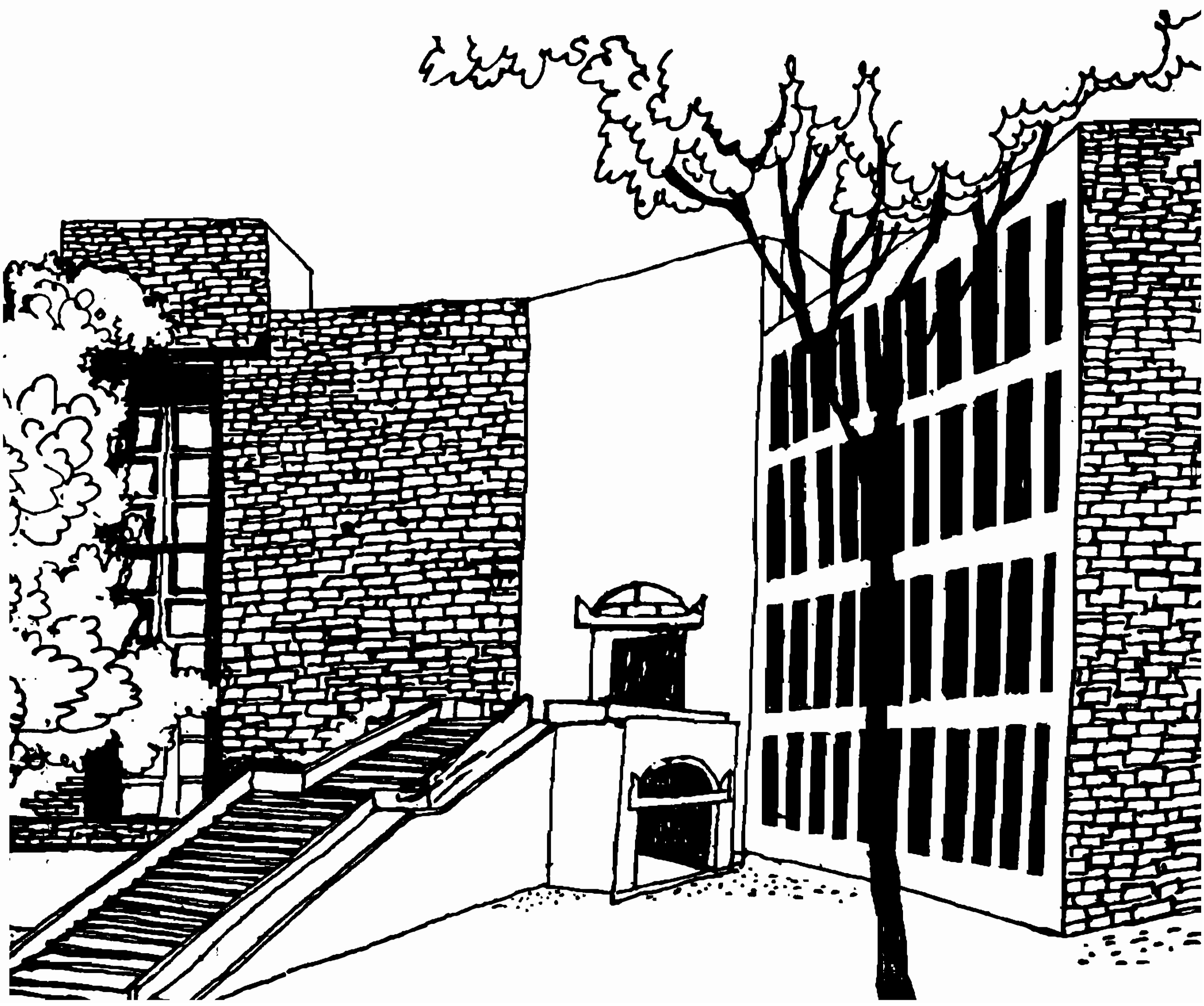
विद्याविनियोगाद्विकासः

IIM

AHMEDABAD

Working Paper



CONGESTION AND COMPLEXITY COSTS IN A PLANT WITH FIXED RESOURCES THAT STRIVES TO MAKE SCHEDULE

By

William S. Lovejoy
Kannan Sethuraman

W.P.No.99-05-03 /1518
May 1999

The main objective of the working paper series of the IIMA is to help faculty members to test out their research findings at the pre-publication stage.

INDIAN INSTITUTE OF MANAGEMENT
AHMEDABAD - 380 015
INDIA

Congestion and Complexity Costs in a Plant with Fixed Resources that Strives to Make Schedule

William S. Lovejoy
Michigan Business School
University of Michigan
Ann Arbor, MI 48109-1234.

Kannan Sethuraman
Indian Institute of Management
Ahmedabad
India.

Revised October, 1998.

SUBMITTED FOR PUBLICATION

Abstract

Current thinking traces complexity costs to the imposition of time on resources, which essentially reduces complexity issues to the more standard congestion issues familiar from queuing logic. This logic is correct in many instances, but cannot explain the phenomenon of making schedule. In a firm that makes schedule, orders are always processed within a fixed time frame. Such firms must relieve congestion by rushing jobs, with potential quality effects. Hence, time and quality are substitutes, a feature that we recognize explicitly in our definition of the firm's capacity. In our model, production yields are not exogenous parameters, but endogenously determined by workers responding to schedule pressures. The model reveals the close relationships among the firm's workforce policies, the integrity of the inspection systems, and the cost performance of the firm as its volume and/or product line expands. Whereas the time effects of complexity will only have cost effects in congested facilities, quality effects are always present. Also in contrast to time effects, quality effects of complexity can be present in non-bottleneck workstations. Hence, the quality consequences of complexity can be as or more important than the time consequences.

A large durables company serving predominantly industrial clients asked us to estimate the potential savings they could enjoy from redesigning their product line using a modular architecture. Currently, their client base is dominated by Fortune 500 firms, for which they provide a wide range of products that would deliver benefits approximating custom work. This resulted in great parts proliferation. Market growth is currently in the smaller, more price-sensitive business segment. The firm felt that because of its more extensive product line, it was at a cost disadvantage relative to competitors in this segment. They wanted to know if they could provide a similar range of benefits to customers, but reduce their manufacturing costs, by redesigning product lines to work off common modules. The redesign effort would be an expensive proposition. To make this business decision rationally, the firm needed to estimate the savings to be gained from their modular proposal, which would essentially decrease the number of different products that each manufacturing center made. Our task was to provide input into a strategic product and process design decision by suggesting what cost savings might arise from moving to modular products. To do this, we had to investigate the cost of product variety (complexity) for this firm. Our report includes insights that neither we, nor the firm's managers expected.

The current literature on the costs of complexity is dominated by time-based logic. In essence, a heterogeneous product line requires the processing of an increased number of transactions, which require time on resources (cf. Miller and Vollman 1985). Likewise, change-over losses between products impose non-productive time on the system. As product variety increases, the plant is handling higher variety with less effective capacity, and queuing delays (shipping delays or inventory costs) accrue. Indeed, to date the only analytical model of complexity that the authors are aware of (Banker et al 1988) uses an M/G/1 queuing model of a bottleneck work station to show how higher variety imposes higher delays and inventory costs. If nonproductive time imposed on resources is the significant consequence of high product variety, then complexity phenomena are conceptually related to the congestion phenomena familiar to us from queueing theory. We began our investigation in our client firm by looking for the imposition of nonproductive time due to product variety, and the presence of delays.

Discussions with principals in the firm revealed some interesting features that could not be explained by this time-based logic, or existing models. This firm has a long history of paternalistic human resource practices; they do not lay people off. Also, their capital stock is relatively fixed in the intermediate term. This is a firm with fixed human and capital resources, and hence their capacity is relatively fixed. Also, plant managers are expected to "make schedule," meaning that all job orders released to the shop are to be completed and ready to ship in a constant amount of time (3 days in most plants, 4 days in some). The precise statement from management would be something like, "97% of all orders through a plant must be complete in 3 days." This mandate is met. Finally, the firm operates in a volatile demand environment. How can a plant with fixed capacity, operating in a variable environment, get products through the plant in a fixed amount of time with high reliability? Queuing logic would suggest that this is impossible, unless the plant enjoys a lot of excess capacity, on average. But, if the plant enjoys a lot of excess capacity, then there would be no time effect of complexity (extra time demanded from non-scarce resources is not costly). Does this plant suffer complexity costs? If so, by what mechanism? In the process of answering these questions, we discovered some interesting relationships among complexity phenomena, congestion, the quality system, labor policy and plant manager incentives.

The next section reviews some of the literature relevant to issues of congestion and complexity. Section II describes our model. Section III considers how scheduled overtime will impact the total costs of the plant, and the incentives facing a plant manager. Section IV presents some cost signatures that calibrate one's intuition, and section V presents our conclusions and recommendations to our client firm.

I. Existing literature

There is an extensive queuing theory literature that relates facility utilizations and the variability in demand rates and processing times to inventory and/or delay consequences. See Heyman and Sobel (1982) for an entree into this body of work. Fundamental in this literature is the fact that idleness is imposed on a facility by asyncronous variability between the arrival of jobs and the facility's processing capabilities. This idleness prevents a perfect match between the mean arrival rate of jobs and the facility's mean capacity to process jobs. The ratio of these two is the utilization of the system, and inventory and delays explode as the utilization approaches unity in a variable environment. This is the classical logic of congestion. One consequence of this logic is that anything that decreases the effective capacity of the system (such as the imposition of nonproductive time) will increase inventory and/or delays in the presence of variability.

Less extensive is the literature on the "cost of complexity," which (in the management literature) refers to the potential diseconomies that attend the performance of a heterogeneous, rather than homogeneous set of tasks. This is sometimes referred to as diseconomies of scope. Financial evidence of these diseconomies include costs rising more than expected as the firm adds products or services to its repertoire. Indeed, it is this type of financial indicator that attracts managerial attention to this topic.

As noted above, Banker et al (1988) use an M/G/1 queueing model of the firm to show that increased product variety increases queueing delays and holding costs, explicitly linking the complexity and congestion literatures. Queueing intuition also underlies our notion of the time-related consequences of complexity, but when the firm makes schedule the cost impact will not be in inventory or delays, as we shall see.

Economists have studied multi-product firms from a theoretical (cf. Panzar and Willig 1981, Teece 1982) and empirical (Caves et al 1979, Pulley and Braunstein 1992) perspective. In this literature, firms add products to better utilize indivisible shared resources. With such resources, a firm may choose to hold excess capacity until it develops new products, so that the conventional model of using an extra hour of labor costing the company the ambient wage rate is not necessarily operative. Utilizing slack resources does not cost the company anything. Hence, there is an important relationship between congestion (utilization) and the opportunity cost for time on resources.

In the management literature, Miller and Vollman (1985) trace complexity costs to fundamental transactions (activities) that must be attended to for the daily operation of a firm. Their notion of breaking down the processing tasks into fundamental activities underlies our view of the time costs of complexity. They trace transaction overload to quality problems, which is explicit in our model. We also allow variety to impact quality directly. The Miller and Vollman article initiated a series of empirical investigations into the existence of complexity costs (cf. Foster and Gupta 1990, Banker et al 1990, Banker et al 1992, Anderson 1995, MacDuffie, Sethuraman and Fisher 1994 and Sethuraman 1994). The conclusions regarding the presence and magnitude of complexity costs are mixed in these papers. This is not surprising when one considers that these costs are contingent upon several root causes and potentially confounding circumstances, all subject to measurement error.

There is also an extensive literature on flexibility and focus (cf. Skinner 1974, Meade 1974, Abernathy and Wayne 1974, Sethi and Sethi 1990) as generic strategies. The fundamental issue is trading off the market gains from variety, and the costs due a system's potential loss of economic effectiveness. Accurate cost estimates are necessary inputs to this strategic decision.

II. The model

Our model is a mathematical reflection of the situation, as we see it, in our client firm. We discuss the model in broad terms here, and then in more detail in following subsections.

We model a plant as a single production station with inspection and rework. Since most production systems are networks of work stations, the model here is most appropriately applied to bottleneck work stations (that is, the single work station that most impacts total system performance). Such bottlenecks can arise in any direct or indirect activity required to complete production.

We presume that the firm is operating with fixed human and machine resources. This is an appropriate model for our client firm in the intermediate term (in the long term everything can vary). Even when resource levels are varied, it is usually because managers believe that performance with current levels is not sufficient. Hence, an analysis with fixed resource levels precedes the decision to adjust those levels. Our model attends to this logical precedent.

In our client firm, plant managers are appraised on meeting schedule. If the backlog of work endangers schedule completion. workers will, in their own words, "work like crazy" to get the products out on time. We call this behavior "rushing," and model it as an implicit worker response to having too much to do in too little time. Managers who do not schedule overtime or allow delayed shipments will invite rushing when schedule completion is threatened. If the job is rushed, there can be task errors and/or omissions. Task errors or omissions will manifest themselves in higher probabilities of producing defectives, so that rushing will decrease the time invested in each unit but also increase the probability of producing defective products. Hence, production yields (% nondefectives produced) in our model are not exogenous parameters but are endogenously determined by workers responding to schedule completion pressures.

