Throughput Matching Algorithm and Stochastic Models for Analysis of Open and Closed Manufacturing Systems

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Abstract

Order release policies such as card-controlled CONWIP policy aim at improving system responsiveness and minimizing system-wide inventory levels. It is not clear if order release policies (without card control) can be equally effective under certain settings of the production system design parameters. In this research, we study the performance of alternate order release (material control policies) under a variety of design parameter settings such as number of stations, station service time characteristics, and the location of bottleneck station using queuing network models. To compare CONWIP policy (closed system) with open control policies (such as deterministic start times), we develop a throughput matching algorithm. The throughput times in CONWIP system is about 2%-50% less in comparison to open control systems; however, we show that the number of stations and the location of the bottleneck station affects the choice of the order release policy, and the benefit of card-controlled policies diminishes for a system with large number of stations (the throughput time percentage benefit using CONWIP in comparison to open control is $1/K$, where K is the number of stations). We also analyze the system for a variety of demand inter-arrival times and check its effect on the expected number of backlogs and system wide expected waiting time.

Keywords: Manufacturing system, tandem stations, order release policies, queuing network models, throughput matching

1 Introduction

Companies today must be responsive and agile enough to react quickly to the fluctuations in customer demand and manage fulfilment operations with little inventory. Manufacturing and fulfilling customers' orders by maintaining large piles of raw-material, work-in-progress and finished goods inventory not only ties up cash for working capital but also causes long lead times and reduces market competitiveness. In a recent survey conducted by Donovan and Inc., 82% of senior executives who responded said that inventory reduction was a major concern.

An effective manufacturing system design includes robust production plans and efficient order release policies (also referred as material control policies). As discussed by Lödding and Lohmann [2012], the goal of production planners is to effectively manage capacity and inventories, which includes identifying the optimal capacity investments in terms of machine and workforce (capabilities/skillsets, timing, sizing) in the short, medium and long term under deterministic as well as stochastic inputs (for example, see Van Mieghem [2003] for a review of literature, also see Wu et al. (2005), Alp and Tan [2008]).

While the key indicator of a production plan performance is robustness, implementing the production plan on the shop-floor falls under the purview of manufacturing control. Manufacturing control typically comprises of three specific tasks: order release, sequencing, and capacity control. While sequencing the jobs refers to establishing the actual sequence of jobs subject to real-time disturbances, capacity control refers to managing short-term capacity using overtime labour or hiring temporary labour in order to match the actual capacity requirements. The 'order release' determines when the processing of a known order can begin on the shop-floor [Wiendahl, 1995]. The focus of this research is to compare the performance of alternate order release policies. There are several order release policies established during the previous decades. The simplest rule is to release the order at the planned start date. However, sometimes coupling the release of orders to match the output of the manufacturing area in order to regulate the expected WIP level is preferred.

Key material control policies that are practiced on the shop floors are:

- Card controlled system: It is a closed manufacturing system in which the flow of jobs in the system is controlled by cards. A card is attached to the job during the manufacturing process. If jobs are waiting to be processed, a new job enters the system as soon as the processing of the previous job is completed. Since a few cards may be waiting to be paired with incoming jobs, the number of cards provides an upper bound on the total number of parts in progress. There are mainly two types of card controlled systems in practice, namely:

    1. Kanban system: This is a pull-based system where cards (known as Kanbans) signal the replenishment of used material from internal or external suppliers to the buffer of inventories. Kanbans reduce outages and shortages of materials and supplies, which improves customer service levels. Kanbans support pull production and continuous flow as the material is not produced at the supplier until a signal to replenish materials is received from the customer. Using this approach, Kanbans reduce overall inventory levels. (Refer Sugimori et al. [1977])

2. CONWIP system: CONWIP (Constant Work-In-Progress) is also a pull-oriented production control system, where the start of each product manufacturing process is triggered by the completion of another at the end of production line. CONWIP is a kind of single-stage Kanban system. While Kanban systems maintain tighter control of system WIP through the individual cards at each workstation, CONWIP systems are easier to implement and adjust since only one set of system cards is used to manage system WIP. No part is allowed to enter the system without a card (authority). After a finished part is completed at the last workstation, a card is transferred to the first workstation and a new part is pushed into the sequential process route. (Refer Spearman et al. [1990])

- Scheduled start times policy: This system behaves as an open system where the jobs enter the line and depart after one pass. Releases into the line are triggered by the material requirements plan without regard to the number of jobs in the line. Hence, the number of jobs in the system vary over time. Throughput (λ) of the system is based on anticipated orders. A job enters the system at a rate, λ. There are three types of scheduled start time policies, namely:

  1. Poisson policy: In this job scheduling policy, the product arrivals follow a Poisson process (the jobs have exponentially distributed inter-arrival times and the CV of job inter-arrival times is 1). (Refer Borthakur and Medhi [1987])
  2. Deterministic policy: In this job scheduling policy, the product inter-arrival times are constant and the CV of inter-arrival times is 0.
  3. Workload regulated system policy: In this process, a job is released into the network whenever the the total amount of remaining work at the bottleneck station falls below a threshold level. (Refer Wein [1990])

- Card controlled with authorized start times system: Paired-cell Overlapping Loops of Cards with Authorization (POLCA) is a variant of Kanban card control policy and is typically suitable for companies that produce high-mix and low-volume products. POLCA divides the work content into flexible and multidisciplinary staffed work-cells. The production cell only makes semi-finished products for receiving work-cells which have free capacity. To assure that, POLCA-cards circulate between the work-cells, which signal if and where there is a capacity for further processing. (Refer Krishnamurthy and Suri [2009])

Since the customer demand is highly variable, the performance of an open manufacturing system (modeled as an open queuing network with tandem stations) with deterministic start times and arrival rate equal to current customer demand rate may be comparable to a closed system (modeled as a closed queuing network) with N CONWIP cards, such that the throughput of the closed system is equal to the current customer demand rate. Though card controlled system has advantages over open system, managing the flow of cards requires more coordination among the stations and adds to the overhead costs. We hypothesize that for a certain range of design parameters, performance of open system policies may be comparable to that of card controlled policy. In the case where raw materials are always available, there is no delay in production due to waiting time for raw material arrival. But raw materials required for production may not be available all the time. Hence, the availability of raw materials becomes one of the factors affecting the output of the system.

In this research, we aim to study the three material control policies - CONWIP, Poisson scheduled start time and Deterministic scheduled start time system under different sets of design parameters. The experimental setup has two model variants - raw materials are always available and raw materials are arriving at some rate. Through this research, we aim to analyze the effect of raw material arrival on expected WIP and expected system lead times. As a separate system, we also analyze the effect of demand arrival parameters on the system performance. The synchronization of the finished goods with the demand arrivals is modeled with a join/ fork station. We study the effects of demand arrival rate and coefficient of variation (CV) of demand inter-arrival times on the expected number of backlogs and expected waiting time for the backlogs to clear. We answer the following research questions for two types of manufacturing systems: a balanced system where all stations have the same utilization and an unbalanced system where the workload varies among the stations.

1. How does the Poisson and Deterministic start time policies perform in comparison to the CONWIP policy for a particular configuration? How does the relative performance change as the number of stations in the network vary?

2. How does the change in mean and CV of service times as well as raw material inter-arrival times affect the relative performance of the three policies?

3. Which material control policy is best suited for a given set of system parameters (such as station utilisation, expected service times and CV of service times, position of the bottleneck station for a unbalanced system)?

4. How does the mean and CV of the demand inter-arrival times affect the expected number of backlogs and expected demand waiting times?

The rest of this paper is organized as follows. We review the relevant literature in Section 2. The system description and analysis of the system is given in Section 3. Numerical results of various analytical models are shown in Section 4. Results and insights from the analytical experiments are shown in Section 5 and conclusions are drawn in Section 6.

2 Literature Review

The existing literature on material control policies can be classified into two areas:
1. Modeling and analysis of material control policies.
2. Comparison of alternate material control polices.

2.1 Modeling and analysis of material control policies

Spearman et al. [1990] introduced a pull alternative to push known as CONWIP system. In this paper, practical advantages of pull system over push were outlined. Theoretical and simulation results were outlined to give insights into the system's performance. Kimura and Terada [1981] divided the production control systems for a multistage production process into two types, namely Push and Pull systems. They formulated the Pull system and gave a model simulation to fluctuations in production and inventory through the whole process in terms of system parameters such as lot size and lead time. Wein [1990] considered the problem of input control and priority sequencing in a multi-station multi-product queueing network with a general service time distribution and a general routing system. The objective was to minimize the long run expected number of customers in the system subject to a constraint on the long run expected output rate. Under heavy load condition, this could be approximated to a control problem involving Brownian motion. Borthakur and Medhi [1987] considered a single server queueing system under control-operating policy (COP) in which the server began service only when the queue-size built up to a preassigned fixed number and in which the interval of time required for startup of servicing, after each idle period, followed a general distribution with finite mean. Such systems generally provide satisfactory models to many realistic situations. Liu and Bin [2010] considered an $M/G/1$ like queue production system, with an unreliable server under N-policy and single vacation. By renewal process and total probability decomposition, they analyzed the system reliability. Economopoulos and Kouikoglou [2008] studied a single-stage, constant work-in-process (CONWIP) production system that produced one product to stock to cope with random demand. The objective was to determine the CONWIP level and the base backlog that maximized the mean profit rate of the system. Numerical results showed that managing inventories and backlogs jointly achieved higher profit than other control policies.

2.2 Comparison of alternate material control policies

Wein [1988] assessed the impact that scheduling can have on the performance of semiconductor wafer fabrication facilities. The performance measure considered here was the mean throughput time for a lot of wafers. The goal was to compare the performance of Poisson, Workload regulated, CONWIP and Deterministic arrivals for a specific design data. It was concluded that when compared to Poisson, Deterministic scheduled start time policy performed better (16%- 20% lead time reduction) than CONWIP policy (8%-16%). Workload regulated system performed better (21%-26% lead time reduction) than Deterministic scheduled start time policy. Whitt [1984] investigated the relation between open and closed queueing network models. He concluded that open queueing network models were analytically more tractable but closed queueing network models were more realistic. Spearman and Zazanis [1992] aimed to show the benefits of Pull system with relation to Push system such as less congestion, more robust to control WIP and WIP is bounded and its variability is less. They concluded that, in CONWIP, with increasing N, the throughput increased and then reached an asymptote. In Push system, if the throughput is increased beyond a certain point, the utilization increased and the WIP built up without a bound. It was also observed that controlling WIP was more robust than controlling throughput. The goal of Duenyas [1994] was to compare CONWIP (generic cards vs. product specific cards) with Work load regulating policy. He concluded that lead time with CONWIP (product specific cards) exceeded the lead time with generic cards by 3%-110% and lead time with CONWIP (product specific cards) exceeded the lead time with Workload regulating policy by 10%-40%. The order of performance for the specific design data: CONWIP (Specific cards)performed better than Workload regulated policy and Workload regulated policy performed better than CONWIP (generic cards).

Krishnamurthy et al. [2004] compared the performance of MRP (push) and kanban (pull) systems for a multi-stage, multi-product manufacturing system. Using simulation experiments, they analyzed system performance under different product mixes. They also studied the impact of design parameters such as safety lead time and safety stock policies on system performance. They concluded that, in certain environments with advance demand information, kanban-based pull strategies can lead to significant inefficiencies. In these environments, MRP-type push strategies yield better performance in terms of inventories and service levels. For low and medium values of system loads, safety lead time policies yield better system performance than safety stock policies. Bondi and Whitt [1986] studied the effect of service time variability on closed queue network. Zahorjan [1983] examined the effect of mean system performance measures on the workload representations chosen. Open and closed representations are compared under the equivalent constraints that they result in identical system throughput or mean system population level for the class being considered. It was shown that open system showed larger response time than the equivalent closed system. Beamon [1998] placed attention on the performance, design, and analysis of the supply chain as a whole and provided a focused review of literature in multi-stage supply chain modeling. Whitt [1983] described Queueing Network Analyzer (QNA)that analyzed multi-nodes queues with FCFS discipline with no capacity constraints. For this queue, the arrivals need not necessarily be Poisson and service time distribution need not necessarily be exponential. Babai and Dallery [2009] considered two approaches of production-inventory control: the future requirements-based approach and the inventory consumption-based approach. The results demonstrated the benefit of using the dynamic policies when compared with the static ones. Lödding and Lohmann [2012] discussed Inventory based Capacity Control (INCAP), a very simple method that allowed inventory levels to be effectively controlled by using short term capacity flexibility in make to stock setting. INCAP was found to be a straightforward but powerful method, able to cope both with uncertainties in production output as well as with varying demand. Ioannidis et al. [2008] investigated four admission policies lost sales, complete backordering, randomized admission, and partial back-ordering. The objective was to determine the inventory level and the maximum number of backorders, as well as the admission probability that maximized the mean profit rate of a system modeled as closed queueing network. Managing inventory levels and sales jointly through partial back-ordering achieved higher profit than other control policies. Thomas et al. [2012] studied two different possible machine allocation policies for a production system consisting of MRP and kanban controlled materials for performance measures like inventory costs, back-order costs and service level. Whenever utilization of the production system was high, the production system segmentation policy was preferable and for medium and low utilization values common machine groups performed best in all scenarios.

