TERM STRUCTURE ESTIMATION IN ILLIQUID GOVERNMENT BOND MARKETS: AN EMPIRICAL ANALYSIS FOR INDIA

Krishnamurthy Vaidyanathan

Corporate Advisory, Credit and Rate Markets, JPMorgan

Singapore 068912

Facsimile: 65-6882-1701

Goutam Dutta *

Indian Institute of Management, Vastrapur, Ahmedabad 380015,India.

Phone (079)-6324828

FAX (079)-6306896

goutam@iimahd.ernet.in

Sankarshan Basu

Indian Institute of Management, Bannerghatta Road, Bangalore 560 076,India.

Phone (080)-6993078

FAX (080)-6584050

sankarshanb@hotmail.com

* Corresponding Author

Abstract

With increasing liquidity of the Indian sovereign debt market from 1997, it has become possible to estimate the term structure in India. However, several frictions that cause individual securities to be priced differently from the "average" pricing in the market characterize the market. In such a scenario, traditional estimation procedures like ordinary least squares using various functional forms do not perform well. In this paper, we find that mean absolute deviation is a better estimation procedure in illiquid markets than the ordinary least square. We further find out a novel liquidity weighted objective function for parameter estimation. We model the liquidity function using the exponential and hyperbolic tangent functions and suggest the most robust model for estimating term structures in India.

Keywords: Finance; Fixed Income Securities; Non-linear Constrained Optimisation

1. Introduction

This paper is an attempt to develop a mathematical model for the term structure of interest rates in India. Since the liberalisation of the Indian economy in the early 1990's and especially since 1997 there has been an increase in liquidity in the sovereign debt market. This has enabled the researchers to develop better estimate of the term structure of interest rates. This has also prompted the National Stock Exchange (NSE) of India to come up with a Zero Coupon yield curve on a daily basis. The methodology, used by NSE is based on the work of Nelson and Seigel (1987). Their model is considered as one of the most useful model in this context. The work is not full proof as it does not take into account illiquidity and other frictions in the market. It is reported that the standard deviation is very when comparison is made.

The Government of India Securities (GOISEC) market is characterized by several frictions that cause individual securities to be priced differently from the "average" pricing in the market as indicated by the term structure of interest rates. One of these frictions is related to the accounting and performance measurement norms prevalent in the market. The performance of a trader is judged by the trading profits as measured by the difference between the purchase and sale prices of a security. The coupon plays no part in performance measurement and is accounted as coupon income in the accounts of the organization. There is an incentive for traders to buy discount bonds to show a trading profit. Hence, the demand for the discount bonds is higher which raises their prices above the economic cost as ascertained from the term structure of interest rates. Another friction is that a substantial portion of trading is concentrated in a handful of bonds. The market perceives these bonds to be liquid. The remaining bonds, which are not perceived as liquid, get traded at prices that incorporate their illiquidity.

In this paper, we start with a comparison of various models available for yield curve generation and their suitability to Indian data. In this process, we try to take into account the various peculiarities of the Indian bond market data - issues like illiquidity, range of data points to be used as well as minimization of errors due to fitting a model. We attempt to produce a framework that can be used to generate accurate model prices of GOISEC.

We are interested in finding out the effect of illiquidity on the estimation on the parameters of term structure. This question, in particular, is quite pertinent to term structure estimation in all emerging markets, which typically exist in developing countries. In fact in emerging markets, in the absence of an established term structure of interest rates, the markets model it on their own. However, the drawback in this is that the market determines the model based only on t liquid securities and hence is only partially complete. The classic example is the Indian market where no specific term structure model of interest rate exists. Market operators devise a proxy of the term structure based on the liquid securities. However, the large number of illiquid securities in the market causes this market-determined proxy to be subject to large errors.

A study by Subramanium (2001) discusses concept of weighted parameter optimisation. The reported results is difficult to replicate. However, the work does not make a comparison of different methods of weighted averages. We extend the work by Subramanium and show that that mean absolute deviation is a better estimation procedure in illiquid markets than the ordinary least square. We also make a comparison of different methods of weights.

