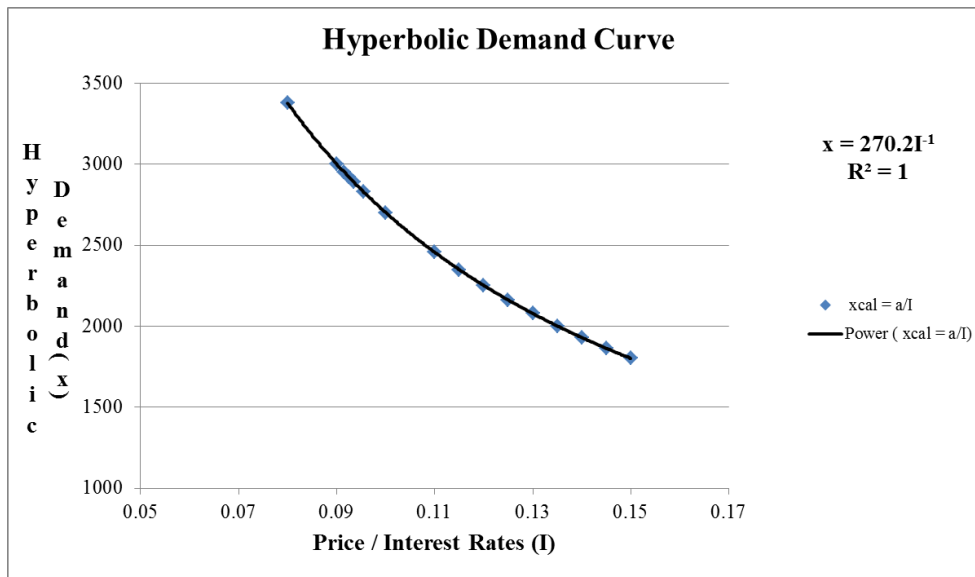


Figure 8: Graph of Hyperbolic Demand function with R^2 value

The following table 2 shows the various demand functions:

Table 2: Demand functions and corresponding equations			
	Actual Equation	Developed Equation	R^2 Value
Linear	$x = a - bI$	$x = 47455 - 339853I$	0.6855
Exponential	$x = a \cdot e^{bI}$	$x = 2 \cdot 10^8 e^{-100.5I}$	0.8654
Rectangular Hyperbola	$x = a / I^a$	$x = 270.2035 / I^1$	0.9756

4.8. Developing the Default Probability

We also collected the default probability values at each interest rate. We then plot the interest rate and the default probability. We find a general trend that when we increase the interest rate, the default probability also increases. The default probability which is directly proportional to the rate of interest is estimated using the following data in Table 3.

Interest Rate	Default Probability
0.0800	0.004149378
0.0900	0.000407258
0.0915	0.001238592
0.0925	0.000983699
0.0955	0.001031034
0.1000	0.00630368
0.1100	0.00482509
0.1150	0.006735141
0.1200	0.010140406
0.1250	0.009452412

The graph of fit of the above to estimate the default probability is as shown in figure 9