The similarities between this paper and previous works include the consideration of exponential service times of station, tandem queue with FCFS discipline, performance evaluation of both balanced and unbalanced system. The dissimilarities include comparison of Poisson scheduled start time policy, Deterministic scheduled start time policy and CONWIP policy, consideration of varied design parameters including small - high number of stations, low - high value of CV of service times, location of bottleneck stations, consideration of the effect of the case when raw material are arriving at some rate, checking the effect of the variation of CV of raw material inter-arrival times on the system performance, consideration of the trade-off between ease of setting the parameters and relative policy performance and checking the effect of demand inter-arrival times and its CV on the expected number of backlogs and expected waiting time of backlogs.

3 System Description and Modeling Approach

We consider a tandem manufacturing system where the jobs proceed to the subsequent manufacturing station sequentially during the manufacturing process. We also assume that there exists sufficient buffer space for the jobs to queue at the stations without any delays.

We use queuing network models to analyze alternate material control strategies. Each manufacturing station is considered as a node and is characterized by the mean and CV of its service time. Product arrival to the queue is characterized by the mean and CV of inter-arrival times. As a product arrives, it is sent to the subsequent station on FCFS basis. If the station is busy, the product waits in a queue till it enters the station. The expected waiting time depends on the expected service time of the station, its CV, utilization of the station and arrival characteristics to the station. The expected service time and the expected waiting time in the queue together constitute the expected throughput time of the station. Summation of the expected throughput times of all the stations gives the system-wide expected throughput time. The relationship between the expected WIP, throughput and the expected throughput time of the system is given by Little's Law.
The design parameter set consists of the mean and CV of station service times, station utilization, the mean and CV of demand inter-arrival times. In addition to these parameters, we also consider the mean and CV of raw material inter-arrival times.

For modeling purposes, the order-fulfillment system is divided into two systems - System I and System II. System I denotes the manufacturing system whereas System II denotes the demand fulfilment process.

System I : System I (manufacturing system) with CONWIP card control policy is modeled using a closed queuing network whereas for Scheduled start time policy, it is modeled with a tandem open queuing network. It consists of K service stations, each characterized by the mean and CV of service times. In this system, the products are sent to the subsequent stations starting from station 1 to K on a first-come first-served (FCFS) basis. Each station k is characterized by the mean (D_k) and CV ($c_{s,k}$) of the service times. The finished goods, after exiting the last station, proceed to System II. In CONWIP policy, the CONWIP card is released from the last station and reaches the first station after the finished goods exit the last station. System I is analyzed for different sets of design parameters and the performance measures such as station expected throughput times and expected work in progress are obtained. With respect to availability of raw materials, two analytical model variants of System I are possible, (1) Raw materials are always available: in this case, the raw materials required for production are always available without any delay. Hence, the orders do not wait for the raw material to arrive, and (2) Raw materials are arriving at some rate: in this case, the raw materials arrive according to a stochastic arrival process, characterized by the mean and CV of inter-arrival times ($\lambda_{p,1}^{-1}$ and $c_{p,1}$). The synchronization between raw materials arrival (p_1) and order arrival buffer (f_1)is modeled as a join/fork station. The order has to wait for the raw materials to arrive. The changes in the CV of raw material inter-arrival times affect the performance measures of the system.

System II : For both CONWIP policy and Scheduled start time policy, System II models the demand fulfilment process using a fork/join station. It consists of a finished products arrival buffer and a demand arrival buffer. The finished goods arrive at a rate equal to the throughput of System I and the CV of inter-arrival times is equal to the CV of inter-departure times from the last station in the manufacturing network. The order arrival is characterized by the mean demand inter-arrival times ($\lambda_{p,2}^{-1}$) and its CV ($c_{p,2}$). System II is analyzed for different values of CV of order inter-arrival times and buffer capacities. The performance measures include expected number of backlogs, which waits at the buffer and the expected waiting time for the backlogs to get fulfilled.

Based on the modeling approach and the system description, the framework for our analysis is summarized in Figure 1. The two analytical model variants of CONWIP policy (raw materials always available and raw materials available at a given rate) are shown in Figures 2 and 3 respectively. Likewise, the two analytical model variants of scheduled start time policy are shown in Figures 4 and 5.

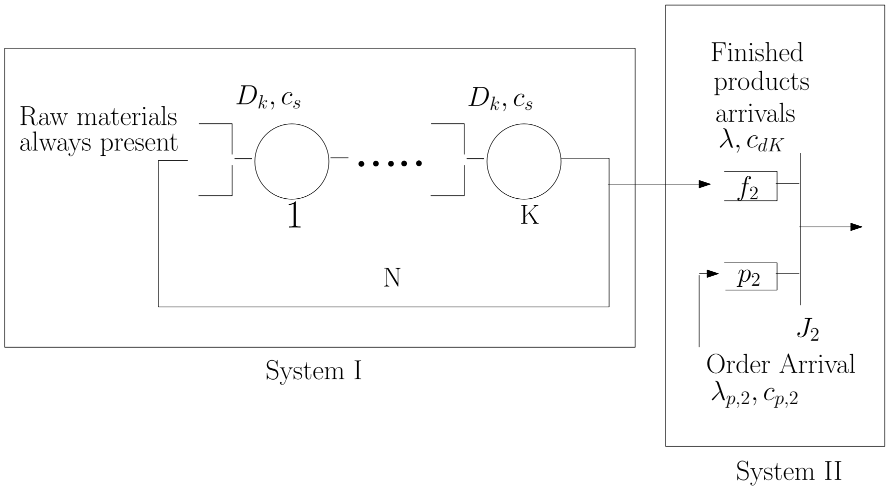

Figure 2: Closed queuing network model for the CONWIP policy when raw materials are always available

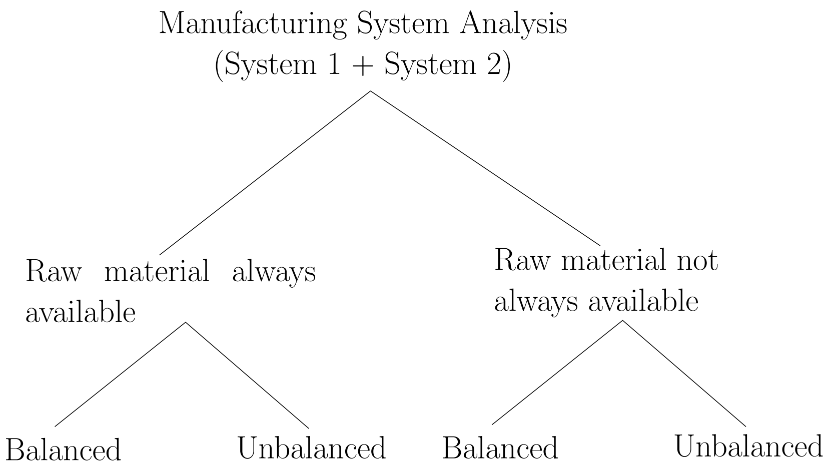

Figure 1: Modeling framework

3.1 Analysis of System I

In this section, we analyze System I with CONWIP, Poisson and Deterministic scheduled start time policies for both balanced and unbalanced systems. We perform separate analysis for the both variants of System I i.e., when raw materials are always available and when raw materials are arriving at some rate.

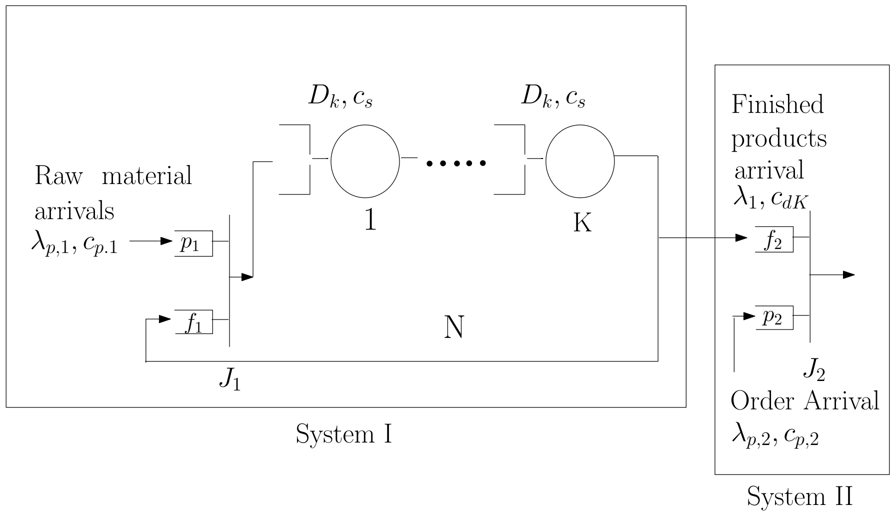

Figure 3: Closed queuing network model for the CONWIP policy when raw materials arrive at rate $\lambda_{p,1}$

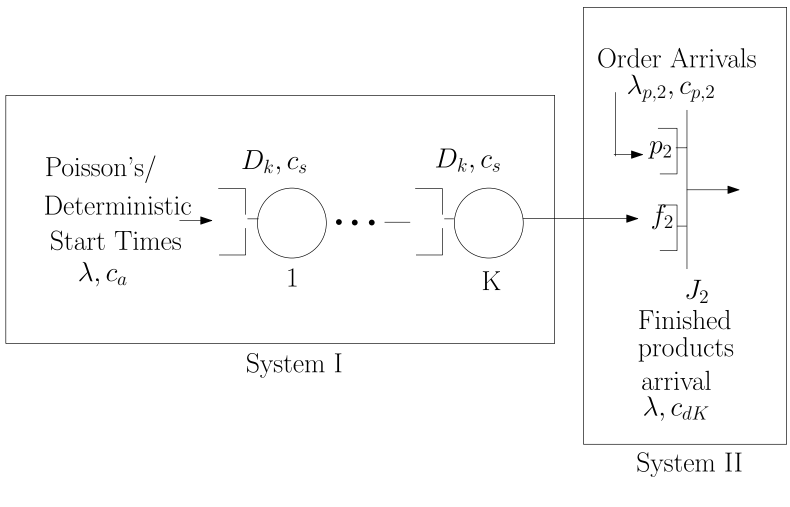

Figure 4: Open queuing network model for the Scheduled start time policy when raw materials are always available

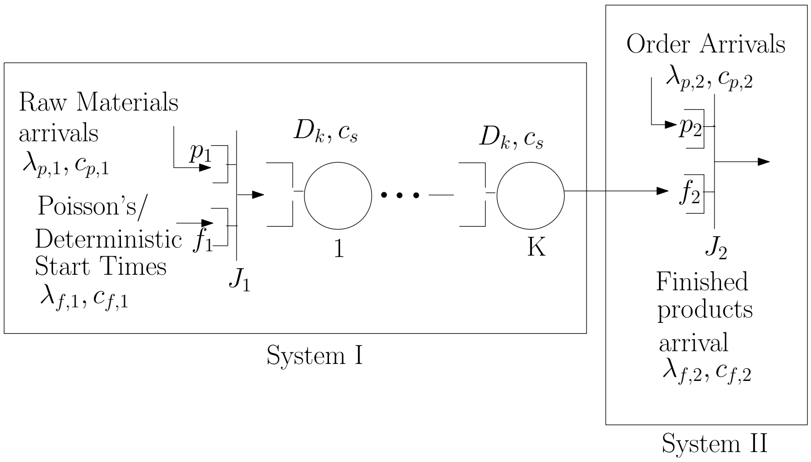

Figure 5: Open queuing network model for the Scheduled start time policy when raw materials arrive at rate $\lambda_{p,1}$

We hypothesize that in a *balanced system* with time-homogeneous customer demands, for lower number of stations, it might be better to run the system with deterministic input (with an arrival rate to the system equal to the current customer demand rate) than a CONWIP system with N cards which has a throughput equal to the current customer demand rate. For a system with a large number of stations, open Poisson input might be suitable. Likewise, for an *unbalanced system* with time-homogeneous customer demand, we hypothesize that for lower number of stations, it might be better to run the system as a CONWIP system with N cards which has a throughput equal to the current customer demand rate than Deterministic scheduled start time policy for both upstream and downstream locations of the bottleneck. For larger number of stations, both Deterministic scheduled start time policy and CONWIP policy may perform similarly for both upstream and downstream locations of the bottleneck station. To compare scheduled start time policies (open queuing network) with CONWIP policy (closed queueing network), we first develop a result that shows that the expected throughput time of an open system is greater than that of the CONWIP system but diminishes as the number of stations increase. Theorem 1 provides a theoretical justification to our hypothesis.