This paper is organized as follows. The next section is devoted to a literature survey of the modelling structure. Section 3 details the methodology adopted in this paper. Section 4 discusses the data used for estimation. Section 5 presents the results and investigates their implications. Section 6 puts forth recommendations and highlights some of the areas for further research.

2. Literature Survey

There are various methods to estimate the term structure; most of them are numerical in nature. The numerical methods basically try to estimate the term structure by taking into account the cash flows from different bonds at one point in time discounted to the same level. This results (like any curve fitting technique) in two possible approaches ; the choice Is thus between the goodness of fit of the curve and the smoothness of the curve. The objective is to arrive at a compromise between the two; this is where Nelson and Siegel (1987) Model and the Nelson, Siegel and Svensson (1995) Model come in most handy. A point that should be made is that in India the Nelson and Siegel (1987) method is the most widely used methodology in estimation of the term structure of interest rates.

The numerical theory underlying the approximation of yield curves using various functional forms is based on the Weierstrass theorem, which holds that a continuously differentiable function can be approximated in some interval (within an arbitrary error) to some polynomial defined over the same interval.

McCulloch's (1971) method tries a spline approximation to yield curve with the Weierstrass theorem as the basis. This method requires the specification of a basis function that is crucial. The discount function is expressed as a linear combination of basis functions, which is estimated by regressing on the price data of the bond. Certain properties of the discount function are mandated:

1. The discount function must be positive
2. The discount function must be monotonically non-decreasing
3. The discount function must be equal to unity at $t = 0$

McCulloch himself suggested a simple polynomial for the basis functions. However, these functions have a uniform resolution. Hence they fit better where the point set is dense as compared to the range where point sets are scarce. McCulloch uses quadratic splines, but this leads to oscillations in forward rate curves,a phenomenon which McCulloch calls "knuckles".

One way to avoid this effect is to increase the order of the estimating function and use (for example) a *cubic spline*. McCulloch (1975) presents the simplest implementation of a cubic spline. It can be quite flexible, as it does not constrain the discount function to be non-increasing; however, the forward rates may turn out to be negative.

Mastronikola (1991) suggests a more complex cubic spline wherein the first and second derivatives of the adjoining functions are constrained to be equal at the knot points. By constraining the end points, each cubic equation is unique and hence the entire curve is unique. The short end of the curve is constrained to have a constant slope while the long end is made flat. But cubic splines too have the disadvantage of producing estimates of forward rates that are rather unstable.

In order to avoid the problem of improbable looking forward curves with cubic splines, a method that uses exponential splines to produce an asymptotically flat forward rate curve is used. This model is as capable of modelling the term structure, as are ordinary polynomial splines. Also there is an added computational burden of estimating the non-linear rather than linear model. Hence use of ordinary splines, rather than exponential splines, is recommended.

But there are important concerns regarding the choice of basis functions as suggested by McCulloch. The contention is that these basis functions can generate a regression matrix with columns that are perfectly collinear, resulting in possible inaccuracies owing to subtraction of large numbers. Use of B-splines as a solution is advocated. These are functions that are identically zero over a large portion of the approximation space and prevent the loss of accuracy because of cancellation. Steeley (1991) suggests the use of B-splines, which he shows to be more convenient and an alternative to the much-involved Bernstein (1926) polynomials. Eom, Subrahmanyam, and Uno (1998) use B-splines successfully to model the tax and coupon effects in the Japanese bond market.

Considering the problems of unbounded forward rates with the above methods, Nelson and Siegel (1987) tried smoothing the forward rate, which they modelled as an exponential polynomial.

Svensson (1995) proposes a modification to Nelson and Siegel's (1987) forward rate model. Bliss (1997) has proposed estimating the Nelson and Siegel model using a non-linear, constrained optimization procedure that accounts for the bid and ask prices of bonds as well.

Adams and Van Deventer (1994), while continuing to focus on the forward rate function, take a fundamentally different approach to estimating the term structure of interest rates. Their criterion for the best fitted curve is in terms of the maximum smoothness for forward rates.