We explicitly recognize that time and quality can be traded off in job performance, and we represent the system "capability" as a time-quality pair. Different throughput rates are possible, with different cost and quality levels. All output is filtered through an inspection process, the performance of which is represented by traditional type I and type II error probabilities. The quality of the output and the integrity of the inspection process will combine to release some product for downstream consumption, and scrap or rework others.

As we add products to this facility we may at first enjoy economies as the resources in place are more fully utilized, generating increased revenues for only modest increases in cost. But, as the time demands on the resources increase, we will reach a point where schedule completion is threatened. The plant manager can, at his or her discretion, schedule overtime up to some limit, and then workers will rush if schedule is still threatened. Rushing has its own consequences for scrap and rework. We will describe these material flows explicitly before turning to their attendant cost signatures.

A. The material flows

We introduce the general model including terms for the time and quality consequences of complexity (γ and h below). By setting these to constants we can eliminate effects due to the number of products produced and focus on pure congestion phenomena. By allowing these terms to vary with the breadth of the product line, we can investigate the additional costs imposed by complexity phenomena. This will become clearer below.

Let n denote the number of products being made in the facility, with an average demand rate for product i of λ_i units/day. The total demand rate over all products is therefore $\lambda := \sum_{i=1}^{n} \lambda_i$, and the long-run proportion of total demand that is for product i is λ_i/λ.

The system capability for product i is represented by a time-quality pair, (t_i^0, p_i^0), representing the nominal time devoted to processing each unit and the anticipated yield (fraction of output of acceptable quality) from that time investment. We assume these quantities are strictly positive. These nominal values can be adjusted in several ways.

The actual mean time it takes to process a unit of product i, t_i, may differ from t_i^0 due to a variety of causes. First, the plant manager can schedule an overtime level, OT (days), effectively increasing the length of a working day. Then, the mean number of working days that each unit of product i requires is decreased by a factor of $1/(1 + OT)$. It is implicit in this formulation that labor is the most constraining resource in production.

The addition of extra products can impose coordination system delays, for example on scheduling systems not designed for high product variety. These include additional machine set ups, information system delays, and other impositions of nonproductive time. To model this we presume that the time required per unit of product i is multiplied by a "nonproductive time" function, $\gamma_i(n)$, of the number of products n. We assume that $\gamma_i > 0$ for all i, and focus intuitively on the case where $\gamma_i(n)$ is greater than or equal to one, and nondecreasing in n. This is most appropriate for build-to-order type environments. This is appropriate in our client firm since it essentially builds to order from the bottleneck work station.

Investing time t_i in production achieves a yield rate, p_i, which may also differ from p_i^0 for several reasons. Research in human factors and ergonomics (Sanders and McCormick 1993, Meister 1976, Cohen 1980) suggests that as the number of stimuli that need to be processed by an individual increases, performance decreases. However, there are a variety of contextual circumstances (e.g. how similar different stimuli are, the presentation rate, frequency of change, etc.) that modify the relationship between number of stimuli and performance. Based on this body of research, we assume that the process yield deteriorates as the number of products that need to be processed by the system increases. Specifically, we use $h_i(n) \leq 1$ to represent the factor by which yield for product i is impacted due to the stimuli load associated with processing n different products. We focus on the case where $h_i(n)$ is decreasing in n. Based on our field application, we intuitively associate $h_i(n)$ with human error, but it can represent other system quality effects, such as information system errors driven by variety.

Workers may need to rush to make schedule, effectively decreasing the time they invest in each unit of product. Let $\theta \in (0, 1]$ denote the factor by which processing time per unit is decreased by rushing. Rushing, however, also decreases the quality of the output. We assume that, as a result of rushing, yields are decreased by a factor θ^δ for some $\delta > 0$. We will assume that these effects are the same for all products. δ less than (greater than) unity implies that yields decrease less than (more than) proportionally to the time saved by rushing.

In summary, the mean time invested to process a unit of product i is $t_i = \frac{t_i^0 \gamma_i(n)\theta}{1+OT}$ working days and the yield achieved by this time investment is $p_i = p_i^0 h_i(n)\theta^\delta$.

After processing, each unit goes into an inspection process. The accuracy of this process is represented by parameters α_i and β_i. α_i is the probability that a quality item of type i is judged to be defective, and β_i is the probability that a defective item of type i is judged to be without defect. In the language of statistical process control, α_i and β_i are the probabilities of type I and type II errors, respectively, by the inspection process for units of product i. We assume that $\alpha_i + \beta_i \leq 1$ for all products i, and we disallow the case with $\alpha = 1$ and $\beta = 0$. This would be the situation in which we scrap or rework everything. In that case, the system would be unstable as jobs arrive but never leave.

A fraction s_i of the units of product i that are judged to be defective by the inspection system is scrapped. The remaining fraction, $(1 - s_i)$, of the output that is judged defective, is reworked. Items that are reworked re-enter the production process. If an item is scrapped, then another job must enter the production process to replace it. From the perspective of time demanded in the production system, there is no difference between replacement (due to being scrapped) and rework jobs. In either case, the system must initiate another processing iteration on the job. The most significant difference between scrap and rework is in the material costs incurred (see below).

Let π_i be the probability that an item of type i, emerging from the production system, is released for consumption, and $1 - \pi_i$ the probability that it is scrapped or reworked. It is clear from the above that

$$\pi_i = p_i(1 - \alpha_i) + (1 - p_i)\beta_i = \beta_i + p_i(1 - \alpha_i - \beta_i).$$

See Figure 1 for a graphical representation of the material flow rates.

B. Cash flows

The material flows just described have attendant cash flows. We assume that the accounting system of the firm collects costs into cost categories, and that these cost figures comprise the bulk of the information that the plant manager has to assess plant performance. The cost categories are labor. material, indirect variable costs, and warranty costs. Revenues will be tracked but the plant manager's actions do not impact that figure (we use warranty costs to capture any revenue loss due to poor quality). Inventory levels (and holding costs) can be tracked with more specific assumptions regarding the statistics of the demand and service time processes, but this will not play a role here (see below). Our cost taxonomy is representative. but others could be assumed without compromising the method of analysis or the resulting qualitative insights.

In the following we provide the long-run average cash flow rates (\$/day) experienced by the firm for a fixed product portfolio. These results can then be used to track how the cost signatures will vary as we broaden the product line. The cash flow expressions are expressed graphically in figure 2, and are described in this section.

Labor costs: With our fixed resource assumption, the labor force is fixed. Let W_F denote the full-time labor wage bill per working day. Direct labor will be charged when this force is productively busy. otherwise the wages are charged to an idle labor account. We assume that direct labor performs setups, etc. so that any time invested in production activities by direct labor personnel is charged to the direct labor account. Different cost taxonomies could be used without qualitatively changing our results. When there is no work to do, the labor costs are charged to an idle labor account. If ρ denotes the system utilization, the direct and idle labor charges (dollars per working day) are $\rho W_F(1 + OT)$ and $(1 - \rho)W_F(1 + OT)$, respectively. Overtime pay increases the base wage by a factor of ξ. The overtime premiums, paid in excess of the base wage, are tracked in a third account at a rate of $OT \times W_F \times \xi$ dollars per working day. Note that the total labor bill (direct plus idle plus overtime) will be $W_F(1 + OT(1 + \xi))$. Without overtime, the total labor bill is constant, and would remain so as we add products (however, costs would move from the idle labor to the direct labor account). The only labor cost consequence of adding products will be the potential use of overtime as increased time demands threaten schedule completion. This is a direct consequence of operating with fixed resources.

Direct material and indirect variable costs: A fraction, s_i, of items of product i that do not pass inspection are scrapped. These require more material to begin production anew. The remaining fraction $(1 - s_i)$ of items that do not pass inspection are reworked, incurring labor time and indirect variable costs but no direct material costs. Recall that $(1 - \pi_i)$ is the probability that a unit of product i finishing production will not pass inspection, and that the total flow rate of product i through the production station (including original and rework flows) will be $\frac{\lambda_i}{\pi_i}$. Hence, considering the original product flows plus rework flows, the total cost rate for material for product i will be $\lambda_i m_i(1 + s_i(\frac{1}{\pi_i} - 1))$. We assume an indirect variable cost I_i per unit for product i that is incurred every time a unit (original or rework) passes through the production process. This captures any variable costs (e.g. supplies) not included in direct material. Hence, the indirect variable cost flow rate for product i will be $\lambda_i I_i / \pi_i$.

Revenues and warranty and goodwill costs: We assume revenues of r_i per unit of product i released for downstream consumption. For each defective item of type i that we release for consumption, we incur a goodwill and warranty cost of ω_i (which may include returned revenues). The flow rate of defective products of type i to customers will be $\frac{\lambda_i}{\pi_i}(1 - p_i)\beta_i$, and the warranty cost flow rate will be ω_i times this number.

Inventory holding costs: Inventory will not exhibit the explosion that usually attends congestion in conventional queueing models. This is because the plant always makes schedule, which bounds the delay experienced by any job in the system. Indeed, in our model average inventory levels will increase at most linearly with sales, and hence inventory cost/sales figures will remain relatively flat. This is in contrast to the model in Banker, et al, in which complexity costs are present only in inventory holding costs. The difference is the phenomenon of making schedule. When a plant always makes schedule, the costs that one intuitively associates with congestion phenomena will not be realized as inventory costs, but will diffuse into other cost categories (see below). Inventory costs will not be considered further.

III. Making schedule

A. Definition and feasibility

We assume that items failing inspection are reworked or restarted immediately. Hence, the number of iterations it takes to produce one good unit of type i is a geometrically distributed random variable. and the expected time between entering the system and being released for consumption will be t_i/π_i for products of type i. Since the long-run fraction of jobs that are type i is λ_i/λ, the long-run average service time per job (averaged over all jobs processed by the system) will be

$$\bar{T} = \sum_{i=1}^{n} \frac{\lambda_i}{\lambda} \frac{t_i}{\pi_i}.$$

With t expressed in "working days," the capacity of the system is $1/\bar{T}$ products/day, and the long-run utilization of the system is $\rho = \lambda\bar{T}$.

We model "making schedule" as follows. Define τ to be the desired maximal total delay between the arrival of a job and its release for consumption. Define $P_D(\tau)$ to be the probability that the delay is less than or equal to τ, and P^{min} to be a specified lower bound on this probability. We model the pressure to "make schedule" as the desire to keep $P_D(\tau) \geq P^{min}$. For example, if plant managers are expected to have 97% of scheduled production complete within 3 days of it being released to the shop floor, we would have $P^{min} = .97$ and $\tau = 3$ days.

Rather than relying on the statistics of specific demand streams and production processes, we will model the desire to make schedule in an indirect, but robust, manner. Specifically, we assume that for any portfolio of products with an aggregate arrival rate of λ units/day, we want to adjust the mean processing time per unit ($\bar{T}$) such that the utilization $\rho = \lambda\bar{T}$ is less than or equal to some specified target $\rho^*(\lambda)$. This should be a generically applicable construction, because all we require is that the probability that an arriving job will be cleared from the system in τ working days or less can be increased to some specified target level by decreasing the utilization. It would be difficult to generate a reasonable queueing model of a firm that does not have this feature. Appendix A demonstrates how one can estimate $\rho^*(\lambda)$ using an M/M/1 queueing approximation.

The plant manager can decrease $\bar{T}$ and ρ by increasing the amount of time available in a working day (that is, use overtime) or decreasing the time needed to process a unit (by forcing the workers to rush jobs). The possibility that rushing breeds poor quality and rework will render some schedules infeasible, a topic we address below. We will assume henceforth that the schedule of target utilizations, $\rho^*(\lambda)$, is known without specifying its form explicitly. Since $\rho = \lambda\bar{T}$, it follows from the above that for any specific product portfolio "making schedule" is equivalent to keeping $\bar{T} \leq T^*$ where $T^* = \rho^*(\lambda)/\lambda$ is a known quantity and $\bar{T}$ is adjusted by rushing and using overtime.

It is apparent that if unlimited overtime is available, then the plant can always make schedule. But, this is never the case. The following proposition articulates those cases in which the plant can "make schedule" for any fixed allocation of overtime.

Proposition 1

a) If $\delta < 1$ or $\beta_i > 0$ for all i, then for any $OT \geq 0$, the $lim_{\theta \to 0} \rho = 0$. That is, starting with any level of overtime the plant can always make schedule by rushing.

b) If $\delta \geq 1$ and $\beta_i = 0$ for some i, then it might not be possible to make schedule.

