Unit Root Tests: Results from some recent tests applied to select Indian macroeconomic variables

Vineet Virmani
vineet@iimahd.ernet.in
(Ph: 079-632-6309)

Indian Institute of Management, Ahmedabad

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Results from newly developed unit roots tests of ERS (1996), PN (1996), NP (2001) and LM (1994) are compared against their traditional counterparts (ADF, PP and KPSS) on select Indian macroeconomic data. Results from ERS, PN and NP are broadly in agreement. However, using the general to specific criterion of Hall (1994) and the Modified Information Criterion (MIC) of NP for lag length selection, it is found that different lag length can lead to different results. Furthermore, results from using these criteria are also sensitive to the 'maximum' lag length. Both KPSS and its modified version, LM, are found to be prohibitively sensitive to the lag length used. Since as of now no theoretical criterion exists for lag length selection for tests which test the null of stationarity, their use should be avoided, even for the purpose of so-called 'confirmation'. Another important finding is that frequency of the data and span covered by the sample size plays an important role and whenever feasible, tests must be conducted with as many different frequencies as the availability of data permits. It is not only a large sample size that is important, but also the span covered, an issue raised long ago by Campbell and Perron (1991).

I. Introduction

Since Schwert (1989) demonstrated poor power properties of the Dickey Fuller (Dickey and Fuller (1979), Said and Dickey (1984); henceforth ADF) test and the poor size properties of the Phillip-Perron (Phillip and Perron (1988), Perron (1988); henceforth PP) tests, there have been a barrage of studies attempting to construct more efficient tests for unit roots (Campbell and Perron (1991), Maddala and Kim (1998); henceforth MK and Phillips and Xiao (1998)). Even though there is still no consensus on any one particular test as 'the most powerful' test, Ng and Perron (2001; henceforth NP) developed M-tests strike out as having the best size adjusted power properties. As described later NP is an extension of the DF-GLS test of Elliott, Rothenberg and Stock (1996; henceforth ERS) and the modified Z tests of Perron and Ng (1996; henceforth PN).[1]

All the tests mentioned above test the null of a unit root against the alternative of stationarity. One test which does otherwise, i.e. which has the null of stationarity, is the test of Kwaitkowski et al (1992; henceforth KPSS). Although in literature KPSS has often been used to 'confirm' results from the ADF and PP tests, MK find in their survey that KPSS test is also plagued by the same poor power and size properties as the traditional ADF and PP tests. Leybourne and McCabe (1994; henceforth LM) modified KPSS test takes account of the possible MA terms in the original data generating process (DGP) and is reported to have better size adjusted power properties than its predecessor.

In this study I compare results from all the aforementioned tests on some select Indian macroeconomic variables. I find that results are highly sensitive to the issues of lag length selection and sampling frequency. Since we have only but a realization of the time series and since a uniformly powerful test do not exist for unit root tests (Dufour and King (1991)) it is suggested that point optimal tests[2] of (the parametric) ERS and (the non-parametric) PN and NP must be used employing alternative lag length selection criteria of Hall (1994) and the Modified Information Criterion (MIC) of NP.

[1] While ERS itself was a modification to the traditional ADF test, the modified Z tests of PN were improvement upon the original PP test.
[2] i.e. optimal under the alternative (in this case with a power of 50%)

Most of the theoretical description that follows has been adapted from the surveys of Campbell and Perron (1991) and MK (1998), and the works of Perron (1988), Schwert (1989), LM (1994), ERS (1996), PN (1996) and NP (2001). Throughout the study $\{y_t\}_1^T$ represents the realization of the time series, T denotes the sample size, L stands for the lag operator such that $Ly_t = y_{t=1}$, *AR (p)* denotes a standard autoregressive process of order *p* and *MA (q)* denotes a standard moving average process of order *q*.

The plan of the study is as follows. In **section II**, to develop a backdrop, I briefly describe the traditional ADF, the PP and the KPSS tests and delineate the issues raised in literature related to their use in checking for nonstationarity. In **section III** I describe how their modified versions tackle some of the problems of the traditional tests and what problems still remain. **Section IV** describes the data. In **section V** I compare results from both the old and the new tests on the Indian data sampled at various frequencies. For reasons described later data on money, income and inflation at monthly and quarterly frequency have been seasonally adjusted prior to testing. **Section VI** concludes.

II. Traditional Unit Root Tests

1. Augmented Dickey-Fuller (ADF) Test

If any of the roots of the polynomial $(1-\phi_1 L-\phi_2 L^2-...-\phi_p L^p)$ of an *AR (p)* stochastic process lie outside the unit circle, the process is said to non-stationary. The traditional ADF way of testing for non-stationarity of an *AR (p)* process involves testing for the null of one unit root $(H_0 : \gamma^* = 0)$ in:

$$\Delta y_t = \gamma^* y_{t-1} + \sum_{j=1}^{p-1} \phi_j \Delta y_{t-j} + \alpha + \beta t + u_t \qquad [1]$$

; $u_t \sim IID(0, \sigma_u^2)$ by construction.

Depending on whether the underlying data generating process (DGP) is assumed to have drift and time trend the specification of deterministic polynomial ($\alpha + \beta t$) in the above equation changes[3]. However, as discussed below, using the ADF tests is plagued with both theoretical and practical problems. A detailed treatment on size and power properties of ADF tests can be found in Schwert (1989) and Campbell and Perron (1991). MK (1998) provide a comprehensive survey.

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[3] Critical values for tests with higher order terms in time trend are provided by see Ouliaris, Park and Phillips (1990)

- *DGP vs. the Estimating Equation*

The original DGP for a macroeconomic time series is hardly, if ever, known. A plausible DGP is assumed and presence of unit root is tested using an estimable form of the DGP.

However, if, for example, we assume

$$y_t = \alpha + \rho y_{t-1} + u_t \qquad [2]$$

as the DGP *and use the same as the estimating equation*, both null and alternative are not nested, because under null, estimating equation reduces to a DGP with a trend, whereas under the alternative it has no time trend, only a drift α (as in equation [2] with $\rho < 1$). To see this, note that if the null of $\rho = 1$ holds the above equation can be recursively expanded to:

$$y_t = \alpha t + y_0 + \varepsilon_t \qquad [3]$$

; ε_t is now a MA on u_t; i.e. [3] has a time trend as opposed where [2] has none if $\rho < 1$.

To take account of this 'problem' Campbell and Perron (1991) argue it is necessary *"...to have as many deterministic components in the trend function of the data generating process.*[4]*"* The situation does not arise, however, if Bhargava (1986) model is used which nests both the null and the alternative in a two-equation state space framework as:

$$y_t = \gamma_0 + \gamma_1 t + u_t \qquad [4a]$$

$$u_t = \rho u_{t-1} + \varepsilon_t \qquad [4b]$$

Bhargava's test, a Lagrange Multiplier statistic test, is a modification to the Sargana and Bhargava (1983) unit root test. Modified Sargana Bhargava (MSB) test enters the discussion later in the context of PP and M-tests.

- *Lag Length Selection Problem and Presence of Negative MA Terms*

A practical problem in using ADF is the selection of lag length in *[1]*. Said and Dickey (1984) suggested modification to the DF test because they noticed that most macroeconomic time series have significant MA terms and, they argued, if unaccounted for, make the DF distributions inapplicable even asymptotically.

To take account of the presence of significant MA terms they suggested using a 'large enough' lag length (they found that the order of $T^{1/3}$ was sufficient). Schwert (1989) also discussed the same issue in his Monte Carlo experiment when the true ARMA order is unknown. He suggested a rule of thumb to account for the unknown MA terms. Selection based on Schwert's formula results in a relatively large lag length in small samples (~ 100) and a modest one when the sample size is large (~ 10, 000), which is reasonable, because one would want to include as large number of a lag terms as feasible in finite samples and not too large when $T \to \infty$.

[4] J.C. Campbell and P. Perron (1991), "Pitfall and Opportunities: What Macroeconomists should know about Unit Roots," *NBER Technical Working Paper # 100*, p. 11

Other suggestions in the literature include using the Akaike's Information Criterion (AIC) and the Schwarz's Bayesian Information Criterion (BIC) to ensure that residual in *[1]* is white noise. The problems with using AIC and BIC to select the lag length, MK note, is that they tend to select too small a length (NP also report similar results) and if there are errors with MA root close to -1, a high order AR process is needed to ensure that unit root tests have good size. But while selecting a large lag length circumvents the problem of MA effect in the residual, increasing the number of regressors reduces the power of the test substantially.