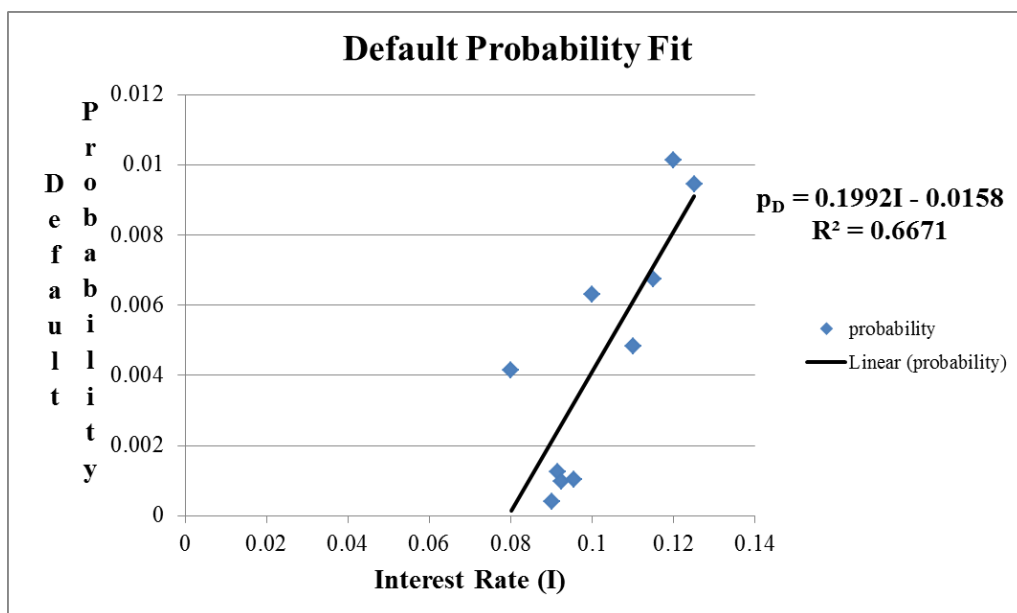


Figure 9: Graph of Default probability with R^2 value

To estimate the default probability, we consider the equation to be $p_D = e + dI$, where p_D is the default probability and I is the interest rate. After running the regression, we estimate the default probability equation to be $p_D = -0.0158 + 0.1992 \cdot I$ where $0.08 \leq I$ with regression coefficient $R^2 = 0.6671$. As the

negative default probability does not make any sense, we only consider all values of interest rate where the default probability is non-negative.

5. Experimentation with the optimization model

We compared the models for optimal dynamic and static pricing. In static pricing we consider the same interest rate for all 180 months by equating all the 180 interest rates. In the dynamic pricing model, the interest rate is constrained with an upper limit and a lower limit of interest rate. In all the cases of demand function, we found that the objective function value of dynamic pricing is much higher than that of static pricing.

To obtain the results, we use AMPL with MINOS solver version 5.5. The above calculations are done for a loan period of $n = 180$ for different values of c . The starting amount that the bank has for home loans is INR 5000 billion and the amount that is given as loan is INR 2.5 million for a period of $n = 180$ months. We first do this with a value of $c=0.5$ and then increase the value of c from 0.5 to 0.9 with an interval of 0.1.

5.1. Linear Demand Function

Table 4 shows the comparison of some of the parameters for the linear demand function.

c	Static Pricing				Optimal Dynamic Pricing				Diffe - rence ($\times 10^{12}$)
	Average Rate of Interest (I)	Total Demand (x_i)	Average Default Prob. (p_D)	Revenue (Q_n) ($\times 10^{12}$)	Average Rate of Interest (I)	Total Demand (x_i)	Average Default Prob. (p_D)	Revenue (Q_n) ($\times 10^{12}$)	
0.5	0.08	3648020	0.00014	5.2058	0.0989	2492520	0.00390	5.9390	0.7332
0.6	0.12	1201080	0.00810	4.7550	0.1027	2261420	0.00465	5.3476	0.5926
0.7	0.12	1201080	0.00810	4.4547	0.1064	2030320	0.00540	4.8141	0.3594
0.8	0.12	1201080	0.00810	4.1544	0.1102	1799220	0.00616	4.3387	0.1843
0.9	0.12	1201080	0.00810	3.8542	0.1140	1568120	0.00691	3.9212	0.0670

In the case of the linear demand function, we observe the following:

- 1) We observe that the optimal revenue from the dynamic pricing model is always more than the optimal revenue from the static pricing model as we move from $c=0.5$ to $c=0.9$.
- 2) In the static pricing model, the average rate of interest, total demand, average default probability are all same for values of $c = 0.6$ to $c = 0.9$ except for $c = 0.5$. The average interest rate for $c = 0.5$ is 0.08 which is the minimum value of the interest rate and for values of $c = 0.6$ to $c = 0.9$, the average interest rate is 0.12 which is the maximum value of the interest rate. As we increase the value of c from 0.6 to 0.9, demand remains the same and the value of Q_1, Q_2, \dots, Q_n decreases, resulting in a decrease in the revenue or the objective function value.
- 3) In dynamic pricing, as c increases, the average rate of interest increases and total demand decreases. However as c increases, the default probability also increases resulting in a decrease in the overall revenue.
- 4) As the interest rates can be dynamically adjusted by optimization, the decrease of revenue as c increases is more in the dynamic pricing model than in the static pricing model.
- 5) The difference between the optimal revenue of the dynamic pricing model and the static pricing model for each value of $c = 0.5$ to $c = 0.9$, decreases systematically as c increases from 0.5 to 0.9.

