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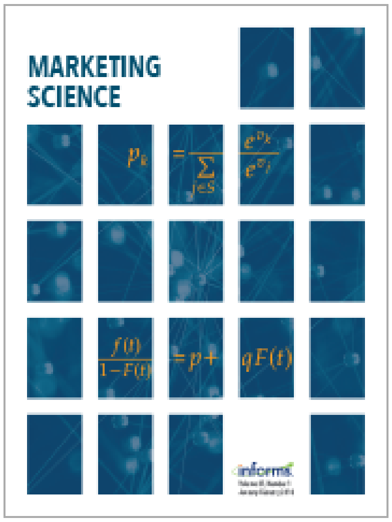

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Research Note

Advertising Competition Under Consumer Inertia

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We construct a multistage game-theoretic model of advertising and price competition in a differentiated products duopoly, in which proportions of consumers exhibit latent inertia in favor of repeat purchase. Advertising simultaneously plays the dual role in reducing such inertia through awareness and enhancing perceived brand value (persuasion). We derive the advertising price cross-effects and provide a theoretical reconciliation of the long-standing debate in the marketing literature regarding the impact of advertising on price sensitivity. We characterize the nature of equilibria under symmetry and show that when a large proportion of consumers exhibit inertial tendencies, then a multiplicity of equilibria exists. Marketing implications and comparative statics are discussed.

Numerical simulations for asymmetric firms are presented, wherein we show that advertising is not a useful competitive tool for small firms. However, advertising spending by the large firm provides a halo effect for the average prices in the category, which has a positive externality on the small firm's profits. In the absence of the small brand advertising, larger brand shares encourage firms to allocate higher expenditures on advertising to enhance the perceived value of their brand, which in turn shore up the average prices in the industry from which all firms benefit.

(*Consumer Inertia; Duopoly; Advertising Competition; Game Theory*)

1. Introduction

Models of advertising competition and its effects on consumer behavior and market performance have two broad schools. One looks at advertising as a channel that provides valuable *information* to consumers, enabling them to make rational choices by reducing informational product differentiation. The other school views advertising as a device that persuades consumers by means of intangible and/or psychic differentiators. It creates differentiation among products (Comanor and Wilson 1974), which at times may not be real (Tirole 1990). This is especially true for most "feel" products, such as beer, cigarettes, soft drinks, perfumes, etc.

However, advertising in the extant literature plays a predominantly unidimensional role. For instance, the large body of literature on informative advertising looks at it as a tool for information dissemination, announcing a brand's existence, available locations, important attributes, price, quality, etc. (namely, Nelson 1970, 1974; Butters 1977; Grossman and Shapiro 1984). On the other hand, the literature on persuasive advertising, which is relatively scarce, uses advertis-

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ing as a device to persuade people to buy a particular product or brand (Koh and Leung 1992).

Although there are situations in which either of the advertising roles (awareness and persuasion) is used separately by marketers (e.g., solely informational features in localized retail advertising, vis-à-vis TV advertising), firms often use advertising as a mix element that informs and persuades simultaneously. For example, a typical beer commercial will contain product/brand-related information (e.g., draft, lager, light), but concurrently the creative strategy contains elements that have nothing to do with the product per se, but helps build the so-called brand effect. In most mature categories in the consumer goods industry, the role of affective persuasion through mass media advertising is an important aspect of the entire marketing strategy. The competition between Coke and Pepsi is a case in point. Hence, it is our goal in this paper to examine competition when advertising *simultaneously* informs as well as persuades consumers.

We examine the cross-effects of advertising on prices, about which the marketing literature has been somewhat equivocal. Some experimental (Moriarty 1983, Bemmaor and Mouchoux 1991) and econometric studies (Bolton 1989 and Popkowski-Leszczyc and Rao 1989) indicate that advertising tends to increase price sensitivity. However, there are some (Staelin and Winer 1976, Ghosh et al. 1983, Krishnamurthi and Raj 1985, Boulding et al. 1994) that indicate the contrary. Kaul and Wittink (1995) suggest that the conflicting evidence depends on the type of the advertisement: price or nonprice. In this paper, we demonstrate that the direction of advertising effect on prices depends on the extent of latent inertia in consumers' purchase behavior. In particular, if a large proportion of consumers is locked into a certain brand, such that they exhibit strong tendencies of inertial (repeat) buying, then the impact of low levels of advertising on prices is negative. However, this impact is positive for high levels of advertising (see, e.g., Kanetkar et al. 1992, Mitra and Lynch 1995). The effect of advertising on prices is governed by the relative magnitudes of the advertising elasticity of consumer consideration and that of persuasive goodwill. This endogenously generates sales, as well as profit functions, that are S-shaped, giving rise to the so-called "threshold effect" in advertising spending. We find that this effect is most pronounced when the cohort of inertial consumers is large.

In our duopoly model, advertising has no defensive role to play. No amount of advertising can prevent consumers of a brand from considering the other brand, if the other brand chooses to go after them by advertising. We show that, even in a symmetric game, there exist asymmetric (advertising) equilibria, wherein only one firm advertises, sets a higher price, and also captures a greater share of the market. An implication of the existence of asymmetric advertising equilibria in our static model seems to suggest that continuous advertising schedules may dominate pulsing policies in markets where consumers exhibit strong tendencies of inertial buying.

The remainder of the paper is organized as follows: §2 describes the model, §3 characterizes the equilibrium in the price-setting subgame, and §4 analyzes the different classes of the advertising equilibria for symmetric firms and discusses some of the comparative static issues. Section 5 provides a discussion on the model with asymmetric firms, and §6 concludes with directions for future research.

2. The Model

We consider a duopolistic market in which two firms, $i=1$ and $i=2$, produce a differentiated product with identical constant marginal cost of production equal to zero. The firms are risk neutral and engage in a two-stage noncooperative game. In Stage 1, the firms simultaneously choose their advertising levels; and in Stage 2, given the first-stage choices of advertising levels, the firms set prices simultaneously.

The firms compete over a market of fixed size, N, of whom σ_1 bought from Firm 1 before, and $\sigma_2 = N - \sigma_1$ bought from Firm 2 before. We could also interpret σ_1/N and σ_2/N as market shares of the respective firms. All consumers have unit demands; they buy one unit of the product as long as the price does not exceed their reservation price. Following Hauser and Wernerfelt (1990), brand choice is described as a two-stage process: First, a consumer forms his/her consideration set (which may be influenced by the firms' advertising efforts) and decides whether to include one or both brands in the set. Then the actual purchase is made, depending on the brands in the consideration set and their respective prices.

Consistent with empirical regularity, our purpose is to analyze the scenario in which some consumers exhibit some amounts of inertia in forming their choice sets. We assume that all consumers who bought from Firm 1 (Firm 2) before will automatically include Brand 1 (Brand 2) in their choice sets. This amounts to saying that consumption of a brand results in some amount of locking in of customers for the following period. Now, whether the inertial consumers will include the brand they did not buy before in their choice sets will depend on the advertising levels of that brand. The choice set response to advertising is described herein.