Splines and the Nelson and Seigel(1987), and Svennson(1995) forms constitute the two polar extremes in the continuum of methods that emphasize flexibility (accuracy) and smoothness respectively. There has been the middle of the road approaches too. Fisher, Nychka, and Zervos (1995) propose using a cubic spline with roughness penalty to extract the forward rate curve. This method tries to model smoothness and flexibility in the same objective function and allows weights to be attached to each. Varying this weight decides the extent of the trade-off required. The roughness penalty is chosen by a generalized cross-validation method to regulate the trade-off. This method performs better than McCulloch's in the medium and long bonds but ends up with excessive smoothing on the short side.

According to Bliss (1997), the method of Fisher *et al.* (1995) tends to mis-price short maturity securities. This is because it attaches the same penalty across maturities. Waggoner (1997) proposes using a variable roughness penalty for different maturities called the VRP (Variable Roughness Penalty) method. This method provides better results than that of Fisher on the short side and performs as well on the medium and long bonds.

All these works concentrate on estimating the yield curves in developed markets where information for securities at different maturities is readily available. However, in less developed markets the government debt market is not liquid and derivatives markets for government debt do not exist. In such a scenario, functional forms discussed earlier can be used but with some modifications. We propose that the effect of liquidity be incorporated into the estimation procedure. Traditional parameter estimation has been done either by minimizing the mean of the squares of the error between the observed and calculated prices or by using maximum likelihood estimation. Bolder and Streliski (1999) use objective functions which penalize to a greater extent the errors on such bonds which fall out of the bid-ask spread. They use maximum likelihood estimation incorporated in the errors by assuming the standard deviation to be equal to half the bid-ask spread. They also use weighted least square estimation. They calculate the weights by dividing the individual security price error with the bid–ask spread and raising the quotient to the power of λ which they call the 'penalty parameter'. Other modified objective functions include incorporating penalties for roughness in the yield curve.

As we have mentioned, a study by Subramanuim (2001), discusses concept of weighted parameter optimisation. We have also found that the results of Subramanium (2001) is difficult to replicate However, in illiquid markets like India where only about a handful of liquid securities get traded in a day, illiquid bonds must also be included in the estimation procedure. Hence the estimation methods must incorporate the effect of liquidity premiums on illiquid bonds. We attempt to estimate the parameters by minimizing the mean absolute deviation between the observed and calculated prices. We also use the weighted least squares and weighted mean absolute deviation to estimate the parameters. The weights have been assigned based on the liquidity of individual securities.

3. Analysis

This section discusses the methodology adopted in our work. Subsection 3.1 explains the various objective functions used for the estimation and the rationale. Subsection 3.2 narrates the functional forms used for estimation.

3.1 Objective Functions

Since the objective of the project is to generate spot interest rates so that the GOISEC can be priced as accurately as possible, the error between the observed price and the price calculated from the model is the basis for Optimisation. Our hypothesis is that the mean absolute deviation is a more appropriate objective function than the mean squared error for the following reasons:

1. The term structure of interest rates enables traders identify over-priced and under-priced securities. The payoff to traders to identify underpriced or overpriced securities in the market and take advantage of them is a linear function of the pricing error in the security. Hence, the function that needs to be optimized must be a linear function in the error. optimisation of the mean squared error, on the other hand, has no economic rationale apart from modelling convenience.

2. In the GOISEC market, as in most other markets, the class of traded securities comprises both liquid ones, which get traded frequently, and illiquid ones that report one-off trades. Liquid securities tend to typically have finer bid-ask spreads compared to illiquid ones. But in an illiquid government debt market the number of liquid securities is low and hence illiquid securities must also be included in estimating the term structure. The market, however, tends to charge a liquidity premium on illiquid securities and hence we expect illiquid securities to be priced more inaccurately by a model that ignores liquidity premiums. Hence we expect the pricing errors to be larger on illiquid securities than on the more liquid ones. In such a scenario, using a squared error criterion tends to accentuate pricing anomalies since large error terms resulting from the presence of liquidity premiums contribute more to the objective function than to the errors on liquid securities.