Theorem 1: Suppose we have a balanced single-class open BCMP network and a closed network with exponential service times. If the throughput of the closed network (λ_c) matches exactly with throughput of the open network (λ_o), then:

$$\frac{R_o - R_c}{R_o} = \frac{1}{K} \tag{1}$$

where:

- R_o is the expected throughput time of the open network
- R_c is the expected throughput time of the closed network
- K is the number of stations in the network

Proof:

Table 1: Notations used in Theorem I

S. No	Symbol	Parameter
1	λ_c	Throughput of closed queueing network
2	λ_o	Throughput of open queueing network
3	K	Number of stations
4	N	Number of CONWIP cards
5	Q	Expected queue length in open queueing network
6	R_c	Expected throughput time in closed queueing network
7	R_o	Expected throughput time in open queueing network
8	D_k	Expected Service time of the station
9	U	Utilization of the station

For a closed queueing network: $\lambda_c = \frac{N}{D_k(N+K-1)}$ ......(1)

Since $\lambda_c = \lambda_o$, $\therefore \frac{N}{D_k(N+K-1)} = \lambda_o$

$\Rightarrow \frac{N+K-1}{N} = \frac{1}{D_k\lambda_o} = \frac{1}{U}$

$\Rightarrow N = \frac{(K-1)U}{1-U}$ ......(2)

By Little's Law:

For an open queueing network:

$Q = \lambda_o R_o = \frac{KU}{1-U}$ ......(3)

For closed queueing network (From (1)):

$N = \lambda_c R_c = \frac{N}{D_k(N+K-1)R_c}$

Since $\lambda_c = \lambda_o$,

$\therefore \frac{R_o - R_c}{R_o} = 1 - \frac{N}{Q}$ ......(4)

From (2),(3) and (4),

$\frac{R_o - R_c}{R_o} = 1 - \frac{\frac{(K-1)U}{1-U}}{\frac{KU}{1-U}} = 1 - \frac{K-1}{K} = \frac{1}{K}$

The notations used for model discussion are presented in Table 2.

3.1.1 Raw materials are always available

In this section, we analyze the models where raw materials are always available for production. Hence there is no delay/ waiting queue involved for the availability of raw materials.

Analysis of scheduled start time policy:

The single product system with K manufacturing stations operating under Scheduled start time policy is modeled with an open queuing network (see Figure 4). Arrival process is either Poisson (CV of inter-arrival times ($c_{a,1}$) is 1) or Deterministic ($c_{a,1} = 0$). (Refer Borthakur and Medhi [1987]) Each station is modeled using a $GI/G/1$ queue and the performance measures are estimated using results from Whitt [1983]. The expected waiting time at each station is determined using Equation 2.

$$W_k = D_k U_k (c_{a,k}^2 + c_{s,k}^2) g / 2(1 - U_k) \tag{2}$$

where

$$g = exp\left[\frac{-2(1-U_k)}{3U_k}\frac{(1-c_{a,k}^2)^2}{c_{a,k}^2 + c_{s,k}^2}\right] \quad (c_{a,k}^2 < 1)$$
$$= 1 \quad (c_{a,k}^2 \geq 1)$$

The CV of inter-departure times ($c_{d,k}$) which equals to the CV of inter-arrival times for next station is given by Equation 3. [Whitt, 1983]

$$c_{d,k}^2 = U_k^2 c_{s,k}^2 + (1 - U_k^2) c_{a,k}^2 \tag{3}$$

Table 2: Symbols and Description

K	Number of stations
U_k	Utilization of the station k
D_k	Expected service time of the station k
$c_{s,k}$	CV of service times at station k
N	Number of CONWIP cards
D_{unb}	Expected service time at bottleneck station
U_{unb}	Utilization of bottleneck station
c_{unb}	CV of service times at bottleneck station
$\lambda_{p,2}$	Demand arrival rate
$c_{p,2}$	CV of demand inter-arrival times
$K_{f,2}$	Maximum queue length of finished goods arrival buffer
$K_{p,2}$	Maximum queue length of demand arrival buffer
λ	Throughput of System I
W_k	Expected waiting time at station k
R_k	Expected throughput time of station k
R	Expected throughput time of the entire system
WIP	Expected work in progress
$L_{p,2}$	Expected number of backlogs
$L_{f,1}$	Expected queue length of product arrival buffer
L_k	Expected queue length at station k
$\lambda_{p,1}$	Raw material arrival rate
$\lambda_{f,1}$	Product arrival rate
$c_{p,1}$	CV of raw material inter-arrival times
$c_{f,1}$	CV of inter-arrival times
$K_{p,1}$	Maximum queue length of raw material buffer
$K_{f,1}$	Maximum queue length of product arrival buffer
OP	Open Poisson start time policy
OD	Open deterministic start time policy
CON	CONWIP Policy
N_b	Expected number of backlogs
W	Expected waiting time for the backlogs to get fulfilled (System II)

The expected throughput time for System I is given by Equation 4. The expected WIP is estimated using Little's Law.

$$R = \sum_{k=1}^{K} R_k \tag{4}$$

where $R_k = D_k + W_k$, $k = 1 \dots K$

Analysis of CONWIP policy:
This section describes the queuing network model of a single product with K manufacturing stations (labeled $k = 1,...,K$) operating under CONWIP policy (see Figure 2). The total number of cards in the system is equal to the number of parts in progress (N). To analyze the CONWIP policy, we adopt the solution approach adopted by Satyam and Krishnamurthy [2008]. The analysis starts with the characterization of the parameters of each station namely the CV of service times($c_{s,k}$), station utilisation (U_k) and the CV of inter-arrival times ($c_{a,k}$) to the station and establishing a relation between these parameters (Refer Equation 5). Then the mean queue length is found in each station which depends on the service times of the station (D_k) and the expected waiting time in that station (W_k) (Refer Equations 6 to 10). Finally, all the station parameters are linked together to give the linkage equation which states that the sum of mean queue lengths at the stations is equal to the number of CONWIP cards (N)(Refer Equation 10).

1. Characteristic Equation:

The CV of inter-arrival times to station $k + 1$ ($c_{a,k+1}$) is given as function of the station utilization (U_k) and the CV of service times ($c_{s,k}$).

$$c^2_{a,k+1} = (1 - U^2_k)c^2_{s,k} + U^2_k c^2_{a,k} \tag{5}$$

2. Analysis of mean queue length at the station k:

Mean queue length at each station is given by:

$$L_k = \lambda(D_k + W_k) \tag{6}$$

where

$$W_k = \left[\frac{\lambda D^2_k}{1 - \lambda D_k}\right]\left[\frac{c^2_{s,k} + c^2_{a,k}}{2}\right] f_m \tag{7}$$

and f_m is a correction factor.

Calculation of f_m:
To determine f_m, a single-class product-form closed queuing network, C is defined. The stations in C correspond to the manufacturing stations of the original model. To ensure that C has a product-form solution, it is assumed that the service times are exponentially distributed for the manufacturing stations in C.

Under this assumption, network C has a product form solution that can be obtained using Mean Value Analysis to determine performance measures such as the throughput, (λ_k), and the expected waiting time, $W_{C,k}$, at each station k in C. Next, consider an $M/M/1$ queue with arrival rate equal to the throughput of the network C, λ_k, and exponential service times D_k. Let $W_{O,k}$ be the expected waiting time in this $M/M/1$ queue. Then, f_m is defined as the ratio of the expected waiting time in C to the expected waiting time in the $M/M/1$ queue. Therefore, f_m is written as:

$$f_m = \frac{W_{C},_k}{W_{O},_k} = W_{C,k}\left(\frac{1-\lambda_k D_k}{\lambda_k D_k^2}\right) \tag{8}$$

Therefore,

$$W_k = \left[\frac{\lambda D_k^2}{1-\lambda D_k}\right]\left[\frac{c_{s,k}^2 + c_{a,k}^2}{2}\right]\left[W_{C},_k\left(\frac{1-\lambda_k D_k}{\lambda_k D_k^2}\right)\right] \tag{9}$$

3. Linkage Equation:

$$\sum_{k=1}^{K} L_k = N \tag{10}$$

Algorithm for throughput matching:
The solution algorithm begins with an initial value of throughput λ_{LB} and progressively updates the estimates of throughput and different traffic process parameters until they converge (within a user-specified tolerance limit) and are consistent with the input parameters. The iterative procedure starts by equating the initial throughput estimate, λ_{LB} ,to the arrival rate, λ, at station 1. To completely specify the arrival process at station 1, an initial choice of $c_{a,1}$ is also made. However, the initial choice of λ and $c_{a,1}$ might not be consistent with each other. Therefore, in next step, the algorithm updates the initial estimate of $c_{a,1}$ till it is consistent with the current choice of λ. To update the estimate of $c_{a,1}$, the algorithm uses the linkage and characterization equations successively at each station in the routing. Subsequently, the departure process parameters at station 1 and K are estimated. Since departures from station K form arrivals to station 1, a new estimate of $c_{a,1}$ is obtained. If the old and the new estimates of $c_{a,1}$ are not within a pre-defined tolerance limit (ε_3), the procedure is repeated with the new estimate of $c_{a,1}$. Once the algorithm obtains a value of $c_{a,1}$ which is consistent with the choice of λ, in

next step the algorithm verifies whether the values of λ and $c_{a,1}$ are consistent with the other input parameters of the network. In particular, the algorithm verifies whether the sum of the mean queue lengths at stations sums up to N. If this equation is satisfied within some user-specified tolerance limit (ε_2), the algorithm proceeds to the next step. Otherwise, the choice of λ is incremented by 0.001 and the above steps are repeated. In the next step, the algorithm verifies if the λ obtained after the above step is equal to the throughput of the open queueing network (λ_{ac}) and it is specified by the user. If this equation is satisfied within some user-specified tolerance limit (ε_1), the algorithm terminates. Otherwise N is incremented by 1 and the above steps are repeated till λ is equal to λ_{ac}. Finally, we get the throughput of the closed system with N cards that matches the arrival rate of the open system, λ.

Algorithm 1 Analysis of CONWIP policy for raw materials always available

Given: $K, \lambda_{ac}, D_k, c_{s,k}$
Define: $\Delta_1 = |\lambda_{ac} - \lambda|$, $\Delta_2 = |L - N|$, $\Delta_3 = |c_{a,1} - c_{a,K}|$
Initialize: $\lambda_1 = \lambda_{LB}$, $N = 1$, $\Delta_1 = \varepsilon_1 + 1$
  while $\Delta_1 > \varepsilon_1$ **do**
Initialize: $\Delta_2 = \varepsilon_2 + 1$
    while $\Delta_2 > \varepsilon_2$ **do**
Initialize: $c_{a,1} = c_{s,1}$, $\Delta_3 = \varepsilon_3 + 1$
      while $\Delta_3 > \varepsilon_3$ **do**
        Compute $c_{a,1}$ and $c_{a,K}$ from Equation 7, compute $\Delta_3 = |c_{a,1} - c_{a,K}|$ and set $c_{a,1} = c_{a,K}$
      end while
      Compute f_m from Equation 10 and mean queue lenghts L_k from Equation 8 and 11, $L = \sum_{k=1}^{K} L_k$
      Compute $\Delta_2 = |L - N|$
      $\lambda = \lambda + 0.001$
    end while
    Compute $\Delta_1 = |\lambda_{ac} - \lambda|$
    $N + +$
  end while

3.1.2 Raw materials are arriving at some rate

In this section, it is considered that the raw materials arrive at some rate. The product arrival buffer and raw material arrival buffer are taken as a join/fork station. We also study the effect of changes in the CV of raw material inter-arrival times on the system throughput and station utilization.

Analysis of scheduled start time policy:
This section describes the queuing network model of a single product system with K manufacturing stations operating under Scheduled start time policy (see Figure 5). A station k is described by three parameters - expected service time (D_k), CV ($c_{s,k}$) of the service time and station utilization (U_k). Product arrival process for Poisson and Deterministic scheduled start time policy is characterized by the mean inter-arrival time, ($\lambda_{f,1}^{-1}$), and the CV of inter-arrival times, $c_{f,1} = 1$ and $c_{f,1} = 0$, respectively. Raw material arrival process is characterized by the mean inter-arrival times ($\lambda_{p,1}^{-1}$) and CV of inter-arrival times ($c_{p,1}$). Raw material arrival buffer and product arrival buffer are considered as join stations.

After obtaining the arrival characteristics from the join station, all the parameters are calculated in the same manner as in the case when raw materials are always available. However, there is an additional waiting time involved due to the queue in product buffer. Hence, expected throughput time is given by:

$$R = \sum_{k=1}^{K}(R_k) + L_{f,1}/\lambda_{f,1} \tag{11}$$

Analysis of CONWIP policy:
This section describes the queuing network model of a single product with K manufacturing stations operating under the CONWIP policy (see Figure 3). Total number of cards in the system is equal to the number of parts in progress (N). Product arrival process is characterized as Poisson if the CV of inter-arrival times ($c_{f,1}$) is 1 or Deterministic if $c_{f,1}$ is 0 and the arrival rate is ($\lambda_{f,1}$). Raw material arrival process is characterized by the mean inter-arrival times ($\lambda_{p,1}^{-1}$) and CV of inter-arrival times ($c_{p,1}$). Raw material arrival buffer and product arrival buffer is considered as a join station.

After obtaining the arrival characteristics from the join station, all the parameters are calculated in the same manner as in the case when raw materials are always available. In addition to the queue length at the stations ($\sum_{k=1}^{K} L_k$), the linkage equation also considers the expected queue length in product arrival buffer ($L_{f,1}$) and finished products arrival buffer ($L_{f,2}$).

$$L_{f,1} + \sum_{k=1}^{K} L_k + L_{f,2} = N \tag{12}$$

The solution approach is identical to the one explained in section 3.1.1.

3.2 Analysis of System II

System II is a join station for both CONWIP policy and scheduled start time policy. Its performance measures are of interest to the customers. System II is analyzed for different values of order-inter arrival times($\lambda_{p,2}^{-1}$), its CV ($c_{p,2}$), and buffer capacities. The performance measures include expected number of backlogs and expected waiting time for the backlogs to get fulfilled. The synchronization between demand arrival and finished products arrival buffer are modeled as a join station (Refer Figure 23). Fork/join stations are used to model synchronization constraints between finished goods and customer demands. It is analyzed according to the formulas given by Satyam and Krishnamurthy [2008], which is mentioned in Appendix A.