An approach, based on the so called LSE methodology, which has found considerable support, is that of Hall (1994). His *general to specific* rule is to start with a large value of p (p_{max}) and reduce p until a significant t value is encountered. A recent development is the Modified Information Criterion (MIC) suggested by NP, which is a modification to AIC and BIC with a sample dependent penalty factor.

2. Phillip-Perron Test

PP test is a non-parametric modification to the standard Dickey-Fuller *t-statistic* to account for the autocorrelation that may be present if the underlying DGP is not *AR (1)*. Instead of adding AR terms in the DGP to account for (possible) MA terms, they modify the test statistic. However, Schwert (1989) showed that PP test suffers from poor size properties if the MA term is large negative. Thus, ADF and PP tests suffer from quite opposite problems. While the ADF test does not suffer from as severe size distortions, it is not as powerful as the PP test.

The other 'problem' with the PP test is that of consistent estimation of the so called *long-run variance* or the variance of the sum of the errors:

$$\sigma^2 = p\lim T^{-1} E[(\sum_{j=1}^{T} \varepsilon_j^2)]^2 \qquad [5]$$

which is different from the variance of errors:

$$\sigma_\varepsilon^2 = p\lim T^{-1} \sum_{j=1}^{T} E(\varepsilon_j^2) \qquad [6]$$

PP involves consistent estimation of both σ^2 and σ_ε^2. Now while consistent estimate of σ_ε^2 is simple,

$$s_\varepsilon^2 = T^{-1} \sum_{t=1}^{T} \varepsilon_t^2 \qquad [7]$$

consistent estimation of σ^2 is problematic. The following heteroskedasticity and autocorrelation consistent (HAC) Newey and West (1987) estimator is normally used, but the 'window'/lag length used for autocovariances is essentially arbitrary. A suggested approach is to check for the sample autocorrelation function of ε_t and select a lag length large enough to take care of residual autocorrelation in the error term. The Newey-West HAC estimator is given as:

$$s^2 = T^{-1}\sum_{t=1}^{T}\varepsilon_t^2 + 2T^{-1}\sum_{\tau=1}^{l}\sum_{t=\tau+1}^{Y} w_{\tau l}\varepsilon_t\varepsilon_{t-j} \qquad ; w_\tau = 1 - \frac{\tau}{l+1} \qquad [8]$$

Although PP test is most popular in its non-parametric modification to the DF *t*-statistic, they proposed three tests, Z_ρ, Z_t and *MSB* whose properties they found both numerically and theoretically similar. Expressions for non-parametric test statistics for the null of unit root for the three standard cases using above estimators of variances are as given (from Perron (1988)) below. Throughout, $t_{\hat{\rho}} = \dfrac{(\hat{\rho}-1)}{s_\varepsilon(\sum y_{t-1}{}^2)^{1/2}}$

- *AR (1) without drift*

$$Z_\rho = T(\hat{\rho}-1) - \frac{1}{2}\frac{s^2 - s_\varepsilon^2}{[T^{-2}\sum_2^T y_{t-1}^2]} \qquad [9a]$$

$$Z_t = \frac{s_\varepsilon}{s}t_{\hat{\rho}} - \frac{1}{2}\frac{s^2 - s_\varepsilon^2}{s[T^{-2}\sum_2^T y_{t-1}^2]^{1/2}} \qquad [9b]$$

$$MSB = [T^{-2}(\sum y_{t-1}{}^2 / s^2)]^{1/2} \qquad [9c]$$

- *AR (1) with only a drift*

$$Z_\rho = T(\hat{\rho}-1) - \frac{1}{2}\frac{s^2 - s_\varepsilon^2}{[T^{-2}\sum_2^T (y_{t-1} - \bar{y}_{-1})^2]} \qquad [10a]$$

$$Z_t = \frac{s_\varepsilon}{s}t_{\hat{\rho}} - \frac{1}{2}\frac{s^2 - s_\varepsilon^2}{s[T^{-2}\sum_2^T (y_{t-1} - \bar{y}_{-1})^2]^{1/2}} \qquad [10b]$$

$$MSB = [T^{-2}\sum_2^T (y_{t-1} - \bar{y}_{-1})^2 / s^2]^{1/2} \qquad [10c]$$

- *AR (1) with a drift and a linear trend*

$$Z_\rho = T(\hat{\rho}-1) - \frac{T^3}{24D_X}(s^2 - s_\varepsilon^2) \qquad [11a]$$

$$Z_t = \frac{s_\varepsilon}{s}t_{\hat{\rho}} - \frac{T^3}{s\sqrt{24D_X}}(s^2 - s_\varepsilon^2) \qquad [11b]$$

$$MSB = [24D_X / T^6 s^2)]^{1/2} \qquad [10c]$$

; D_X is the determinant of the matrix *[1 t y_{t-1}]* ; *1* denotes the column vector forming the intercept (the drift term), *t* the time trend, and y_{t-1} the vector of lagged y_t. Also, in above equations Z_ρ and Z_t are related as $Z_t = MSB \times Z_\rho$. To facilitate comparisons with the ADF and its modified versions Z_t is used.

3. *KPSS Test*[5]

KPSS is the only popularly used test in which the null of stationarity is tested against a non-stationary alternative. In particular, the KPSS specification is:

$$y_t = \delta t + \varsigma_t + \varepsilon_t \qquad [12a]$$

$$\varsigma_t = \varsigma_{t-1} + u_t \qquad [12b]$$

and the null hypothesis is $H_0 : \sigma_u^2 = 0$

The above specification is special case of the model discussed by Nabeya and Tanaka (1988) with the null of parameter constancy against the alternative that the parameters follow a random walk:

$$y_t = \beta_t x_t + \gamma^{'} z_t + \varepsilon_t$$

$$\beta_t = \beta_{t-1} + u_u$$

; $u_t \sim IID(0, \sigma_u^2)$ with the test statistic given as:

$$LM = \sum_{t=1}^{T} S_t^2 / s_\varepsilon^2 \qquad [13]$$

; S_t is the partial sum of the error terms defined as $S_t = \sum_{j=1}^{t} \varepsilon_j$

KPSS modified the above test statistic, because as they argued, the above test statistic is valid only if the errors (ε_t) are *IID*. KPSS consider the general case and suggest a modification to (denominator of) the above LM statistic. Instead of using the error variance, they suggest using the Newey-West HAC estimator of long run variance (discussed in the context of PP earlier). The KPSS test statistic, then, is:

$$LM = \sum_{t=1}^{T} S_t^2 / s^2 \qquad [14]$$

It is often suggested that KPSS, in which the null is that of stationarity, can be used to 'confirm' results from the ADF and PP. But, as MK discuss at length, not only the inference from KPSS test very sensitive to the lag length *(l)* used in estimation of the HAC variance, it also has the same poor power properties of the ADF.

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[5] Most of this section has been adapted from MK (1998)

Thus, not only are all of the above three traditional tests sensitive to the issue of lag length and presence of (large) negative MA terms, they are also characterized by problems of poor power and size. Since the study of Schwert (1989) literature on unit root tests has been abounding. Throughout the '90s there have been many developments in the area. Here we restrict ourselves to the discussion of modification of the above tests, because despite the problems they are the ones that are still used most often and communicating modifications to these tests may be far easier than moving on to completely new tests. Except the modified version of the KPSS test that we discuss next, modifications to ADF and PP have now been accepted in the literature as primary tests for unit roots, and further theoretical developments are being centered on them (see Phillips and Xiao (1998), Xiao and Phillips (1997), MK (1998), NP (2001) etc.).

III. Modified ADF, PP and KPSS tests

- *Elliott, Rothenberg and Stock (1996) DF-GLS test*

Exploiting the Dufour and King (1991) result that uniformly powerful test do not exist for unit root tests, ERS modify the ADF test and show that their DF-GLS test has the limiting power function close to the point optimal test. Note that when we test for non-stationarity the alternative hypothesis is $\rho < 1$ and we are not testing against *any value of* ρ. Under these circumstances we have a power envelope covering the continuous set of each possible value of ρ under the alternative. ERS propose a family of tests whose power functions, they show, is tangent to the power envelope at one point and never below. They call these tests $P_T(0.5)$, signifying that the tests are optimal at the 50% power. They then go on to show that their DF-GLS has the limiting power function close to $P_T(0.5)$.