Proof: Substituting the detailed expressions for π_i and t_i into the expression for ρ yields

$$\rho = \sum_{i=1}^{n} \lambda_i \frac{\theta t_i^0 \gamma_i(n)}{1+OT} \frac{1}{\beta_i + p_i^0 h_i(n) \theta^\delta (1-\alpha_i-\beta_i)}.$$

Define ρ_i as follows:

$$\rho_i = \lambda_i \frac{\theta t_i^0 \gamma_i(n)}{1+OT} \frac{1}{\beta_i + p_i^0 h_i(n) \theta^\delta (1-\alpha_i-\beta_i)}$$

so that $\rho = \sum_{i=1}^{n} \rho_i$. Define $S_\beta = \{i|\beta_i > 0\}$, then

$$\rho = \sum_{i \in S_\beta} \rho_i + \sum_{i \in S_\beta^c} \rho_i$$

where S_β^c denotes the complement of the set S_β. Note that regardless of δ, $lim_{\theta \to 0} \rho_i = 0$ for $i \in S_\beta$. Now suppose that $\delta < 1$. Then for $i \in S_\beta$ the result holds as just shown. For $i \in S_\beta^c$ we have that ρ_i will be proportional to $\theta^{1-\delta}$ (recall that we disallow $\alpha_i + \beta_i = 1$ when $\beta_i = 0$ because that would characterize an unstable system). Hence, again the limit of ρ_i as we drive θ to zero is zero. This completes the proof of (a). For part (b), note that if $\delta \geq 1$ and $S_\beta^c \neq \emptyset$ then for $i \in S_\beta^c$ the limit of ρ_i as θ goes to zero is proportional to $\frac{1}{\theta^{\delta-1}}$. This term is either constant (if $\delta = 1$) or increasing as we decrease θ, so that we might not be able to drive the utilization down to the target ρ^*. In particular, if $\delta > 1$ and $\beta_i = 0$ for all i. then ρ is increasing as we decrease θ, so that if we cannot make schedule using overtime only, we cannot make schedule at all. **QED**.

Rushing saves us time, but decreases the quality of the output. If poor quality product is recycled ($\beta_i = 0$), we will not save time overall by rushing if $\delta \geq 1$. Hence, if we cannot make schedule with overtime alone, we may never be able to make schedule. If $\beta_i > 0$ for all i, then we release some bad product for consumption. The total production rate of poor quality product increases as we rush (decrease θ), and a constant fraction, β_i, of these leave the facility and are not reworked. Hence, we can make schedule by releasing an increasing quantity of poor quality goods from the facility. Goodwill and warranty costs will suffer, but we will not incur delays. Intuitively, if we observe a plant that always makes schedule as we add products, then we can reasonably suspect that either the plant enjoys a lot of excess capacity on average, uses a lot of overtime, or is allowing an increasing amount of poor quality items to reach market.

This emphasizes the point that in this model there is not one single capacity figure that limits the firm's output capabilities. Instead, there is a range of output rates available, each with different quality and cost consequences. The optimal cost level of any given level of output is achieved by rationally trading off labor costs (incurred by increasing the working time per day) and quality costs (both internal rework costs and external failure costs). How should a plant manager behave (normatively) in this environment? In the following we determine the optimal level of overtime required to meet schedule, given the anticipated reactions of workers. Recall that for a given λ and $T^* = T^*(\lambda)$, making schedule requires that $\rho = \sum_{i=1}^{n} \lambda_i \frac{t_i}{\pi_i} \leq \rho^*(\lambda) = T^*\lambda$, or equivalently

$$\bar{T}(OT,\theta) = \sum_{i=1}^{n} \frac{\lambda_i}{\lambda} \frac{\theta t_i^0 \gamma_i(n)}{1+OT} \frac{1}{\beta_i + p_i^0 h_i(n)\theta^\delta(1-\alpha_i-\beta_i)} \leq T^*.$$

We assume that both the plant manager and the workers desire to make schedule, but that the plant manager wishes to minimize total costs and the workers wish to minimize their effort (subject to making schedule). We model this by assuming that the plant manager chooses an overtime level OT, and then the workers respond by rushing just enough to make schedule. That is, the workers choose

$$\theta = f(OT) := max\{\theta \in (0,1] : \bar{T}(OT,\theta) \leq T^*\}.$$

Total costs as a function of overtime and rushing are therefore

$$TC(OT,\theta) = \sum_{i=1}^{n} [\frac{I_i}{\pi_i} + \frac{1-p_i}{\pi_i}\beta_i\omega_i + m_i(1+\frac{1-\pi_i}{\pi_i}s_i)]\lambda_i + W_F[1+OT(1+\xi)]. \quad (1)$$

The plant manager's problem is to choose an overtime level (between zero and the maximal feasible amount of overtime, OT_{max}) that minimizes TC, given worker reactions. That is, the plant manager wishes to solve the following problem, which we call problem PM:

$$\text{Minimize}_{OT\in[0,OT_{max}]} TC(OT, f(OT)).$$

Problem PM is not easy to analyze directly, but in the following we generate an equivalent form of the problem that yields simpler results. The next Lemma is a standard result in functional analysis (cf. Hoffman 1975).

Lemma 1: If $f : R \to R$ is continuous and strictly monotone on an interval $I \subset R$, then $f(I)$ is an interval, f^{-1} exists on $f(I)$ and the set $\{(x, f(x)) : x \in I\} = \{(f^{-1}(y), y) : y \in f(I)\}$.

Assume that either $\delta < 1$, or $\delta = 1$ and $\beta_i > 0$ for all i, then it follows from Proposition 1 that for any OT the workers have a feasible response, meaning that there is a $\theta \in (0, 1]$ such that schedule is made. Hence, $f(OT)$ is well defined. Under these same conditions, it can be verified that $\bar{T}(OT, \theta)$ is strictly monotone in $\theta > 0$ so that the workers' response to overtime level OT will be $f(OT) = 1$ if $\bar{T}(OT, \theta) < T^*$, and otherwise will be the unique θ such that $\bar{T}(OT, \theta) = T^*$. Since overtime is costly, the plant manager would never choose a level of overtime greater than any OT for which $\bar{T}(OT, 1) \leq T^*$. To do so would cost money, but invite no change in worker response, which would remain at $\theta = 1$. Hence, if $\bar{T}(0, 1) \leq T^*$, then $(OT, f(OT)) = (0, 1)$ is the optimal solution to PM. If $\bar{T}(0, 1) > T^*$, then define $\hat{OT}$ to be the unique $OT > 0$ such that $\bar{T}(OT, 1) = T^*$. Define $OT' = \text{minimum}\{\hat{OT}, OT_{max}\}$.

Proposition 2: If $\delta < 1$, or $\delta = 1$ and $\beta_i > 0$ for all i, then

a) If $\bar{T}(0, 1) \leq T^*$ then the plant manager's problem (PM) is solved by $(OT, \theta) = (0, 1)$.

b) If $\bar{T}(0. 1) > T^*$ then the plant manager's problem (PM) is equivalent to

$$\text{minimize}_{\theta \in J} TC(OT(\theta), \theta)$$

where

$$J = \text{the interval } f([0, OT']) \subset R^+$$

$$OT(\theta) = \sum_{i=1}^{n} \frac{\lambda_i}{\lambda} \frac{t_i^0 \gamma_i(n) \theta}{T^*} \frac{1}{\beta_i + p_i^0 h_i(n) \theta^\delta (1 - \alpha_i - \beta_i)} - 1.$$

Proof: Part (a) follows from the above comments. So, assume that $\bar{T}(0, 1) > T^*$. It also follows from the above comments that we can substitute $I := [0, OT']$ for $[0, OT_{max}]$ as the search interval in problem PM. Under the stated asssumptions $f(OT)$ is strictly increasing and continous on I. Since for any $OT \in I$ there exists a $\theta > 0$ such that schedule is made (Proposition 1) it follows that $f(0) > 0$ so $I \subset R^{+}$. The result would follow from Lemma 1 if $f^{-1}(\theta) = OT(\theta)$. But for $OT \in I$, $f(OT)$ is the unique θ such that $\bar{T}(OT, \theta) = T^{*}$. Rearranging this equation so that OT is the dependent variable yields the appropriate expression for f^{-1}. **QED**

We can write $TC(\theta) = TC(OT(\theta), \theta)$ by substituting the appropriate expression for $OT(\theta)$ into the total cost equation. For notational convenience define the following terms:

$$V_i := I_i + m_i s_i + \beta_i \omega_i.$$

$$Q_i := p_i^0 h_i(n) \beta_i \omega_i.$$

$$L_i := \frac{W_F(1+\xi) t_i^0 \gamma_i(n)}{\lambda T^{*}}.$$

$$R_i := p_i^o h_i(n)(1 - \alpha_i - \beta_i).$$

V_i, Q_i, L_i, and R_i are related, respectively, to variable costs, external quality costs, labor costs, and release probabilities for product i. Direct substitution generates

$$TC(\theta) = -W_F \xi + \sum_{i=1}^{n} \lambda_i m_i (1 - s_i) + \sum_{i=1}^{n} \lambda_i \frac{V_i - Q_i \theta^{\delta} + L_i \theta}{\beta_i + R_i \theta^{\delta}}.$$

Problem PM has an easily stated solution if $\delta = 1$, found by inspecting the derivative of $TC(\theta)$ with respect to θ.

Corollary 2.1: If $\delta = 1$ and $\beta_i L_i \geq R_i V_i + \beta_i Q_i$ for all i, then the plant manager will choose $\theta_{TC}^{*} = f(0)$, or equivalently $OT_{TC}^{*} = 0$. The reverse inequality will imply $\theta_{TC}^{*} = f(OT')$, or equivalently $OT_{TC}^{*} = OT'$.

Corollary 2.1 shows that when $\delta = 1$, intuitive notions of what should occur do occur. If variable costs and external failure costs are large relative to labor related costs, overtime is preferred to induced rushing. If the reverse is true, then overtime is avoided and it is economic to encourage workers to rush to make schedule. But, things are very interdependent, and in particular the decision of how much overtime to authorize is not as simple as, for example, looking at the % of the cost of sales that is attributable to labor vs (other) variable costs, and using that as a barometer. The integrity of the quality control system, and the fraction of material judged defective that is scrapped versus reworked all play a role. For $\delta = 1$ Corollary 2.1 gives us an exact answer, using a comparison of costs that include all relevant parameters. Numerical trials (also see section IV below) indicate that for reasonable ranges of parameters the comparison in Corollary 2.1 provides good guidance down to a δ of .7 or so, and that in most cases OT is preferred to rushing. By the time δ drops to .5 we begin to favor rushing as a means to make schedule. Obviously, if $\delta = 0$ we always rush, as there is no quality consequence for working faster. So, one critical feature in this decision is how seriously working faster impacts quality.

It is worth commenting on the situation where $\delta > 1$; when quality deterioration due to rushing is more than proportional to time savings. First, it may not be possible to make schedule if it cannot be made by overtime alone. So, problem PM may not be well defined. However, suppose that the plant can make schedule with some combination of overtime and rushing. $\bar{T}$ will be unimodal but not necessarily monotone in θ, so that a unique inverse will not exist. Problem PM is then well defined, but not as tractable analytically. Also, one is tempted to assume that if problem PM solves at $\theta = 1$ (no rushing) for $\delta = 1$, certainly it will not call for rushing at $\delta > 1$. That is, we would suspect that amplifying the quality deterioration that results from rushing can only make us more reluctant to rush. While this is true in most practical circumstances, there are extreme situations where this claim is false. Suppose that we cannot make schedule, even with maximal overtime, and that the amount of rushing needed to make schedule decreases the process yields to very low values. Since yields are bounded below by zero, increased rushing will only decrease yields modestly from its already low value. However, every unit of rushing saves overtime costs. Once this very bad situation is entered, increased rushing does not harm quality significantly, but does save labor costs, so that the firm will choose to replace all overtime with rushing. Examples of this behavior are presented in section IV. Such a perverse situation may not be likely in practice, but this example does illustrate the potentially counter-intuitive effects of δ.

B. The plant report and incentives

It is common in many companies (including our client) for warranty costs to be accumulated at the corporate level, and excluded from plant reports. In such companies, the plant manager has a local agenda that differs from the minimization of total costs. Define $C(\theta)$ to be $TC(\theta)$ with $\omega_i = 0$ for all products i. If warranty costs are not included on the plant report, then a plant manager will solve problem PM using C as his/her relevant cost function, and not TC. We relate these two problems, and provide some additional insights into optimal overtime allocations, in the following corollary. Let θ^*_{TC} (OT^*_{TC}) denote the optimal θ (OT) in the problem PM using cost function TC, and θ^*_C (OT^*_C) those values using cost function C. The following corollary follows directly from these definitions and the fact that

$$\frac{\partial TC}{\partial \theta} = \sum_{i=1}^{n} \frac{\lambda_i}{(\beta_i + R_i\theta^\delta)^2}\{\beta_i L_i - \delta(R_i V_i + \beta_i Q_i)\theta^{\delta-1} + R_i L_i \theta^\delta(1-\delta)\}.$$

Corollary 2.2: If $\delta < 1$, or $\delta = 1$ and $\beta_i > 0$ for all i, then:

a) $\frac{\partial TC}{\partial \theta} = \frac{\partial C}{\partial \theta} - \sum_{i=1}^{n} \frac{\lambda_i}{\pi_i^2}\delta\beta_i Q_i \theta^{\delta-1} < \frac{\partial C}{\partial \theta}$ and hence $\theta^*_C \leq \theta^*_{TC}$ and $OT^*_C \leq OT^*_{TC}$.

b) $Lim_{\beta_i \downarrow 0 \text{ for all } i} TC(\theta) = C(\theta)$.

Part (a) notes that if external failure costs are not included in plant reports, the plant manager will have an incentive to use less than an optimal level of overtime. One resolution to this incentive incompatibility, of course, is to include the external costs in the manager's performance reports. This is not done in many cases because these external costs are often not realized until long after production is complete, and matching the costs to specific plant schedules (while conceptually straightforward) is perceived as difficult. Another resolution to the incentive problem is suggested by part (b) above. If the inspection system is made more stringent, then all potential external costs will be internalized in plant costs, and C and TC become the same.