4 Numerical Results

In this section, we analyze the models described in the previous sections for various sets of input parameters described in each experiment. In Section 4.1 we numerically demonstrate the results for the theorem stated in Section 3.1.1. In Sections 4.2 and 4.3, the numerical results for the two cases of raw material always available and raw materials arrival at some rate are provided. In Section 4.4, we validate the model results using simulation results.

4.1 Theorem

In this section, we consider a balanced single-class open BCMP network and a closed network with exponential service times. Expected service times at the stations (D_k) is 1 second i.e., $D_1 = D_2 = D_3 = D_4 = D_5 = D_6 = D_7 = D_8 = D_9 = D_{10} = 1$ second, station utilization $U_k = 80\%$ and throughput of the system = 0.8 jobs per second. (See Table 3)

From Table 4, we see that the benefit of the closed card controlled system in terms of expected throughput times decreases with the increasing number of stations i.e. $\frac{R_o - R_c}{R_o} = \frac{1}{3}$ with three stations and $\frac{R_o - R_c}{R_o} = \frac{1}{10}$ with 10 stations.

4.2 Raw materials are always available

In this section, we numerically analyze the models where raw materials are always available for production. Hence there is no delay/ waiting times involved for the availability of raw materials. For the performance evaluation of System II, demand arrival rate is taken to be 1.2 jobs per second. The effect of changes in the CV of

Table 3: Design parameters

S. No	Parameter	Value
1	U_k	80%
2	D_k (sec)	1.0
3	λ(per sec)	0.8

Table 4: Comparison of expected throughput times

K	R_o	R_c	$\frac{R_o-R_c}{R_o}$	$\frac{1}{K}$
3	15	8.75	0.40	0.33
10	50	43.75	0.12	0.10

demand inter-arrival times is analyzed by changing the CV value to 0.5, 1 and 2.5 for each experiment. The experiments are described in Appendix B.

4.2.1 Balanced System

In this section, we consider that the stations of the system have same expected service times ($D_k = 1$) and we consider two values of the CV of service times (0.3 and 1). The experimental details are summarized in Table 5. The comparison of expected WIP and expected throughput times are given in Table 6.

4.2.2 Unbalanced System

In this section we consider that the system has one bottleneck station with higher service times than the rest of the stations. ($D_{unb} = 1$ sec and $D_k = 0.8$ sec). We consider two positions for the bottleneck station - the first station and the last station. These two cases are analyzed separately. The comparison of expected WIP and expected throughput times are shown in Table 6.

Analysis of System II:

Balanced System:

The value of expected number of backlogs and the expected waiting time for the backlogs to get fulfilled in a balanced system are calculated for number of stations $K = 3$ and 10, the CV of service times of the station $c_{s,k} = 0.3$ and 1.0 respectively, the demand arrival rate $\lambda_{p,2} = 1.2$ per sec and the CV of demand inter-arrival times $c_{p,2} = 0.5$, 1 and 2.5 (see Table 7).

Table 5: Design parameters for the case when raw materials are always available

Experiment No.	K	U_k	D_k (sec)	$c_{s,k}$	U_{unb}	D_{unb} (sec)	Position of bottleneck
1	3	80%	1	0.3	-	-	-
2	3	80%	1	1.0	-	-	-
3	10	80%	1	0.3	-	-	-
4	10	80%	1	1.0	-	-	-
5	3	75%	0.8	0.3	94%	1.0	first station
6	3	75%	0.8	0.3	94%	1.0	last station
7	3	75%	0.8	1.0	94%	1.0	first station
8	3	75%	0.8	1.0	94%	1.0	last station
9	10	75%	0.8	0.3	94%	1.0	first station
10	10	75%	0.8	0.3	94%	1.0	last station
11	10	75%	0.8	1.0	94%	1.0	first station
12	10	75%	0.8	1.0	94%	1.0	last station

Table 6: Comparison of expected WIP and Expected throughput times when raw materials are always available

			Comparison of expected WIP					Comparison of Expected throughput times				
Experiment No.	λ	N	OP	CON	%dec.	OD	%dec.	OP	CON	%dec.	OD	%dec.
1	0.8	3	5.20	3.00	-42.30	2.65	-49.50	6.50	3.75	-42.31	3.31	-49.08
2	0.8	7	12.00	7.00	-41.66	9.33	-22.50	15.00	8.75	-41.67	11.66	-22.27
3	0.8	11	11.84	11.00	-7.00	9.15	-22.70	14.80	13.75	-7.09	11.44	-22.70
4	0.8	35	40.00	35.00	-12.00	37.21	-7.00	50.00	43.75	-12.50	46.51	-7.00
5	0.94	3	10.90	3.00	-72.00	3.00	-72.00	11.6	3.19	-72.5	3.19	-72.5
6	0.94	3	6.70	3.00	-55.22	3.45	-48.50	7.13	3.19	-55.25	3.67	-48.53
7	0.94	11	21.73	11.00	-49.00	13.36	-36.21	23.11	11.70	-49.37	14.21	-37.50
8	0.94	11	21.73	11.00	-49.00	18.42	-15.23	23.11	11.70	-49.37	19.60	-15.19
9	0.94	9	16.62	9.00	-45.85	8.77	-47.23	17.68	9.57	-45.87	9.33	-47.22
10	0.94	9	11.51	9.00	-21.81	9.34	-17.11	12.24	9.57	-21.81	9.93	-18.87
11	0.94	34	42.96	34.00	-20.80	35.05	-18.40	45.70	36.17	-20.85	37.29	-18.40
12	0.94	34	42.96	34.00	-20.80	40.65	-5.38	45.70	36.17	-20.85	43.24	-5.38

Unbalanced System:

The value of the expected number of backlogs and the expected waiting time for the backlogs to get fulfilled in an unbalanced system are calculated for number of stations $K = 3$ and 10, the CV of service times of the station $c_{s,k} = 0.3$ and 1.0 respectively, demand arrival rate $\lambda_{p,2} = 1.2$ per sec and the CV of demand inter-arrival times $c_{p,2} = 0.5$, 1 and 2.5 (see Table 8).

Table 7: Analysis of System II (Balanced System)

K	$K_{p,2}$	$K_{f,2}$	$c_{f,2}$	$c_{s,k}$	N_b	W (sec)
3	3	100	0.5	0.3	1.64	1.37
10	7	100	0.5	1.0	5.19	4.32
3	3	100	1.0	0.3	1.59	1.32
10	7	100	1.0	1.0	5.12	4.26
3	3	100	2.5	0.3	1.45	1.21
10	7	100	2.5	1.0	4.6	3.83
3	11	100	0.5	0.3	9.28	7.73
10	18	100	0.5	1.0	16.23	13.53
3	11	100	1.0	0.3	9.15	7.62
10	18	100	1.0	1.0	16.00	13.33
3	11	100	2.5	0.3	8.25	6.88
10	18	100	2.5	1.0	14.40	12.00

Table 8: Analysis of System II for unbalanced system

K	$c_{s,k}$	$K_{p,2}$	$K_{f,2}$	$c_{f,2}$	N_b	W (sec)
3	0.3	3	100	0.5	1.15	0.96
10	0.3	9	100	0.5	5.96	4.97
3	0.3	3	100	1.0	1.14	0.95
10	0.3	9	100	1.0	5.87	4.89
3	0.3	3	100	2.5	1.02	0.85
10	0.3	9	100	2.5	5.26	4.38
3	1.0	11	100	0.5	7.74	6.45
10	1.0	34	100	0.5	30.84	25.70
3	1.0	11	100	1	7.63	6.36
10	1.0	34	100	1	30.38	25.32
3	1.0	11	100	2.5	6.83	5.69
10	1.0	34	100	2.5	27.20	22.67

4.3 Raw materials are arriving at some rate

In this section, the numerical experiments are performed for the case where raw materials arrive at some rate. The card/authorization signal arrival buffer and raw material arrival buffer are taken as a join/fork station. We also study the effect of changes in CV for the raw material inter-arrival times on the system throughput and station utilization. The experiments are described in Appendix C.

4.3.1 Balanced System

In this section, we consider that the stations of the system have same expected service times ($D_k = 1$) and we consider two values of CV of service times (0.3 and 1). The comparison of expected WIP and expected throughput time (inclusive of the expected waiting time for the backlog), value of expected number of backlogs and the expected waiting time for the backlogs to get fulfilled are shown in Table 10.

4.3.2 Unbalanced System

In this section, we consider that the system has one bottleneck station with higher service times than the rest of the stations. ($D_{unb} = 1$ and $D_k = 0.8$). We consider two positions for the bottleneck stations - first station and last station. These two cases are analyzed separately. The comparison of expected WIP and expected throughput time (inclusive of the waiting time of the backlog), value of expected number of backlogs and the expected waiting time for the backlogs to get fulfilled have been shown in Table 10.

Table 9: Design parameters for the case when raw materials are arriving at some rate

	System I parameters								System II parameters			
Experiment No.	$\lambda_{p,1}$(per sec)	$c_{p,1}$	K	D_k(sec)	$c_{s,k}$	D_{unb}	c_{unb}	Bottleneck's Position	$\lambda_{p,2}$(per sec)	$c_{p,2}$	$K_{p,2}$	$K_{f,2}$
1	0.8	0.5	3	1.0	0.3	-	-	-	0.8	0.5	100	3
2	0.8	1.0	3	1.0	0.3	-	-	-	0.8	1.0	100	3
3	0.8	2.5	3	1.0	0.3	-	-	-	0.8	2.5	100	3
4	0.8	0.5	3	1.0	1.0	-	-	-	0.8	0.5	100	11
5	0.8	1.0	3	1.0	1.0	-	-	-	0.8	1.0	100	11
6	0.8	2.5	3	1.0	1.0	-	-	-	0.8	2.5	100	11
7	0.8	0.5	10	1.0	0.3	-	-	-	0.8	0.5	100	7
8	0.8	1.0	10	1.0	0.3	-	-	-	0.8	1.0	100	7
9	0.8	2.5	10	1.0	0.3	-	-	-	0.8	2.5	100	7
10	0.8	0.5	10	1.0	1.0	-	-	-	0.8	0.5	100	35
11	0.8	1.0	10	1.0	1.0	-	-	-	0.8	1.0	100	35
12	0.8	2.5	10	1.0	1.0	-	-	-	0.8	2.5	100	35
13	0.94	0.5	3	0.8	0.3	1.0	0.3	first station	0.94	0.5	200	12
14	0.94	0.5	3	0.8	0.3	1.0	0.3	last station	0.94	0.5	200	12
15	0.94	1.0	3	0.8	0.3	1.0	0.3	first station	0.94	1.0	200	12
16	0.94	1.0	3	0.8	0.3	1.0	0.3	last station	0.94	1.0	200	12
17	0.94	2.5	3	0.8	0.3	1.0	0.3	first station	0.94	2.5	200	12
18	0.94	2.5	3	0.8	0.3	1.0	0.3	last station	0.94	2.5	200	12
19	0.94	0.5	3	0.8	1.0	1.0	1.0	first station	0.94	0.5	200	16
20	0.94	0.5	3	0.8	1.0	1.0	1.0	last station	0.94	0.5	200	16
21	0.94	1.0	3	0.8	1.0	1.0	1.0	first station	0.94	1.0	200	16
22	0.94	1.0	3	0.8	1.0	1.0	1.0	last station	0.94	1.0	200	16
23	0.94	2.5	3	0.8	1.0	1.0	1.0	first station	0.94	2.5	200	16
24	0.94	2.5	3	0.8	1.0	1.0	1.0	last station	0.94	2.5	200	16
25	0.94	0.5	10	0.8	0.3	1.0	0.3	first station	0.94	0.5	200	20
26	0.94	0.5	10	0.8	0.3	1.0	0.3	last station	0.94	0.5	200	20
27	0.94	1.0	10	0.8	0.3	1.0	0.3	first station	0.94	1.0	200	20
28	0.94	1.0	10	0.8	0.3	1.0	0.3	last station	0.94	1.0	200	20
29	0.94	2.5	10	0.8	0.3	1.0	0.3	first station	0.94	2.5	200	20
30	0.94	2.5	10	0.8	0.3	1.0	0.3	last station	0.94	2.5	200	20
31	0.94	0.5	10	0.8	1.0	1.0	1.0	first station	0.94	0.5	200	35
32	0.94	0.5	10	0.8	1.0	1.0	1.0	last station	0.94	0.5	200	35
33	0.94	1.0	10	0.8	1.0	1.0	1.0	first station	0.94	1.0	200	35
34	0.94	1.0	10	0.8	1.0	1.0	1.0	last station	0.94	1.0	200	35
35	0.94	2.5	10	0.8	1.0	1.0	1.0	first station	0.94	2.5	200	35
36	0.94	2.5	10	0.8	1.0	1.0	1.0	last station	0.94	2.5	200	35

Table 10: Comparison of expected WIP, expected throughput times and System II analysis for the case when raw materials are arriving at some rate