Having argued that the mean absolute deviation is a better Optimisation parameter, we go one step further. Errors are caused for two reasons: (a) curve fitting and (b) presence of liquidity premium. Now, the errors due to curve fitting arise from the calculations and should be avoided. But the error due to the presence of liquidity premium is reflective of market conditions and one does not want to ignore them. Assigning equal weights to both types of errors will give undue weight to the kind of error that creeps in due to curve fitting. Hence, we hypothesize that a weighted error function, with weights based on liquidity, would lead to better estimation than that using equal weights. However, note that this is valid only when equal weights bias the results and hence using weights based on liquidity premium is better. On the other hand, if equal weights do not bias the results, assigning weights based on liquidity premium also does not bias the results. Thus, as an overall picture, it is better to use weights based on the liquidity premium.

A reciprocal of the bid-ask spread is ideally the best liquidity function to use. However, available data do not report the bid-ask spreads in individual securities. Hence we try to model the liquidity using a function with two factors: the volume of trade in a security and the number of trades in that security. We highlight the necessity to use both factors through the following example. Consider two securities each reporting a trade volume of INR 500 million. One of them reports a single trade of INR 500 million and the other reports 10 trades of INR 50 million each. The second security is more liquid and hence would have a smaller liquidity premium associated with it than the first one. Hence, its price would be more reflective of the market prices than the first one. To quantify the liquidity weights, we use two variations:

The weight of the i^{th} security W_i is given by

$$W_i = \frac{\left(\left(1 - e^{(-v_i/v_{max})}\right) + \left(1 - e^{(-n_i/n_{max})}\right)\right)}{\sum_i Wi}$$

$$W_i = \frac{\left(\tanh\left(- v_i/v_{max}\right) + \tanh\left(- n_i/n_{max}\right)\right)}{.\sum_i Wi} \qquad \textbf{(1)}$$

where v_i and n_i are the volume of trade and the number of trades in the i^{th} security while v_{max} and n_{max} are the maximum volume of trades and the maximum number of trades among all the securities traded for the day respectively.

We use the exponential and the hyperbolic tangent function to incorporate asymptotic behaviour in the liquidity function. The relatively liquid securities would have v_i/v_{max} and n_i/n_{max} close to 1 and hence the weights of liquid securities would not be significantly different. However, the weights would fall at a fast rate as liquidity decreases. This is the behaviour that we wish liquidity function to accomplish.

Thus we use the following six objective functions:

- Mean squared error (RMSE)
- Mean absolute deviation (MAD)
- Liquidity weighted mean squared error using the hyperbolic tangent function (LRMSE-T)
- Liquidity weighted mean squared error using the exponential function (LRMSE-E)
- Liquidity weighted mean absolute deviation using the hyperbolic tangent function (LMAD-T)

- Liquidity weighted mean squared error using the exponential function (LMAD-T)

3.2 Model Selection

The basis for judging model performance is linked to the expectations from the model. Our expectations are:

1. Accurate pricing of GOISEC
2. Robustness of the model in producing stable spot interest rate curves

Given the lack of interest rate derivative markets in India from which forward rates can be obtained, the general equilibrium models of Vasicek (1977), Cox, Ingersoll and Ross (1985), Brennan, and Schwartz (1979) are difficult to implement. Hence we do not consider this class of models.

Among the models discussed in Section 2, we use the Nelson, Seigel, and Svennson model (the generalized version of the Nelson Siegel model) with a premium on smoothness, cubic B-splines, and cubic splines with VRP. B-spline models are superior to cubic splines models. Hence, in the class of models emphasizing flexibility, we use B-splines. We also test smoothing splines with the VRP method to include the class of models that use a compromise between accuracy and smoothness.

3.2.1 Nelson-Siegel-Svensson Model

The Nelson, Siegel, and Svensson model derives the forward rate in a functional form and determines the discount function from it to avoid oscillations in the forward rate. This method has the advantage of estimating lesser number of parameters and ensures a smooth forward curve.