DF-GLS proceeds by first detrending the series as

$$y_t^d = y_t - \hat{\beta}_0 - \hat{\beta}_1 t \qquad [15]$$

; $\hat{\beta}_0$ and $\hat{\beta}_1$ are obtained by regressing $\bar{y}$ on $\bar{z}$

$$\bar{y} = [y_1, (1-\alpha L)y_2, ..., (1-\alpha L)y_T]' \qquad [16a]$$

$$\bar{z} = [z_1, (1-\alpha L)z_2, ..., (1-\alpha L)z_T]' \qquad [16b]$$

; y_t is the original time series and z_t is $[1,t]'$, $\alpha = 1 + (\bar{c}/T)$ and $\bar{c}$ takes the value *-7* or *-13.5* depending upon whether the original DGP is assumed to have a drift or drift and trend both. ERS determine the values of $\bar{c}$ using simulations so as to result in power close to point optimal tests. NP confirm that other values of $\bar{c}$ do not lead to any improvement, and they also stick to the same values.

After the series has been suitably detrended the DF-GLS proceeds on similar lines as the traditional ADF test i.e. the null of a unit root $(H_0 : \gamma^* = 0)$ can be tested in:

$$\Delta y_t^d = \gamma^* y_{t-1}^d + \sum_{j=1}^{p-1} \phi_j \Delta y_{t-j}^d + u_t \qquad [17]$$

Critical values for both $\bar{c}$ = *-7* and $\bar{c}$ = *-13.5* have been provided by ERS. Note that although DF-GLS has much better power properties the issue of lag length selection still remains. NP find that ERS has poor size properties when the underlying DGP has large negative *MA* terms. They augment their test by using MIC and they show it overcomes this problem, *if* indeed there are large negative MA terms in the underlying DGP.

- *Perron and Ng (1996) Modified Z tests*

As mentioned earlier, PP is characterized by poor size properties. PN modify the Z statistics proposed by PP to correct for this problem. Their modified Z statistics are:

$$MZ_t = MSB \times MZ_\rho \qquad [18]$$

where MSB follows from equations *[9c] to [11c]*, and

$$MZ_\rho = Z_\rho + \frac{T}{2}(\hat{\rho} - 1)^2 \qquad [19]$$

where Z_ρ for the three cases follow from the equations *[9a] to [11a]*

Unlike in PP test, however, PN do not propose the Newey-West HAC estimator for the *long run variance* (autoregressive spectral density estimator at frequency zero). Instead, they suggest using

$$s_{AR}^2 = \frac{S_{\varepsilon e}^2}{[1 - \hat{b}(1)]^2} \qquad [20]$$

$$; S_\varepsilon^2 = (T-k)^{-1} \sum_{t=k+1}^{T} \hat{\varepsilon}_t^2 \text{ and } \hat{b}(1) = \sum_{j=1}^{k} \hat{b}_j \qquad [21]$$

and $\hat{b}(j)$ and $\hat{\varepsilon}_t$ for the drift/drift and trend case to be obtained from the following autoregression[6]:

$$\Delta y_t^d = \alpha + b_0 y_{t-1}^d + \sum_{j=1}^{k} b_j \Delta y_{t-j}^d + \varepsilon_t \qquad [22]$$

[6] The intercept α does not occur when there is neither drift nor trend; also, data is not detrended in that case

Here, y_t^d refers to de-meaned data in the only drift case and detrended data if the underlying DGP is assumed to contain both drift and trend terms. Although the above modification may seem rather complicated, it is essentially the long variance estimator which PN show to be inefficient in the context of PP. They show that the modified *Z* statistic when used with any other estimator of long variance yields no improvement in the size properties. PN further show that the modified *Z* statistic is able to maintain good power while correcting for the size problems of PP.

The lag length problem, however, still comes to fore in the form of estimating equation *[22]*. However, since the test is not really *based* on the above autoregressive equation, the issue is less serious. The PN test remains essentially a non-parametric modification to the DF test statistic. However, this practical problem still needs to be addressed. NP show that using MIC to select *k* in *[22]* results in better size adjusted power properties.

- *Ng and Perron (2001) M-tests and the Modified Information Criterion*

M-tests of NP are an extension of the ERS to the modified Z tests (MZ_ρ, MZ_t and *MSB* discussed above) developed in PN. By detrending data using GLS and using MIC for lag length selection, NP show that size adjusted power properties of the *MZ* tests increase significantly. They show using Monte Carlo experiments that even in the case of DF-GLS developed by ERS, if lag length is selected using MIC, the power improvements are significant especially when there are MA terms in the underlying DGP.

For long run variance to be used in *MZ* statistic of PN, NP suggest using GLS detrended data in the autoregression *[22]*. They find that the long run variance estimation using GLS detrended data results in higher power (than *MZ*). They find using simulation studies that for most 'practical' ARMA cases while $\text{DF-GLS}^{\text{MIC}}$ outscores MZ^{MIC} on power, on the size criterion it is the other way round.

Without going into the details, MIC is basically a modification of AIC[7] which depends upon the *sample* value of the parameter (b_0) tested under null and the sample size. For lag *k* and sample size *T* and the value of the coefficient on y_{t-1} (b_0), MIC is given as:

$$MIC(k) = \ln(s^2) + \frac{2(\tau(k)+k)}{T-k_{\max}} \qquad [23]$$

$$;\ \tau(k) = (s^2)^{-1}\hat{b}_0^2 \sum_{t=k_{\max}+1}^{T} y_{t-1}^2 \text{ and } s^2 = (T-k_{\max})^{-1}\sum_{t=k_{\max}+1}^{T}\hat{\varepsilon}_t^2$$

$\hat{\varepsilon}_t$ are to be obtained from the autoregression as in *[22]*. Note that the y_t here denotes the appropriately GLS de-meaned/detrended data and not the original time series. NP find that the theoretical and numerical properties of the three M-tests (MZ_ρ, MZ_t and *MSB*) are quite similar and they illustrate using MZ_ρ. In this study to facilitate comparison with the traditional PP test, MZ_t has been used after appropriately detrending the data.

[7] They also modify the BIC but show that AIC has superior theoretical properties

- *Leybourne and McCabe (1994) Modified KPSS test*

LM modification to KPSS is analogue of ADF to DF (although KPSS *per se* is closer to PP in spirit). Their modification to KPSS was an answer to the sensitivity of the KPSS to the value of the lag *l* used in the estimation of the Newey-West HAC estimator of long run variance.

LM modify equations *[12a]* and *[12b]* used by KPSS to:

$$\phi(L)y_t = \delta t + \varsigma_t + \varepsilon_t \qquad [24a]$$

$$\varsigma_t = \varsigma_{t-1} + u_t \qquad [24b]$$

It can be shown that *[23a]* and *[23b]* can be reduced to the following *reduced-form model*:

$$\phi(L)(1-L)y_t = \delta + (\varepsilon_t - \varepsilon_{t-1} + u_t) \qquad [25]$$

They write $(\varepsilon_t - \varepsilon_{t-1} + u_t)$ as $(1-\theta L)\eta_t$ and derive relations between the second moments of η_t, ε_t and u_t. Then they test for the null of stationarity as:

$H_0 : \theta = 1$ (or equivalently $H_0 : \sigma_u^2 = 0$)

The procedure for implementing their test starts by estimating by Maximum Likelihood (ML) *ARIMA (p, 1, 1)* y_t as:

$$\Delta y_t = \delta + \sum_{j=1}^{p-1} \phi_j \Delta y_{t-j} + \eta_t - \theta\eta_{t-1} \qquad [26]$$

and retrieve $\hat{\phi}_j$ [8]. Their test statistic is:

$$\hat{s}_\delta = \hat{\varepsilon}' W \hat{\varepsilon} \Big/ \hat{\sigma}_\varepsilon^2 T^2 \; ; \; \sigma_\varepsilon^2 = \hat{\varepsilon}' \times \hat{\varepsilon} / T \text{ provides a consistent estimate } \sigma_\varepsilon^2 \qquad [27]$$

;*W* is a *T X T* matrix where $W_{ij} = min\ (i, j)$ and the residuals, $\hat{\varepsilon}$ are retrieved by regressing y_t^* on an intercept and a time trend, and

$$y_t^* = y_t - \sum_{j=1}^{p} \hat{\phi}_j y_{t-j} \qquad [28]$$

LM go on to show that the asymptotic distribution of $\hat{s}_\delta$ is the same as that corresponding distribution derived by KPSS and thus the same critical values can be used for testing for the null of stationarity.