IV. Cost signatures

In this section we document the cost consequences of adding products using the above model of a firm that strives to make schedule with fixed resources. The approach is experimental, in which a spectrum of different cost scenarios are modeled. In each, we look at optimal overtime policies and the resulting cost signatures as we add products. We assume that the cost parameters are constant across all products so that we can isolate pure congestion and complexity effects from the effects of changing the product/cost mix. Hence, we will drop the subscript i in this discussion. In this context, if $h(n) = \gamma(n) = 1$ we are essentially adding higher volume of the same product and will run into pure congestion effects. When these functions differ from unity, there are true complexity effects because time, quality or both are impacted directly by the number of products produced. Complexity effects will manifest themselves in costs via direct quality consequences and/or the time-quality-rework-time dynamics moderated by plant labor policies, as discussed above. We assume that across all scenarios tested, $r = \$25$, $s = 33\%$, $\xi = .5$, $t^0 = 1$, $p^0 = .98$, and we will allow overtime up to 50% of regular time capacity ($OT_{max} = .5$). The parameters that are allowed to vary from one scenario to another are the inspection system integrity (α and β), the degree of quality deterioration with rushing (δ), the material, indirect and warranty costs (m, I, and ω), the full time labor cost (W_F), and the extent of complexity impacts on time and quality ($\gamma(n)$ and $h(n)$). Specifically, these parameters vary across the following set of values:

$\alpha = .02$ or $.10$; $\beta = .02$ or $.10$; $\delta = .5$ or 1.0 or 1.5;
$m = 2$ or 6; $I = 2$ or 6; $\omega = 10$ or 50;
$W_F = 10$ or 50;
$h(n) = 1$ always, or equals $1 - .1n$;
$\gamma(n) = 1$ always, or equals $1 + .1n$.

There are $3 \times 2^8 = 768$ different possible scenarios using these data values. We chose an orthogonal fractional factorial design using Addelman's (1962) tables, to construct an experiment with 16 scenarios. The parameter values in each scenario are listed in table 1. We conducted two sets of experiments. The first tracked the cost signatures as we add products, where additional products add to both the total volume in the plant and the number of different products produced. Specifically, we assume that $\lambda_i = 1$ for all i, so that $\sum_{i=1}^{n} \lambda_i = \lambda = n$. In this set of experiments the total firm costs include both congestion and complexity effects.

In the second set of experiments, we assume that the total *base utilization*

$$\rho^0 = \sum_{i=1}^{n} \frac{\lambda_i t_i^0}{p_i^0(1-\alpha_i)+(1-p_i^0)\beta_i} = [\frac{t^0}{p^0(1-\alpha)+(1-p^0)\beta}] \sum_{i=1}^{n} \lambda_i = [\frac{t^0}{p^0(1-\alpha)+(1-p^0)\beta}]\lambda$$

remains constant as we add products. To do this, it is easy to see that we must keep λ constant, so that if $\lambda_1 = \lambda = K$ when one product is in the plant we must have $\lambda_i = K/n$ for each i when there are n products in the plant. So, adding products requires that the production of existing products be cut back. By keeping the base utilization constant, we are essentially controlling for volume effects and can interpret the cost signatures as representing complexity effects.

We computed the upper utilization bound needed to make schedule $(\rho^*(\lambda))$ using the M/M/1 queuing approximation described in appendix A.

Within each set of experiments, we considered each of the 16 scenarios in turn, and tracked the optimal overtime policy and resulting costs, cost/sales ratios, profit (revenues minus the costs considered here, ignoring any allocated expenses not included in the model) and profit margin (profit/sales) as we add up to five products to the facility. We discuss the results in the following subsections.

A. Costs of congestion and complexity

In this set of scenarios we assume that $\lambda_i = 1$ for all i, and hence we increase both scale and scope as we add products. For each of the 16 scenarios, we began with one product in the plant and sequentially added more, one at a time, up to five. At each step, we found the optimal combination of overtime and rushing, and recorded the costs at that optimal solution. The results for each of the 16 scenarios are shown in table 2. These results demonstrate the following.

• Naturally and intuitively, the profit margin typically increases and then decreases as products are added. That is, a typical scenario shows a profit margin that is nonlinear and non-monotone in sales, increasing as long as the facility is not stressed due to congestion and/or complexity phenomena, but then decreasing as additional products stress the facility's ability to make schedule.

• No overtime or rushing is needed as long as $\rho \leq \rho^*$. After that an optimal combination of overtime and rushing is used to make schedule (keeping $\rho = \rho^*$). Overtime increases labor costs, and rushing increases failure costs. Hence, labor costs are substituted for failure costs by OT policy. That is, the costs of stresssing a facility can be deliberately shunted into, or out of, the labor category by using, or not, OT to make schedule.

• Without overtime, the labor cost/sales ratio declines as the firm adds products, because the constant labor cost is spread over an increasing number of units. Once OT begins, this cost ratio can rise. However, once the maximal OT is reached the total labor expense again becomes fixed and extra units will again cause this ratio to decline. For example, in scenario 9 the labor cost/sales declines from product 1 to 2 as the same labor is used to make twice as many units. This ratio then increases when overtime is first used to handle product 3. But, the maximal overtime of 50% increase over regular time hours is nearly reached for this product, so that for products 4 and 5 the overtime level remains relatively constant and the labor cost/sales declines again.

• The materials/sales, indirect/sales, and warranty/sales ratios will remain constant as long as there is no deterioration in quality (no rushing effects or quality effects of complexity). However, these ratios can leap upward as products are added if rushing effects or $h(n)$ effects are significant. For example, in scenarios 2 and 4 there is no $h(n)$ effect and these ratios remain constant as long as OT (only) is used to make schedule. They increase when rushing is introduced. In scenario 5, these ratios increase before resorting to rushing due to the $h(n)$ effects on quality. In cases where these ratios would increase significantly there is a natural tendency to use OT instead of rushing to temper the effect. The most significant leaps in these ratios occur when cost considerations make OT the preferred choice for making schedule, but the plant reaches an OT cap which forces costly rushing behaviors.

• The integrity of the inspection system determines whether failure costs are realized internally or externally, consistent with cost of quality logic. For example, with β equal to zero no defectives will ever be released and all of the congestion and complexity costs will be realized internally. The actual cost signatures in the scenarios, however, do not show a simple relationship between external costs and β, because of the confounding effects of product economics (how expensive are external failures relative to internal failures?), the consequences of rushing (how significant is δ?), and on optimal overtime policies.

• In this test series, the comparison in Corrolary 2.1 suggests overtime in all 16 scenarios. In most instances this predicts the optimal policy. In scenario 14, however, rushing is preferred to overtime. In scenarios 1, 6, 7 and 15 mixed solutions are optimal. Naturally, in each of these scenarios δ equals either .5 or 1.5 (if $\delta = 1$ Corrolary 2.1 will give exact results and we would see no deviation from the suggested policy).

The nonlinear manner in which variables interact make optimal labor policy predictions based on one or a few variables difficult. Table 3 isolates the input parameters for these five rushing-oriented cases. The mixed results in scenarios 1, 7 and 14 are not surprising. Even when Corollary 2.1 suggests overtime instead of rushing with $\delta = 1$, we would expect that it might be optimal to depart from that suggestion if $\delta < 1$ (with $\delta = 0$ we would always rush), because lower δ's decrease the quality consequences of rushing. However, scenarios 6 and 15 are surprising. As noted in section III.A above, we might expect that if overtime is optimal with $\delta = 1$, it would remain optimal as we increase δ. Scenarios 6 and 15 violate this intuition. Both of these scenarios exhibit a willingness to use overtime at first, but then abandon this policy in favor of rushing as we continue to add products. The reasons for this have been described above. In these cases, even at maximal overtime the rushing levels needed to make schedule result in very low quality. Once this unfortunate situation is in place, yields are already extremely low and, being bounded below by zero, increased rushing does not significantly decrease the already low quality. For example, in scenario 6 with 3 products the yield is already down to $p = .98 \times (.8) \times (.347)^{1.5} = 16\%$. With four products the yields would drop to 4% even with maximal overtime. This cannot drop much farther as we increase rushing, but rushing can save us overtime costs. In fact, yields only drop to 2% as all overtime is replaced by rushing, so it is optimal to do this and save the overtime premiums.

B. Costs of complexity

In the second set of experiments, we again begin with one product in the plant and sequentially add more, one at a time, up to five. However, we keep the base utilization constant at .5 as we added products. Different scenarios have different α and β values, meaning that the total demand rate λ consistent with $\rho^0 = .5$ will vary from one scenario to another. Consequently, the "making schedule" utilization $\rho^*(\lambda)$ will also differ from one scenario to another. In all cases, however, $\rho^* > .5$ so that no scenarios required overtime or rushing when there was just one product in the plant.

As we added products, we have to reduce the demand rate per product to maintain $\rho_0 = .5$. As before, we add products, solve for the optimal overtime/rushing combination, and record the cost and profit figures. These results, shown in table 4, demonstrate the following.

• When h and γ are constant there are no complexity effects, and we will see no deterioration in performance as we add products. Intuitively, we are controlling for congestion by keeping the base utilization constant, and declaring that complexity effects are not present by keeping h and γ constant, so neither congestion nor complexity effects are being experienced. Hence, the model predicts no cost consequences. Scenarios 1, 2, 13 and 14 have this feature.

• When γ is increasing in n but h is constant at 1, there is only a time effect for complexity. This will not affect performance until schedule is threatened. After that, the additional time expenditures to build n versus 1 product hurt following standard congestion logic. Scenarios 3, 4, 15 and 16 have this feature. Looking at either the absolute cost or cost/sales figures, we see no deterioration until the fourth product is added, at which point the time costs of complexity begin to hurt. In each of these scenarios, it is optimal to use overtime rather than rushing to make schedule. Hence, only labor costs are affected.

• When $\gamma = 1$ but h is decreasing in n, we see a deterioration in materials, indirect variable and warranty costs even before schedule is threatened. Scenarios 5, 6, 9 and 10 have this feature. Even though in each of these cases it is optimal to use overtime instead of rushing to make schedule, our internal and external failure costs suffer as we add products. This illustrates the fact that the quality costs of complexity are always there, regardless of how congested the facility is and regardless of labor policy. In contrast, the time consequences of complexity will only impact costs when schedule completion is threatened.

• Overtime is suggested by Corrolary 2.1 in each of these scenarios, except for scenario 6 with $n = 5$. In that case, Corollary 2.1 suggests rushing, but it is optimal to use overtime. This is because $\delta = 1.5$, making rushing less desirable. If in this scenario we decreased δ to 1, then rushing would be optimal as it must be from Corollary 2.1. In the rest of the cases, Corrolary 2.1 suggests overtime as the optimal response to schedule pressures. This correctly anticipates the optimal policy in all but two cases (scenarios 7 and 12). In both of these cases $\delta = .5$. Hence, in these cases the quality effects of rushing are less severe than in the case analyzed in Corollary 2.1. Two other cases (1 and 14) had $\delta = .5$, but both of these also had $h = \gamma = 1$ and so never faced schedule stress as products were added. Hence, there was no need to consider either overtime or rushing.

V. Conclusions and Recommendations

One of the most significant benefits of articulating our model to the management of our client firm was the conceptual framing of complexity issues, and issues facing a plant that makes schedule, that it presents. We were also able to give the firm some specific feedback on the original question posed.

Conceptual deliverables

There is a quality "vent" for congestion and complexity pressures. The cost consequences of this are that congestion and complexity costs can appear in cost categories other than inventories and delay. The quality level in a plant at any one time is not exogenously supplied by basic resource capabilities, but is (at least in part) endogenously determined by workers responding to schedule pressures. Under these conditions, there is not one single capacity figure that limits a firm's output capabilities, but instead a range of output rates that it can achieve, with varying levels of quality. That is, "capacity" is a time-quality pair.

When rushing adversely affects quality, and poor quality items are reworked, the plant can enter a rush-rework cycle from which it cannot make schedule. Under these conditions, either overtime is extensively used or the plant makes schedule by shipping poor quality items out. Intuitively, if we see a plant that always makes schedule in a variable environment, then either it has a lot of excess capacity on average, makes extensive use of overtime, or it is making schedule by allowing poor quality items to reach market.

The plant's labor policies and inspection system determine where the costs of congestion and complexity will be realized. If overtime is allowed to relieve schedule pressures, then these costs will appear in labor costs. If overtime is not allowed, then they will appear in quality costs. If inspection is strict, quality costs will appear internally in labor, materials and overhead. If inspection is not strict, they will appear externally in warranty and goodwill losses.

If a plant has an imperfect inspection system, then a plant manager that is not accountable for external quality costs will have an incentive to use a suboptimal amount of overtime. Two ways to align plant manager incentives with firm goals are to (1) tighten up the inspection system so that all failure costs are internal, and/or (2) include external failure costs on plant reports.

All of the above holds for congestion pressures alone, even without complexity effects. In this paper we considered two fundamental complexity effects, time and quality. When there is only a time consquence of complexity there will be no cost performance impact as we add products, until the nonproductive time imposed on the plant due to product heterogeneity is sufficient to threaten schedule completion. Even then, when overtime is an optimal response to an endangered schedule there will only be a labor cost of complexity.