5.2. Exponential Demand Function

Table 5 shows the comparison of some of the parameters for the exponential demand function.

c	Static Pricing				Optimal Dynamic Pricing				Diffe - rence (x10 ¹²)
	Average Rate of Interest (I)	Total Demand (x _i)	Average Default Prob. (p _D)	Revenue (Q _n) (x10 ¹²)	Average Rate of Interest (I)	Total Demand (x _i)	Average Default Prob. (p _D)	Revenue (Q _n) (x10 ¹²)	
0.5	0.08000	11603100	0.000136	5.6546	0.0991	6158930	0.00394	9.0819	3.4273
0.6	0.08047	11071200	0.000229	2.8554	0.1029	5082760	0.00470	7.6796	4.8242
0.7	0.08056	10970800	0.000247	1.3191	0.1067	4006580	0.00545	6.5483	5.2292
0.8	0.08059	10932000	0.000254	-2.5840	0.1104	2930400	0.00620	5.6880	8.2720
0.9	0.08061	10915800	0.000257	-5.3017	0.1142	1854230	0.00695	5.0987	10.4004

In the case of the exponential demand function, we observe the following:

- 1) We observe that the optimal revenue from the dynamic pricing model is always more than the optimal revenue from the static pricing model as we move from $c=0.5$ to $c=0.9$.
- 2) In the static pricing model, the average rate of interest and the average default probability goes on increasing as we move from values of $c = 0.5$ to $c = 0.9$. But the total demand goes on decreasing exponentially as we increase the value of c from 0.5 to 0.9 and the value of Q_1, Q_2, \dots, Q_n decreases resulting in a drastic decrease in the revenue or the objective function value. Due to the exponential decrease in the optimal revenue we get negative values of the optimal revenue at $c = 0.8$ and $c = 0.9$ resulting in a loss.
- 3) In dynamic pricing, as c increases, the average rate of interest increases and total demand decreases drastically. However as c increases, the default probability also increases resulting in a decrease in the overall revenue.
- 4) As the interest rates can be dynamically adjusted by optimization, the decrease of revenue as c increases is much more in the static pricing model than in the dynamic pricing model.
- 5) The difference between the optimal revenue of the dynamic pricing model and the static pricing model for each value of $c = 0.5$ to $c = 0.9$, increases drastically as c increases from 0.5 to 0.9.

5.3. Hyperbolic Demand Function

Table 6 shows the comparison of some of the parameters for the hyperbolic demand function.

c	Static Pricing				Optimal Dynamic Pricing				Difference ($\times 10^{12}$)
	Average Rate of Interest (I)	Average Demand (x_i)	Average Default Prob. (p_D)	Revenue (Q_n) ($\times 10^{12}$)	Average Rate of Interest (I)	Average Demand (x_i)	Average Default Prob. (p_D)	Revenue (Q_n) ($\times 10^{12}$)	
0.5	0.0800	607958	0.00014	5.0343	0.0989	512261	0.00390	5.0943	0.0600
0.6	0.0864	563058	0.00141	4.8901	0.1027	493121	0.00465	4.9686	0.0785
0.7	0.0878	553670	0.00170	4.7533	0.1064	473982	0.00540	4.8476	0.0943
0.8	0.0884	550101	0.00181	4.6173	0.1100	455968	0.00611	4.7313	0.1140
0.9	0.0887	548585	0.00186	4.4811	0.1138	436829	0.00686	4.6198	0.1387

In the case of the hyperbolic demand function, we observe the following:

- 1) We observe that the optimal revenue from the dynamic pricing model is always more than the optimal revenue from the static pricing model as we move from $c=0.5$ to $c=0.9$.
- 2) In the static pricing model, the average rate of interest and average default probability goes on increasing as we move from values of $c = 0.5$ to $c = 0.9$. But the total demand goes on decreasing as we increase the value of c from 0.5 to 0.9 and the value of Q_1, Q_2, \dots, Q_n decreases, resulting in a decrease in the revenue or the objective function value.
- 3) In the dynamic pricing model, as c increases, the average rate of interest increases and total demand decreases. However as c increases, the default probability also increases resulting in a decrease in the overall revenue.
- 4) As the interest rates can be adjusted dynamically by optimization, the decrease of revenue as c increases is more in the static pricing model than in the dynamic pricing model.
- 5) The difference between the optimal revenue of the dynamic pricing model and the static pricing model for each value of $c = 0.5$ to $c = 0.9$, increases as c increases from 0.5 to 0.9.