The forward rate function, *F(m),* is modelled as follows:

$$F(m) = \beta_0 + \beta_1 \exp^{\left(-m/\tau_1\right)} + \beta_2 \left[\left(-m/\tau_1\right)\exp^{\left(-m/\tau_1\right)}\right] + \beta_3 \left[\left(-m/\tau_2\right)\exp^{\left(-m/\tau_2\right)}\right] \quad \textbf{(2)}$$

The parameters:

β_0 is positive and is the asymptotic value of f(m).

β_1 determines the starting value of the curve in terms of deviation from the asymptote. It also defines the basic speed with which the curve tends towards its long-term trend.

τ_1 must be positive and specifies the position of the first hump or the U-shape on the curve.

β_2 decides the magnitude and direction of the hump. If this is positive, a hump occurs at τ_I whereas if it is negative, the U-shape occurs at τ_I.

τ_2 must also be positive and defines the position of the second hump or the U-shape on the curve.

β_3 like β_2, determines the magnitude and direction of the hump.

3.2.2 B-splines

We define the following with respect to B- splines:

n = number of control points
m = number of elements in knot vector
p = degree

T is a knot vector; $\mathbf{T} = \{ t_0, t_1, \ldots t_m \}$; where **T** is a non-decreasing sequence with $t_i \varepsilon [0,1]$, and defines control points $\mathbf{P_0, \ldots, P_n}$.

We also define the degree as $p \equiv m - n - 1$.

Based on above terms, the basis function, $N_{i,o}(t)$ is defined as follows:

$$N_{i,0}(t) = \begin{matrix} 1 & t_i \le t < t_{i+1} \\ 0 & otherwise \end{matrix} \quad \textbf{(3)}$$

$$N_{i,p}(t) = \left({(t - t_i)} \big/ {(t_{i+p} - t_I)} \right) N_{i,p-1}(t) + \left({(t_{i+p+1} - t)} \big/ {(t_{i+p+1} - t_i + 1)} \right) N_{i+1,p-1}(t) \quad \textbf{(4)}$$

Then the curve defined by

$\mathbf{C}(t) = \sum \mathbf{P}_i N_{i,p}(t)$ is a B-spline.

The discount function between any two-knot points s_j and s_{j+1} is defined as:

$$P(m) = \sum_j a_j B_j^g(m) \quad \textbf{(5)}$$

where B^g is a g-order B-spline.

To obtain a smooth forward curve a spline function of at least order three must be used.

3.2.3 Variable Roughness Penalty (Smoothing Splines)

The smoothing splines approach is an extension of the splines approach suggested by McCulloch (1971).

Cubic splines generate oscillations in the forward rate term structure. Since oscillation in the forward rate is an unexpected behaviour, this method is not acceptable.

Since the forward curve is well behaved in the short maturity segment, the penalty is less. In the long maturity segment, the penalty is the highest.
The objective is to minimize the function:

$$\sum_j (P_i - P_i\hat{})^2 + \lambda \int_0^k [f'(t)dt$$

(6)

The first term represents the goodness of fit (using ordinary least squares) while the second term is the roughness penalty. The values of λ suggested by McCulloch (1971) are:

$$\lambda(t) = \begin{matrix} 0.1 & 0 \le t < 1 \\ 100 & 1 \le t < 10 \\ 100000 & 10 \le t \end{matrix}$$

(7)

where t is measured in years. These values of λ are somewhat close to the square of the number of years.

4. Data

In India, two primary sources of data are traded prices of GOISEC in the subsidiary general ledger (SGL) available from Reserve Bank of India (RBI) and the daily trade data, released by the National Stock Exchange (NSE). Subramanium (2001) discusses a comparison of the relative advantages of data. Interested readers are requested to go through his work.

The data set is similar to the data set used by Subramanium (2001) though our data set covers a period larger than that covered by Subramanium (2001).

We must note that the market lot for trade is Rs. 50 million. Hence all trades which are less than Rs. 50 million or are not multiples are ignored. This is because these trades, being odd lot trades, would distort the price information.