Firms advertise in Stage 1 for two purposes. First, advertising allows consumers who did not buy the firm's product before to consider its brand as relevant. This endogenously partitions the market into a loyal segment (those who consider only one brand: a monopoly market for the brand) and a competitive segment (those who consider both brands: a duopoly market). If Firm 1 spends a_1 dollars on advertising, then a fraction $K(a_l)$ of the (σ_2) Brand 2 consumers will include Brand 1 in their consideration set. We will call $K(a_i)$ the consideration function. Therefore, $(1 - K(a_l))\sigma_2$ will be the number of consumers who will continue to have only Brand 2 in their consideration set.[1] Firm 2's advertising levels will have a similar effect. Note that this part of our model is similar in spirit to Hauser and Wernerfelt (1989). Therefore, we define the various segments in the market in the following way: $M_i = (1 - K(a_j))\sigma_i =$ the number of consumers with only Brand i in their consideration set (i.e., a monopoly segment for Firm i) and similarly for Brand j; and $D = K(a_2)\sigma_1 + K(a_1)\sigma_2 =$ the number of consumers with both brands in their choice set (i.e., a duopoly segment).[2]

Second, advertising plays a persuasive role by way of scaling up the "perceived utility" attached to the consumption of one unit of the product. We model the persuasive role of advertising on the duopoly segment using the standard linear spatial model. For the consumers with both products in their choice set, the two firms are located at the extremes of a linear segment of unit distance. The consumers are distributed uniformly along the segment between the two firms, with density D. The perceived utility (gross of price paid) that a particular consumer attaches to consuming one unit of product i is given by:

$$\text{Perceived utility of Brand } i = (r - tx_i)f(a_i), \quad i = 1, 2, \tag{1}$$

where $r > 0$ is a constant taken as a proxy for product quality, $0 < x_i < 1$ is the distance of the consumer from Firm i, and t is the transport cost per unit distance. Throughout the remainder of the paper, we assume $r > t = 1$, so that perceived utility in (1) is positive for all x. Note that the persuasiveness of advertising is relevant only for the duopoly segment, because the consumers with only one brand in their choice set will buy the product anyway (as long as the price does not exceed their reservation price).

Because we derive our results using numerical simulation, we will use specific forms to characterize the two advertising functions, $K(a_i)$ and $f(a_i)$. Following Koh and Leung (1992), we characterize the "persuasion" function of advertising, as:

$$f(a_i) = 1 + (a_i)^w, \quad \text{where } w \in (0, 1]. \tag{2}$$

Note that $f'(a_i) > 0$; $f''(a_i) < 0\ \forall a_i > 0$; and $f(0) = 1$, $i = 1, 2$. On the other hand, the consideration function is described as:

$$K(a_i) = 1 - \frac{1}{d + (a_i)^s}, \text{ where } d > 1 \text{ and } s \in (0, 1). \tag{3}$$

Note that $K'(a_i) > 0$; $K''(a_i) < 0\ \ \forall a_i > 0$; $0 < K(0) < 1$; and $\lim_{a_i \to \infty} K(a_i) = 1$, $i = 1, 2$. In other words, $K(\cdot)$ is

[1] If N is large, we could convert this model into a one consumer model, where $K(a_i)$ will be the probability that a consumer who previously purchased Brand j now considers Brand i.

[2] If we interpret $K(\cdot)$ purely as an information transmission process, then M_i is the number of people who are only informed about Brand i, $i = 1, 2$; and D is the number of people who are informed about both brands.

strictly concave (i.e., exhibits diminishing returns to advertising) and is asymptotic at 1 (i.e., it costs a firm infinite amount of advertising dollars to get the entire market to consider its brand). Furthermore, $K(\cdot)$ has an intercept, $(d-1)/d$, which is a positive fraction. We shall call d the "inertia" parameter, such that $K(0)\to 0$ or $d\to 1$ implies extreme inertia in that the consumers do not consider the nonpurchased brand in the absence of advertising by the latter. Similarly, $K(0)\to 1$ or $d\to\infty$ reduces the model into a standard spatial model without any locked-in consumers. Because $d>1$ in our model, there is always a cohort of consumers who will consider both brands, even in the absence of advertising by either firm.

Although our results will continue to hold with no further restrictions on $f(\cdot)$ other than strict concavity as in (2), it is intuitive that $f(\cdot)$ needs to be sufficiently concave relative to $K(\cdot)$ to prevent the perceived value of a brand for a large segment of the market from being driven to infinity by large advertising spending. For this purpose, in what follows, we state our results by pegging $w=0.5$ and by assuming $s\le w$.

In summary, when a firm advertises in Stage 1, two effects are simultaneously in action. First, the density of the duopoly segment (i.e., consumers with both brands in their consideration sets) is determined. At the same time, the perceived values of the brands for the consumers therein are influenced. In Stage 2, the firms simultaneously set prices to maximize profits without being able to engage in any price discrimination.

3. The Price-Setting Subgame

In what follows, we characterize the set of perfect equilibria in the multistage game for symmetric firms described, using the standard technique of backward induction. Hence, we examine the pricing subgame first.

Let us define $L_i>r$ as the maximum price that any consumer is willing to pay for Brand i, $i=1,2$. It may be argued that L_i should realistically be a function of a_i. However, in the context of our model, this would only complicate the algebra without having an effect on our main results.

First, note that Firm i can sell to its entire monopoly segment (of size M_i) as long as $p_i\le L_i$. Now let us consider the duopoly segment. Using the standard solution concept for horizontal production differentiation (à la Hotelling) models, the sales for Firm 1 is obtained by the location (x^D) of the consumer who is indifferent to purchasing either of the two brands. For the indifferent customer: $(r-x_1)f(a_1)-p_1=[r-(1-x_1)]f(a_2)-p_2$, which when solved for x_1 yields Firm 1's share of sales to the duopoly segment as:

$$x^D=\frac{p_2-p_1+r[f(a_1)-f(a_2)]+f(a_2)}{f(a_1)+f(a_2)}. \tag{4}$$

Note that x^D is a fraction, provided $p_1\le p_2+r[f(a_l)-f(a_2)]+f(a_2)$, that becomes larger (smaller) as a_1 (a_2) increases, or p_1 (p_2) decreases (holding all other variables constant). If $p_1\ge p_2+r[f(a_l)-f(a_2)]+f(a_2)$, then Firm 1 has no sales in the duopoly segment. Also, if $p_1\le p_2+r[f(a_1)-f(a_2)]-f(a_1)$, then Firm 1 is undercutting Firm 2's price enough to serve the entire duopoly market in addition to its M_1 locked-in customers.

Given that the marginal cost of production is assumed to be zero and that advertising expenditure is sunk in Stage 2, we can write Firm 1's profit function as follows:

$$\begin{aligned}
\Pi_1 &= p_1M_1 \\
&\qquad \text{if } L_1\ge p_1\ge p_2+r[f(a_l)-f(a_2)]+f(a_2) &\text{(5a)}\\
&= p_1[M_1+x^D\cdot D] \\
&\qquad \text{if } L_1\ge p_2+r[f(a_l)-f(a_2)]+f(a_2) \\
&\qquad\quad \ge p_1\ge p_2+r[f(a_l)-f(a_2)]-f(a_l) &\text{(5b)}\\
&= p_1[M_1+D] \\
&\qquad \text{if } L_1\ge p_2+r[f(a_l)-f(a_2)]-f(a_l)\ge p_1 &\text{(5c)}
\end{aligned}$$

It is important to note here that the three distinct zones in Equations (5a), (5b), and (5c) make the profit function non–quasi-concave in prices. It can be shown that, in the pricing subgame, the best response function of Firm 1 (and similarly, Firm 2) is of the following form:

$$\Psi_1(p_2)=\begin{cases} L_1 & \text{if } p_2\le \tilde{p}_2 \quad (6a)\\ R_1(p_2) & \text{if } \tilde{p}_2\le p_2\le p_1 + r[f(a_2)-f(a_1)]+f(a_1) \quad (6b)\\ p_2+r[f(a_1)-f(a_2)]-f(a_1) & \text{if } L_2\ge p_2\ge p_1 + r[f(a_2)-f(a_1)]+f(a_1) \quad (6c)\end{cases}$$

where

$$R_1(p_2) = \frac{1}{2}p_2+\frac{1}{2}\left[f(a_2)+r(f(a_1)-f(a_2)) + \frac{(f(a_1)+f(a_2))M_1}{D}\right] \tag{7}$$

and $\tilde{p}_2=\{\max p_2 : M_1L_1=[M_1+Dx^D(R_1(p_2),p_2)]R_1(p_2)\}$ (see Appendix for proof).