When there is a quality consequence for complexity, costs will be impacted immediately as we add products. even to a lightly utilized facility. These costs will manifest themselves in by workers responding to schedule pressures. Under these conditions, there is not one single capacity figure that limits a firm's output capabilities, but instead a range of output rates that it can achieve, with varying levels of quality. That is, "capacity" is a time-quality pair.

When rushing adversely affects quality, and poor quality items are reworked, the plant can enter a rush-rework cycle from which it cannot make schedule. Under these conditions, either overtime is extensively used or the plant makes schedule by shipping poor quality items out. Intuitively, if we see a plant that always makes schedule in a variable environment, then either it has a lot of excess capacity on average, makes extensive use of overtime, or it is making schedule by allowing poor quality items to reach market.

The plant's labor policies and inspection system determine where the costs of congestion and complexity will be realized. If overtime is allowed to relieve schedule pressures, then these costs will appear in labor costs. If overtime is not allowed, then they will appear in quality costs. If inspection is strict, quality costs will appear internally in labor, materials and overhead. If inspection is not strict, they will appear externally in warranty and goodwill losses.

If a plant has an imperfect inspection system, then a plant manager that is not accountable for external quality costs will have an incentive to use a suboptimal amount of overtime. Two ways to align plant manager incentives with firm goals are to (1) tighten up the inspection system so that all failure costs are internal, and/or (2) include external failure costs on plant reports.

All of the above holds for congestion pressures alone, even without complexity effects. In this paper we considered two fundamental complexity effects, time and quality. When there is only a time consquence of complexity there will be no cost performance impact as we add products, until the nonproductive time imposed on the plant due to product heterogeneity is sufficient to threaten schedule completion. Even then, when overtime is an optimal response to an endangered schedule there will only be a labor cost of complexity.

When there is a quality consequence for complexity, costs will be impacted immediately as we add products, even to a lightly utilized facility. These costs will manifest themselves in internal and/or external failure costs, the exact location being determined by the integrity of the inspection system. When and if the increased rework load threatens schedule, the usual overtime and rushing responses will weigh in with their own cost consequences.

This is the situation when resources are assumed to be held at one fixed level. If resource levels can vary, then there is another category of complexity costs that captures the inherent capacity levels needed to deal with the expected level of product variety. These costs were not analyzed in this paper. Instead, we provide insight into how to best operate with a given resource level, which is a significant precedent to deciding what level is required to operate efficiently.

Because rushing causes rework, which takes time and causes more rushing, the time and quality consequences of complexity are difficult to unravel. Hence, it is very difficult to uncover root causes of complexity costs using traditionally collected data. To justify investments of the type considered by our firm, special investigations will have to take place. These are inevitably costly, and one will be faced with the question of where to invest a finite budget in time and investigative dollars? Our results suggest that, unless the firm is fundamentally congested (even absent complexity influences), a task force may prefer looking into the quality costs of complexity, rather than the time costs (also see Lovejoy and Sethuraman 1998). This is because the quality costs are always there, even in lightly loaded facilities. Also, when loads get heavy enough to threaten schedule, quality issues will exacerbate the time stress through rework. So in all cases the quality costs of complexity are important. The time costs can also be significant but only in a heavily loaded facility.

Also, we have presumed that the single queuing station in our model represents the bottleneck workstation in the plant. However, the above suggests that the quality costs of complexity will can be felt at any (even non-bottleneck) stations, whereas this is not true for the time costs of complexity. Hence, looking into the quality costs of complexity may surface significant cost consequences throughout the facility.

In the data ranges represented in the 16 scenarios in table 1, overtime is preferred to rushing (at optimality) for all cases except those with low quality consequences for rushing.

We suspect that for many realistic situations overtime should be the preferred means to schedule completion.

Recommendations

Here we discuss our report in response to the original question posed, which was to assess complexity cost consequences of a modular product redesign. The investigative tactics suggested by the model are simple. Do you believe the firm is fundamentally congested (cannot make schedule reliably even at (t^0, p^0)) or not? Prior beliefs on this point can be calibrated by plant tours and some rapid data assessment. If the plant is overloaded, then the common tactic of looking into time-based phenomena (e.g. change-over losses) will bear fruit in understanding the costs of complexity in the plant. If the plant is not overloaded, then the greatest return on invested effort will be had by looking into the quality effects of complexity (e.g. defectives, scrap and rework issues).

If the firm is fundamentally congested, then remedies for the costs of complexity will be the same as those for congestion-related problems. Set up reductions, enhanced raw capacity, or demand reduction/prioritization are common tactics. If the firm is not congested, then fool-proofing and other quality system upgrades are recommended.

It was our prior suspicion, based on plant tours, that our client firm has excess theoretical capacity and does not suffer from a fundamental congestion problem. A student team worked in parallel to us, and under our guidance. Our initial instructions to the team, given before our understanding of complexity issues had matured to its current form, was to look for time drivers via a standard utilization study. They set about estimating, through employee interviews and observations, the theoretical throughput rate of the various work stations and indirect activities in a representative plant, and the downtime imposed by changing tasks. They compared this to the actual load placed on the facility. The students confirmed our suspicions that the plant has excess theoretical capacity, and in the process confirmed that we would have benefitted from the model's insights, had they been available, in retargeting their efforts toward quality issues.

Time remains an issue in the plant, and workers must rush upon occasion, but these time issues arise predominantly via random bursts of demand and through quality problems. Our priors are similar for other plants. So, our first conclusion is that, with its resource base fixed, this firm will learn more about its complexity cost structure by looking into issues of scrap, rework and external failures than issues of change over losses or number of transactions.

Our next conclusion builds on this, and speaks to the advisability of a modular product design. Quality-related complexity costs derive from stimulus overload; asking workers to process too many different types of information. It is often easy to reduce this stimulus load in cost effective ways. For example, color coding can reduce the complex task of recognizing a part shape to the simpler task of recognizing colors. We went back to the shop floor to seek anecdotes to support, or not, our suspicions. The feedback was that, yes, quality costs are potentially very large and in many cases these costs can be avoided with simple system upgrades. The bottleneck work station in this plant is the paint cell. One quality-related complexity issue is hanging the wrong part to be painted. This robs the paint cell of capacity, threatens schedule and generates rework. But, simple color- or bar-coding can prevent this. Another issue raised was the many different shades of grey offered in end products. It may not be detected that one part, for example, was painted the wrong shade of grey until it is assembled into the final product and its contrast with other parts is apparent. This can be remedied by color charting upsteam in the process, or paint gun verification by bar code. These stories were only revealed when we asked the right questions, prompted by our more complete model of complexity in this firm. Also, it is almost certain that a quality audit of current systems, with fool-proofing remedies, would be less expensive than a modular product redesign.

So, our second conclusion is that if there are complexity costs in this firm with its current resources, they are quality-based but can be reduced significantly with initiatives that are less costly than a modular product redesign. Hence, the modular architecture initiative cannot be justified based on the current reources and product line. Modular products may still be desirable for this firm, but managers will have to root the cost justification for this initiative in the capital cost savings for future tool sets, and not cost savings with current resources.

We also wanted to report something about plant incentives and overtime policy in this firm. External failure costs are not currently included on plant reports. We recommend that they include these. Also, while we did not estimate firm cost parameters precisely, we did ask a controller for ballpark estimates and then tested our results over a range of parameters allowed by these estimates. Standard data such as the material and labor components of the cost of sales were provided. Reliable quality data in this firm is not yet available. However, the controller believed that β is "low," but external failures (which involve on-site rework by the firm) are very expensive. We tested β values ranging from 0 to .05, and external failure costs from 0 to 50% of the cost of sales. Developing a credible range for δ was difficult, but we did have piece work data that indicated that work rates could, at least periodically, increase by as much as 15% without noticeable quality losses. If we presume quality deteriorations of 10% or more would be noticed, this suggests that $\delta \geq .76$. We tested δ values as low as .5. Over the entire range of parameters tested, overtime is preferred to rushing. We add that over this same range of parameters the cost comparison in Corollary 2.1 suggests the appropriate overtime policy. That is, the conclusions from Corollary 2.1, rigorously valid only for $\delta = 1$, in this case hold for δ values as low as .5. Under these conditions, we are confident in recommending that overtime be used to make schedule in the plant, and in fact spending additional energy gathering more complete information, if it is only to inform this labor policy, is not recommended.

As a postscript, after the original version of this manuscript was prepared a second student team was assigned to the plant and told to look carefully at quality issues. They found 15% rework and 7% scrap rates in the first section of the plant they investigated. About 20% of these problems could be directly traced to complexity issues (e.g. working on the wrong parts, incorrect colors being applied). Quality issues are now a top priority in this firm. We cannot claim sole credit for this management perspective, but neither were our voices inconsequential.

Acknowledgements: The authors would like to thank Michigan professors Peter Lenk, Marty Young, Shannon Anderson and Bill Lanen for helpful advice on estimation theory, the cost of complexity and accounting systems. We also thank Michigan students Jennifer Brown, Charlie Choi, Bill Reeves, Kirk Schell, Ryan Schmidt and Chris Spears for their on-site work and insights.

References

Addelman, S. Orthogonal Main-Effect Plans for Assymetrical Factorial Experiments. *Technometrics* 4, 1962, 21-46.

Anderson, Shannon, Measuring the Impact of Product Mix Heterogeneity on Manufacturing Overhead Cost, *The Acconting Review* 70, 1995, 363-387.

Banker, R., S. Datar and S. Kekre, Relevant Costs, Congestion and Stoshasticity in Production Environments. *Journal of Accounting and Economics* 10, 1988, 171-197.

Banker, R., S. Datar, S. Kekre, and T. Mukhopadhyay, Costs of Product and Process Complexity, in R. Kaplan (ed), *Measures of Manufacturing Excellence*, Harvard Business School Press. Boston, MA 1990.

Banker. R., G. Potter and R. Schroeder, An Empirical Analysis of Manufacturing Overhead Cost Drivers, *Journal of Accounting and Economics* 19, 1995, 115-137.

Caves. D., L. Christiansen and M. Tretheway, Flexible Cost Functions for Multiproduct Firms, *Review of Economics and Statistics*, 1979, 477-481.

Cohen, S., Aftereffects of stress on human performance and social behavior: A review of research and theory, *Psychological Bulletin*, 88, 1980, 82-108.

Foster, G. and M. Gupta, Manufacturing Overhead Cost Driver Analysis, *Journal of Accounting and Economics* 12, 1990, 309-337.

Heyman. D. and M. Sobel. Stochastic Models in Operations Research, vol. II. McGraw Hill, N.Y. 1982.

Lovejoy, W. and K. Sethuraman. A Note on Information Gathering in Complexity Studies. Michigan Business School working paper. University of Michigan, 1998.

MacDuffie, John Paul, Sethuraman, Kannan, and Fisher, Marshall, Product Variety and Manufacturing Performance: Evidence from the International Automotive Assembly Plant Sutdy, *Management Science* 42, 1996, 350-368.

Meister, D., Behavioral Foundations of System Development, Wiley 1976.

Miller, J. and and T. Vollman. The Hidden Factory, Harvard Business Review, Sep-Oct 1985.

Panzar, J. and R. Willig, Economies of Scope, *American Economic Review*, 71, 1981, 268-272.

Pulley, L. and Y. Braunstein, A Composite Cost Function for Multiproduct Firms with an Application to Economies of Scope in Banking, *Review of Economics and Statistics* 74, 1992, 221-230.

Saunders, M. and E. McCormick. Human Factors in Engineering and Design, McGraw-Hill, 1993.

Sethi, A. and S. Sethi, Flexibility in Manufacturing: A Survey, *International Journal of Flexible Manufacturing Systems* 2, 1990, 289-328.

Sethuraman, Kannan, The Impact of Product Variety on Manufacturing Performance:An Empirical Investigation of the World Automobile Industry, Unpublished Ph.D. Thesis, Wharton School, University of Pennsylvania, 1994.

Teece, D., Towards an Economic Theory of the Multiproduct Firm, *Journal of Economic Behavior and Organization*, 3, 1982, 39-63.

Appendix A: Using an M/M/1 approximation to compute T^* and $\rho^*(\lambda)$

As described in the text, we model "making schedule" as keeping $P_D(\tau) \geq P^{min}$, where τ is the desired maximal total delay between the arrival of a job and its release for consumption, $P_D(\tau)$ is the probability that the delay is less than or equal to τ, and P^{min} is a specified lower bound on this probability. At our industrial client's facility, plant managers were expected to have 97% of scheduled production complete within 3 days of it being released to the shop floor, which suggests $P^{min} = .97$ and $\tau = 3$ days.

Recall that $\lambda = \sum_{i=1}^{n} \lambda_i$ is the aggregate demand arrival rate, and $\bar{T} = \sum_{i=1}^{n} \frac{\lambda_i}{\lambda} \frac{t_i}{\pi_i}$ is the expected total service time (including rework) experienced by the next arriving job. Hence. the system utilization is $\rho = \lambda \bar{T}$.