5.4. Comparison of all three demand functions

We observe the following:

- 1) In the case of all three demand functions, the optimal revenue obtained from the dynamic pricing model is more than the static pricing model.
- 2) In the case of the dynamic pricing model, the exponential demand curve gives the maximum total demand resulting in maximum overall revenue, followed by the linear demand curve and lastly the hyperbolic demand curve. But as the value of c increases from 0.5 to 0.9, the decrease in the total demand and hence a decrease in the overall revenue, is much lesser using the hyperbolic demand curve followed by the linear demand curve and lastly the exponential demand curve with drastic difference in revenue.
- 3) In the case of the static pricing model, as c value increases, the increase in the rate of interest is much slower in the case of the exponential and hyperbolic demand curve as compared to the linear demand curve. Again the total demand in the case of the exponential demand

curve is much higher compared to the linear and hyperbolic demand curve, which is the least. This results in a loss in overall revenue using the exponential demand curve for higher values of c .

- 4) The difference in the overall revenue using the static pricing model and the dynamic pricing model is highest in the case of the exponential demand curve, as the value of c rises from 0.5 to 0.9. This difference has an increasing trend in the case of exponential and hyperbolic demand curve but has a decreasing trend in the case of the linear demand curve.

5.5. Other Experiments:

We now perform some sensitivity analysis for all the three demand functions.

- 1) For the linear demand function, we consider increasing the values of b from 1% to 10% to check the sensitivity on the average interest rate, total demand and overall revenue at the end of n period for $c = 0.9$. The results are as shown in the following table 7:

Table 7: Linear demand curve model with increasing b value						
Percentage Increase in b value	c value	Average Rate of Interest (I)	Average Demand (x_i)	Average Default Prob. (p_D)	Revenue (Q_n) ($\times 10^{12}$)	Difference from original revenue
1%	0.9	0.1140	1498380	0.00691	3.99163	0.07045
2%	0.9	0.1140	1428640	0.00691	4.06208	0.14090
3%	0.9	0.1140	1358900	0.00691	4.13254	0.21136
4%	0.9	0.1140	1289170	0.00691	4.20299	0.28181
5%	0.9	0.1140	1219430	0.00691	4.27345	0.35227
6%	0.9	0.1140	1149690	0.00691	4.34390	0.42272
7%	0.9	0.1140	1079950	0.00691	4.41435	0.49317
8%	0.9	0.1140	1010210	0.00691	4.48481	0.56363
9%	0.9	0.1140	940476	0.00691	4.55526	0.63408
10%	0.9	0.1140	870738	0.00691	4.62572	0.70454
12%	0.9	0.1140	731262	0.00691	4.76662	0.84544
14%	0.9	0.1140	591787	0.00691	4.90753	0.98635
16%	0.9	0.1140	452311	0.00691	5.04844	1.12726

The sensitivity analysis done by changing the value of the parameter b in the linear demand function gives the following observations:

- 1) In the case of the linear demand function, the value of the parameter b cannot be increased to more than 16% of the original value because beyond a 16% increase, the expected demand becomes negative.
- 2) We observe that increasing the value of b does not alter the value of default ratio and average interest rate.
- 3) Increase in the value of b , results in a decreasing value of the total demand but increasing value of the expected revenue, which is as shown in the figures below.