Assuming that L_i is never large enough to deter firms from competing for the duopoly segment and sell only to their respective monopoly segments, the pricing subgame has a unique equilibrium in pure strategies (p_1^*, p_2^*), which is obtained by solving $R_1(p_2)=R_2(p_1)$ and is characterized by:

$$p_i^* = \frac{1}{3}\left\{(1+r)f(a_i)+(2-r)f(a_j) + \frac{f(a_i)+f(a_j)}{D}(M_j+2M_i)\right\}; \tag{8}$$

$i=1,2;\ j=3-i$.

Now we can show by differentiating Equation (8) with respect to a_i and rearranging terms that:

$$\frac{\partial p_i^*}{\partial a_i} = \frac{1}{3}[f(a_i)+f(a_j)] \cdot \left\{\left(r+\frac{3-K(a_j)}{K(a_i)+K(a_j)}\right)\varepsilon_v^i - \frac{3-K(a_j)}{K(a_i)+K(a_j)}\varepsilon_c^i\right\}, \tag{9}$$

where $\varepsilon_v^i=(\partial f(a_i)/\partial a_i)/(f(a_i)+f(a_j))$ is the "persuasion elasticity," whereas $\varepsilon_c^i=(\partial K(a_i)/\partial a_i)/(K(a_i)+K(a_j))$ is the "consideration elasticity" of advertising for Firm i. Note that the first term involving ε_v^i in parentheses in (9) is positive, whereas the last term involving ε_c^i is negative. Hence, the sign of the cross-effect of advertising on prices may not be unidirectional and will be determined by the interplay of the elasticities of the two advertising functions, ε_v^i and ε_c^i.

Proposition 1. *Given any arbitrary a_j, $\partial p_i^*/\partial a_i>0$, $\partial^2 p_i^*/\partial a_i^2<0$, when d is large. However, $\partial p_i^*/\partial a_i<0$ in the neighborhood of $a_i=0^+$, when d is small. Generally, equilibrium prices of firms increase (at a decreasing rate) with its own advertising spending, except when d is small, wherein prices initially decline with low levels of advertising.*

Given that $K(\cdot)$ has an asymptote at 1, $\varepsilon_c^i\to 0$ for large a_i, whereas ε_v^i is positive. Therefore, when a_i is large, p_i^* is increasing in a_i in (9). Furthermore, concavity of $f(\cdot)$ by (3) ensures that $\partial^2 p_i^*/\partial a_i^2<0$ for large a_i. However, an examination of (9) in the neighborhood of $a_i = 0^+$ yields an inflection in the price elasticity such that $\partial p_i^*/\partial a_i<0$. Using the candidate functions in (2) and (3), it can be shown that

$$\frac{\partial p_i^*}{\partial a_i}<0, \text{ if } \frac{\varepsilon_c^i}{\varepsilon_v^i} = \frac{s a_i^s(2+a_i^w+a_j^w)}{w a_i^w (d+a_i^s)^2\left(2-\frac{1}{d+a_i^s}-\frac{1}{d+a_j^s}\right)} > 1+\frac{r(K(a_i)+K(a_j))}{3-K(a_j)} = 1+\frac{r\left(2-\frac{1}{d+a_i^s}-\frac{1}{d+a_j^s}\right)}{2+\frac{1}{d+a_j^s}}. \tag{10}$$

This inequality holds when $d\to 1$ and $a_i\to 0^+$. Figure 1 illustrates the claim in the proposition for representative values of rival advertising levels. Note that the critical (small) value of d that generates the inflection in the neighborhood of $a_i=0^+$ is increasing in the rival firm's advertising.

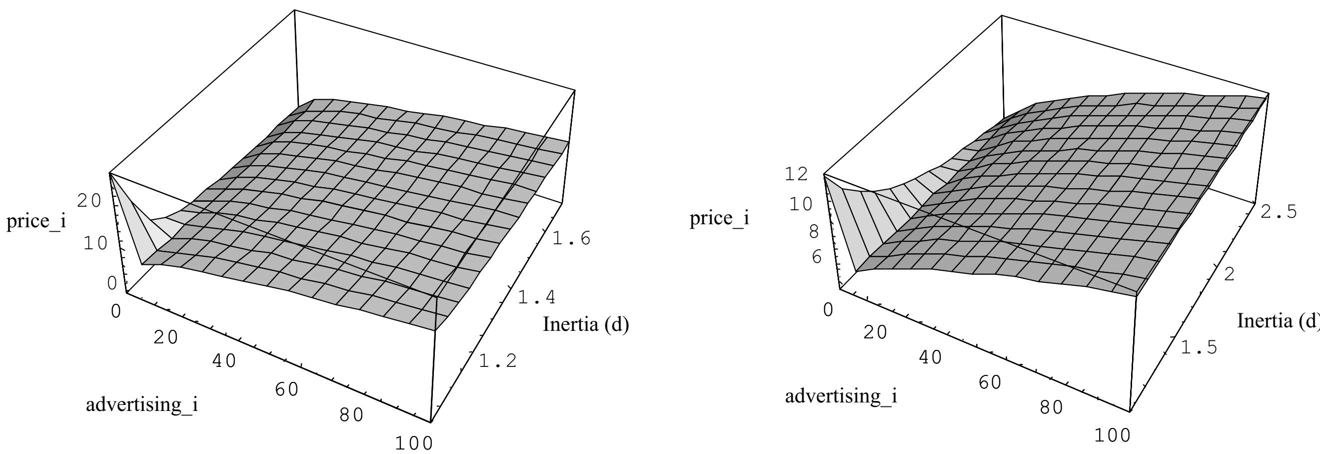
Proposition 2. *Given any arbitrary a_i, $\partial p_i^*/\partial a_j>0$, $\partial^2 p_i^*/\partial a_j^2<0$, when d is large. However, $\partial p_i^*/\partial a_j<0$ in the neighborhood of $a_j=0$, when d is small.*

Differentiating (8) with respect to a_j and rearranging terms, we get

$$\frac{\partial p_i^*}{\partial a_j} = \frac{1}{3}[f(a_i)+f(a_j)] \cdot \left\{\left(\frac{3+K(a_i)}{K(a_i)+K(a_j)}-r\right)\varepsilon_v^j - \frac{3+K(a_i)}{K(a_i)+K(a_j)}\varepsilon_c^j\right\}. \tag{11}$$

The claim is established by proceeding similarly as in Proposition 1 by examining (11) when $d\to 1$ and