					Comparison of expected WIP					Comparison of expected throughput times					System II Analysis	
Experiment No.	U_{unb} %	U_k %	λ (/sec)	N	OP	CON	%dec.	OD	%dec.	OP	CON	%dec.	OD	%dec.	N_b	W(sec)
1	-	59.00	0.59	3	3.48	2.78	-20.11	2.77	-20.40	5.90	4.72	-20.00	4.70	-20.33	1.35	1.69
2	-	58.00	0.58	3	3.54	2.78	-21.46	2.77	-21.75	6.10	4.80	-21.3	4.78	-21.64	1.40	1.75
3	-	58.00	0.58	3	4.23	3.72	-12.06	3.60	-14.89	7.30	6.42	-12.00	46.20	-15.00	1.51	1.89
4	-	71.00	0.71	7	13.21	12.18	-7.80	12.21	-7.61	18.60	17.15	-7.80	17.20	-7.50	5.35	6.69
5	-	70.00	0.70	7	13.39	12.45	-7.02	12.53	-6.69	19.13	17.78	-7.00	17.9	-6.73	5.69	7.11
6	-	68.00	0.68	7	14.96	13.48	-8.98	14.00	-6.41	22.00	19.82	-10.00	20.60	-6.80	6.32	7.9
7	-	69.00	0.69	11	8.83	7.25	-17.89	6.17	-30.00	12.28	10.51	-14.41	8.95	-27.11	2.99	3.74
8	-	68.00	0.68	11	8.70	7.34	-15.63	6.24	-28.17	12.8	10.8	-15.6	9.19	-28.2	3.04	3.8
9	-	67.00	0.67	11	9.65	7.82	-18.96	7.28	-24.53	14.40	11.68	-18.88	10.87	-24.51	3.2	4
10	-	77.00	0.77	35	48.00	42.37	-11.73	42.53	-11.38	62.35	55.03	-11.74	55.24	-11.40	16.22	20.27
11	-	77.00	0.77	35	48.51	42.75	-11.87	43.13	- 11.09	63.00	55.52	-11.87	56.01	-11.09	16.3	20.40
12	-	76.00	0.76	35	51.93	45.53	-12.32	45.86	-11.69	68.33	59.91	-12.32	60.34	-11.69	18.54	23.18
13	90.00	72.00	0.90	12	8.14	6.20	-23.83	6.49	-20.27	9.04	6.89	-23.78	7.21	-20.24	3.48	3.70
14	90.00	72.00	0.90	12	7.43	6.20	-16.55	6.67	-10.23	8.26	6.89	-16.58	7.41	-10.29	3.48	3.70
15	90.00	72.00	0.90	12	9.80	7.01	-28.47	7.94	-18.98	10.89	7.79	-28.46	8.82	-19.00	3.98	4.23
16	90.00	72.00	0.90	12	9.00	7.01	-22.21	8.08	-10.22	10.01	7.79	-22.18	8.98	-10.28	3.98	4.23
17	89.00	71.00	0.89	12	14.77	11.75	-20.48	12.71	-13.96	16.41	13.05	-20.47	14.12	-14.03	4.80	5.12
18	89.00	71.00	0.89	12	14.24	11.75	-17.48	13.30	-6.60	15.82	13.05	-17.51	14.78	-6.57	4.80	5.12
19	90.00	72.00	0.90	16	17.83	14.83	-16.82	15.32	- 14.08	19.81	16.48	-16.80	17.02	-14.08	5.65	6.01
20	90.00	72.00	0.90	16	17.33	14.83	-14.42	16.30	- 6.00	19.26	16.48	-14.43	18.11	-6.00	5.65	6.01
21	90.00	72.00	0.90	16	18.93	15.62	-17.51	16.42	- 13.28	21.03	17.35	-17.49	18.24	-13.26	6.08	6.47
22	90.00	72.00	0.90	16	18.11	15.62	-13.73	17.17	- 5.18	20.12	17.35	-13.88	19.08	-5.18	6.08	6.47
23	89.00	71.00	0.89	16	23.22	19.58	-15.68	20.79	- 10.46	25.80	21.75	-15.69	23.10	-10.46	7.68	8.17
24	89.00	71.00	0.89	16	21.79	19.58	-10.14	20.43	- 6.24	24.21	21.75	-10.18	22.70	-6.24	7.68	8.17
25	91.00	73.00	0.91	20	17.05	13.85	-18.76	13.90	-18.50	18.94	15.39	-18.74	15.44	-18.47	5.84	6.21
26	91.00	73.00	0.91	20	16.10	13.85	-13.99	14.61	-9.27	17.89	15.39	-13.97	16.23	-9.28	5.84	6.21
27	91.00	73.00	0.91	20	19.21	16.07	-16.34	16.02	-16.57	21.01	17.66	-15.94	17.61	-16.18	6.97	7.41
28	91.00	73.00	0.91	20	18.33	16.07	-12.31	17.08	-6.77	20.14	17.66	-12.31	18.78	-6.75	6.97	7.41
29	89.00	71.00	0.89	20	25.06	21.99	-12.24	22.08	-11.88	28.16	24.71	-12.25	24.81	-11.90	8.94	9.51
30	89.00	71.00	0.89	20	24.03	21.99	-8.49	22.73	-5.41	27.00	24.71	-8.50	25.54	-5.40	8.94	9.51
31	92.00	74.00	0.92	35	47.11	41.69	-11.51	42.01	-10.82	52.34	46.32	-11.50	46.68	-10.86	16.57	17.73
32	92.00	74.00	0.92	35	46.59	41.69	-10.52	45.50	-2.33	51.77	46.32	-10.53	50.56	-2.34	16.57	17.73
33	91.00	73.00	0.91	35	48.84	43.33	-11.27	43.52	-10.90	53.67	47.62	-11.27	47.82	-11.89	17.81	18.95
34	91.00	73.00	0.91	35	48.03	43.33	-9.78	45.96	-4.30	52.78	47.62	-9.78	50.51	-4.30	17.81	18.95
35	88.00	70.00	0.88	35	52.53	46.85	-10.81	47.18	-10.19	59.68	53.24	-10.79	53.61	-10.16	19.78	21.04
36	88.00	70.00	0.88	35	50.94	46.85	-8.03	49.49	-2.84	57.89	53.24	-8.28	56.24	-2.85	19.78	21.04

4.4 Comparison with Simulation

Table 11: Design of experiment for model validation

Expected Absolute Error %				Percentage Error Range			
R	WIP	N_b	W	R	WIP	N_b	W
3.97	3.96	3.80	3.79	(0.55,6.87)	(0.62, 6.90)	(0.76, 6.52)	(0.66,6.53)

This section describes the set of design experiments conducted to validate the analytical model results. The analytical results are validated using the models developed using Simulation with Arena software. There are 144 scenarios including both the model variants namely (i) raw materials always present and (ii) raw materials arriving at some rate together. For each scenario, 20 replications were run for 24 hours with a warm up period of 5 hours. The expected absolute error percentage is obtained for system parameters - expected work in progress (WIP) for System I, expected throughput time (R) for System I, expected number of backlogs in System II and expected waiting time for the backlogs to get cleared in System II using the formula ((|A-S|)/S)x100, where A is the result obtained from analytical model and S is the result obtained from simulation model. The distribution of percentage error of the above mentioned parameters is given in Appendix A. For the four parameters, the absolute error percentage range and expected values are given in Table 11. The frequency distribution of absolute percentage of error for expected throughput time of System I, expected work in progress in System I, expected number of backlogs in System II and expected waiting time for the backlog to get cleared in System II is given in Appendix A.

5 Results and Insights

As expected, from the numerical results, it can be concluded that both Deterministic start time policy and CONWIP policy provided improved performance over Poisson input, by reducing the expected throughput times and expected work in progress (WIP).

Effect of change in number of stations on expected WIP:
From the experiments, it is observed that as the number of stations increases, the expected WIP of the system increases almost linearly. For both balanced and unbalanced system, as the number of stations increases, CONWIP policy performs better than both Poisson and Deterministic scheduled start time policies. Figure 6 illustrates the variation in expected WIP for Poisson scheduled start time policy,

Deterministic scheduled start time policy and CONWIP policy.

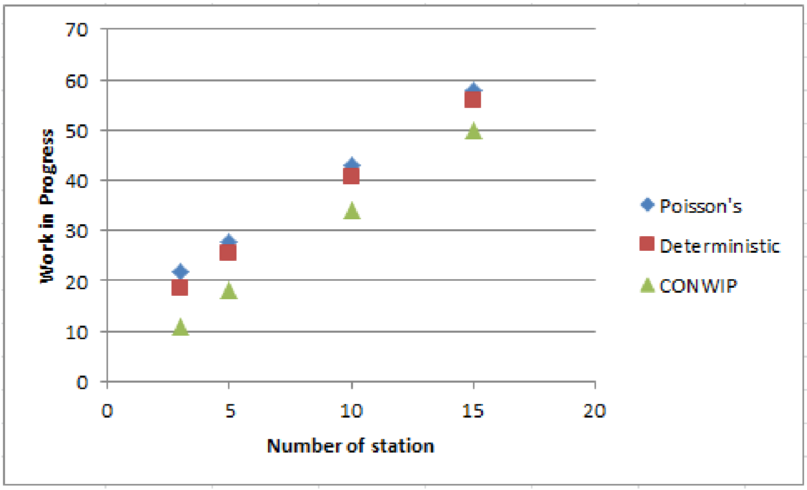

Figure 6: Expected WIP vs Number of stations when raw materials are always available (Experiments 8 and 12)

Effect of CV of service times on expected WIP:
From the experiments, it is observed that as the CV of service times of stations increases, the expected WIP of the system increases exponentially. As the CV increases, CONWIP policy becomes more efficient than both open Poisson and Deterministic scheduled start time policies. Figure 7 illustrates the variation in expected WIP for Poisson scheduled start time policy, Deterministic scheduled start time policy and CONWIP policy for balanced system and Figure 8 represents the same for an unbalanced system.

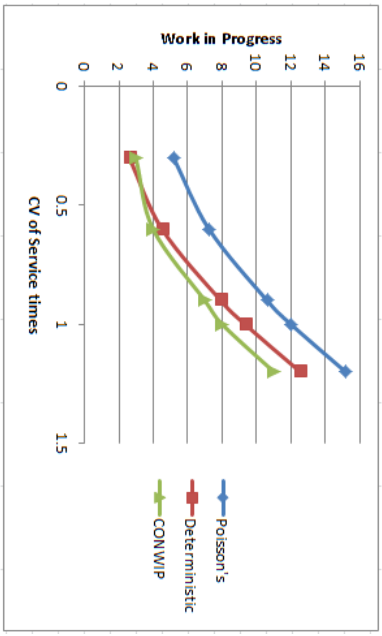

Figure 7: Expected WIP vs CV of service times - Balanced System (Experiments 1 and 2) - Raw materials are always available

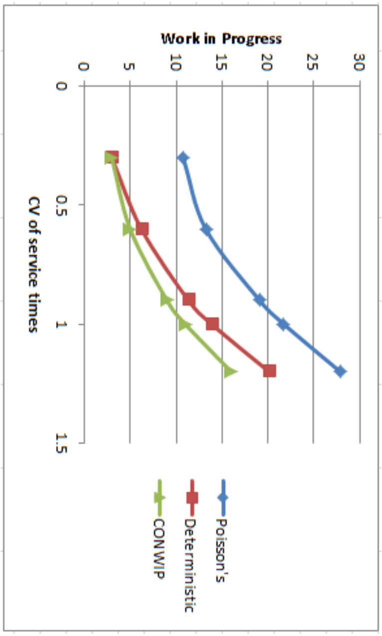

Figure 8: Expected WIP vs CV of service times - Unbalanced System (Experiments 5 and 7)- Raw materials are always available

Effect of location of bottleneck on WIP:
From the experiments, it is observed that Poisson scheduled start time policy performs better when the bottleneck station is placed in the downstream location. Deterministic start time policy performs better when the bottleneck station is placed in the upstream location. Position of the bottleneck does not have any effect on CONWIP policy.

The following Figures 9, 10 and 11 illustrate the variation in expected WIP for Poisson scheduled start time policy, Deterministic scheduled start time policy and CONWIP policy respectively.