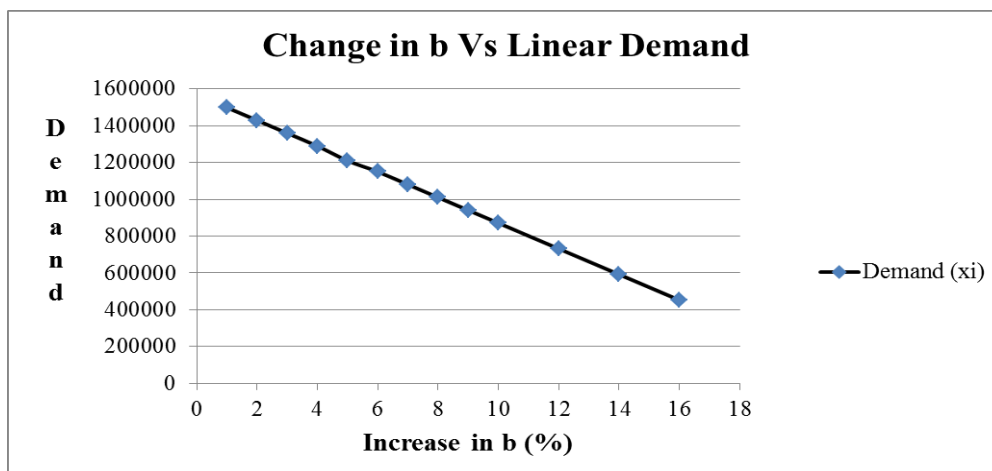


Figure 10: Demand Trend line for increasing b value in a linear demand function

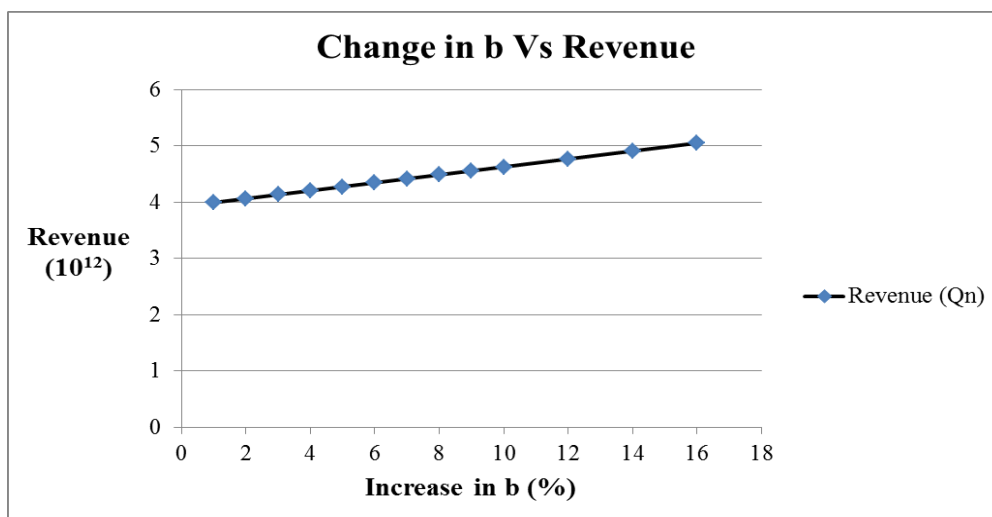


Figure 11: Revenue Trend line for increasing b value in a linear demand function

- 2) For the exponential demand function, we apply sensitivity by increasing the value of b from 1% to 10%. The results are as in the following table 8:

Percentage Increase in b value	c value	Average Rate of Interest (I)	Average Demand (x_i)	Average Default Prob. (p_D)	Revenue (Q_n) ($\times 10^{12}$)	Difference from original revenue
1%	0.9	0.1142	1704500	0.00695	5.09849	0.00023
2%	0.9	0.1142	1567080	0.00695	5.09744	0.00128
3%	0.9	0.1142	1440930	0.00695	5.09573	0.00299
4%	0.9	0.1142	1325100	0.00695	5.09349	0.00523
5%	0.9	0.1142	1218730	0.00695	5.09084	0.00788
6%	0.9	0.1142	1121040	0.00695	5.08787	0.01085
7%	0.9	0.1142	1031290	0.00695	5.08467	0.01405
8%	0.9	0.1142	948835	0.00695	5.08131	0.01741
9%	0.9	0.1142	873066	0.00695	5.07785	0.02087
10%	0.9	0.1142	803430	0.00695	5.07434	0.02438
20%	0.9	0.1142	351659	0.00695	5.04230	0.05642
30%	0.9	0.1142	155013	0.00695	5.02163	0.07709
40%	0.9	0.1142	68665	0.00695	5.01049	0.08823
50%	0.9	0.1142	30518	0.00695	5.00494	0.09378
60%	0.9	0.1142	20364	0.00695	5.00336	0.06536

The sensitivity analysis done by changing the value of the parameter b in the exponential demand function gives the following observations:

- 1) In the case of the linear demand function, the value of the parameter b cannot be increased to more than 60% of the original value because beyond a 60% increase, the expected demand becomes less than one or zero.
- 2) We observe that there is no change in the value of default ratio and average interest rate remains, as the value of b increases.