Figure 1 Impact of *d* on Price Sensitivity from Own Advertising



(a) advertising _ j = 0 (b) advertising _ j = 1000

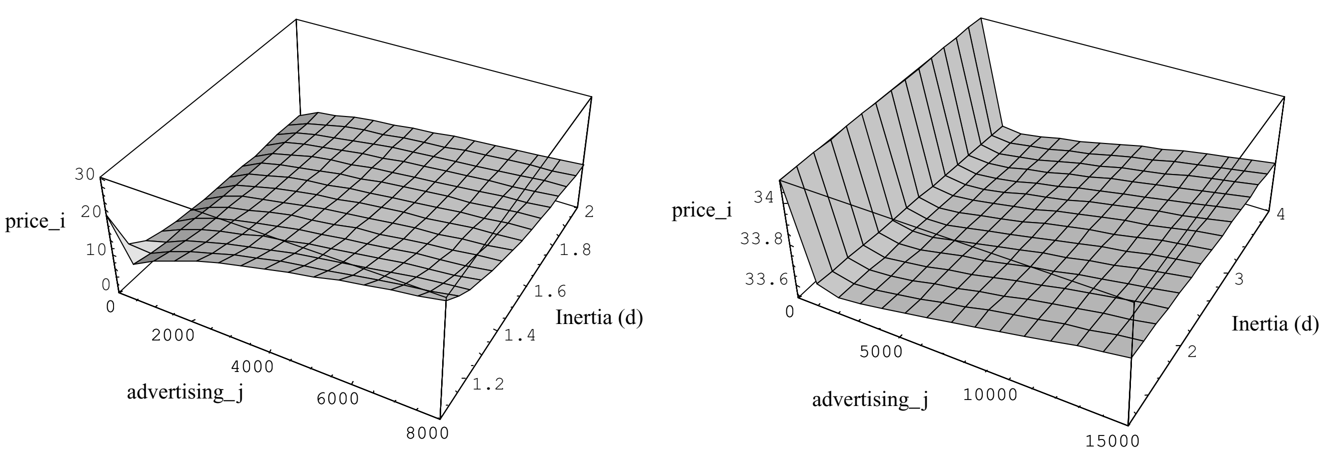
$a_j \to 0^+$. Figure 2 illustrates the effects of the rival firm's advertising on the prices of firm i for representative values of its own advertising levels.

Compared with models with no consumer inertia, our model yields higher prices. However, the individual impacts of the two advertising functions on prices are somewhat different. The persuasive elements of advertising per se exert upward pressures on prices by communicating the brand differentiators more effectively. However, the "consideration" element of advertising increases the density of the duopoly segment at the expense of the monopoly segments, thereby leading to more aggressive pricing.

4. The Advertising Competition

With the pure strategy equilibrium prices characterized as functions of advertising levels in Equation (8), we now fold the game backward to solve for the perfect Nash equilibrium in advertising under conditions of symmetry ($\sigma_i = \sigma_j$). In Stage 1, Firm i faces the

Figure 2 Impact of *d* on Price-Sensitivity from Rival Advertising



(a) advertising _ i = 0 (b) advertising _ i = 1000 problem:

$$\operatorname*{Max}_{a_i} \Pi_i(a_i, a_j) = p_i^*(a_i, a_j) \times q_i(p_i^*(a_i, a_j), p_j^*(a_i, a_j), a_i, a_j) - a_i,$$

where $\Pi_i(\cdot)$ is the profit and $q_i(\cdot)$ is sales at the end of Stage 2. Using p_i^* and p_j^* in (8), and the expression for sales from (5b), it can be shown that $q_i(p_i^*, p_j^*, a_i, a_j) = Ap_i^*$, where $A = D/(f(a_i) + f(a_j))$. This reduces the firm's problem to:

$$\operatorname*{Max}_{a_i} \Pi_i(a_i, a_j) = Ap_i^{*2} - a_i. \tag{12}$$

Since the nonlinear advertising functions, (2) and (3), enter the profit equation in (12) in a complex way, it is cumbersome to solve for the advertising equilibrium explicitly. Hence, in what follows, we implicitly derive the equilibria in the advertising game.

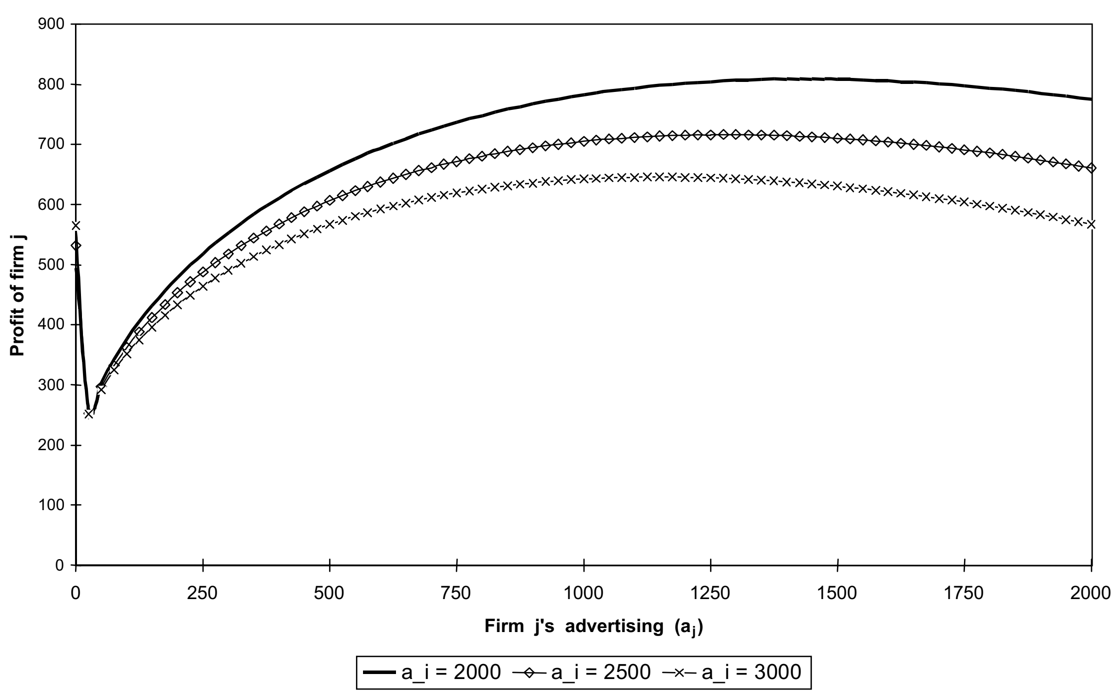
Proposition 3. *The profit function in advertising levels* (*Equation* (12)) *has the following properties.* (a) *The intercept of the profit function is positive* (*i.e., firms make positive profits even without advertising spending, regardless of the rival firm's advertising level*). (b) *The profits of a firm are decreasing in the rival's advertising levels.* (c) *Except for an inflection in the neighborhood of $a_i = 0^+$, the profit function is concave, such that interior maximizer in advertising exists. However, when d is small, the profit function has an S-shape, wherein profits initially decline* (*i.e., in the neighborhood of $a_i \to 0^+$*) *and reach a minimum, and then begin to increase.*

The claims in (a) and (b) follow from the construction of the model and the fact that $d > 1$. To establish the claim in (c), note that differentiating (12) with respect to a_i yields:

$$\frac{\partial \Pi_i}{\partial a_i} = 2Ap_i^* \frac{\partial p_i^*}{\partial a_i} + p_i^{*2} \frac{\partial A}{\partial a_i} - 1. \tag{13}$$

First, it can be shown that $\partial A/\partial a_i = A(\varepsilon_c^i - \varepsilon_v^i)$. Now, consider (13) when d is large. From Proposition 1, $\partial p_i^*/\partial a_i > 0$ implies that $\partial A/\partial a_i < 0$. Also note that $\partial^2 p_i^*/\partial a_i^2 < 0$ ensures that (13) will have a unique maximizer in a_i. However, by Proposition 1, $\partial p_i^*/\partial a_i < 0$

Figure 3 **Profit Functions in Own Advertising Levels**



Note. For $d = 1.1$, $s = 0.2$, $w = 0.5$, $r = 2$, $\sigma_1 = \sigma_2 = 50$

in the neighborhood of $a_i = 0^+$, when d is small. It is seen that, although $\partial A/\partial a_i > 0$ in this range of parameter values, $|2Ap_i^* \partial p_i^*/\partial a_i - 1| > |p_i^{*2} \partial A/\partial a_i|$, such that the profit function has the claimed inflection in the neighborhood of $a_i = 0^+$, when $d \to 1$, that gives it the S-shape. Therefore, the shape of the profit function is also determined by the interplay of the two advertising elasticities, ε_c^i and ε_v^i, as in the price function in (8), with a dampened effect from the cost of advertising. Figure 3 illustrates the profit function for representative values of all the relevant parameters.