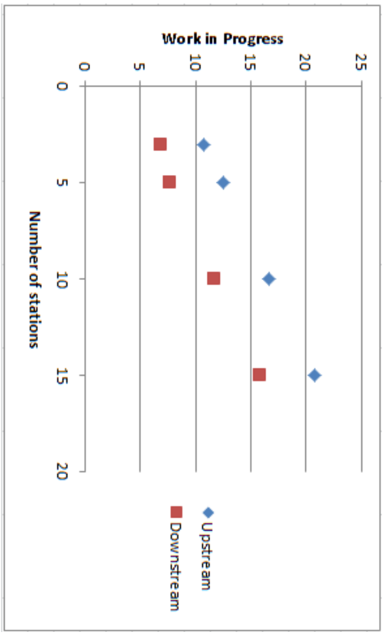

Figure 9: Expected WIP vs Number of stations - Poisson scheduled start time policy (Experiments 5,6,9 and 10)- Raw materials are always available

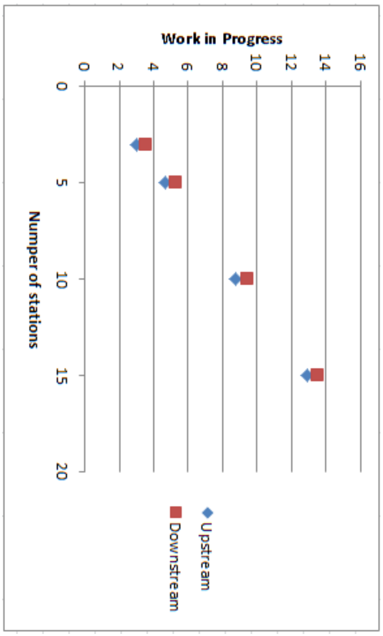

Figure 10: Expected WIP vs Number of stations - Deterministic scheduled start time policy (Experiments 5,6,9 and 10)- Raw materials are always available

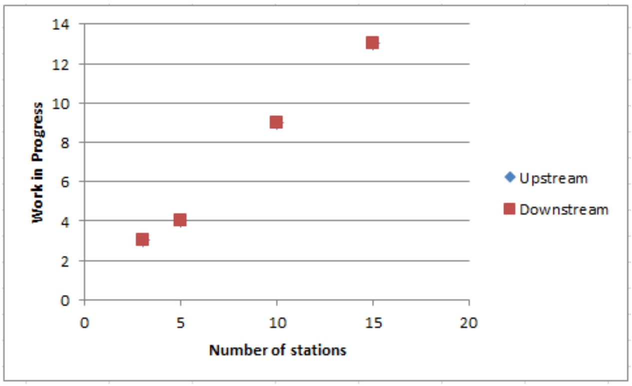

Figure 11: Expected WIP vs Number of stations - CONWIP policy (Experiments 5,6,9 and 10)- Raw materials are always available

Effect of CV of raw material inter-arrival times on expected throughput time:
From the experiments, it is observed that as the CV of raw material inter-arrival time increases, the expected system throughput time increases for all the three

policies. The following Figures 12, 13 and 14 illustrate the variation in expected WIP for Poisson scheduled start time policy, Deterministic scheduled start time policy and CONWIP policy respectively.

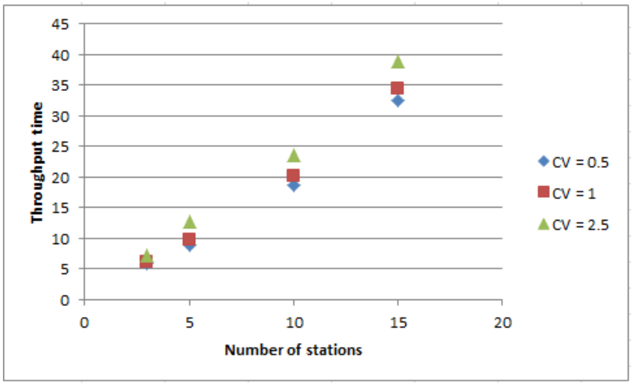

Figure 12: Expected throughput time vs Number of stations : CV of raw material inter-arrival times for Poisson scheduled start time policy (Experiments 1,2,3,7,8 and 9) - Raw materials are arriving at some rate

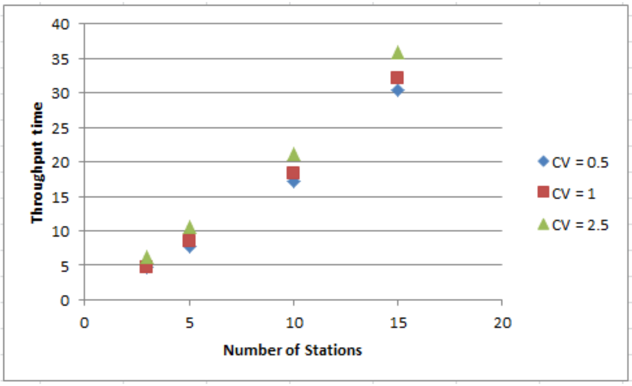

Figure 13: Expected throughput time vs Number of stations : CV of raw material inter-arrival times for Deterministic scheduled start time policy (Experiments 1,2,3,7,8 and 9) - Raw materials are arriving at some rate

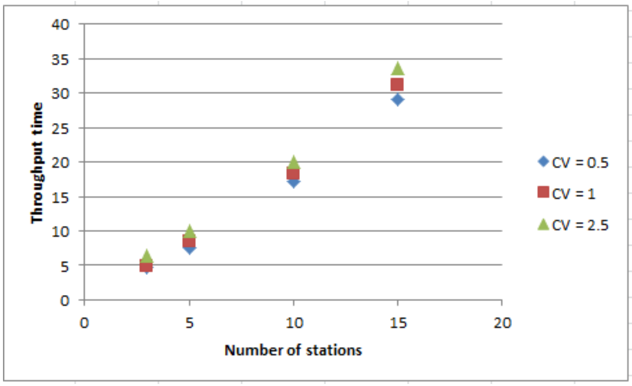

Figure 14: Expected throughput time vs Number of stations : CV of raw material inter-arrival times for CONWIP policy (Experiments 1,2,3,7,8 and 9) - Raw materials are arriving at some rate

Effect of CV of raw material inter-arrival time on expected number of backlogs:
From the experiments, it is observed that as the CV of raw material inter-arrival time increases, the expected number of backlogs and the expected waiting time for the backlogs to get fulfilled also increases. This relationship has been illustrated in Figure 15.

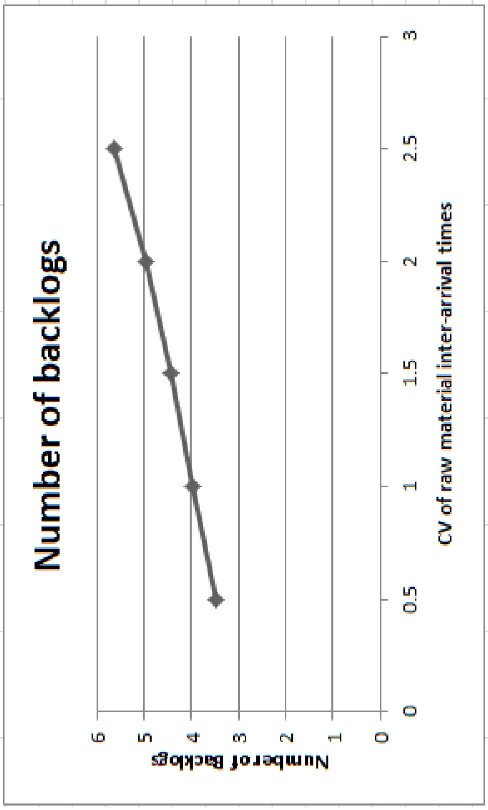

Figure 15: Expected number of backlogs vs CV of raw material inter-arrival times for an unbalanced system with CV of service times 0.3 and number of stations 3 - Raw materials are arriving at some rate

Case wise Summary:

Balanced System
From the experiments conducted on balanced system for various design input parameters, it can be concluded that for low CV of service time and lower number of stations - CONWIP performs similar to Deterministic scheduled start time policy and both perform considerably better than Poisson scheduled start time policy. For low CV of service time and higher number of stations - CONWIP performs better than Deterministic scheduled start time policy and both perform better than Poisson scheduled start time policy (Illustrated in Figure 16). For high CV of service time and lower number of stations - CONWIP performs similar to Deterministic scheduled start time policy and both perform considerably better than Poisson scheduled start time policy. For high CV of service time and higher number of stations - CONWIP performs similar to Deterministic scheduled start time

policy and both perform considerably better than Poisson scheduled start time policy (Illustrated in Figure 17). When raw materials are arriving at some rate, as the CV of raw material inter-arrival time increases, the expected throughput time and expected WIP increases.

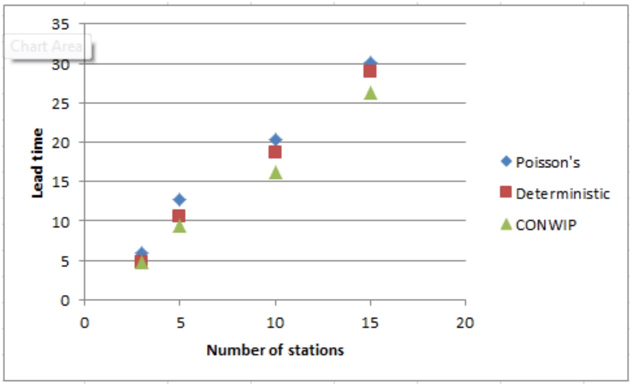

Figure 16: Expected throughput time vs Number of stations (CV =0.3)(Experiments 1 and 7) - Raw materials are arriving at some rate

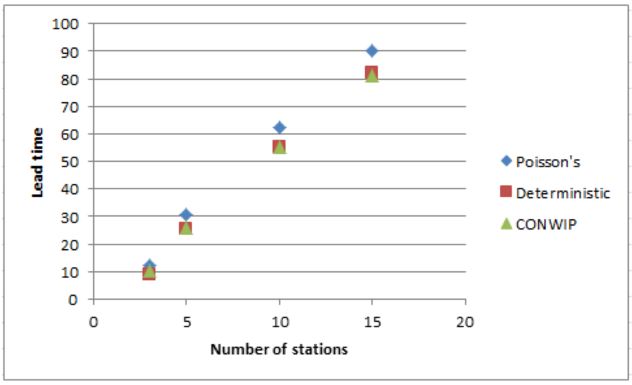

Figure 17: Expected throughput time vs Number of stations (CV =1) (Experiments 4 and 10) - Raw materials are arriving at some rate

No. of Stations \ Service time variability	Low (0.3)	High (1)
Low (3)	CONWIP ≅ Deterministic	CONWIP ≅ Deterministic
High (10)	CONWIP > Deterministic	CONWIP ≅ Deterministic

Figure 18: Performance comparison of CONWIP Policy and Deterministic scheduled start time policy for balanced system

Unbalanced System

From the experiments conducted on unbalanced systems, it can be concluded that CONWIP performs better than Deterministic scheduled start time policy giving high reduction in expected throughput times and expected WIP for upstream location of the bottleneck and lower number of stations as well as for downstream location of the bottleneck for both higher and lower number of stations. Performance of CONWIP policy is approximately equal to that of Deterministic scheduled start time policy for upstream location of the bottleneck and higher number of stations. In all the cases, both Deterministic scheduled start time policy and CONWIP policy perform better than Poisson scheduled start time policy (Illustrated in Figures 19 and 20). For raw material arriving at some rate, as the CV of raw material inter-arrival time increases, the expected throughput time and expected WIP increases.

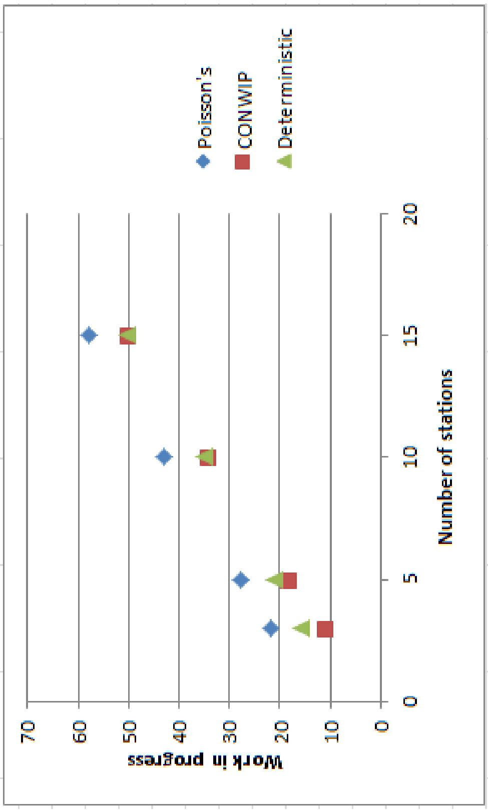

Figure 19: Expected WIP vs Number of stations (Upstream position of bottleneck)(Experiments 7 and 11) - Raw materials are always available

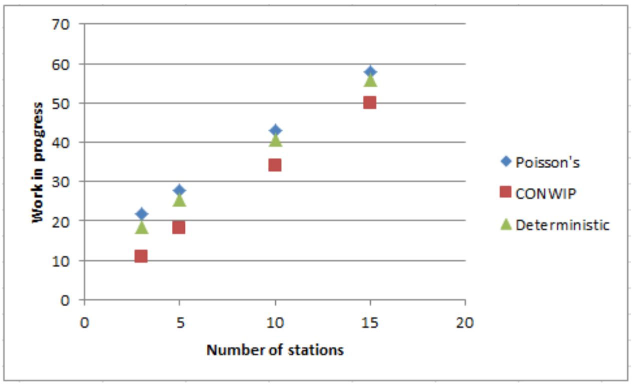

Figure 20: Expected WIP vs Number of stations (Downstream position of bottleneck)(Experiments 8 and 12) - Raw materials are always available

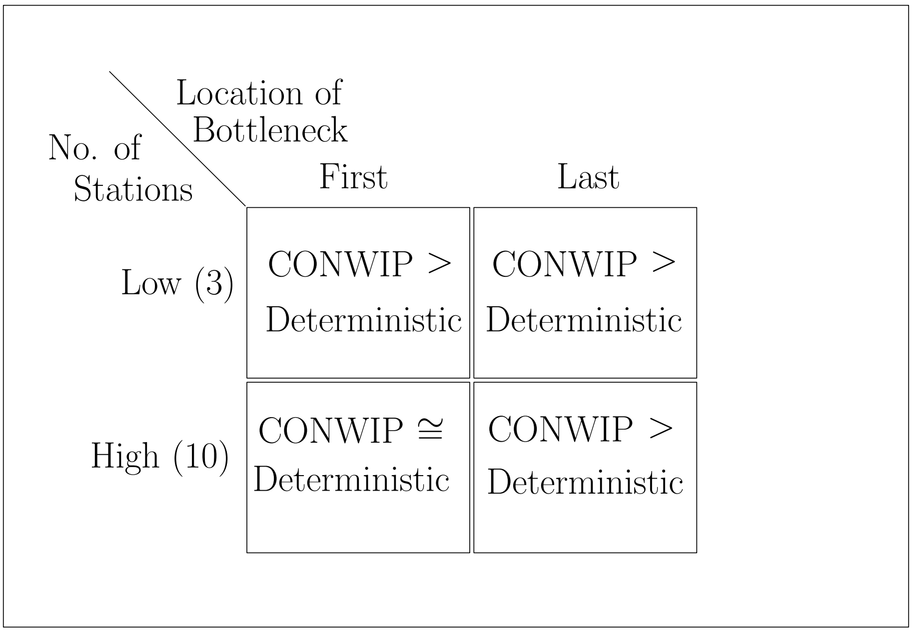

Figure 21: Performance Comparison of CONWIP Policy and Deterministic scheduled start time policy for unbalanced system

Analysis of System II

When the raw materials are always available, the finished products inter-arrival times and its CV does not vary with the changes in CV of demand inter-arrival times as they are independent of each other. Hence, when there is large variations in order arrivals (increasing CV of demand inter-arrival times), the number of finished products stored in the inventory increases which reduces the number of backlogs to be fulfilled. Hence as the CV of demand inter-arrival times increases, the expected number of backlogs and the expected waiting time for the backlogs to get fulfilled decreases. This has been illustrated in Figure 22.