- 3) We observe decrease in the value of the total demand and expected revenue with increasing value of b , as shown in the figures below.
- 4) Compared to the linear demand function, the change in the overall revenue in the case of the exponential demand function is very slow.

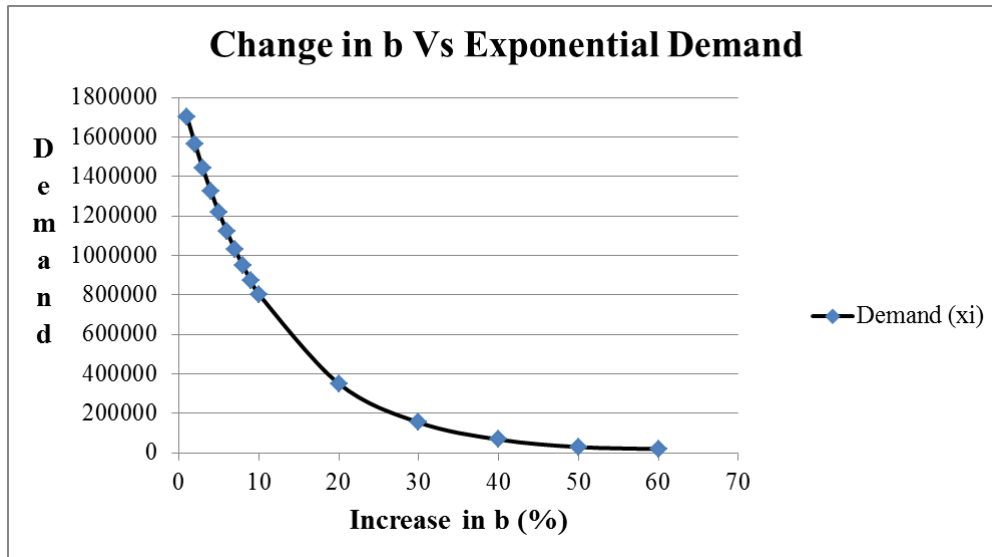


Figure 12: Demand Tread line for increasing b value in an exponential demand function

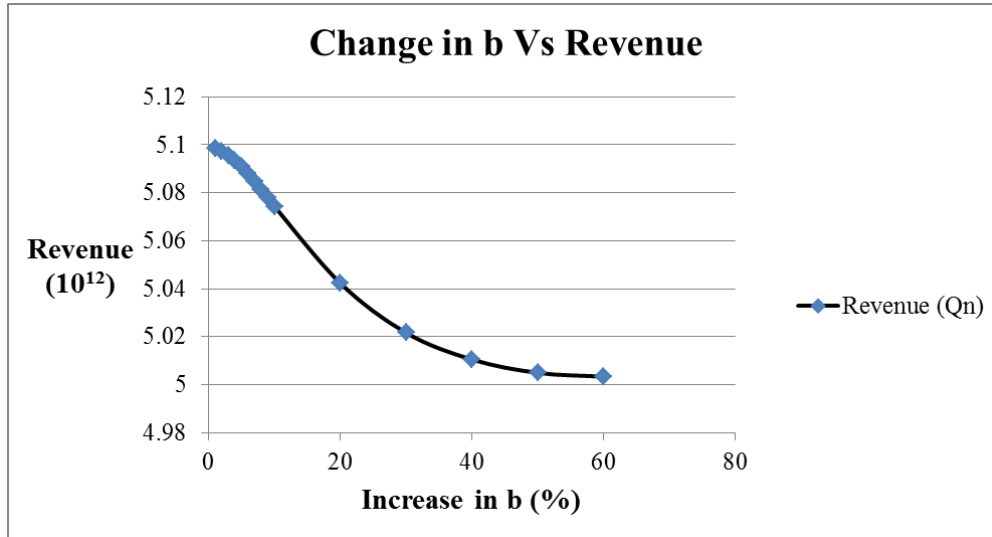


Figure 13: Revenue Tread line for increasing b value in an exponential demand function

- 3) In the case of the hyperbolic demand function, we consider different values of α . We show the results for the hyperbolic demand function with different values of α in the following table 9:

α value	c value	Average Rate of Interest (I)	Average Demand (x_i)	Average Default Prob. (p_D)	Revenue (Q_n) ($\times 10^{12}$)	Difference from original revenue
1.2	0.9	0.1138	281568	0.00686	4.75492	0.13484
1.4	0.9	0.1138	181491	0.00686	4.84203	0.22225
1.6	0.9	0.1138	116984	0.00686	4.89818	0.27840
1.8	0.9	0.1138	75405	0.00686	4.93437	0.31459
2.0	0.9	0.1138	48604	0.00686	4.95769	0.33791
2.2	0.9	0.1138	31329	0.00686	4.97273	0.35295
2.4	0.9	0.1138	20194	0.00686	4.98242	0.36264
2.6	0.9	0.1138	13016	0.00686	4.98867	0.33889
2.8	0.9	0.1138	8390	0.00686	4.99270	0.37292
3.0	0.9	0.1138	5408	0.00686	4.99529	0.37551
3.2	0.9	0.1138	3486	0.00686	4.99697	0.37719
3.4	0.9	0.1138	2247	0.00686	4.99804	0.37826
3.6	0.9	0.1138	1617	0.00686	4.99859	0.37881

In the case of the hyperbolic demand function, the sensitivity analysis is done by increasing the value of α , where we observe the following:

- 1) In the case of the hyperbolic demand function, the value of α cannot be increased to more than 3.6, beyond which the value of the parameter α remains the same with $\alpha = 1$.
- 2) We observe that increasing the value of α does not affect the values of the default ratio and the average interest rate.
- 3) We observe a decreasing trend in the value of the total demand and a slow increasing trend in the value of the expected revenue, as we increase the value of α , which is as shown in the figures below.
- 4) Compared to the linear demand function, the change in the overall revenue in the case of the hyperbolic demand function is very slow.

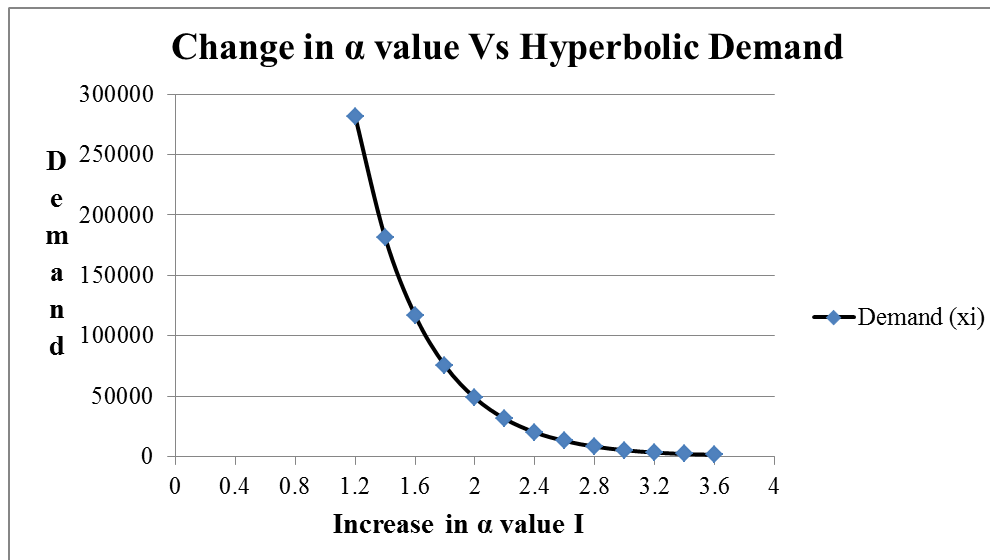


Figure 14: Demand Tread line for increasing α value in a hyperbolic demand function