Given the properties of the profit functions as in Proposition 3, the first-order conditions for the profit maximization problem in (12) yield a pair of best-response functions in advertising, for $i = 1, 2$. The Nash equilibrium in pure strategies in advertising levels (a_i^*, a_j^*) can be obtained from the solution(s) to the best response functions, i.e., by solving:

$$2Ap_i^* \frac{\partial p_i^*}{\partial a_i} + p_i^{*2} \frac{\partial A}{\partial a_i} = 2Ap_j^* \frac{\partial p_j^*}{\partial a_j} + p_j^{*2} \frac{\partial A}{\partial a_j}. \tag{14}$$

Let $\tilde{a}_i$ denote firm i's advertising level, which solves Equation (14) for the range of a_j, such that $\Pi_i(\tilde{a}_i) > \Pi_i(0)$. We know (refer to Figure 1) that such an $\tilde{a}_i$ exists because $\Pi_i(a_i, a_j)$ is locally concave by Proposition 3, and $\max_{a_i} \Pi_i(a_i, a_j) > \Pi_i(0, a_j)$ for a range of $a_j \geq 0$. Therefore, $\tilde{a}_i(a_j)$ partially defines Firm i's best-response function in advertising.

Lemma 1. *The best-response function in advertising is downward sloping, with a discontinuous segment, given as*:

$$T_i(a_j = \begin{cases} \tilde{a}_i(a_j) & \text{if } a_j \leq \hat{a}_j \\ 0 & \text{otherwise}, \end{cases} \tag{15}$$

where $\hat{a}_j$ is the level of Firm j's advertising, such that $\Pi_i(\tilde{a}_i \cdot (\hat{a}_j), \hat{a}_j) = \Pi_i(0, \hat{a}_j)$ (see Appendix for proof).

Proposition 4. *In the advertising game, equilibrium in pure strategy always exists. Given the advertising functions in* (2) *and* (3), *there exists a d^* such that,*

(a) *when $d \leq d^*$, the advertising game has three pure strategy equilibria, namely, a symmetric equilibrium $(a_i^* = a_j^* > 0)$, and two asymmetric equilibria $(a_i = 0, a_j = \tilde{a}_j(0) > 0;\ i, j = 1, 2, i \neq j)$; and*

(b) *when $d > d^*$, the symmetric equilibrium $(a_i^* = a_j^* > 0)$ is the only equilibrium of the game in pure strategies.*

The proposition is best explained using Figure 4, which illustrates Scenario (a).[3] Numerical computations show that $\tilde{a}_i(0)$ is decreasing in d (i.e., Firm i's optimal advertising level in the absence of the rival brand advertising falls as inertia in consumer-buying behavior reduces). On the other hand, $\hat{a}_i$ is seen to be increasing in d (i.e., Firm i's advertising level for which Firm j's best-response function will jump down to zero level of advertising increases as consumers become less inertial (see Table 1)). Now a small d generates a regime, where $\tilde{a}_i(0) > \hat{a}_j \geq \tilde{a}_i(\hat{a}_j)$, that yields Case (a) in the proposition (see Figure 4). However, a large d generates a regime, where $\hat{a}_j > \tilde{a}_i(0) \geq \tilde{a}_i(\hat{a}_j)$, which supports Case (b) in the proposition.[4] Table 1 exhibits how $\tilde{a}_i(0), \hat{a}_i$, and $\Pi_i(\tilde{a}_i(0))$ change for different values of the inertia parameter, d, and traces d^* given various configurations of the advertising parameters, s and w.

Although the intuition for the symmetric equilibrium is obvious, the existence of the asymmetric equilibria (in which only one firm advertises) is caused by the S-shaped nature of the profit function when d is small. Note that there is a locked-in (monopoly) segment always available to the firm to milk. Also, there will always be a finite group of consumers who are preferentially close to a brand (given the heterogeneity in the duopoly segment). Therefore, the optimal response to heavy rival advertising will be not to advertise.

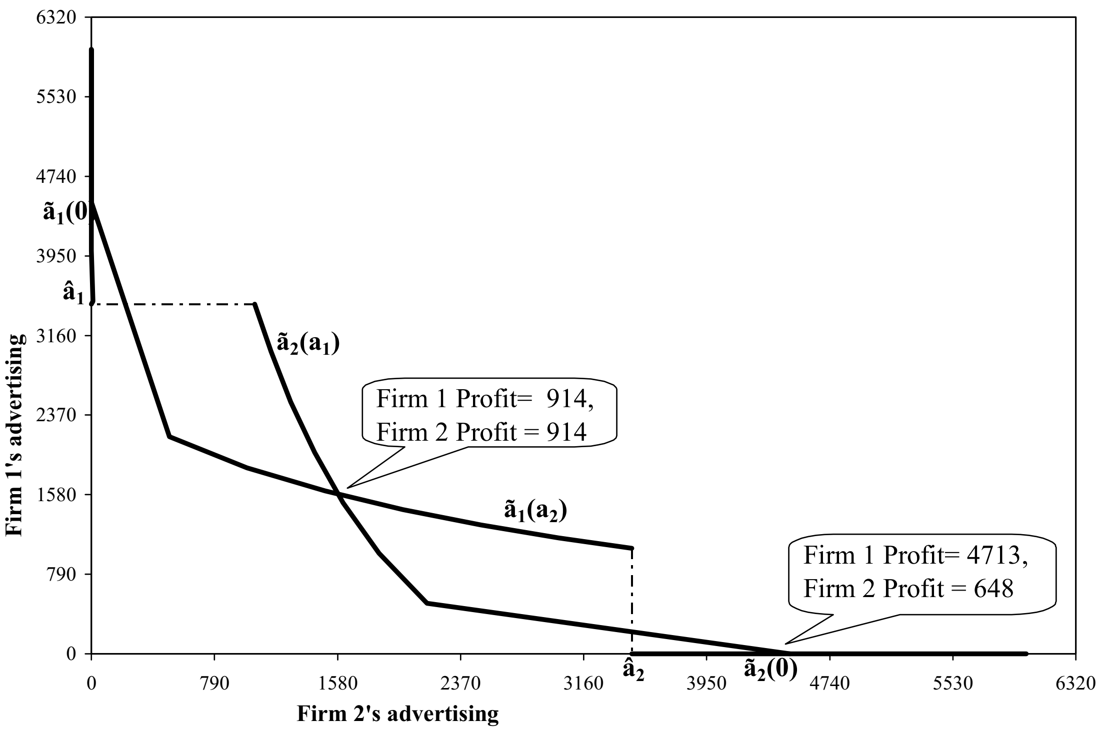
Proposition 5. *The firm that spends more in advertising charges a higher price and also sells more in equilibrium.*

Using (8), we can show that $p_i^* - p_j^* = \frac{1}{3}\{(2r-1) \cdot (f(a_i) - f(a_j)) + D^{-1}(f(a_i) + f(a_j))(M_i - M_j)\}$. Clearly, $p_i^* > p_j^*$ if $a_i > a_j$. Also, given sales, $q_i^* = Ap_i^*$, $q_i^* - q_j^* =$

[3] The existence of equilibrium for symmetric firms is guaranteed because the best-response functions in (15) are downward sloping for $a_j \leq \hat{a}_j$ and the fact that $\tilde{a}_i = 0$ for $a_j \geq \hat{a}_j$.