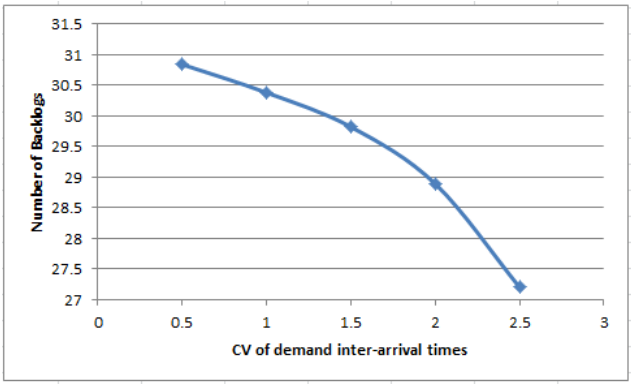

Figure 22: Expected number of backlogs vs CV of demand inter-arrival times for an unbalanced system with CV of service time is 1 and number of stations 10 - Raw materials are always available

6 Conclusions and Future work

In this research, we analyze the performance and design trade-offs among three material control policies namely CONWIP policy, Poisson scheduled start time policy, and Deterministic scheduled start time policy under certain design parameters. The design parameters include station utilization (U_k), expected service times of the station (D_k), CV of service times of the station ($c_{s,k}$), location of bottleneck for an unbalanced system, demand arrival rate, CV of inter-arrival times, raw material arrival rate, and CV of raw materials inter-arrival times (for the case where raw materials are arriving at some rate). From the numerical results, various insights were developed. As the number of stations increases, the expected throughput time and expected WIP increases almost linearly. With increase in CV of service times, CONWIP performs better than the other two systems for fixed number of service stations. As the CV of demand inter-arrival times increases, the expected number of backlogs and the expected waiting time for the backlogs to get fulfilled decreases. With increases in CV of raw material inter-arrival times (for the case where raw materials are arriving at some rate), the throughput of the system decreases and the expected throughput time, the expected number of backlogs and the expected waiting time for the backlogs to get fulfilled increases. As expected, CONWIP and deterministic scheduled start time policies perform better than Poisson scheduled policies in all the cases. For a balanced system, CONWIP performs better than deterministic scheduled start time policy only when CV of service times is low, and the number of stations is high. For all the other cases (for low CV of service times and less number of stations, high CV of service times and any number of stations), their performance is similar. In an unbalanced system, CONWIP policy performs similar to deterministic scheduled start time policy only for upstream location of bottleneck and higher number of stations. For all the other cases (for upstream location of the bottleneck and lower number of stations and downstream location of bottleneck and any number of stations), CONWIP performs better than Deterministic scheduled start time policy. From the above observations it can be concluded that though CONWIP is a better material control policy in most cases and could be adopted on the shop floor for all the design parameters considered in this research, Deterministic policy is comparable with CONWIP in some cases and hence can be adopted as an alternate start time policy.

This research focuses on evaluation of a single class system. We have analyzed the system for various design parameters. In future, a multiple class system (more than one product class) could be analyzed for the various design parameters. For an unbalanced system, instead of just one bottleneck station, a combination of two or more bottleneck stations could be considered and the impact of its location on the system parameters can be evaluated. More material control policies such as POLCA and Workload regulated policies could be considered for performance analysis.

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Appendix A

Analysis of Join/fork station:
Arrival process to buffer f_r is characterized by the arrival rate ($\lambda_{f,r}$), CV ($c_{f,r}$) of inter-arrival times (Refer Figure 23). Arrival process to buffer p_r is characterized by - arrival rate ($\lambda_{p,r}$) and CV of inter-arrival times($c_{p,r}$). Maximum queue length of p_r buffer is $K_{p,r}$ and maximum queue length of f_r buffer is $K_{f,r}$. The mean inter-departure rate is given by λ_r and CV $c_{a,r}$. The mean queue lengths are given by $L_{p,r}$ and $L_{f,r}$ at the buffers p_r and f_r respectively.
(Note: $'r'$ is 1 for raw material/ product arrival fork station and 2 for finished products/ demand arrival fork station)
(Refer Satyam and Krishnamurthy [2008])

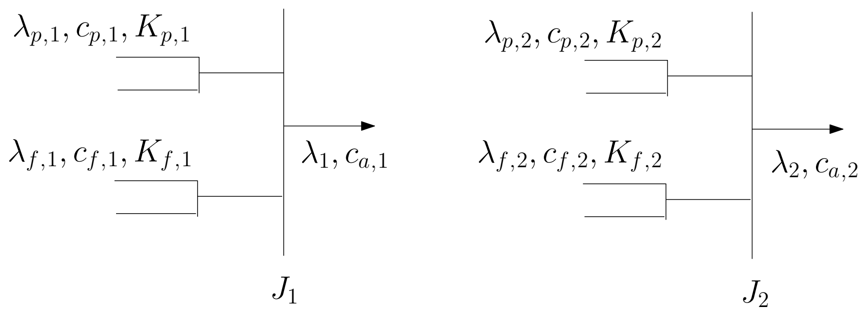

Figure 23: Characterization of fork/join synchronization stations

Analysis of fork station for arrival process

$w_r = \lambda_{p,r} / \lambda_{f,r}$

$c_r = 0.5({c_{p,r}}^2 + {c_{f,r}}^2)$

$v_r = \frac{(1-w_r)w_r^4}{(1+w_r)(1+w_r^8)}$

$K_{a,r} = K_{p,r} + K_{f,r}$

If $w_r \neq 1$

$$\lambda_r = \lambda_{p,r}\left(\frac{1-w_r^{K_{a,r}}}{1-{w_r}^{K_{a,r}+1}}\right)\left(1-0.5(c_r-1)\left(\frac{(1-w_r){w_r}^{K_{a,r}}}{1-{w_r}^{2k_{a,r}+1}}\right)\right) \tag{13}$$

$$c_{j,r}^{2}=\left[\left(\frac{w_r^5 c_{f,r}{}^2}{w_r{}^5+1}\right)+\left(\frac{c_{p,r}{}^2}{w_r{}^5+1}\right)\right]\left[\frac{K_{a,r}(K_{a,r}+1)-1}{(K_{a,r}+1)^2}\right](1+w_r{}^2)^{-0.5} \tag{14}$$

$$L_{p,r}=\left[\left(\frac{w_r^{K_{f,r}}}{1-w_r}\right)\left(\frac{1-w_r{}^{K_{p,r}}}{1-w_1{}^{K_{a,r}+1}}\right)-\left(\frac{K_{p,r}w_r{}^{K_{a,r}+1}}{1-w_r{}^{K_{a,r}+1}}\right)\right](1+v_r(c_r-1)) \tag{15}$$

$$L_{f,r}=\left[\left(\frac{K_{f,r}}{1-w_r{}^{K_{a,r}+1}}\right)-\left(\frac{w_r}{1-w_r}\right)\left(\frac{1-w_r{}^{K_{f,r}}}{1-w_r{}^{K_{a,r+1}}}\right)\right](1-v_r(c_r-1)) \tag{16}$$

If $w_r = 1$

$$\lambda_r=\lambda_{p,r}\left[\frac{K_{a,r}}{K_{a,r}+1}\right]\left[1-0.5\left(\frac{c_r-1}{2K_{a,r}+1}\right)\right] \tag{17}$$

$$c_{j,r}^{2}=\left[\frac{c_{p,r}{}^2+c_{f,r}{}^2}{2\sqrt{2}}\right]\left[\frac{K_{a,r}(K_{a,r}+1)-1}{(K_{a,r}+1)^2}\right] \tag{18}$$

$$L_{p,r}=\frac{K_{p,r}}{2}\left(\frac{K_{p,r}+1}{K_{a,r}+1}\right) \tag{19}$$

$$L_{f,r}=\frac{K_{f,r}}{2}\left(\frac{K_{f,r}+1}{K_{a,r}+1}\right) \tag{20}$$

Results of model validation:
Figures 24, 25, 26 and 27 show the frequency distribution of the absolute percentage of error for expected throughput time of System I, expected work in progress in System I, expected number of backlogs in System II and the expected waiting time for the backlogs to get cleared in System II.

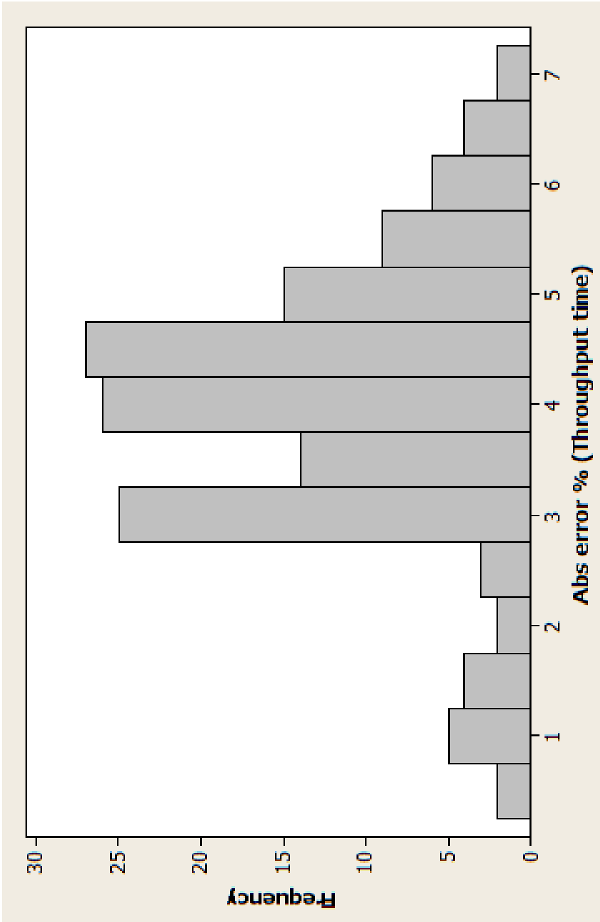

Figure 24: Abs error % (Expected throughput time of System I)

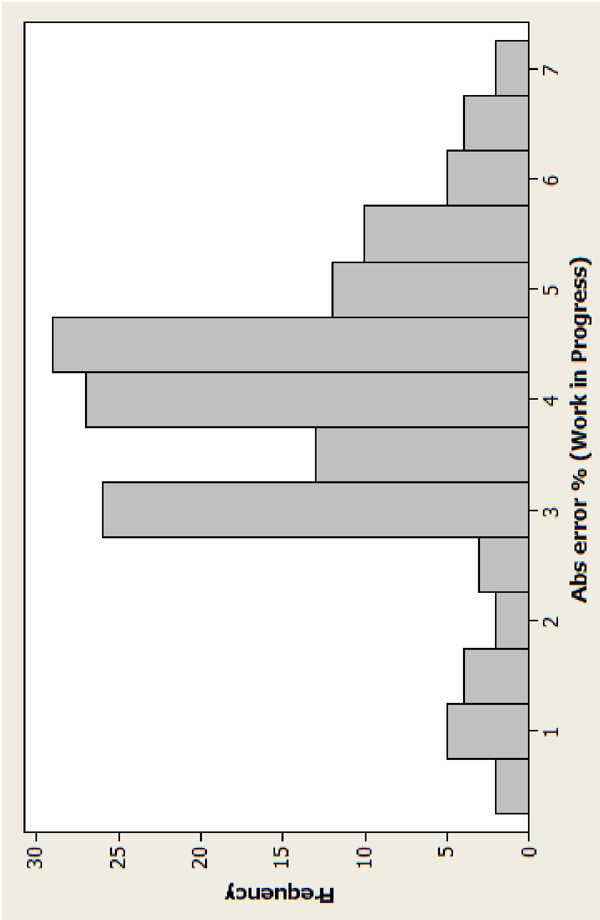

Figure 25: Abs error % (Expected Work in Progress in System I)