5. Results

We employ in-sample tests using the mean absolute error between the observed prices and calculated prices as the criterion. We calculate the mean absolute error and the standard deviation in the absolute errors for each of the days for which the models are fitted. The B-splines and the VRP method are found to be quite unstable for estimation as they produced very large errors while they fitted the price data quite well on other days. Hence these two methods were not found to be robust for the purposes of estimation.

Hence, we pursue our estimation tests using the NSS method. We use the paired *t*-test to test for significance in the difference in errors and standard deviations using the various Optimisation methods. All tests are carried out at 95% level of confidence. The results are shown in Tables 1-9.

We use the following notations in Tables 1-9:

MAD	Mean absolute Deviation
RMSE	Root Mean Squared Error
WMAD-E	Weighted Mean Absolute Deviation – Exponential
WMAD-HT	Weighted Mean Absolute Deviation – Hyperbolic Tangent
WRMSE-E	Weighted Root Mean Squared Error-Exponential
WRMSE-HT	Weighted Root Mean Squared Error - Hyperbolic Tangent

In Table 1, we find that that MAD estimation gives smaller absolute errors and smaller standard deviations than RMSE estimation. Similarly Tables 2 and 3 show that MAD gives smaller error and smaller standard deviation than WMAD-E and WMAD-HT. In Table 4 we find a comparison of WMAD-E and WMAD-HT. The values of mean and standard deviation are very close to each other and the differences in the mean and standard deviation are in the fourth decimal places. In Tables 5 and 6, we find that RMSE gives smaller mean and smaller variance than WRMSE-E and WRMSE-HT. Table 7 makes a paired two-sample comparison of WRMSE-E and WRMSE-HT. In this case we find that WRMSE-HT has smaller mean and standard deviation than WRMSE-HT. Table 8 denotes a paired comparison between WRMSE-HT and WMAD-HT. In Table 9, we show paired comparison between WRMSE-E and WMAD-E. In both the tables we find that WMAD-E is better than RMSE-E and WMAD-HT is better than WRMSE-HT. From these observations, can conclude than MAD gives better estimation than RMSE. In case we assign weights, WMAD-HT is slightly better than WMAD-E.

There are two sources of error. First, NSE data are not comprehensive and do not capture all the trades. Second, the prices are weighted averages over the day and do not consider intra-day fluctuations. Using RBI SGL data, which are available from May 13, 2000 onwards, would remove the two errors and hence would definitely improve the pricing efficiency. However, for continuity, NSE data are used as the period of study also includes a period earlier to May 13, 2000. For any future studies, only RBI SGL data should be used.

6. Conclusions and Extensions

In a Government bond market with a limited number of liquid bonds, the emphasis needs to be on the modelling of the liquidity or illiquidity of the various bonds available In the market. There are various ways in which this can be done - one method (as suggested by Subramanian (2001)) could be based on a function that takes into account the volume of trades in a particular security and the total volume of liquid or illiquid securities. Estimation using this weighted objective function ensures that liquid bonds in the market are priced efficiently. We also find that in illiquid markets, minimizing the mean absolute deviation is a better estimation procedure than minimizing the root mean squared error.

For modelling the liquidity of individual securities, we find the hyperbolic tangent function to be a better approximation than an exponential function. Further work in this area can be done to include modelling of liquidity premiums in individual securities.

The work could be extended in many other ways. The similar research questions that can be addressed in the corporate in bond markets.

1) Is liquidity weighted function a better estimation of the term structure of interest rate of bonds
2) Is MAD better estimate of term structure of interest rate than mean squared error?

A comparison similar to our comparison of different methods can be done in bond markets.

7. References

Adams, K. J. and van Deventer, D. (1994); "Fitting Yield Curves and Forward Rate Curves with Maximum Smoothness," Journal of Fixed Income, **4(1**), 52-62.

Bernstein, S. N. (1926); *Lecons sur Les Proprieties Extremales et la Meillure Approximation des Fonctions Analytiques d'une Variable Reete*, Gauthier-Villars, Paris; L'Approximation, Chelsa, New York, 1970.