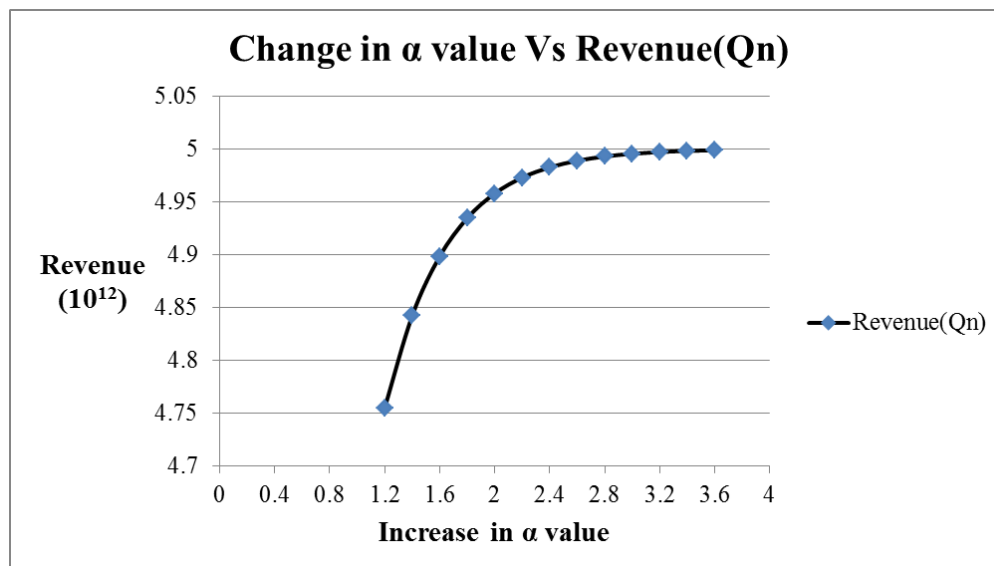


Figure 15: Revenue Tread line for increasing α value in a hyperbolic demand function

6. Conclusion

In this paper, we have shown how dynamic pricing concepts can be applied to home loan pricing in a bank, for providing loans to customers who may have the ability to buy a home in one single instalment. We have made several simple assumptions to develop this model. We consider a complex function consisting of the term of the loan, the rate of interest, the amount borrowed, and the default ratio that constitute the profitability of a home loan. Optimizing loan pricing is different and somewhat more

difficult than retail or airline pricing because of the existence of nonlinearity in profitability functions, complex behaviour of customers, unfavourable selection, and regulatory constraints. All these factors are not easy to model mathematically and make the optimization of the model a tough task. We have shown that dynamic pricing generates more revenue than static pricing in all the three types of demand price relationships (namely the linear, the exponential and the rectangular hyperbola). Next, we have conducted experiments to show the effect of changing the slope of the demand curve on the revenue. For exponential demand, we have shown the effect of increase on the revenue with the changes in the value of b . Similarly we have shown the effect of increase of revenue on the parameter α of the hyperbolic demand curve.

The managerial implications drawn from this model is as follows:

1. This model helps the banks to know when to offer what price in order to increase its revenue.
2. This model brings about consistency in terms of pricing methodology within the bank and puts an end to uncertain approaches by various lenders.
3. It enables the bank to compare returns by studying the various demand patterns of the customers along with their defaulting ratios.
4. It aids the management's disagreement over a deal, which may be attractive or not.
5. It enables in managing a better customer relationship.
6. As the model takes into account the constraints on minimum and maximum rate of interest, it gives an idea about the minimum and maximum volumes of home loans sold.
7. Unlike the lost room price in a hotel or the price of a lost airline seat, lost quote loan data is easily accessible and important.

7. Future Work

The research related to home loans pricing is a new topic and several extensions are possible.

1. The research could be extended to multiple products. We have considered only one product in our analysis. We can extend the paper with several products, where we need to develop a demand price relationship and an optimization problem for each product. It is possible that the bank may have a constraint on the total loan amount for products. How we allocate this amount to different products along with the existing constraints will be an interesting problem. We can also consider the demand correlation between various products.
2. This research can be applied to other consumer finance sectors like car loans, educational loans, commercial loans, credit cards, etc.

3. We have done this for only one state in India. As the willingness to pay is different in different states, the demand price relationship and the default probability relationship may be different in different states, it will be interesting to know how this bank actually uses the home loan portfolio in different geographic or demographic segments.
4. Every customer will have a different willingness to pay for home loans. This research can be extended by considering the study on how to segment customers according to their willingness to pay and apply dynamic pricing to these segments.
5. We can use various demand forecasting methods to forecast the demand price relationship. Various other demand price relationships can be applied to the dynamic pricing model.
6. Higher capital and liquidity requirements have an economic cost and also have an impact on the borrowing cost of funds for banks. The loan rates have to take into account the effects of these regulatory changes on the cost of capital and other sources of funding.
7. We have considered the home loan problem under certain assumptions. This model can be further extended by finding other factors that affect the home loan business. We can consider an integrated model for multiple model segments.

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