If we are willing to assume that the interarrival times and generic service times are exponentially distributed, then we know from M/M/1 queuing logic (c.f. Heyman and Sobel 1982) that the long-run probability of there being k jobs in the system is $(1-\rho)\rho^k$. The delay experienced by an arriving job, conditional on there being k jobs in the system is the sum of $(k+1)$ exponential service times (one for each of the k jobs in the system, and the arriving job). This sum is gamma distributed, with

$$Pr\{Delay \leq \tau | k\} = 1 - e^{-\tau/\bar{T}} \sum_{j=0}^{k} \frac{(\tau/\bar{T})^j}{j!}.$$

So,

$$P_D(\tau) = \sum_{k=0}^{\infty} (1-\rho)\rho^k \left[1 - e^{-\tau/\bar{T}} \sum_{j=0}^{k} \frac{\left(\frac{\tau}{\bar{T}}\right)^j}{j!}\right],$$

which can be increased to unity by reducing $\bar{T}$ (and hence ρ) to zero.

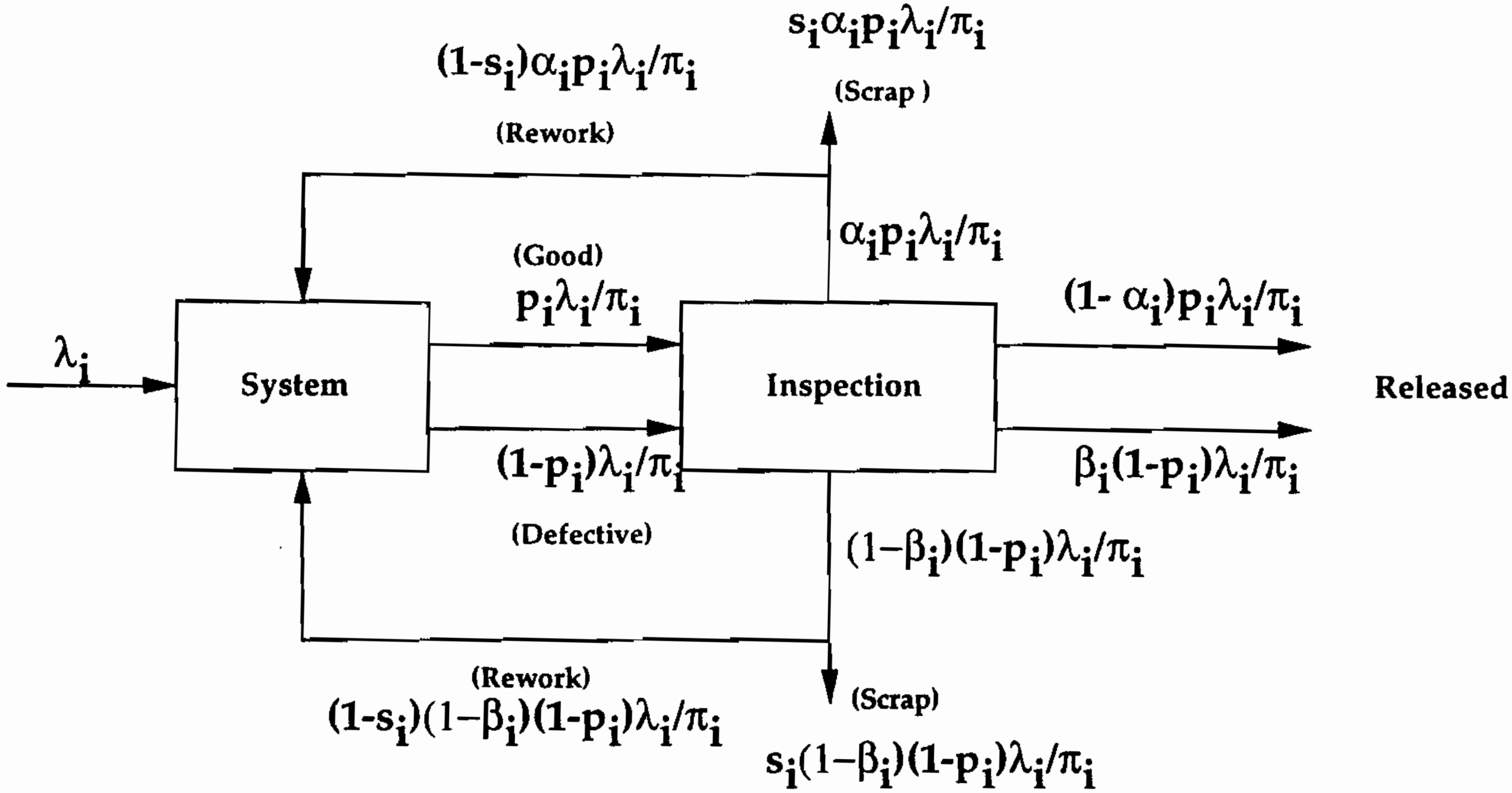
In our formulation P^{min} and τ are parameters that remain fixed as we adjust demand and capacity. For a given aggregate arrival rate λ, define $T^*(\lambda)$ as the maximum $\bar{T}$ such that $P_D(\tau) = P^{min}$. Making schedule is equivalent to ensuring that $\bar{T} \leq T^*(\lambda)$. The plant manager can decrease $\bar{T}$ by increasing the amount of time available in a working day (that is, use overtime) or decreasing the time needed to process a unit (by forcing the workers to rush jobs).

Table A.1 shows T^* and ρ^* as a function of λ using the above M/M/1 approximation with $\tau = 3$ days and $P^{min} = .97$.

λ	$T^*(\lambda)$	$\rho^*(\lambda)$
1	.460	.460
2	.315	.630
3	.240	.720
4	.194	.776
5	.162	.810
...	...	...
10	.0895	.895
...	...	...
20	.0472	.945

Table A.1
$T^*(\lambda)$ and $\rho^*(\lambda)$ from M/M/1 approximation

Figure 1: Mathematical Model



λ_i = Demand for Product i

p_i = Probability of producing a good unit per processing iteration for product i

α_i, β_i = Inspection parameters

s_i = Scrap rate for product i

$\pi_i = p_i(1-\alpha_i) + \beta_i(1-p_i)$

Figure 2: Costs Incurred

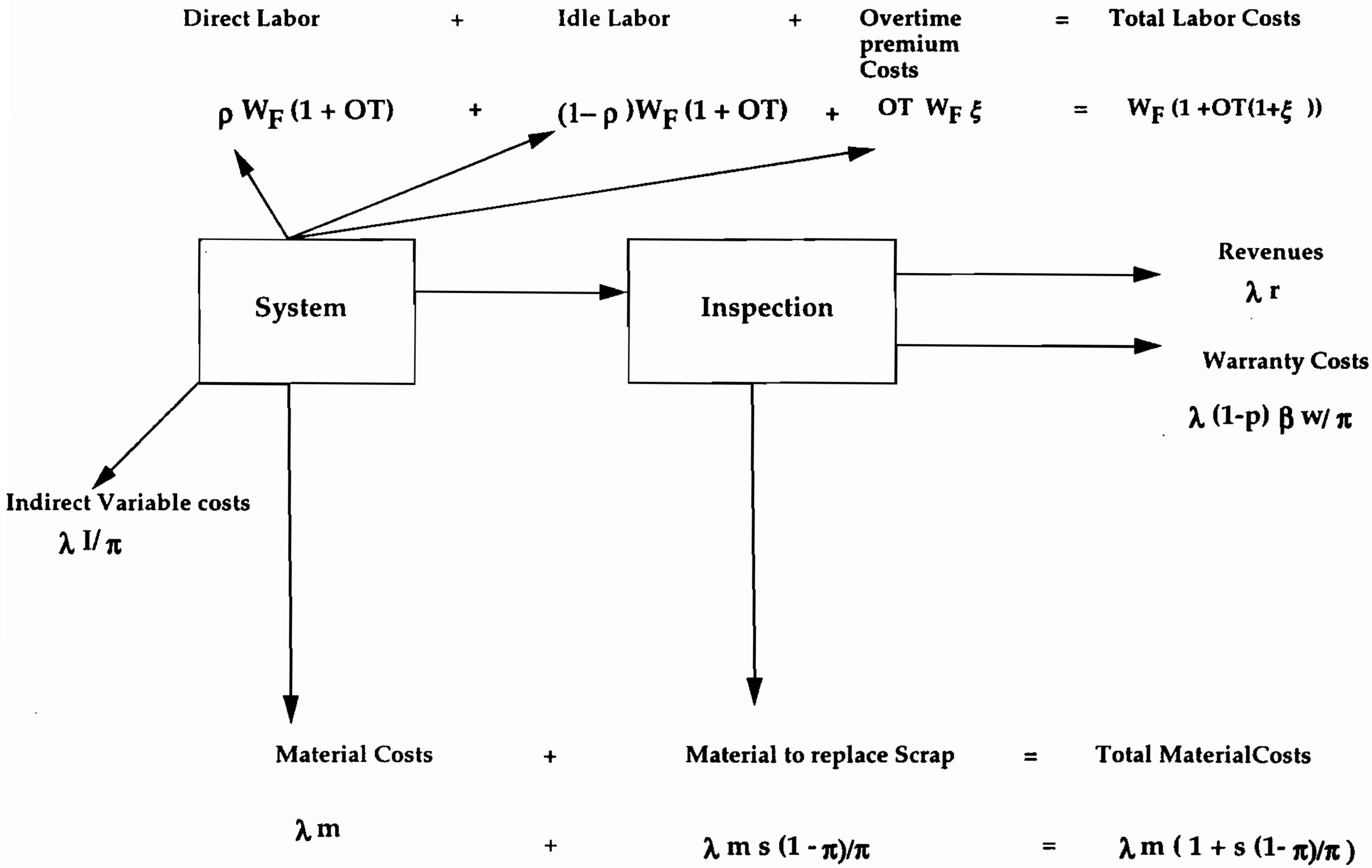


Table 1: Parameter values for numerical experiments

Scenario	α	β	δ	$h(n)$	$\gamma(n)$	m	W_F	ω	I
1	.02	.02	.50	C	C	2	10	10	2
2	.02	.02	1.0	C	C	6	50	10	6
3	.02	.02	1.0	C	L	2	50	50	2
4	.02	.02	1.5	C	L	6	10	50	6
5	.02	.10	.10	L	C	2	10	10	6
6	.02	.10	1.5	L	C	6	50	10	2
7	.02	.10	.50	L	L	2	50	50	6
8	.02	.10	1.0	L	L	6	10	50	2
9	.10	.02	1.5	L	C	2	10	50	2
10	.10	.02	1.0	L	C	6	50	50	6
11	.10	.02	1.0	L	L	2	50	10	2
12	.10	.02	.50	L	L	6	10	10	6
13	.10	.10	1.0	C	C	2	10	50	6
14	.10	.10	.50	C	C	6	50	50	2
15	.10	.10	1.5	C	L	2	50	10	6
16	.10	.10	1.0	C	L	6	10	10	2