[8] Note the equivalence between [24] and [25]. Also, note that the error term in [24] is by construction an MA, thus mimicking the fact that most economic time series exhibit significant (negative) MA terms

Summary and Notation

To wrap up the discussion I summarize the tests used in the study in the following table along with the notation used and the criteria of lag length selection.

Table 1

Test	Notation	Long Run Variance Estimator	Lag Length Criteria
ADF	ADF	-	Hall's**, MIC** and SACF[9]
PP	Z	Newey West HAC	MIC and SACF
KPSS	KPSS	Newey West HAC	MIC and SACF
DF-GLS of ERS	DF-GLS	-	Hall's, MIC and SACF
Modified PP of PN	MZ-OLSD	S_{AR}* based on OLS detrending	Hall's, MIC and SACF
Modified PP GLSD of	MZ-GLSD	S_{AR} based on GLS detrending	MIC and SACF
LM	MKPSS	S_{AR} based on ARIMA (p, 1, 1)	Schwert (1989) and SACF

** - S_{AR} is the long run variance (autoregressive spectral density estimator at frequency zero) as calculated in equations [20] - [22]*
*** - For both Hall and MIC k_{max} has been alternatively chosen as T/4 and Schwert's l_{12}*

Thus, in all, I conduct 27 tests for each variable sampled at various frequencies as discussed in the next section on data.

IV. Data

The series tested for nonstationarity are:

- *GDP at factor cost (Y)*: Sampled at annual (1951/52 – 2001/02), quarterly (1983/84Q1 – 2001/02Q4) and monthly (1983/84M4 – 1998/99M3) frequency. For details on estimation of quarterly and monthly GDP-FC see author's earlier paper (IIMA WP # 2003-10-03). Natural logarithm (*y*) of the variable is tested. Also, monthly and quarterly data are seasonally adjusted (using TRAMO/SEATS of EUROSTAT[10]). Plot in **Figure 1**.

- *Money Supply (M)*: Definitions *M1* and *B*, sampled at annual (1971 – 2003), quarterly (1983Q1 – 2001Q4) and monthly (1983M4 – 2001M3) frequency. Natural logarithm (*m, b*) of the variable is tested. Also, monthly and quarterly data are seasonally adjusted (using TRAMO/SEATS of EUROSTAT). Plot in **Figure 2** for *M1*.

- *Call Rate (R)*: Sampled at quarterly (1992Q1 – 2001Q4) and monthly (1992M4 – 2002M3) frequency. Variable is tested both in levels (*R*) and in natural logarithm (*r*). Plot in **Figure3**.

- *Inflation based on the Wholesale Price Index All Commodities Index* (π_w): Sampled at quarterly (1982/83Q1 – 2001/02Q4) and monthly (1982/83M4 – 2001/02M3) frequency at 1993-94 prices (and seasonally adjusted using TRAMO/SEATS of EUROSTAT). A note on the generation of the series at the new base (1993-94 = 100) is provided in **Appendix 1**.

[9] SACF used as criterion here refers to starting from lag zero till all lags till *T/4* shows zero autocorrelation in residuals in *[1]* at 5% significance

[10] Adjustment was also performed using the popular X-11-ARIMA of US Census Bureau; results were similar

- *Inflation based on the 49/50% Optimal Trimmed Mean (π_c):* Sampled at quarterly (1982/83Q1 – 2001/02Q4) and monthly (1982/83M4 – 2001/02M3) frequency at (1993-94 = 100) prices. For selection of optimal trimmed mean see author's earlier paper (IIMA WP # 2003-12-02). Plot for both inflation series in **Figure 4**