[4] Note that there exists another theoretical possibility, namely $\tilde{a}_i(0) > \tilde{a}_i(\hat{a}_j) > \hat{a}_j$, under which the two asymmetric equilibria $(a_i = 0, a_j = \tilde{a}_j(0) > 0;\ i, j = 1, 2, i \neq j)$ will be the only two pure strategy equilibria in advertising. However, computations using (2) and (3) and our parameter restriction $s \leq w = 0.5$ show that this scenario does not occur.

Figure 4 Advertising Equilibria



Note. $\sigma_1 = \sigma_2 = 50$; $d = 1.1$

$A(p_i^* - p_j^*)$; i.e., the higher-priced firm sells more in equilibrium.

In summary, when a large percentage of consumers exhibit strong tendencies of inertial buying (d close to 1), our advertising game has two asymmetric equilibria in addition to a symmetric equilibrium (we ignore possibilities of mixed strategies). It is difficult to predict which of the above class of equilibria will actually emerge. It appears that, instead of simultaneous choice of advertising, if firms were to move sequentially (à la Stackelberg), then the asymmetric equilibrium will be the likely outcome. The firm moving first will advertise enough to position its product vis-à-vis the other's to make it unprofitable for the rival firm to choose advertising as a competitive tool. However, even in the asymmetric equilibria, the firm that does the advertising does not spend in a manner to drive the other firm out of the market.

On the other hand, when d is reasonably large (i.e., a sizeable portion of the consumers does not have any latent inertia in favor of repeat purchase), the advertising equilibrium will be uniquely determined. In such a case, both firms will choose the same level of advertising, charge the same price, and split the market in half.

The two different classes of the market outcome in the advertising game are driven by the nature of the profit function. In particular, local nonconcavities in the profit functions that are brought about by the so-called threshold effects of advertising response will generate the different kinds of equilibria. We claim that such threshold effects in revenue or profit response to advertising are caused by the interaction of the multidimensional effects of advertising, namely, getting consumers to include a brand in the consideration sets and subsequently influencing the brand valuation through persuasive communication.

5. Discussion: Asymmetric Firms

We showed in the preceding section that multiple equilibria in advertising competition exist even under the assumption of symmetry of market shares (i.e.,

Table 1 Effect of Change in d on the Nature of Advertising Equilibria, for Various Values of s and w

	$s=0.1$; $w=0.5$; $\sigma_i=\sigma_j=50$; $r=2$					
d	1.1	1.125	1.15*	1.2	1.3	1.4
$\tilde{a}_i(0)$	4835	4691	4561	4338	3999	3755
$\hat{a}_i$	4131	4458	4792	5483	6949	8530
$\Pi_i(\tilde{a}_i,0)$	5115.95	4948.92	4799.63	4544.41	4159.59	3884.80
	$s=0.2$; $w=0.5$; $\sigma_i=\sigma_j=50$; $r=2$					
d	1.1	1.15	1.2*	1.3	1.4	1.5
$\tilde{a}_i(0)$	4485	4285	4120	3860	3665	3515
$\hat{a}_i$	3469	3860	4255	5064	5895	6755
$\Pi_i(\tilde{a}_i,0)$	4712.92	4481.85	4290.88	3994.43	3775.72	3608.24
	$s=0.3$; $w=0.5$; $\sigma_i=\sigma_j=50$; $r=2$					
d	1.1	1.2	1.25	1.3*	1.4	1.5
$\tilde{a}_i(0)$	4386	4056	3929	3821	3644	3507
$\hat{a}_i$	3065	3601	3866	4129	4652	5174
$\Pi_i(\tilde{a}_i,0)$	4557.11	4190	4049.96	3929.47	3734.33	3583.37
	$s=0.4$; $w=0.5$; $\sigma_i=\sigma_j=50$; $r=2$					
d	1.1	1.2	1.3	1.35*	1.4	1.5
$\tilde{a}_i(0)$	4359	4041	3812	3721	3641	3508
$\hat{a}_i$	2845	3267	3671	3867	4059	4437
$\Pi_i(\tilde{a}_i,0)$	4494.94	4151.07	3904.39	3805.69	3719.39	3575.75
	$s=0.5$; $w=0.5$; $\sigma_i=\sigma_j=50$; $r=2$					
d	1.1	1.2	1.3	1.35	1.4*	1.5
$\tilde{a}_i(0)$	4355	4035	3810	3720	3640	3510
$\hat{a}_i$	2730	3100	3445	3610	3775	4085
$\Pi_i(\tilde{a}_i,0)$	4469.24	4134.68	3894.32	3798.02	3713.75	3573.37

*The (approximately) minimum value of d (i.e., d^*) for which the symmetric advertising equilibrium is the unique equilibrium

$\sigma_1=\sigma_2$). It is worthwhile at this stage to reexamine the model under conditions of asymmetry ($\sigma_1\neq\sigma_2$). We report our findings in Table 2 for a representative set of parameter values and to provide a discussion.

It is evident that asymmetry in the shares of firms may change the nature of equilibria in our model. In particular, we find that there exist two scenarios that are in contrast to the symmetric firms case. First, for σ_i sufficiently larger than σ_j, we may have two advertising equilibria in pure strategies: one in which both firms spend positive (but different) amounts in advertising and another in which only the large firm advertises (Table 2, Regime a). When the difference in σ_i and σ_j is significantly more pronounced, the game has an unique equilibrium in which only the large firm advertises and the small one does not (Table 2, Regime b). Figure 5 illustrates Regime a.

To explain the result that a small share brand does not advertise in equilibrium, we revisit the price elasticity expressions (9) and (12) under asymmetry (i.e., when $\sigma_1\neq\sigma_2$). Defining $\gamma=\sigma_1/\sigma_2$, it can be shown that:

$$\frac{\partial p_i^*}{\partial a_i} = \frac{1}{3}\left[f(a_i)+f(a_j)\right]\left\{\left(r+\frac{(2-K(a_j))\gamma+1}{K(a_i)+\gamma K(a_j)}\right)\varepsilon_v^i - \frac{(2-K(a_j))\gamma+1}{K(a_i)+\gamma K(a_j)}\cdot\frac{K'(a_i)}{K(a_i)+\gamma K(a_j)}\right\}, \tag{16}$$

$$\frac{\partial p_i^*}{\partial a_j} = \frac{1}{3}\left[f(a_i)+f(a_j)\right]\left\{\left(\frac{1+K(a_i)+2\gamma}{K(a_i)+\gamma K(a_j)}-r\right)\varepsilon_v^j - \frac{1+K(a_i)+2\gamma}{K(a_i)+\gamma K(a_j)}\cdot\frac{\gamma K'(a_j)}{K(a_i)+\gamma K(a_j)}\right\}. \tag{17}$$