C = constant model

L = linear model

TABLE 2: Numerical trials for congestion and complexity

Scenario				OPTIMAL		SALES	ABSOLUTE COSTS				COSTS/SALES				PROFIT	PROFIT
number	n	rho*	rho	Theta	OT	$	Labor	Material	Indirect	Warranty	Labor	Material	Indirect	Warranty		MARGIN
1	1	0.46	0.26	1	0	40	10	2.02693	2.0816	0.00416	0.25	0.0507	0.052	0.0001	25.8873106	0.64718276
	2	0.63	0.52	1	0	80	10	4.05386	4.1632	0.00833	0.125	0.0507	0.052	0.0001	61.7746211	0.77218276
	3	0.72	0.72	0.854	0	120	10	6.24663	6.74737	0.06379	0.0833	0.0521	0.0562	0.00053	96.9422005	0.80785167
	4	0.776	0.776	0.564	0	160	10	8.99523	11.0158	0.2912	0.0625	0.0562	0.0688	0.00182	129.697732	0.81061082
	5	0.81	0.81	0.586	0.222	200	13.328	11.1579	13.5088	0.33736	0.0666	0.0558	0.0675	0.00169	161.66795	0.80833975
2	1	0.46	0.26	1	0	40	50	6.08078	6.2448	0.00416	1.25	0.152	0.1561	0.0001	-22.3297419	-0.5582435
	2	0.63	0.52	1	0	80	50	12.1616	12.4896	0.00833	0.625	0.152	0.1561	0.0001	5.34051624	0.06675645
	3	0.72	0.72	1	0.084	120	56.312	18.2423	18.7344	0.01249	0.4693	0.152	0.1561	0.0001	26.6983264	0.22248605
	4	0.776	0.776	1	0.341	160	75.593	24.3231	24.9792	0.01665	0.4725	0.152	0.1561	0.0001	35.0883133	0.21930196
	5	0.81	0.81	0.227	0.5	200	87.481	62.5194	128.544	3.33239	0.4374	0.3126	0.6427	0.01666	-81.8765599	-0.4093828
3	1	0.46	0.26	1	0	40	50	2.02693	2.0816	0.02082	1.25	0.0507	0.052	0.00052	-14.1293422	-0.3532336
	2	0.63	0.572	1	0	80	50	4.05386	4.1632	0.04163	0.625	0.0507	0.052	0.00052	21.7413156	0.27176644
	3	0.72	0.72	1	0.301	120	72.575	6.08078	6.2448	0.06245	0.6048	0.0507	0.052	0.00052	35.0370358	0.2919753
	4	0.776	0.776	0.113	0.499	160	87.404	26.2609	63.336	28.1611	0.5463	0.1641	0.3959	0.17601	-45.1621483	-0.2822634
	5	0.81	0.81	0.04	0.5	200	87.5	63.9247	173.408	83.302	0.4375	0.3196	0.867	0.41651	-208.13474	-1.0406737
4	1	0.46	0.26	1	0	40	10	6.08078	6.2448	0.02082	0.25	0.152	0.1561	0.00052	17.6536053	0.44134013
	2	0.63	0.572	1	0	80	10	12.1616	12.4896	0.04163	0.125	0.152	0.1561	0.00052	45.3072107	0.56634013
	3	0.72	0.72	1	0.301	120	14.515	18.2423	18.7344	0.06245	0.121	0.152	0.1561	0.00052	68.4458285	0.5703819
	4	0.776	0.776	0.02	0.5	160	17.495	364.508	1055.84	175.473	0.1093	2.2782	6.599	1.09671	-1453.31669	-9.0832293
	5	0.81	0.81	0.015	0.5	200	17.5	475.363	1379.58	229.513	0.0875	2.3768	6.8979	1.14756	-1901.96054	-9.5098027
5	1	0.46	0.26	1	0	40	10	2.02579	6.23441	0.02078	0.25	0.0506	0.1559	0.00052	21.7190191	0.54297548
	2	0.63	0.571	1	0	80	10	4.18657	13.6961	0.26936	0.125	0.0523	0.1712	0.00337	51.8479401	0.64809925
	3	0.72	0.72	1	0.319	120	14.78	6.52658	22.7871	0.82034	0.1232	0.0544	0.1899	0.00684	75.085479	0.62571233
	4	0.776	0.776	0.392	0.5	160	17.5	13.2021	71.2922	8.68682	0.1094	0.0825	0.4456	0.05429	49.3188047	0.30824253
	5	0.81	0.81	0.196	0.5	200	17.5	23.1026	149.115	21.9948	0.0875	0.1155	0.7456	0.10997	-11.7120789	-0.0585604
6	1	0.46	0.26	1	0	40	50	6.07736	2.07814	0.02078	1.25	0.1519	0.052	0.00052	-18.176276	-0.4544069
	2	0.63	0.571	1	0	80	50	12.5597	4.56538	0.26936	0.625	0.157	0.0571	0.00337	12.6055442	0.1575693
	3	0.72	0.72	0.347	0.5	120	87.487	36.6875	24.8762	10.4424	0.7291	0.3057	0.2073	0.08702	-39.4930098	-0.3291084
	4	0.776	0.776	0.09	0	160	50	84.1301	68.7374	33.7288	0.3125	0.5258	0.4296	0.2108	-76.5962984	-0.4787269
	5	0.81	0.81	0.071	0	200	50	110.245	91.0552	45.0194	0.25	0.5512	0.4553	0.2251	-96.319262	-0.4815963

Scenario number	n	T*	rho*	OPTIMAL Theta	OT	SALES $	ABSOLUTE COSTS Labor	Material	Indirect	Warranty	COSTS/SALES Labor	Material	Indirect	Warranty	PROFIT	PROFIT MARGIN
7	1	0.46	0.26	1	0	40	50	2.02579	6.23441	0.10391	1.25	0.0506	0.1559	0.0026	-18.3641064	-0.4591027
	2	0.63	0.628	1	0	80	50	4.18657	13.6961	1.34679	0.625	0.0523	0.1712	0.01683	10.7705113	0.13463139
	3	0.72	0.72	0.45	0	120	50	7.5367	31.97	12.6236	0.4167	0.0628	0.2664	0.1052	17.8696831	0.14891403
	4	0.776	0.776	0.234	0	160	50	12.0943	61.2211	34.0878	0.3125	0.0756	0.3826	0.21305	2.59672679	0.01622954
	5	0.81	0.81	0.203	0.317	200	73.739	16.6059	90.0535	55.1633	0.3687	0.083	0.4503	0.27582	-35.5618981	-0.1778095
8	1	0.46	0.26	1	0	40	10	6.07736	2.07814	0.10391	0.25	0.1519	0.052	0.0026	21.7405985	0.54351496
	2	0.63	0.628	1	0	80	10	12.5597	4.56538	1.34679	0.125	0.157	0.0571	0.01683	51.5281154	0.64410144
	3	0.72	0.72	0.697	0.5	120	17.498	22.286	10.3293	11.7121	0.1458	0.1857	0.0861	0.0976	58.1742214	0.48478518
	4	0.776	0.776	0.195	0.5	160	17.494	52.4826	36.7703	79.6447	0.1093	0.328	0.2298	0.49778	-26.3912041	-0.164945
	5	0.81	0.81	0.108	0.5	200	17.493	83.5458	64.0866	150.014	0.0875	0.4177	0.3204	0.75007	-115.139007	-0.575695
9	1	0.46	0.283	1	0	40	10	2.08796	2.26655	0.02267	0.25	0.0522	0.0567	0.00057	25.6228286	0.64057072
	2	0.63	0.628	1	0	80	10	4.33796	5.02412	0.29642	0.125	0.0542	0.0628	0.00371	60.3415032	0.75426879
	3	0.72	0.72	1	0.467	120	17.01	6.80905	8.45166	0.91278	0.1417	0.0567	0.0704	0.00761	86.8169957	0.72347496
	4	0.776	0.776	0.026	0.5	160	17.5	122.326	354.442	176.703	0.1094	0.7645	2.2153	1.10439	-510.970458	-3.1935654
	5	0.81	0.811	0.021	0.5	200	17.5	159.657	463.507	231.339	0.0875	0.7983	2.3175	1.15669	-672.002303	-3.3600115
10	1	0.46	0.283	1	0	40	50	6.26388	6.79964	0.02267	1.25	0.1566	0.17	0.00057	-23.0861831	-0.5771546
	2	0.63	0.628	1	0	80	50	13.0139	15.0723	0.29642	0.625	0.1627	0.1884	0.00371	1.61735531	0.02021694
	3	0.72	0.72	1	0.467	120	85.048	20.4271	25.355	0.91278	0.7087	0.1702	0.2113	0.00761	-11.7424994	-0.0978542
	4	0.776	0.776	0.078	0.5	160	87.5	133.807	356.749	56.264	0.5469	0.8363	2.2297	0.35165	-474.320277	-2.9645017
	5	0.81	0.81	0.039	0.5	200	87.5	266.126	745.533	121.398	0.4375	1.3306	3.7277	0.60699	-1020.55661	-5.1027831
11	1	0.46	0.283	1	0	40	50	2.08796	2.26655	0.00453	1.25	0.0522	0.0567	0.00011	-14.359039	-0.358976
	2	0.63	0.63	1	0.097	80	57.24	4.33796	5.02412	0.05928	0.7155	0.0542	0.0628	0.00074	13.3386514	0.16673314
	3	0.72	0.72	0.14	0.5	120	87.5	21.0532	51.6156	4.59705	0.7292	0.1754	0.4301	0.03831	-44.7658529	-0.3730488
	4	0.776	0.776	0.039	0.5	160	87.5	66.0058	183.775	17.8861	0.5469	0.4125	1.1486	0.11179	-195.167107	-1.2197944
	5	0.81	0.81	0.022	0.5	200	87.5	112.424	320.374	31.6292	0.4375	0.5621	1.6019	0.15815	-351.92695	-1.7596347
12	1	0.46	0.283	1	0	40	10	6.26388	6.79964	0.00453	0.25	0.1566	0.17	0.00011	16.9319492	0.42329873
	2	0.63	0.63	1	0.097	80	11.448	13.0139	15.0723	0.05928	0.1431	0.1627	0.1884	0.00074	40.4064956	0.50508119
	3	0.72	0.72	0.733	0.5	120	17.5	21.7892	29.4824	0.32327	0.1458	0.1816	0.2457	0.00269	50.9050807	0.42420901
	4	0.776	0.776	0.327	0.5	160	17.5	37.7647	65.7113	1.33107	0.1094	0.236	0.4107	0.00832	37.6929669	0.23558104
	5	0.81	0.81	0.156	0.5	200	17.5	64.2747	133.863	3.42714	0.0875	0.3214	0.6693	0.01714	-19.0645169	-0.0953226
13	1	0.46	0.283	1	0	40	10	2.08661	6.78733	0.11312	0.25	0.0522	0.1697	0.00283	21.0129412	0.52532353
	2	0.63	0.566	1	0	80	10	4.17321	13.5747	0.22624	0.125	0.0522	0.1697	0.00283	52.0258824	0.65032353
	3	0.72	0.72	1	0.178	120	12.675	6.25982	20.362	0.33937	0.1056	0.0522	0.1697	0.00283	80.3634842	0.6696957
	4	0.776	0.776	1	0.458	160	16.866	8.34643	27.1493	0.45249	0.1054	0.0522	0.1697	0.00283	107.185365	0.66990853
	5	0.81	0.81	0.409	0.5	200	17.5	14.5507	71.3704	35.6597	0.0875	0.0728	0.3569	0.1783	60.9191315	0.30459566

Scenario				OPTIMAL		SALES	ABSOLUTE COSTS				COSTS/SALES				PROFIT	PROFIT
number	n	T*	rho*	Theta	OT	$	Labor	Material	Indirect	Warranty	Labor	Material	Indirect	Warranty		MARGIN
14	1	0.46	0.283	1	0	40	50	6.25982	2.26244	0.11312	1.25	0.1565	0.0566	0.00283	-18.6353846	-0.4658846
	2	0.63	0.566	1	0	80	50	12.5196	4.52489	0.22624	0.625	0.1565	0.0566	0.00283	12.7292308	0.15911538
	3	0.72	0.72	0.746	0	120	50	19.7033	7.72051	2.96393	0.4167	0.1642	0.0643	0.0247	39.6122599	0.33010217
	4	0.776	0.776	0.514	0	160	50	28.0473	12.0881	8.99788	0.3125	0.1753	0.0756	0.05624	60.8667238	0.38041702
	5	0.81	0.81	0.377	0	200	50	37.1378	17.2099	17.153	0.25	0.1857	0.086	0.08576	78.4992177	0.39249609
15	1	0.46	0.283	1	0	40	50	2.08661	6.78733	0.02262	1.25	0.0522	0.1697	0.00057	-18.8965611	-0.472414
	2	0.63	0.622	1	0	80	50	4.17321	13.5747	0.04525	0.625	0.0522	0.1697	0.00057	12.2068778	0.15258597
	3	0.72	0.72	1	0.414	120	81.052	6.25982	20.362	0.06787	0.6754	0.0522	0.1697	0.00057	12.2582805	0.10215234
	4	0.776	0.776	0.068	5E-07	160	50	28.5385	210.714	34.5088	0.3125	0.1784	1.317	0.21568	-163.761071	-1.0235067
	5	0.81	0.809	0.05	0	200	50	37.0148	275.589	45.423	0.25	0.1851	1.3779	0.22712	-208.027291	-1.0401365
16	1	0.46	0.283	1	0	40	10	6.25982	2.26244	0.02262	0.25	0.1565	0.0566	0.00057	21.4551131	0.53637783
	2	0.63	0.622	1	0	80	10	12.5196	4.52489	0.04525	0.125	0.1565	0.0566	0.00057	52.9102262	0.66137783
	3	0.72	0.72	1	0.414	120	16.21	18.7795	6.78733	0.06787	0.1351	0.1565	0.0566	0.00057	78.1549321	0.6512911
	4	0.776	0.776	0.301	0.5	160	17.5	39.6636	23.8218	8.39975	0.1094	0.2479	0.1489	0.0525	70.6149193	0.44134325
	5	0.81	0.81	0.152	0.5	200	17.5	65.2063	45.5619	19.3786	0.0875	0.326	0.2278	0.09689	52.3532353	0.26176618

Table 3: Parameter values for rushing-oriented cases (congestion and complexity experiment)

Scenario	α	β	δ	$h(n)$	$\gamma(n)$	m	W_F	ω	I
1	.02	.02	.50	C	C	2	10	10	2
6	.02	.10	1.5	L	C	6	50	10	2
7	.02	.10	.50	L	L	2	50	50	6
14	.10	.10	.50	C	C	6	50	50	2
15	.10	.10	1.5	C	L	2	50	10	6

C = constant model

L = linear model

TABLE 4: Numerical trials for complexity only with base utilization held constant