Figure 1

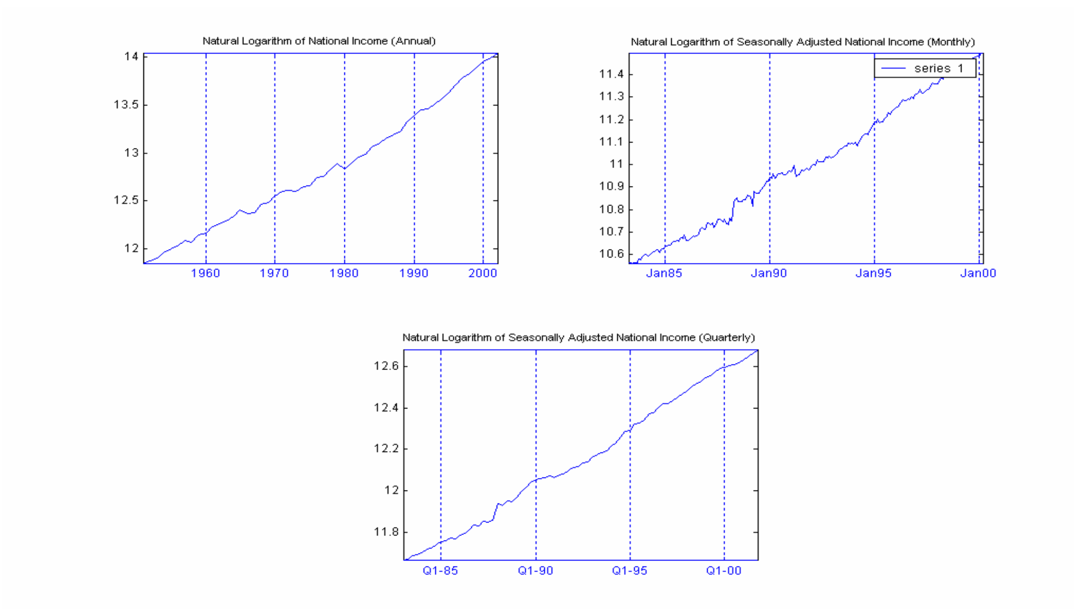


Figure 2

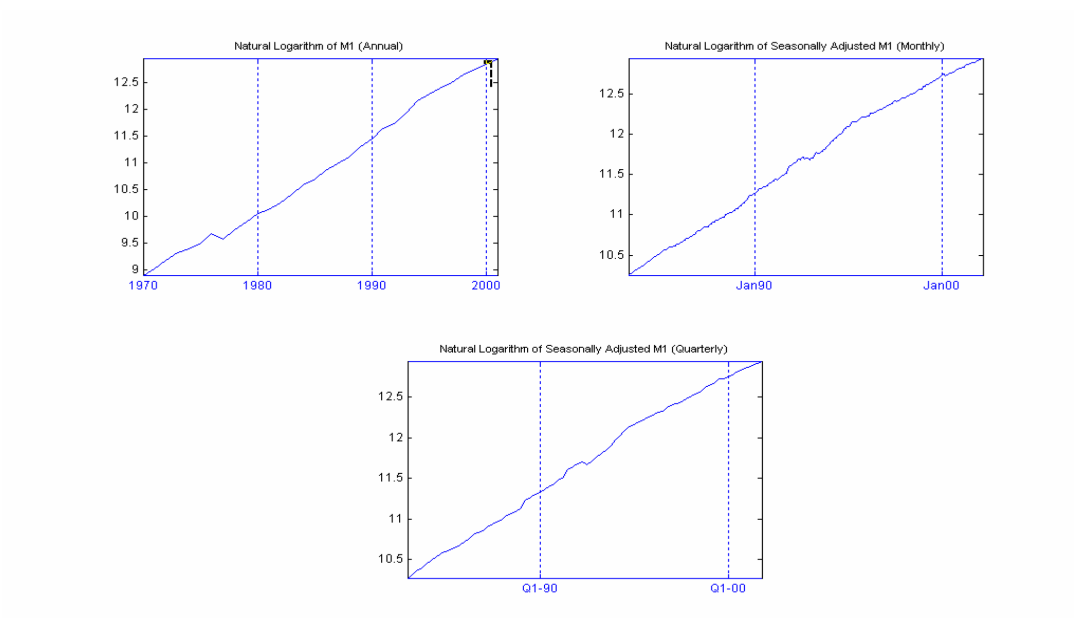


Figure 3

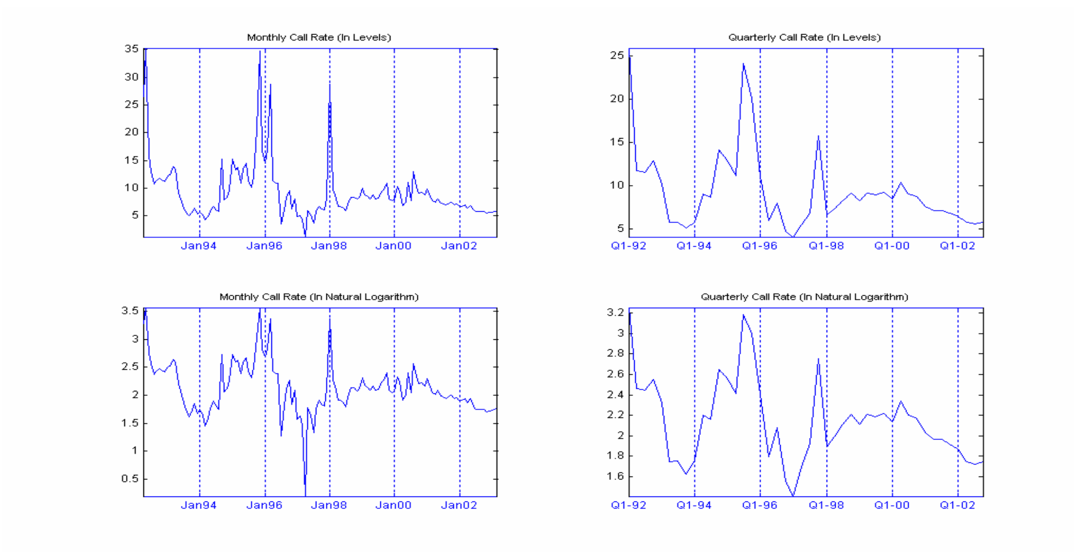
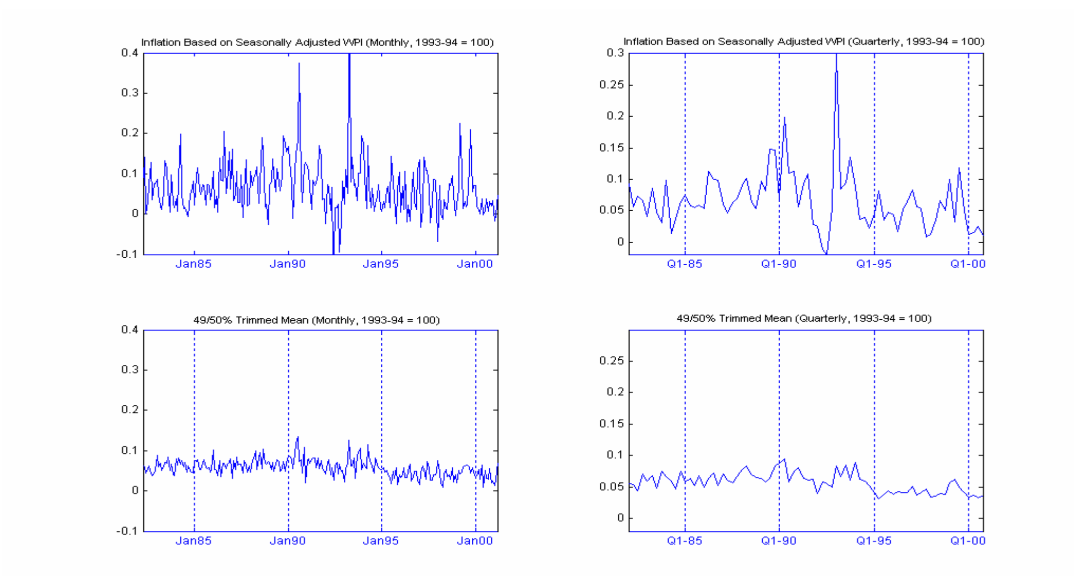


Figure 4



Following Schwert (1989) and others, seasonally adjusted data has been used to preclude the presence of a seasonal unit root. Also, it was noticed that the autocorrelation in residuals from *[1]* for unadjusted data did not die for as large a lag length as *T/4.*

The idea of using the same data with different frequency is motivated by the suggestion of Campbell and Perron (1991) who note that power of unit root tests *"…depends very little on the number of observations per se but is rather influenced…by the span of the data.*[11]*"* Testing on the same data sampled at different frequency helps in validation.

[11] Campbell and Perron (1991), *ibid,* p. 13

V. Results and Discussion

Results are presented in **Tables 2 to 9**. To emphasize the importance of using alternative testing procedures and using data spanning different time period but identical sample size, results for output **(Table 2)** are discussed in some detail.

Using data sampled at annual frequency while all but two combinations point towards a non stationary output series, test using monthly data tells that the series is stationary. Also, using quarterly data, agreement is more in favour of non-stationarity than stationarity, with four combinations pointing towards a stationary series. Now what do we conclude from such results? Campbell and Perron (1991) argue that tests on data spanning a larger time period have more power. Intuitively, data spanning larger time period gives us a chance to look at the data *for much longer* than using monthly realizations. Also, not much can be read on the evidence of stationarity based on the KPSS test given its sensitivity to the lag length and poor size adjusted power properties. All in all, evidence is more in favour of output being a non-stationary series. Clearly, there is aplenty in here to satisfy any critique's vanity. Degree of confidence in inference from the tests must depend on the problem at hand, but results here are a strong indication that looking at high frequency data to check for unit roots may give misleading results. Campbell and Perron (1991) argue in their survey that to enhance our trust in unit root test results, we must check using data with different sampling frequency but identical sample size

Similar patterns are found for monetary base, *B* and *M1*, albeit the evidence is more conclusive regarding its non-stationarity. As would be intuitively expected inflation and interest rate series (how often interest rate and inflation move beyond the range of 2-10%, barring the times of hyperinflation, which can be treated as a *jump shift* – a significant *jump up* for a finite time and a *jump down* back to normalcy) both point towards a stationary series. Again, we need to be cautious in our inferences. This suggests a further area of research for Indian data; checking if there has been a structural break around early '90s, c.f. Perron (1990).

Before moving ahead, note that results from ADF and PP here are provided only for comparison. As has been argued rather forcefully, they should not be used to test for unit roots. Better alternatives exist in their modified versions, which are less plagued by size and power problems. Regarding the specific tests, as seen from the tables, results from the three more powerful tests of ERS, PN and NP are more or less in agreement, with lag length selected using MIC with k_{max} selected as *T/4* finding stationarity' more often than Hall (1994).

Also, more often than not, KPSS and LM tests 'do not confirm' results, not that they are supposed to do so. Even LM is sensitive to the lag *p* in *ARIMA (p, 1, 1)* used to retrieve $\hat{\phi}_j$. As can be seen in the last row of all tables for various frequencies, *MA (1)* term is highly sensitive to changes in *p*, resulting in drastically different values of the test statistic. The use of both KPSS and LM is not recommended. Note that in practice a suitable *ARMA (p, 1)* formulation for the first difference can be identified and that can be used to retrieve $\hat{\phi}_j$, but as is well known, since there is no unique *ARMA (p, q)* for a time series, different *ARMA* specifications may lead to different conclusions. Monte Carlo exercise can be conducted to further investigate the sensitivity of the LM test to the value of *p* and ascertain the size and power dependence to the chosen *p*.