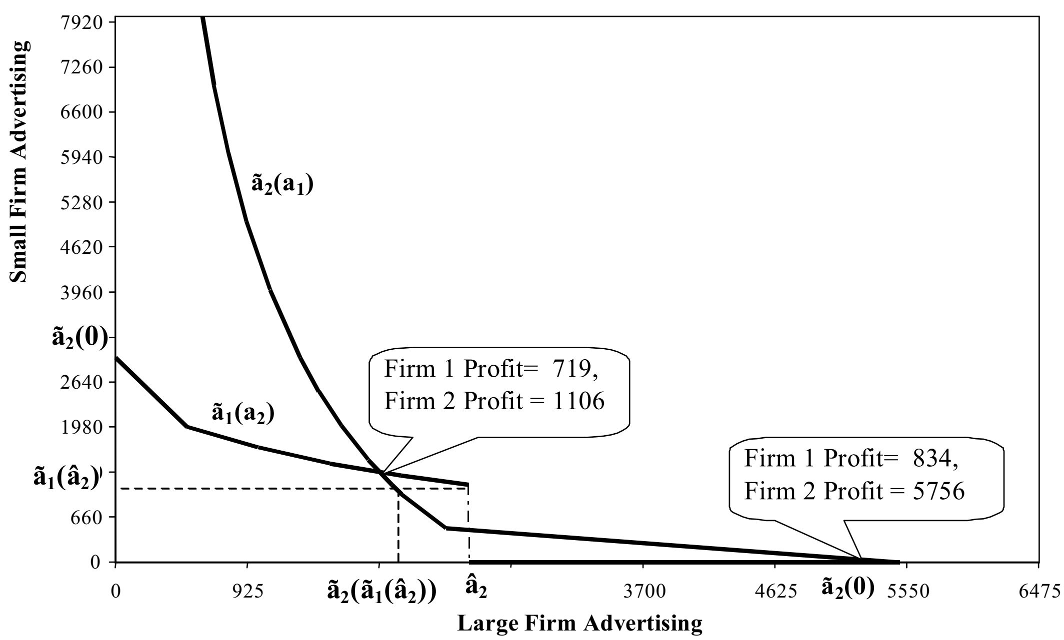
We find that $\lim_{\gamma\to\infty}\partial p_i^*/\partial a_i>0$ and also that $\lim_{\gamma\to 0}\partial p_i^*/\partial a_j>0$, indicating that the large brand's advertising spending unambiguously tends to increase not only its own price, but also that of the rival (small) firm. On the other hand, we find that $\lim_{\gamma\to 0}\partial p_i^*/\partial a_i<0$ and also $\lim_{\gamma\to\infty}\partial p_i^*/\partial a_j<0$ in the neighborhood of $a_i=0$ and $a_j=0$, respectively, for $d\to 1$ (similar to the inflection effects in the symmetric case in §3). This indicates that the initial advertising of the small firm immediately triggers a heightened price competition wherein both firms cut prices. Thus, when firms' shares of the market are sufficiently different (Regime b in Table 2), we find that advertising is never a viable competitive tool for the small firm. Although the small brand's advertising is proportionately more effective in the consideration stage, the prices are driven down to levels in which the incremental revenue from advertising does not cover the incremental costs of advertising for the small firm.

This finding supports the claim that a competitive tool like advertising does not serve the same purpose even for firms in the same industry (Sutton 1991). In oligopolistic industries comprised of large national brands, as well as small private labels, larger firms use advertising to reinforce their brand proposition among the value-conscious customer franchise. In contrast, the smaller players tend to compete on the basis of prices. This finding is consistent with

Table 2 Illustrations of Asymmetric Firms Equilibria (for $s=0.2$, $w=0.5$, $d=1.3$, $r=2$)

		$\tilde{a}_1(0)$	$\hat{a}_1$	$\tilde{a}_2(0)$	$\hat{a}_2$	$\tilde{a}_2(\tilde{a}_1(\hat{a}_2))$	$\Pi_1(0,\tilde{a}_2(0))$	$\Pi_2(0,\tilde{a}_2(0))$
Regime a: Two advertising equilibria $\{(0,\tilde{a}_2(0)\}$ $\{a_2^* > a_1^* > 0\}$	$\sigma_1=40$ $\sigma_2=60$	3,370	7,464	4,530	3,530	1,940	583	4,722
	$\sigma_1=30$ $\sigma_2=70$	3,004	11,686	5,498	2,477	1,936	834	5,756
	$\sigma_1=25$ $\sigma_2=75$	2,853	15,261	6,156	2,071	1,939	1,013	6,450
Regime b: One advertising equilibrium $\{0,\tilde{a}_2(0)\}$	$\sigma_1=20$ $\sigma_2=80$	2,720	20,950	6,990	1,725	1,945	1,246	7,316
	$\sigma_1=15$ $\sigma_2=85$	2,601	31,300	8,065	1,429	1,952	1,558	8,424
	$\sigma_1=10$ $\sigma_2=90$	2,495	55,005	9,498	1,174	1,962	1,987	9,881
	$\sigma_1=5$ $\sigma_2=95$	2,399	146,390	11,480	955	1,975	2,601	11,863

Figure 5 Advertising Equilibria



Note. $\sigma_1=30$, $\sigma_2=70$, $d=1.3$

most product markets in the consumer packaged-goods industry.

A comparison of the small firm's profits in the equilibrium, where only the large firm is advertising, deserves emphasis. Note that, in Table 2, $\Pi_1(0, \tilde{a}_2)$ is increasing as σ_1 becomes smaller and smaller (i.e., the smaller the small firm, the better the profitability). The implication of this finding is that when large national brands advertise extensively in mature markets, the primary focus is on persuasive brand communication that also creates a halo around the "perceived value" of the category. This in turn increases the reference price umbrella, which is more efficient for the purpose of profit maximization, not only for themselves but also for the small firms. This assertion is further supported by the fact that the total spending on advertising by the large firm at the corner equilibrium exceeds the combined spending of both firms at the interior equilibrium, whenever it exists (see Figure 5). Our result explains why private label brands that never advertise in categories, such as beer continue to thrive in markets in which large entrenched national brands command a high share of the consumers' mind (*Wall Street Journal* 1994).

6. Conclusion

Several studies in the marketing literature have postulated that advertising competition has to contend with the so-called threshold effect (wherein small advertising spending is are unprofitable) caused by the S-shaped response function. In this paper, we construct a model of advertising competition that endogenously generates such an S-shaped advertising response function. We show that such a shape of the profit-and-sales response function arises when advertising influences consumers in multidimensional ways that alter the proportion of consumer segments in a manner that creates opposing cross-effects on prices. We show that prices will increase or decrease from advertising, depending on the relative magnitudes of the consideration elasticity (encroaching into competitors' customer franchise) and the persuasion elasticity (enhancing value perception of a brand) of advertising. In the face of substantial media weight of the rival, the potential profitability from increased consumer consideration and goodwill through insubstantial advertising outlays is dissipated away through more intense price competition for the duopoly or competitive segment. It may be hypothesized that small spending on advertising has a competitive "commodification effect" ("just another beer"), which exerts downward pressures on prices. Consumers register the brand differentiators only when the advertising spending is substantial, and this in turn allows brands to earn a higher price premium. Our paper, therefore, provides a theoretical reconciliation of the longstanding debate in the marketing literature regarding the impact of advertising on prices (see also Kaul and Wittink 1995).

Consumer inertia or lock-in is a critical parameter that drives the market outcome in our model of advertising price competition. When a large proportion of consumers exhibit latent inertia against brand switching, then a multiplicity of market outcomes may be supported as equilibria in our simultaneous move game. Although it is difficult to predict which of the equilibrium outcomes would actually occur, we can conclude that, given the equilibrium payoffs, there exists a strong first-mover advantage for competing (symmetric) firms. In particular, if the firms' advertising decisions were sequential, then the firm moving first may advertise in such a way as to force an equilibrium, where advertising as a competitive tool is not viable for the follower firm.