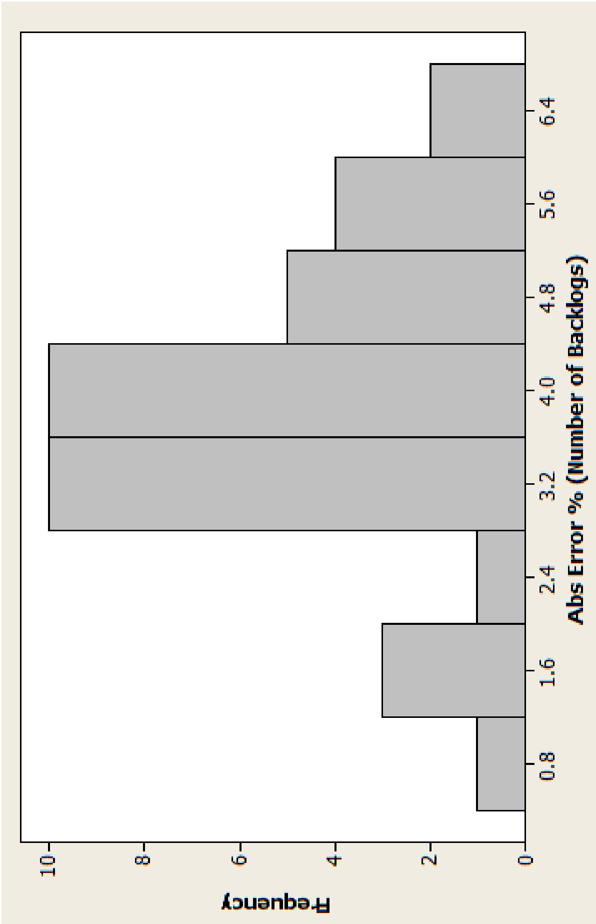

Figure 26: Abs error % (Expected number of backlogs in System II)

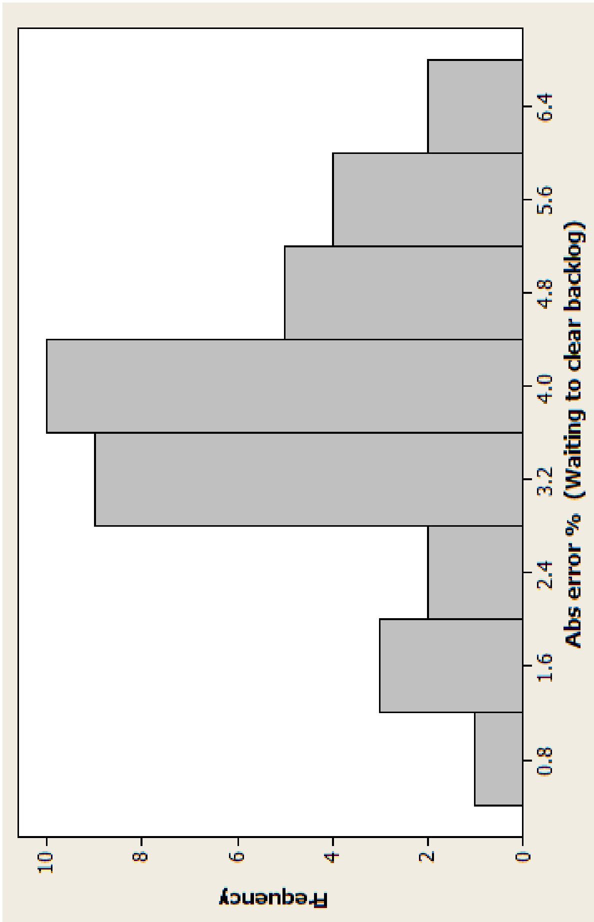

Figure 27: Abs error % (Expected waiting time for the backlogs to get cleared in System II)

Appendix B : Experiments for the case where raw materials are always available

Experiment 1
In this experiment, the performance of a network with 3 manufacturing stations is analyzed. Each product visits manufacturing stations k = 1, 2 and 3 sequentially. Expected service time at station (D_k) $D_1 = D_2 = D_3 = 1$ sec, CV of the service times $c_{s,k} = 0.3$ and utilization $U_k = 80\%$.

Experiment 2
In this experiment, all the design parameters are similar to Experiment 2 except CV of service times $c_{s,k}$ is 1.0.

Experiment 3
In this experiment, the performance of a network with 10 manufacturing stations is analyzed. Each product visits manufacturing stations k = 1, 2,..,10 sequentially. Expected service time at station (D_k) $D_1 = D_2 = D_3 = D_4 = D_5 = D_6 = D_7 = D_8 = D_9 = D_{10} = 1$ sec, CV of service times $c_{s,k} = 0.3$ and utilization $U_k = 80\%$.

Experiment 4
In this experiment, all the design parameters are similar to Experiment 3 except CV of service times $c_{s,k}$ is 1.0.

Experiment 5
In this experiment, the performance of a network with three manufacturing stations is analyzed. Each product visits manufacturing stations k = 1, 2 and 3 sequentially. Expected service time at bottleneck station (D_{unb}) $D_1 = 1$ sec, CV $c_{unb} = 0.3$ and utilization $U_{unb} = 94\%$. Expected service time at non-bottleneck stations (D_k) $D_2 = D_3 = 0.8$ sec, CV $c_{s,k} = 0.3$ and utilization $U_k = 75\%$. Bottleneck station is located in the upstream **first station**.

Experiment 6
In this experiment, all the design parameters are similar to Experiment 5 except that the position of bottleneck which is located at **last station**.

Experiment 7
In this experiment, all the design parameters are similar to Experiment 5 except that the CV of service times $c_{s,k}$ is 1.0.

Experiment 8

In this experiment, all the design parameters are similar to Experiment 7 except that the bottleneck is located at the **last station**.

Experiment 9

In this experiment, the performance of a network with 10 manufacturing stations is analyzed. Each product visits manufacturing stations $k = 1, 2,...,10$ sequentially. Expected service time at bottleneck station (D_{unb}) $D_1 = 1$ sec, CV $c_{unb} = 0.3$ and utilization $U_{unb} = 94\%$. Expected service time at non-bottleneck station (D_k) $D_2 = D_3 = D_4 = D_5 = D_6 = D_7 = D_8 = D_9 = D_{10} = 0.8$ sec, CV $c_{s,k} = 0.3$ and utilization $U_k = 75\%$. Bottleneck station is located in the upstream **first station**.

Experiment 10

In this experiment, all the design parameters are similar to Experiment 9 except that the bottleneck station is located at **last station**.

Experiment 11

In this experiment, all the design parameters are similar to Experiment 9 except that the CV of service times $c_{s,k}$ is 1.0.

Experiment 12

In this experiment, all the design parameters are similar to Experiment 11 except that the position of bottleneck is located at the **last station**.

Appendix C : Experiments for the case where raw materials are arriving at some rate

Experiment 1
In this experiment, the performance of a network with 3 manufacturing stations is analyzed. Each product visits manufacturing stations $k = 1$, 2 and 3 sequentially. Expected service time at station (D_k) $D_1 = D_2 = D_3 = 1$ sec, CV $c_{s,k} = 0.3$. Raw material arrival rate and Demand arrival rate are set at 0.8 per sec, CV of raw material inter-arrival times and demand inter-arrival times are equal to 0.5.

Experiment 2
In this experiment, all the design parameters are similar to Experiment 1 except that the CV of raw material and demand inter-arrival times are equal to 1.0.

Experiment 3
In this experiment, all the design parameters are similar to Experiment 1 except that the CV of raw material and demand inter-arrival times are equal to 2.5.

Experiment 4
In this experiment, all the design parameters are similar to Experiment 1 except that the CV of service times at each station $c_{s,k}$ is equal to 1.0.

Experiment 5
In this experiment, all the design parameters are similar to Experiment 4 except that the CV of raw material and demand inter-arrival times are equal to 1.0.

Experiment 6
In this experiment, all the design parameters are similar to Experiment 4 except that the CV of raw material and demand inter-arrival times are equal to 2.5.

Experiment 7
In this experiment, the performance of a network with 10 manufacturing stations is analyzed. Each product visits manufacturing stations $k = 1, 2, ..., 10$ sequentially. Expected service time at station (D_k) $D_1 = D_2 = D_3 = D_4 = D_5 = D_6 = D_7 = D_8 = D_9 = D_{10} = 1$ sec, the CV $c_{s,k} = 0.3$. Raw material and the Demand arrival rates are equal to 0.8 per sec. CV of raw material inter-arrival times and the demand inter-arrival times are equal to 0.5.

Experiment 8
In this experiment, all the design parameters are similar to Experiment 7 except

that the CV of raw material and demand inter-arrival times are equal to 1.0.

Experiment 9
In this experiment, all the design parameters are similar to Experiment 7 except that the CV of raw material and demand inter-arrival times are equal to 2.5.

Experiment 10
In this experiment, all the design parameters are similar to Experiment 7 except that the CV of service times at each station $c_{s,k}$ are equal to 1.0.

Experiment 11
In this experiment, all the design parameters are similar to Experiment 10 except that the CV of raw material and demand inter-arrival times are equal to 1.0.

Experiment 12
In this experiment, all the design parameters are similar to Experiment 10 except that the CV of raw material and demand inter-arrival times are equal to 2.5.

Experiment 13
In this experiment, the performance of a network with three manufacturing stations is analyzed. Each product visits manufacturing stations k = 1,2 and 3 sequentially. Expected service time at non-bottleneck station (D_k) D_2 = D_3= 0.8 sec, CV $c_{s,k}$ = 0.3. Expected service time at bottleneck station (D_{unb}) D_1 = 1, CV c_{unb} = 0.3. Raw material and the Demand arrival rates are equal to 0.94 per sec. CV of raw material inter-arrival times and the demand inter-arrival times are equal to 0.5. Bottleneck station is located at **first station**.

Experiment 14
In this experiment, all the design parameters are similar to the Experiment 13 except that the position of bottleneck is located at the **last station**.

Experiment 15
In this experiment, all the design parameters are similar to Experiment 13 except that the CV of raw material and demand inter-arrival times are equal to 1.0.

Experiment 16
In this experiment, all the design parameters are similar to Experiment 15 except that the position of bottleneck is located at the **last station**.

Experiment 17

In this experiment, all the design parameters are similar to Experiment 13 except that the CV of raw material and demand inter-arrival times are equal to 2.5.

Experiment 18
In this experiment, all the design parameters are similar to Experiment 17 except that the position of bottleneck is located at the **last station**.

Experiment 19
In this experiment, all the design parameters are similar to Experiment 13 except that the CV of service times at station is equal to 1.0.

Experiment 20
In this experiment, all the design parameters are similar to Experiment 19 except that the position of bottleneck which is located at the **last station**.

Experiment 21
In this experiment, all the design parameters are similar to Experiment 19 except that the CV of raw material and demand inter-arrival times is equal to 1.0.

Experiment 22
In this experiment, all the design parameters are similar to Experiment 21 except that the position of bottleneck is located at the **last station**.

Experiment 23
In this experiment, all the design parameters are similar to Experiment 19 except that the CV of raw material and demand inter-arrival times are equal to 2.5.

Experiment 24
In this experiment, all the design parameters are similar to Experiment 24 except that the position of bottleneck is located at the **last station**.

Experiment 25
In this experiment, the performance of a network with 10 manufacturing stations is analyzed. Each product visit manufacturing stations sequentially. Expected service time at non-bottleneck station (D_k) D_2= D_3= D_4= D_5= D_6= D_7= D_8= D_9= D_10= 0.8 sec, CV $c_{s,k} = 0.3$. Expected service time at bottleneck station (D_{unb}) $D_1 = 1$ sec, CV $c_{unb} = 0.3$. Raw material and the Demand arrival rates are equal to 0.94 per sec, CV of raw material inter-arrival times and demand inter-arrival times are equal to 0.5. Bottleneck station is located at the **first station**.

Experiment 26
In this experiment, all the design parameters are similar to the Experiment 25 except that the position of bottleneck is located at the **last station**.

Experiment 27
In this experiment, all the design parameters are similar to Experiment 25 except that the CV of raw material and demand inter-arrival times are equal to 1.0.

Experiment 28
In this experiment, all the design parameters are similar to Experiment 27 except that the position of bottleneck is located at the **last station**.

Experiment 29
In this experiment, all the design parameters are similar to Experiment 25 except that the CV of raw material and demand inter-arrival times are equal to 2.5.

Experiment 30
In this experiment, all the design parameters are similar to Experiment 29 except that the position of bottleneck is located at the **last station**.

Experiment 31
In this experiment, all the design parameters are similar to Experiment 25 except that the CV of service times at station is equal to 1.0.

Experiment 32
In this experiment, all the design parameters are similar to Experiment 31 except that the position of bottleneck is located at the **last station**.

Experiment 33
In this experiment, all the design parameters are similar to Experiment 31 except that the CV of raw material and demand inter-arrival times are equal to 1.0.

Experiment 34
In this experiment, all the design parameters are similar to Experiment 33 except that the position of bottleneck is located at the **last station**.

Experiment 35
In this experiment, all the design parameters are similar to Experiment 31 except that the CV of raw material and demand inter-arrival times are equal to 2.5.

Experiment 36
In this experiment, all the design parameters are similar to Experiment 35 except that the position of bottleneck which is located at the **last station**.