Table 2

y		*FREQUENCY: ANNUAL (T = 52)*							
TEST	t_{CR}	*HALL 1* †		*HALL 2*		*MIC* ††		*SACF*	
		t	*k*	*t*	*k*	*k1 = 11*	*k2 = 10*	*t*	*k*
ADF	-3.50	0.698	4	0.698	4	-1.14	-0.49	-1.85	0
Z	-3.50	*NA*		*NA*		-0.72	-0.70	-1.25	0
DF-GLS‡	-3.19	-1.66	2	-1.66	2	-1.34	-0.95	-2.99	1
MZ-OLSD	-3.50	-1.16	0	-1.16	0	0.47	-0.89	-1..16	0
MZ-GLSD	-3.19	-2.84	2	-2.84	2	**-4.07***	**-6.42***	**-3.25***	1
KPSS‡‡	0.145	*NA*		*NA*		0.15	0.16	1.11	0
MKPSS	0.145	l_{12} = 10; MA(1)** = -0.958; t = 0.9433				l_4 = 3; MA(1) = 0.841; **t =0.07***			
y		*FREQUENCY: MONTHLY (T = 204)*							
TEST	t_{CR}	*HALL 1*		*HALL 2*		*MIC*		*SACF*	
		t	*k*	*t*	*k*	*k1 = 1*	*k2 = 1*	*t*	*k*
ADF	-3.45	-2.49	24	-3.27	1	-3.27	-3.27	-3.04	23
Z	-3.45	*NA*		*NA*		**-5.62***	**-5.62***	**-6.82**	**23***
DF-GLS	-2.93	**-18.77***	**0***	**-18.77***	**0***	**-11.14***	**-11.14***	**-1.46**	**27***
MZ-OLSD	-3.45	**-21.02**	**24**	-3.28	1	**-4.17***	**-4.17***	**-17.34**	**23***
MZ-GLSD	-2.93	**-4.38**	**0**	**-4.38**	**0**	**-4.39***	**-4.39***	-1.32	27
KPSS	0.146	*NA*		*NA*		0.91	0.91	**0.129***	**23***
MKPSS	0.146	l_{12} = 14; MA(1) = -0.946; t = 1.9365				l_4 = 4 ; MA(1) =0.651; t =1.397			
y		*FREQUENCY: QUARTRELY (T = 76)*							
TEST	t_{CR}	*HALL 1*		*HALL 2*		*MIC*		*SACF*	
		t	*k*	*t*	*k*	*k1 = 7*	*k2 = 7*	*t*	*k*
ADF	-3.50	-2.08	4	-2.08	4	-3.22	-3.22	-2.08	4
Z	-3.50	*NA*		*NA*		-4.39	-4.39	-4.28	5
DF-GLS	-3.19	-2.76	4	-2.76	4	-2.02	-2.02	-2.76	4
MZ-OLSD	-3.50	-3.39	0	-3.39	0	**-9.21***	**-9.21***	-3.02	4
MZ-GLSD	-3.19	-2.74	4	-2.74	4	-2.51	-2.51	-2.74	4
KPSS	0.145	*NA*		*NA*		**0.114***	**0.114***	**0.136***	**5***
MKPSS	0.145	l_{12} = 11; MA(1) = -0.729; t = 0.4885				l_4 = 3; MA(1) = -0.818; t =0.5403			

* - Denotes rejection at 5% level; c.f. critical values t_{CR} in column 2; for KPSS and LM tests * denotes *acceptance*
** - Denotes the coefficient of the MA term in ARIMA (p, 1, 1) y_t. Note the sensitivity of the coefficient to *p*
‡ - Denotes that the tests are conducted taking both drift and the trend in the estimating equation (c = -13.5). For M1 and B also trend has been taken in the estimating equation. For inflation and interest rate data, however, only a drift term (c = -7) has been assumed.
‡‡ - Finite sample critical values for KPSS provided by Hornok and Larsson (2000) used for both KPSS & MKPSS
† - *Hall 1* uses k_{max} as *T/4* and *Hall 2* uses k_{max} as $l_{12} = int\{12(T/100)^{1/4}\}$
†† - *MIC k1* uses k_{max} as *T/4* and *MIC k2* uses k_{max} as $l_{12} = int\{12(T/100)^{1/4}\}$

Table 3 [Variable: M1]

m						*FREQUENCY: ANNUAL (T = 32)*			
TEST	t_{CR}	*HALL 1*		*HALL 2*		*MIC*		*SACF*	
		t	*k*	*t*	*k*	*k1 = 5*	*k2 = 5*	*t*	*k*
ADF	-3.60	-2.05	0	-2.05	0	-2.78	-2.78	-2.05	0
Z	-3.60	*NA*		*NA*		-2.96	-2.96	-3.05	0
DF-GLS	-3.19	**-4.56***	**0***	**-4.56***	**0***	-1.50	-1.50	**-4.56***	**0***
MZ-OLSD	-3.60	-2.42	0	-2.42	0	**-5.64***	**-5.64***	-2.42	0
MZ-GLSD	-3.19	-2.91	0	-2.91	-2.95	-2.95	-2.91	-2.91	0
KPSS	0.145	*NA*		*NA*		0.143	0.143	**0.572***	**0***
MKPSS	0.145	*l_{12} = 9; MA(1) = 0.09; t = 0.2824*				*l_4 = 3; MA(1) = 0.532;* ***t = 0.172****			
m						*FREQUENCY: MONTHLY (T = 228)*			
TEST	t_{CR}	*HALL 1*		*HALL 2*		*MIC*		*SACF*	
		t	*k*	*t*	*k*	*k1 = 10*	*k2 = 10*	*t*	*k*
ADF	-3.43	0.15	24	-0.01	14	0.31	-1.27	-0.26	18
Z	-3.43	*NA*		*NA*		-0.86	-1.14	-0.92	42
DF-GLS	-2.93	-0.62	56	-2.70	14	-0.69	-2.63	-1.99	17
MZ-OLSD	-3.43	-0.44	14	-0.44	14	0.39	-2.97	-0.62	18
MZ-GLSD	-2.93	-0.87	56	**-3.54***	**14***	-1.21	-2.54	-2.04	17
KPSS	0.146	*NA*		*NA*		0.125	**0.347***	**0.137***	**42***
MKPSS	0.146	*l_{12} = 14; MA(1) = -0.33; t = 3.5812*				*l_4 = 4 ; MA(1) = -0.37; t =3.383*			
m						*FREQUENCY: QUARTRELY (T = 76)*			
TEST	t_{CR}	*HALL 1*		*HALL 2*		*MIC*		*SACF*	
		t	*k*	*t*	*k*	*k1 = 19*	*k2 = 11*	*t*	*k*
ADF	-3.50	-0.52	0	-0.52	0	-0.36	-0.53	-0.97	2
Z	-3.50	*NA*		*NA*		-0.76	-0.86	-0.76	0
DF-GLS	-3.19	**-5.46***	**0***	**-5.46***	**0***	-0.80	-0.92	**-5.46***	**0***
MZ-OLSD	-3.50	-0.72	0	-0.72	0	0.36	**-12.19***	-1.74	2
MZ-GLSD	-3.19	**-3.54***	**0***	**-3.54***	**0***	**-3.51***	-1.76	**-3.43***	**1***
KPSS	0.145	*NA*		*NA*		**0.122***	**0.144***	1.04	0
MKPSS	0.145	*l_{12} = 11; MA(1) = 0.99;* **t = 0.0756***				*l_4 = 3; MA(1) = -0.83; t =1.043*			

* - Notes for the table same as Table 2

Table 4 [Variable: B]