Scenario number	n	rho*	rho	OPTIMAL Theta	OT	Sales $	ABSOLUTE COSTS Labor	Material	Indirect	Warranty	COSTS/SALES Labor	Material	Indirect	Warranty	PROFIT	PROFIT MARGIN
1	1	0.622	0.5	1	0	76.864	10	3.89494	4	0.008	0.1301	0.0507	0.052	0.0001	58.961056	0.76708285
	2	0.622	0.5	1	0	76.864	10	3.89494	4	0.008	0.1301	0.0507	0.052	0.0001	58.961056	0.76708285
	3	0.622	0.5	1	0	76.864	10	3.89494	4	0.008	0.1301	0.0507	0.052	0.0001	58.961056	0.76708285
	4	0.622	0.5	1	0	76.864	10	3.89494	4	0.008	0.1301	0.0507	0.052	0.0001	58.961056	0.76708285
	5	0.622	0.5	1	0	76.864	10	3.89494	4	0.008	0.1301	0.0507	0.052	0.0001	58.961056	0.76708285
2	1	0.622	0.5	1	0	76.864	50	11.6848	12	0.008	0.6505	0.152	0.1561	0.0001	3.171168	0.04125687
	2	0.622	0.5	1	0	76.864	50	11.6848	12	0.008	0.6505	0.152	0.1561	0.0001	3.171168	0.04125687
	3	0.622	0.5	1	0	76.864	50	11.6848	12	0.008	0.6505	0.152	0.1561	0.0001	3.171168	0.04125687
	4	0.622	0.5	1	0	76.864	50	11.6848	12	0.008	0.6505	0.152	0.1561	0.0001	3.171168	0.04125687
	5	0.622	0.5	1	0	76.864	50	11.6848	12	0.008	0.6505	0.152	0.1561	0.0001	3.171168	0.04125687
3	1	0.622	0.5	1	0	76.864	50	3.89494	4	0.04	0.6505	0.0507	0.052	0.00052	18.929056	0.24626686
	2	0.622	0.55	1	0	76.864	50	3.89494	4	0.04	0.6505	0.0507	0.052	0.00052	18.929056	0.24626686
	3	0.622	0.6	1	0	76.864	50	3.89494	4	0.04	0.6505	0.0507	0.052	0.00052	18.929056	0.24626686
	4	0.622	0.622	1	0.046	76.864	53.427	3.89494	4	0.04	0.6951	0.0507	0.052	0.00052	15.5023101	0.20168493
	5	0.622	0.622	1	0.126	76.864	59.46	3.89494	4	0.04	0.7736	0.0507	0.052	0.00052	9.46948349	0.1231979
4	1	0.622	0.5	1	0	76.864	10	11.6848	12	0.04	0.1301	0.152	0.1561	0.00052	43.139168	0.56124022
	2	0.622	0.55	1	0	76.864	10	11.6848	12	0.04	0.1301	0.152	0.1561	0.00052	43.139168	0.56124022
	3	0.622	0.6	1	0	76.864	10	11.6848	12	0.04	0.1301	0.152	0.1561	0.00052	43.139168	0.56124022
	4	0.622	0.622	1	0.046	76.864	10.685	11.6848	12	0.04	0.139	0.152	0.1561	0.00052	42.4538188	0.55232383
	5	0.622	0.622	1	0.126	76.864	11.892	11.6848	12	0.04	0.1547	0.152	0.1561	0.00052	41.2472535	0.53662642
5	1	0.622	0.5	1	0	76.992	10	3.89923	12	0.04	0.1299	0.0506	0.1559	0.00052	51.052768	0.66309185
	2	0.622	0.549	1	0	76.992	10	4.02916	13.1812	0.25923	0.1299	0.0523	0.1712	0.00337	49.5224576	0.64321563
	3	0.622	0.609	1	0	76.992	10	4.18746	14.6202	0.52633	0.1299	0.0544	0.1899	0.00684	47.6580019	0.6189994
	4	0.622	0.622	1	0.099	76.992	11.489	4.38455	16.412	0.85889	0.1492	0.0569	0.2132	0.01116	43.8479119	0.56951257
	5	0.622	0.622	1	0.253	76.992	13.792	4.63671	18.7043	1.28436	0.1791	0.0602	0.2429	0.01668	38.5749402	0.5010253
6	1	0.622	0.5	1	0	76.992	50	11.6977	4	0.04	0.6494	0.1519	0.052	0.00052	11.254304	0.14617498
	2	0.622	0.549	1	0	76.992	50	12.0875	4.39372	0.25923	0.6494	0.157	0.0571	0.00337	10.2515757	0.13315118
	3	0.622	0.609	1	0	76.992	50	12.5624	4.8734	0.52633	0.6494	0.1632	0.0633	0.00684	9.02990052	0.11728362
	4	0.622	0.622	1	0.099	76.992	57.443	13.1537	5.47067	0.85889	0.7461	0.1708	0.0711	0.01116	0.06560308	0.00085208
	5	0.622	0.622	1	0.253	76.992	68.958	13.9101	6.23478	1.28436	0.8957	0.1807	0.081	0.01668	-13.3955656	-0.1739865

Scenario number	n	rho*	rho	OPTIMAL Theta	OT	Sales $	ABSOLUTE COSTS Labor	Material	Indirect	Warranty	COSTS/SALES Labor	Material	Indirect	Warranty	PROFIT	PROFIT MARGIN
7	1	0.622	0.5	1	0	76.992	50	3.89923	12	0.2	0.6494	0.0506	0.1559	0.0026	10.892768	0.14147922
	2	0.622	0.604	1	0	76.992	50	4.02916	13.1812	1.29615	0.6494	0.0523	0.1712	0.01683	8.4855401	0.11021327
	3	0.622	0.622	0.753	8E-05	76.992	50.006	4.39788	16.5332	4.40692	0.6495	0.0571	0.2147	0.05724	1.648259	0.02140819
	4	0.622	0.622	0.541	0	76.992	50	4.91434	21.2283	8.7641	0.6494	0.0638	0.2757	0.11383	-7.91469729	-0.102799
	5	0.622	0.622	0.391	4E-08	76.992	50	5.57795	27.261	14.3627	0.6494	0.0724	0.3541	0.18655	-20.2096992	-0.2624909
8	1	0.622	0.5	1	0	76.992	10	11.6977	4	0.2	0.1299	0.1519	0.052	0.0026	51.094304	0.66363134
	2	0.622	0.604	1	0	76.992	10	12.0875	4.39372	1.29615	0.1299	0.157	0.0571	0.01683	49.2146583	0.63921782
	3	0.622	0.622	1	0.175	76.992	12.626	12.5624	4.8734	2.63164	0.164	0.1632	0.0633	0.03418	44.2984156	0.57536388
	4	0.622	0.622	1	0.429	76.992	16.435	13.1537	5.47067	4.29447	0.2135	0.1708	0.0711	0.05578	37.6379733	0.48885564
	5	0.622	0.622	0.489	0.5	76.992	17.5	18.5333	10.9047	19.4232	0.2273	0.2407	0.1416	0.25228	10.6307704	0.1380763
9	1	0.601	0.5	1	0	70.592	10	3.68483	4	0.04	0.1417	0.0522	0.0567	0.00057	52.867168	0.7489116
	2	0.601	0.554	1	0	70.592	10	3.82781	4.43328	0.26156	0.1417	0.0542	0.0628	0.00371	52.0693424	0.73760968
	3	0.601	0.601	1	0.033	70.592	10.5	4.00554	4.97183	0.53696	0.1487	0.0567	0.0704	0.00761	50.5780161	0.71648368
	4	0.601	0.601	1	0.176	70.592	12.643	4.23241	5.65931	0.88851	0.1791	0.06	0.0802	0.01259	47.1688748	0.66819009
	5	0.601	0.601	1	0.365	70.592	15.474	4.53208	6.56743	1.35289	0.2192	0.0642	0.093	0.01916	42.6656416	0.60439769
10	1	0.601	0.5	1	0	70.592	50	11.0545	12	0.04	0.7083	0.1566	0.17	0.00057	-2.502496	-0.0354501
	2	0.601	0.554	1	0	70.592	50	11.4834	13.2998	0.26156	0.7083	0.1627	0.1884	0.00371	-4.45284568	-0.0630786
	3	0.601	0.601	1	0.033	70.592	52.498	12.0166	14.9155	0.53696	0.7437	0.1702	0.2113	0.00761	-9.37536398	-0.1328106
	4	0.601	0.601	1	0.176	70.592	63.214	12.6972	16.9779	0.88851	0.8955	0.1799	0.2405	0.01259	-23.1861428	-0.3284528
	5	0.601	0.601	1	0.365	70.592	77.37	13.5963	19.7023	1.35289	1.096	0.1926	0.2791	0.01916	-41.429199	-0.5868824
11	1	0.601	0.5	1	0	70.592	50	3.68483	4	0.008	0.7083	0.0522	0.0567	0.00011	12.899168	0.18272847
	2	0.601	0.601	1	0.014	70.592	51.014	3.82781	4.43328	0.05231	0.7227	0.0542	0.0628	0.00074	11.2645179	0.15957216
	3	0.601	0.601	1	0.24	70.592	67.998	4.00554	4.97183	0.10739	0.9633	0.0567	0.0704	0.00152	-6.4907356	-0.0919472
	4	0.601	0.601	0.623	0.5	70.592	87.5	5.30337	8.90465	0.50961	1.2395	0.0751	0.1261	0.00722	-31.6256339	-0.4480059
	5	0.601	0.601	0.12	0.5	70.592	87.5	16.5887	43.1026	4.00713	1.2395	0.235	0.6106	0.05676	-80.6063595	-1.1418625
12	1	0.601	0.5	1	0	70.592	10	11.0545	12	0.008	0.1417	0.1566	0.17	0.00011	37.529504	0.53163962
	2	0.601	0.601	1	0.014	70.592	10.203	11.4834	13.2998	0.05231	0.1445	0.1627	0.1884	0.00074	35.5535901	0.503649
	3	0.601	0.601	1	0.24	70.592	13.6	12.0166	14.9155	0.10739	0.1927	0.1702	0.2113	0.00152	29.9529233	0.42431045
	4	0.601	0.601	0.945	0.485	70.592	17.28	12.8517	17.446	0.19366	0.2448	0.1821	0.2471	0.00274	22.8210039	0.32328031
	5	0.601	0.601	0.628	0.5	70.592	17.5	15.2183	24.6177	0.43815	0.2479	0.2156	0.3487	0.00621	12.8178253	0.18157617
13	1	0.602	0.5	1	0	70.72	10	3.68912	12	0.2	0.1414	0.0522	0.1697	0.00283	44.83088	0.63392081
	2	0.602	0.5	1	0	70.72	10	3.68912	12	0.2	0.1414	0.0522	0.1697	0.00283	44.83088	0.63392081
	3	0.602	0.5	1	0	70.72	10	3.68912	12	0.2	0.1414	0.0522	0.1697	0.00283	44.83088	0.63392081
	4	0.602	0.5	1	0	70.72	10	3.68912	12	0.2	0.1414	0.0522	0.1697	0.00283	44.83088	0.63392081
	5	0.602	0.5	1	0	70.72	10	3.68912	12	0.2	0.1414	0.0522	0.1697	0.00283	44.83088	0.63392081

Scenario number	n	rho*	rho	OPTIMAL Theta	OT	Sales $	ABSOLUTE COSTS Labor	Material	Indirect	Warranty	COSTS/SALES Labor	Material	Indirect	Warranty	PROFIT	PROFIT MARGIN
14	1	0.602	0.5	1	0	70.72	50	11.0674	4	0.2	0.707	0.1565	0.0566	0.00283	5.45264	0.07710181
	2	0.602	0.5	1	0	70.72	50	11.0674	4	0.2	0.707	0.1565	0.0566	0.00283	5.45264	0.07710181
	3	0.602	0.5	1	0	70.72	50	11.0674	4	0.2	0.707	0.1565	0.0566	0.00283	5.45264	0.07710181
	4	0.602	0.5	1	0	70.72	50	11.0674	4	0.2	0.707	0.1565	0.0566	0.00283	5.45264	0.07710181
	5	0.602	0.5	1	0	70.72	50	11.0674	4	0.2	0.707	0.1565	0.0566	0.00283	5.45264	0.07710181
15	1	0.602	0.5	1	0	70.72	50	3.68912	12	0.04	0.707	0.0522	0.1697	0.00057	4.99088	0.0705724
	2	0.602	0.55	1	0	70.72	50	3.68912	12	0.04	0.707	0.0522	0.1697	0.00057	4.99088	0.0705724
	3	0.602	0.6	1	0	70.72	50	3.68912	12	0.04	0.707	0.0522	0.1697	0.00057	4.99088	0.0705724
	4	0.602	0.602	1	0.08	70.72	55.98	3.68912	12	0.04	0.7916	0.0522	0.1697	0.00057	-0.98864838	-0.0139798
	5	0.602	0.602	1	0.163	70.72	62.209	3.68912	12	0.04	0.8796	0.0522	0.1697	0.00057	-7.21784287	-0.1020623
16	1	0.602	0.5	1	0	70.72	10	11.0674	4	0.04	0.1414	0.1565	0.0566	0.00057	45.61264	0.64497511
	2	0.602	0.55	1	0	70.72	10	11.0674	4	0.04	0.1414	0.1565	0.0566	0.00057	45.61264	0.64497511
	3	0.602	0.6	1	0	70.72	10	11.0674	4	0.04	0.1414	0.1565	0.0566	0.00057	45.61264	0.64497511
	4	0.602	0.602	1	0.08	70.72	11.196	11.0674	4	0.04	0.1583	0.1565	0.0566	0.00057	44.4167343	0.62806468
	5	0.602	0.602	1	0.163	70.72	12.442	11.0674	4	0.04	0.1759	0.1565	0.0566	0.00057	43.1708954	0.61044818