The extension of our analysis for asymmetric firms (in terms of their market shares) suggests that advertising is generally a nonviable competitive tool for smaller firms. In particular, we establish that, when the market shares of the firm are significantly different, then the unique equilibrium has only the large firm advertising. More significantly, our numerical calculations show that when firm sizes are sufficiently asymmetric, the smaller the size of the small firm, the better is its profitability in equilibrium in which only the large firm advertises. In the absence of the small brand advertising, larger brand shares encourage firms to allocate higher expenditures on advertising to enhance the perceived brand value of their brand, which in turn shore up the average prices in the industry from which both the firms benefit.

Our results suggest that the existence of consumer heterogeneity, which slices the market into discontinuous segments, introduces local convexities in the firms' objective functions. These often lead to outcomes and marketing implications, which cannot really be inferred from marketing models that rely on conceptualizing consumer heterogeneity on a continuum.

Ours is a simple static model that examines advertising price cross-effects. It may be worthwhile to examine advertising scheduling decisions in a dynamic version of our model of advertising competition in oligopolistic markets with high levels of latent consumer inertia. We hypothesize that a continuous advertising schedule may outperform pulsing policies in such a model. Another useful extension of our model would be to allow firms to choose awareness and persuasive advertising separately, and then engage in price competition.

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Appendix

The best-response function of Firm 1 (and similarly, Firm 2) is of the following form:

$$\Psi_1(p_2)=\begin{cases} L_1 & \text{if } p_2\le\tilde{p}_2 \\ R_1(p_2) & \text{if } \tilde{p}_2\le p_2\le p_1+r[f(a_2)-f(a_1)]+f(a_1) \\ p_2+r[f(a_1)-f(a_2)]-f(a_1) & \\ & \text{if } L_2\ge p_2\ge p_1+r[f(a_2)-f(a_1)]+f(a_1), \end{cases}$$

where, $R_1(p_2)=\frac{1}{2}p_2+\frac{1}{2}[f(a_2)+r(f(a_1)-f(a_2))+(f(a_1)+f(a_2))\cdot M_1/D]$; and $\tilde{p}_2=\{\max p_2: M_1L_1=[M_1+Dx^D(R_1(p_2),p_2)]R_1(p_2)\}$.

Proof. For an arbitrary p_2, the shape of the profit function of Firm 1 (given in Equations (6a), (6b), and (6c)) is not quasi-concave in p_1. Given any p_2, Firm 1's profit function is not differentiable at $p_1=p_2+r[f(a_l)-f(a_2)]-f(a_2)$ and at $p_1=p_2+r[f(a_1)-f(a_2)]-f(a_l)$. Therefore, we need to analyze three different cases to specify completely the firms' best-response functions in prices.

First, since Firm 1 (Firm 2) has a segment of locked-in consumers whose choice sets contain only Brand 1, it is assured of the monopoly profit it can make from them. Hence, if $L_1>0$ is the maximum price the captive consumers are willing to pay for Brand 1, then for very low prices of Firm 2 (such as in the range of prices given in (6a)), Firm 1's best response may be to charge L_1 and make the maximum possible profit from its locked-in customers. This implies that there will be a discontinuity in the two firms' best-response functions in the p_1-p_2 space.

At the other extreme, in the range of prices given in (6c), Firm 1 is selling to the entire competitive segment in addition to its locked-in customers; hence, assuming all consumers buy, its best response is to set $p_1=p_2+r[f(a_l)-f(a_2)]-f(a_l)$.

Finally, in the range of prices given in (6b), Firm 1's profit function is strictly concave in its price. This is the range of prices in which some consumers from the duopoly segment buy from Firm 1, whereas others buy from Firm 2. Therefore, in this case, we can get Firm 1's best-response to Firm 2's prices by looking at the first-order condition for profit maximization. Thus, for prices as in (6c), $\partial\Pi_1/\partial p_1=0$ yields:

$$R_1(p_2) = \frac{1}{2}p_2+\frac{1}{2}\bigg[f(a_2)+r(f(a_1)-f(a_2)) + \frac{(f(a_1)+f(a_2))(1-K(a_2))\sigma_1}{K(a_1)\sigma_2+K(a_2)\sigma_1}\bigg]. \tag{A1}$$

The second-order condition for a global maximum holds, since

$$\frac{\partial^2 p_i}{\partial p_i^2}=-2\frac{D}{f(a_1)+f(a_2)}<0.$$

Similar analysis for Firm 2 yields the following first-order condition:

$$R_2(p_1) = \frac{1}{2}p_1+\frac{1}{2}\bigg[f(a_1)+r(f(a_2)-f(a_1)) + \frac{(f(a_1)+f(a_2))(1-K(a_1))\sigma_2}{K(a_1)\sigma_2+K(a_2)\sigma_1}\bigg]. \tag{A2}$$

As noted previously, Equations (A1) and (A2) only partially describe a pair of reaction functions in the p_1-p_2 space. We now need to look at the point of discontinuity in the firms' price reaction functions. For that purpose, let us define:

$$\tilde{p}_2=\{\max p_2: M_1L_1=[M_1+Dx^D(R_1(p_2),p_2)]R_1(p_2)\}. \tag{A3}$$

In other words, $\tilde{p}_2$ is that price of Firm 2 for which Firm 1 is indifferent to selling only to its captive customers at price L_1 and going after part of the duopoly segment (along with its captive customers) by its best alternative (i.e., $R_1(p_2)$). Therefore, only when $p_2>\tilde{p}_2$ will Firm 1 attempt to sell to a portion of the duopoly segment. Now we can specify Firm 1's reaction function in prices as follows (ignoring the case when $p_2>L_2$, because such a price is strictly dominated):

$$\Psi_1(p_2)=\begin{cases} L_1 & \text{if } p_2\le\tilde{p}_2 \\ R_1(p_2) & \text{if } \tilde{p}_2\le p_2\le p_1+r[f(a_2)-f(a_1)]+f(a_1) \\ p_2+r[f(a_1)-f(a_2)]-f(a_1) & \\ & \text{if } L_2\ge p_2\ge p_1+r[f(a_2)-f(a_1)]+f(a_1). \end{cases} \tag{A4}$$

Similar analysis for Firm 2 will yield its corresponding reaction function $\Psi_2(p_1)$ with the point of discontinuity being $\tilde{p}_1$, defined similar to (A3). □

Lemma. *The best-response function in advertising is downward sloping, with a discontinuous segment, given as*:

$$T_i(a_j) = \begin{cases} \tilde{a}_i(a_j) & \text{if } a_j \le \hat{a}_j \\ 0 & \text{otherwise}, \end{cases}$$

where $\hat{a}_j$ is defined as the level of Firm j's advertising such that $\Pi_i(\tilde{a}_i(\hat{a}_j), \hat{a}_j) = \Pi_i(0, \hat{a}_j)$.

Proof. We know that Firm i's profits are decreasing in Firm j's advertising levels by Proposition 3a. Now by Proposition 3c in the previous text, and the fact that $\Pi_1(0) > 0$, there exists an $\hat{a}_j$ such that $\Pi_i(\tilde{a}_i(\hat{a}_j),\ \hat{a}_j) = \Pi_i(0, \hat{a}_j)$. Given that profits of firms decrease in rival's advertising by Proposition 3b, for all $a_j > \hat{a}_j$, we have $\Pi_i(0) > \Pi_i(\tilde{a}_i(a_j))$. Hence, whenever Firm j's advertising level exceeds $\hat{a}_j$, Firm i's best response is to choose $a_i = 0$ and earn profit $\Pi_i(0, a_j)$. This establishes the claim in the lemma. □

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