b						*FREQUENCY: ANNUAL (T = 32)*			
TEST	t_{CR}	*HALL 1*		*HALL 2*		*MIC*		*SACF*	
		t	*k*	*t*	*k*	*k1 = 6*	*k2 = 6*	*t*	*k*
ADF	-3.60	-1.64	0	-1.64	0	-0.48	-0.48	-1.64	0
Z	-3.60	*NA*		*NA*		-2.75	-2.75	-2.44	0
DF-GLS	-3.19	**-3.86***	**0***	**-3.86***	**0***	-1.58	-1.58	**-3.86***	**0**
MZ-OLSD	-3.60	-1.36	0	-1.36	0	**-123.7***	**-123.7***	-1.36	0
MZ-GLSD	-3.19	-2.01	0	-2.01	0	**-3.23***	**-3.23***	-2.01	0
KPSS	0.145	*NA*		*NA*		**0.104***	**0.104***	0.259	0
MKPSS	0.145	*l_{12} = 9; MA(1) = 0.99;* **t = 0.0818***				*l_4 = 3; MA(1) = -0.0478; t =0.306*			
b						*FREQUENCY: MONTHLY (T = 228)*			
TEST	t_{CR}	*HALL 1*		*HALL 2*		*MIC*		*SACF*	
		t	*k*	*t*	*k*	*k1 = 53*	*k2 = 13*	*t*	*k*
ADF	-3.43	-0.52	0	-0.52	0	-0.36	-0.53	-0.61	8
Z	-3.43	*NA*		*NA*		-2.58	-1.89	-1.85	12
DF-GLS	-2.93	-0.47	27	-1.04	12	-0.52	-1.25	-1.48	8
MZ-OLSD	-3.43	-0.77	3	-0.77	3	-3.22	-2.18	-1.95	8
MZ-GLSD	-2.93	**-3.63***	**27***	-2.64	12	**-5.86***	-2.48	-2.41	7
KPSS	0.146	*NA*		*NA*		0.156	0.397	0.424	12
MKPSS	0.146	*l_{12} = 14; MA(1) = -0.27; t = 4.411*				*l_4 = 3; MA(1) = 0.95; t =0.1695*			
b						*FREQUENCY: QUARTERLY (T = 76)*			
TEST	t_{CR}	*HALL 1*		*HALL 2*		*MIC*		*SACF*	
		t	*k*	*t*	*k*	*k1 = 17*	*k2 = 11*	*t*	*k*
ADF	-3.50	-1.09	0	-1.09	0	0.14	-1.09	-1.09	0
Z	-3.50	*NA*		*NA*		-1.01	-1.10	-1.57	0
DF-GLS	-3.19	-1.04	4	-1.04	4	-0.48	-0.76	-3.57	1
MZ-OLSD	-3.50	-1.52	0	-1.52	0	**-7.66***	**-4.59***	-1.52	0
MZ-GLSD	-3.19	**-4.29***	**4***	**-4.29***	**4***	**-7.38***	**-4.23***	**-4.48**	1
KPSS	0.145	*NA*		*NA*		0.154	0.187	1.51	0
MKPSS	0.145	*l_{12} = 11; MA(1) = -0.5; t = 1.2671*				*l_4 = 3; MA(1) = -0.11; t =1.48*			

* - Notes for the table same as Table 2

Table 5 [Variable: R]

R	FREQUENCY: MONTHLY (T =132)								
TEST	t_{CR}	HALL 1 *t*	*k*	HALL 2 *t*	*k*	MIC *k1 = 3*	*k2 = 3*	SACF *t*	*k*
ADF	-2.93	-3.48*	3*	-3.48*	3*	-3.48*	3*	-3.48*	3*
Z	-2.93	NA		NA		-781.1*	-781.1*	-791.5*	1*
DF-GLS	-1.95	-14.83*	0*	-14.83*	0*	-6.34*	-6.34*	-9.16*	2*
MZ-OLSD	-2.93	-430.5*	3*	-430.5*	3*	-430.5*	-430.5*	-430.5*	3*
MZ-GLSD	-1.95	-337.5*	0*	-337.5*	0*	-305.2*	-305.2*	-307.4*	1*
KPSS	0.463	NA		NA		0.615	0.615	0.926	1*
MKPSS	0.463	l_{12} = 12; MA(1) = 0.99; t = 0.032*				l_4 = 4; MA(1) = 0.99; t =0.009*			

R	FREQUENCY: QUARTRELY (T = 44)								
TEST	t_{CR}	HALL 1 *t*	*k*	HALL 2 *t*	*k*	MIC *k1 = 2*	*k2 = 2*	SACF *t*	*k*
ADF	-3.00	-4.93*	0*	-4.93*	0*	-2.73	-2.73	-4.93*	0*
Z	-3.00	NA		NA		-216.5*	-216.5*	-217.1*	0*
DF-GLS	-1.95	-8.00*	0*	-8.00*	0*	-3.15*	-3.15*	-8.00*	0*
MZ-OLSD	-3.00	-153.1*	0*	-153.1*	0*	-152.2*	-152.2*	-153.1*	0*
MZ-GLSD	-1.95	-118.9*	0*	-118.9*	0*	--119.5*	-119.5*	-118.9*	0*
KPSS	0..396	NA		NA		0.37*	0.37*	0.65	0
MKPSS	0.396	l_{12} = 9; MA(1) = 0.99; t = 0.113*				l_4 = 3; MA(1) = 0.253; t =0.403			

* - Notes for the table same as in Table 2

Table 6 [Variable: r]

r	FREQUENCY: MONTHLY (T = 132)								
TEST	t_{CR}	HALL 1 *t*	*k*	HALL 2 *t*	*k*	MIC *k1 = 2*	*k2 = 2*	SACF *t*	*k*
ADF	-2.93	-4.52*	1*	-4.52*	1*	-3.43*	-3.43*	-5.19*	0*
Z	-2.93	NA		NA		-646.9*	-646.9*	-657.3*	1*
DF-GLS	-1.95	-15.19*	0*	-15.19*	0*	-7.82*	-7.82*	-11.34*	1*
MZ-OLSD	-2.93	-419.2*	1*	-419.2*	1*	-386.9*	-386.9*	-462.5*	0*
MZ-GLSD	-1.95	-328.9*	0*	-328.9*	0*	-311.9*	-311.6*	-306.6*	1*
KPSS	0.463	NA		NA		0.555*	0.555*	0.745*	0*
MKPSS	0.463	l_{12} = 12; MA(1) = -0.32; t = 0.799				l_4 = 4; MA(1) = 0.99; t =0.043*			

r	FREQUENCY: QUARTRELY (T = 44)								
TEST	t_{CR}	HALL 1 *T*	*k*	HALL 2 *t*	*k*	MIC *k1 = 1*	*k2 = 1*	SACF *t*	*k*
ADF	-3.00	-3.81*	0*	-3.81*	0*	-2.87	-2.87	-3..81*	0
Z	-3.00	NA		NA		-169.2	-169.2*	-167.7*	0
DF-GLS	-1.95	-7.54*	0*	-7.54*	0*	-4.59	-4.59*	-7.54*	0
MZ-OLSD	-3.00	-114.5*	0*	-114.5*	0*	-116.2	-116.2*	-114.5*	0
MZ-GLSD	-1.95	-101.9*	0*	-101.9*	0*	-102.1	-102.1*	-101.9*	0
KPSS	0.396	NA		NA		0.366*	0.366*	0.569	0
MKPSS	0.396	l_{12} = 9; MA(1) = 0.99; t = 0.274*				l_4 = 3; MA(1) = -0.273; t =0.411			

* - Notes for the table same as in Table 2

Table 7 [Variable: π_w]

π_w	FREQUENCY: MONTHLY (T = 228)								
TEST	t_{CR}	HALL 1 *t*	*k*	HALL 2 *t*	*k*	MIC *k1 = 8*	*k2 = 8*	SACF *t*	*k*
ADF	-2.89	-1.67	31	-12.25*	0*	-3.65*	-3.65*	-1.67	4
Z	-2.89	NA		NA		-2869.7*	-2869.7*	-2787.6*	1*
DF-GLS	-1.95	-24.29*	0*	-24.29*	0*	-7.48*	-7.48*	-10.94*	4*
MZ-OLSD	-2.89	-271.0*	31*	-931.5*	0*	-606.9*	-606.9*	-866.9*	4*
MZ-GLSD	-1.95	-620.1*	0*	-620.1*	0*	-477.1*	-477.1*	-1113.4*	4*
KPSS	0.463	NA		NA		0.329*	0.329*	0.494	1
MKPSS	0.463	l_{12} = 14; MA(1) =-0.90; t = 2.855				l_4 = 4; MA(1) = 0.99; t =0.3283			

π_w	FREQUENCY: QUARTERLY (T = 76)								
TEST	t_{CR}	HALL 1 *t*	*k*	HALL 2 *t*	*k*	MIC *k1 = 5*	*k2 = 5*	SACF *t*	*k*
ADF	-2.93	-6.24*	0*	-6.24*	0*	-2.91	-2.91	-6.24*	0*
Z	-2.93	NA		NA		-478.5*	-478.5*	-473.6*	0*
DF-GLS	-1.95	-12.95*	0*	-12.95*	0*	-4.89*	-4.89*	-9.31*	1*
MZ-OLSD	-2.93	-174.1*	0*	-174.1*	0*	-187.5*	-187.2*	-174.1*	0*
MZ-GLSD	-1.95	-124.9*	0*	-124.9*	0*	-413.8*	-413.8*	-115.7*	1
KPSS	0.396	NA		NA		0.291*	0.291*	0.493	0
MKPSS	0.396	l_{12} = 11; MA(1) = 0.789; t = 1.301				l_4 = 3; MA(1) = 0.99; t = 0.0642			