Bliss, R. R. (1997); "Testing Term Structure Estimation Methods," Advances in Futures and Options Research, **9**, 197-232.

Bolder, D. J. and Streliski, D. (1999); "Yield Curve Modelling at the Bank of Canada," Bank of Canada Technical Report.

Brennan, M. J., and Schwartz, E. S. (1979); "A Continuous Time Approach to the Pricing of Bonds," Journal of Banking and Finance, **3**, 133-155.

Cox, J. C., Ingersoll (Jr.), J. E, and Ross, S. A. (1985); "A Theory of Term Structure of Interest Rates," Econometrica, **53(2)**, 385 - 407

Eom, Y. H., Subrahmanyam, M. G., and Uno, J. (1998); "Coupon effects and the Pricing of Japanese Government Bonds: An Empirical Analysis', Journal of Fixed Income, **September**, 69-86.

Fisher, M., Nychka, D., and Zervos, D. (1995); "Fitting the Term Structure of Interest Rates with Smoothing Splines," Working Paper **95-1**, Finance and Economics Discussion Series, Federal Reserve Board.

Mastronikola, K. (1991); "Yield Curves for Gilt-Edged Stocks: A New Model," Technical Series 49, Bank of England Discussion Paper.

McCulloch, J. H. (1971); "Measuring the Term Structure of Interest Rates," Journal of Business, **34**, 19-31.

McCulloch, J. H. (1975); "The Tax-Adjusted Yield Curve', Journal of Finance **30**, 811-829.

Nelson, C. R. and Seigel, A. F. (1987); "Parsimonious Modelling of Yield Curves," Journal of Business, **60**, 473-489.

Svensson, L. E. O (1995); "Estimating Forward Interest Rates with the Extended Nelson and Seigel Method', Sveriges Riksbank Quarterly Review, **3**, 13-26.

Steeley, J. M. (1991); "Estimating the gilt-edged term structure: Basis splines and confidence intervals", Journal of Business, Finance and Accounting, **18**, 512 – 529.

Subramanian, K. V. (2001); "Term Structure Estimation In Illiquid Markets", Journal of Fixed Income, Vol. 11, No. 1, 77 - 86.

Vasicek, O. (1977); "An Equilibrium Characterization of the Term Structure," Journal of Financial Economics, **19**, 351 – 372.

Waggoner, F. D. (1997); "Spline Methods for Extracting Interest Rate Curves from Coupon Bond Prices," Working Paper **97-10**, Federal Reserve Bank of Atlanta.

Glossary

Yield rate : Yield rates or the spot interest rate, also called the Yield to Maturity (YTM) is the true rate of return an investor would receive if the security were held until maturity) expressed as a function of maturity is known as the ***term structure of interest rates***.

Yield Curve: The graphical plotting of yield ratefunction is known as the ***Yield Curve***. The yield curve is one of the most important indicators of the level and changes in the interest rates in the economy and hence the interest in studying it as well as accurately modeling it.

Splines: These can be thought of as a number of polynomials joined smoothly at the point of join. These join points are called knot points and smooth means that at these points the first and second derivatives of the curve exist.

Table 1
t-Test: Paired Two Sample for Means (MAD and RMSE)

	MAD	RMSE
Mean	0.137320233	0.167163307
Variance	0.003281801	0.006848428
Observations	270	270
Pearson Correlation	0.549153875	
Hypothesised Mean Difference	0	
Df	269	
t Stat	-4.460757003	
P(T<=t) one-tail	9.97407E-06	
t Critical one-tail	1.968819561	
P(T<=t) two-tail	1.99481E-05	
t Critical two-tail	2.254018909	

Table 2
t-Test: Paired Two Sample for Means (MAD and WMAD-E)

	MAD	***WMAD-E***
Mean	0.137320233	0.141179955
Variance	0.003281801	0.003538834
Observations	270	270
Pearson Correlation	0.951584003	
Hypothesised Mean Difference	0	
Df	269	
t Stat	-2.212253972	
P(T<=t) one-tail	0.014517398	
t Critical one-tail	1.968819561	
P(T<=t) two-tail	0.029034797	
t Critical two-tail	2.254018909	