* - Notes for the table same as in Table 2

Table 8 [Variable: π_c]

π_c	FREQUENCY: MONTHLY (T = 228)								
TEST	t_{CR}	HALL 1 *t*	*k*	HALL 2 *t*	*k*	MIC *k1 = 6*	*k2 = 6*	SACF *t*	*k*
ADF	-2.89	-2.16	6	-2.16	6	-2.16	-2.16	-3.14*	4*
Z	-2.89	NA		NA		-2641.9*	-2641.9*	-2833.8*	9*
DF-GLS	-1.95	-2.71*	28*	-26.68*	0*	-8.31*	-8.31*	-11.34*	4*
MZ-OLSD	-2.89	-378.2*	6*	-378.2*	6*	-378.2*	-378.2*	-535.2*	4*
MZ-GLSD	-1.95	267.6*	28*	-517.3*	0*	-9056.6*	-9056.6*	-1152.1*	4*
KPSS	0.463	NA		NA		1.358	1.358	1.038	9
MKPSS	0.463	l_{12} = 14; MA(1) = 0.955; t = 1.956;				l_4 = 4; MA(1) = 0.673; t =6.58			

π_c	FREQUENCY: QUARTERLY (T = 76)								
TEST	t_{CR}	HALL 1 *t*	*k*	HALL 2 *t*	*k*	MIC *k1 = 1*	*k2 = 1*	SACF *t*	*k*
ADF	-2.93	-2..31	1	-2.31	1	-2.31	-2.31	-2.31	1
Z	-2.93	NA		NA		-294.6*	-294.6*	-294.6*	1*
DF-GLS	-1.95	-1.78	12	-14.74*	0*	-7.84*	-7.84*	-7.84*	1*
MZ-OLSD	-2.93	-135.9*	1*	-135.9*	1*	-135.9*	-135.9*	-135.9*	1*
MZ-GLSD	-1.95	-49.33*	12*	-113.0*	0*	-111.9*	-111.9*	-111.9*	1*
KPSS	0.396	NA		NA		1.539	1.539	1.539	1
MKPSS	0.396	l_{12} = 11; MA(1) = 0.17; t = 4.364				l_4 = 3; MA(1) = 0.904; t =1.329			

* - Notes for the table same as in Table 2

VI. Conclusion

As Hamilton (1994) noted, *"…although it might be very interesting to know whether a time series has a unit root…the question is inherently unanswerable on the basis of a finite sample of observations.*[12]*"*

The main problem is the near observational equivalence of a unit root and a covariance stationary process. Given a sample size, for any unit root process, there exists a covariance stationary process with identical features (see Hamilton (1994) for details) and vice versa. Notwithstanding this difficulty, the hypothesis that $\{y_t\}$ is an *AR (1)* process with a unit root is certainly testable. The new unit root tests explored in the study are better in doing just that. They have better size adjusted power to local alternatives in parsimonious autoregressive models.

But as seen from survey of the literature and the results in this study, it is hard to conclude which test is the best. Monte Carlo studies in ERS (1996), PN (1996) and NP (2001) show that their traditional counterparts must now be discarded. Results in this study on Indian data show that although conclusions from three tests are roughly the same, they too are sensitive to the lag length selected. The sensitivity stems from the requirement to select/specify the 'maximum' lag length in all of these tests.

Also, of no less importance is the problem of sampling frequency of the data. Note that if abscissa in **Figures 1 to 4** were hidden, it would be virtually impossible to identify between monthly, quarterly and annual data, especially if sample size for each sampling frequency were the same. But that said, the plots do indicate that income and money supply are more likely candidates to follow a random walk than either inflation or the interest rate. The ideal thing when checking for unit roots would be to use data sampled at different frequency covering different time periods, but having the same sample size. Since that luxury is not available with most Indian macroeconomic data, following Campbell and Perron (1991), results from tests that use data sampled at a lower frequency, annually or quarterly, should be considered more reliable than from data sampled at a higher frequency, say monthly or weekly. An area of future work on Indian data spanning pre-liberalization period would be to test for unit root tests accounting for structural breaks, c.f. Perron (1990).

As Dufour and King (1991) showed, a uniformly powerful test does not exist for identifying the unit root. Point-optimal tests of ERS, PN and NP offer a second-best solution; their tests have power close to point optimal tests with power 50%. Thus, when checking for unit roots these tests should be used with data sampled at as many different sampling frequency as feasible. After all what we have is just a snapshot of the temporal evolution of data, and to draw conclusions about it following a random walk with/without drift/trend one must look at the 'walk' not only as closely as possible but also for as long as possible.

Eighteenth century British writer, Samuel Johnson said, *prudence keeps life safe, but does not often make it happy.* When testing for unit roots, prudence is certainly a more desirable virtue. Who said conservatism is dead?

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[12] J. D. Hamilton (1994) *Time Series Analysis,* Princeton University Press, Princeton, New Jersey, pp. 444-445.

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Appendix 1

Note on creating WPI Series at 1993-94 prices

The data on WPI in India is compiled and published on a weekly basis by the Office of Economic Advisor (OEA), Ministry of Commerce and Industry. In the year 2000, in keeping with the changes in the structure of the economy, the base year was shifted from 1981-82 to 1993-94. However, a long back series of the WPI at the new base year is still not available from any of the official sources (publications of the Center for Monitoring Indian Economy (CMIE), the Central Statistical Organisation (CSO), OEA and the Reserve Bank of India (RBI)). Farthest back the monthly disaggregate series is available from any official source is starting April 1990 from the Business Beacon Electronic Database of the CMIE. The series with the old base is available till the end of 1996 from the Monthly Abstract of Statistics, published by the CSO. This enables splicing of the index to arrive at a series with a common base. As far as level of disaggregation goes, we are constrained by the level till which the data is available at 1993-94 prices, which is 'Level 1' comprising 38 sub-components *(see Table A)*.

Splicing

Splicing Factor (S_{FM}) can be defined as the ratio $\frac{I_{NEW-M}}{I_{OLD-M}}$ where I_{NEW-M} is the index value at the new base for the common month M for which the data is available at the old base. Clearly, the selection of the month would have a bearing on the 'accuracy' of the splicing factor. To be as 'correct' in our splicing method, we use a different splicing factor for each month, and use the average of the splicing factors for the years 1990, 1991 and 1992 to arrive at the final splicing factor, i.e. our Splicing Factor for month M is derived as

$$S_{FM} = \frac{1}{3}\left[\frac{I_{NEW-M-1991}}{I_{OLD-M-1991}} + \frac{I_{NEW-M-1992}}{I_{OLD-M-1992}} + \frac{I_{NEW-M-1993}}{I_{OLD-M-1993}}\right]$$

Further, an issue often ignored is the treatment of the monthly index values *for the base year*. If we leave the value of the index numbers to 100 for all months for the base year, it creates a kind of 'degeneracy' in the sample, as in it creates distortion in the month-to-month changes in the price index for that year. We use the information available from the 1981-82 base to get around that problem. We 're-base' the monthly values to ensure consistency with the month-to-month inflation from the old-base data, while leaving the 'year' base value at 100. The sample period is April 1982 - April 2003 (253 observations) and the weights in our study correspond to the base year of 1993-94.

Table A

Sr. No.	Category
1	Potatoes
2	Other Vegetables
3	Other Fibres
4	Tea and Coffee
5	Cotton
6	Metallic Minerals
7	Raw Cotton
8	Fruits
9	Other Oil Minerals
10	Other Cereals
11	Kerosene
12	Groundnut
13	Other Sugar Items
14	Other Minerals
15	Wheat
16	Sugar Group
17	Spices
18	Pulses
19	Wood
20	Edible Oils
21	Eggs
22	Leather
23	Sugarcane
24	Coal
25	Textile
26	Other Mineral Oils
27	Rice
28	Milk
29	Non Metallic Minerals
30	Other Food Group Items
31	Electrical Machinery
32	Rubber
33	Machines
34	Paper and Pulp
35	Beverages
36	Transport
37	Chemicals
38	Basic Metals