Table 3
t-Test: Paired Two Sample for Means (MAD and WMAD-HT)

	MAD	***WMAD-HT***
Mean	0.137320233	0.141177129
Variance	0.003281801	0.003526343
Observations	270	270
Pearson Correlation	0.955459833	
Hypothesised Mean Difference	0	
Df	269	
t Stat	-2.307059123	
P(T<=t) one-tail	0.011470519	
t Critical one-tail	1.968819561	
P(T<=t) two-tail	0.022941038	
t Critical two-tail	2.254018909	

Table 4
t-Test: Paired Two Sample for Means (WMAD-E and WMAD-HT)

	WMAD-E	***WMAD-HT***
Mean	0.141179955	0.141177129
Variance	0.003538834	0.003526343
Observations	270	270
Pearson Correlation	0.988771369	
Hypothesised Mean Difference	0	
Df	269	
t Stat	0.003328035	
P(T<=t) one-tail	0.49867535	
t Critical one-tail	1.968819561	
P(T<=t) two-tail	0.9973507	
t Critical two-tail	2.254018909	

Table 5
t-Test: Paired Two Sample for Means (RMSE and WRMSE-E)

	RMSE	***WRMSE-E***
Mean	0.167163307	0.290252015
Variance	0.006848428	0.022421391
Observations	270	270
Pearson Correlation	0.294077267	
Hypothesised Mean Difference	0	
Df	269	
t Stat	-8.707317274	
P(T<=t) one-tail	1.8685E-14	
t Critical one-tail	1.968819561	
P(T<=t) two-tail	3.737E-14	
t Critical two-tail	2.254018909	

Table 6
t-Test: Paired Two Sample for Means (RMSE and WRMSE-E)

	RMSE	***WRMSE-HT***
Mean	0.167163307	0.213281377
Variance	0.006848428	0.019014788
Observations	270	270
Pearson Correlation	0.293167075	
Hypothesised Mean Difference	0	
Df	269	
t Stat	-3.493256714	
P(T<=t) one-tail	0.000345304	
t Critical one-tail	1.968819561	
P(T<=t) two-tail	0.000690608	
t Critical two-tail	2.254018909	

Table 7 t-Test: Paired Two Sample for Means (WRMSE-E and *WRMSE-HT*)		
	WRMSE-E	***WRMSE-HT***
Mean	0.290252015	0.213281377
Variance	0.022421391	0.019014788
Observations	270	270
Pearson Correlation	0.476931138	
Hypothesised Mean Difference	0	
Df	269	
t Stat	5.474984786	
P(T<=t) one-tail	1.41472E-07	
t Critical one-tail	1.968819561	
P(T<=t) two-tail	2.82944E-07	
t Critical two-tail	2.254018909	

Table 8 t-Test: Paired Two Sample for Means (WRMSE-HT and WMAD-HT)		
	WRMSE-HT	***WMAD-HT***
Mean	0.213281377	0.141177129
Variance	0.019014788	0.003526343
Observations	270	270
Pearson Correlation	0.604336353	
Hypothesised Mean Difference	0	
Df	269	
t Stat	6.725395826	
P(T<=t) one-tail	4.19751E-10	
t Critical one-tail	1.968819561	
P(T<=t) two-tail	8.39501E-10	
t Critical two-tail	2.254018909	

Table 9 t-Test: Paired Two Sample for Means (WRMSE-E and WMAD-E)		
	WRMSE-E	***WMAD-E***
Mean	0.290252015	0.141179955
Variance	0.022421391	0.003538834
Observations	270	270
Pearson Correlation	0.320922935	
Hypothesised Mean Difference	0	
Df	269	
t Stat	10.98894806	
P(T<=t) one-tail	1.16954E-19	
t Critical one-tail	1.968819561	
P(T<=t) two-tail	2.33908E-19	
t Critical two-tail	2.254018909	