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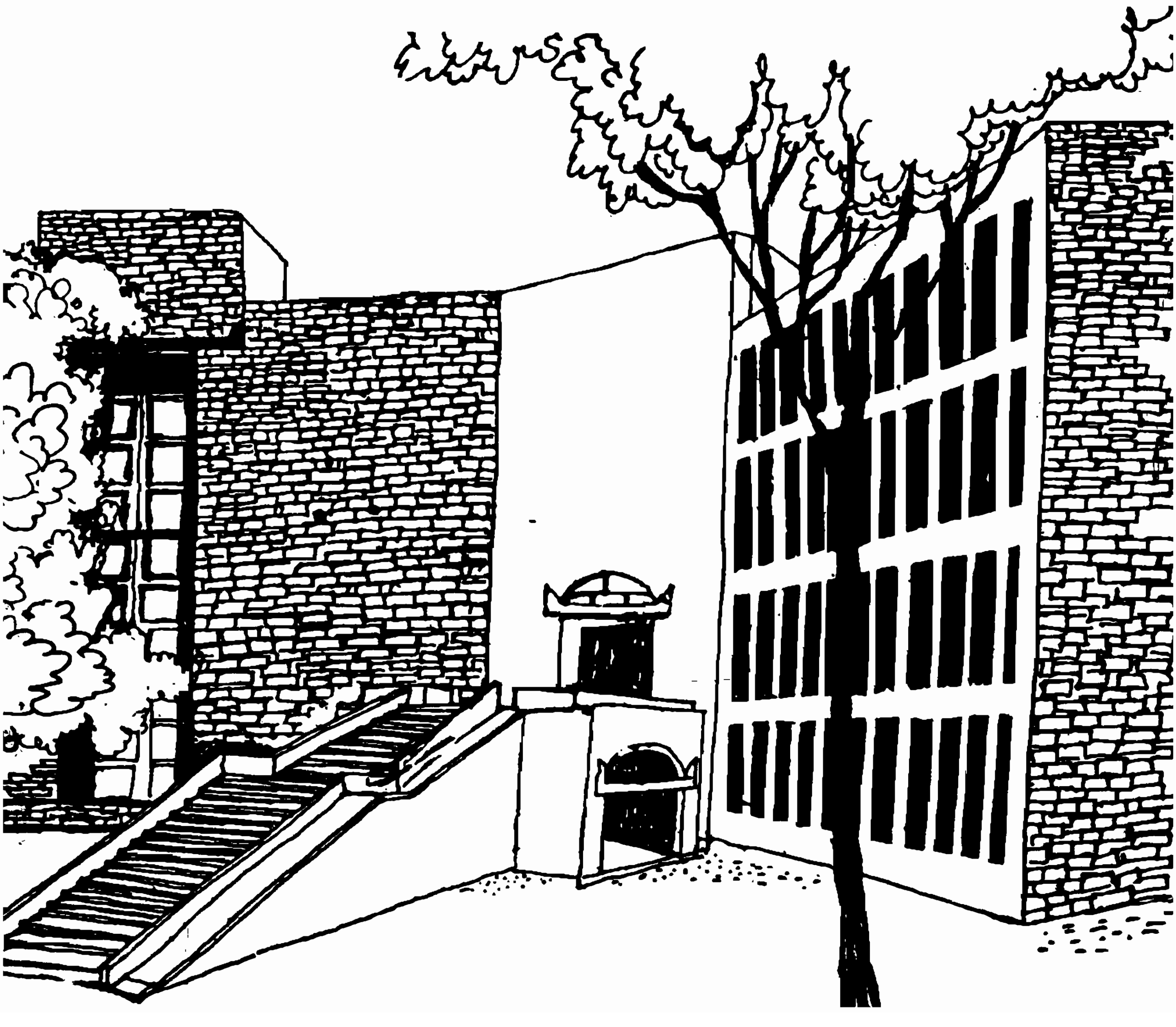

RATIONAL CHOICE CORRESPONDENCES IN SOCIAL CHOICE PROBLEMS

By

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Abstract

This paper grew out of a reading of an interesting exposition by Danilov and Sotskov [1997], where general one to one correspondences are established between binary relations and choice correspondences. Our purpose in this paper is to establish similar and more specific bijections in choice situations that arise in social choice theory. The kind of domain we consider here, has not been considered by Danilov and Sotskov [1997].

We consider social choice problems of the type discussed in the literature on axiomatic bargaining. Instead of choice functions, we consider choice correspondences i.e. multivalued solutions. Expositions of the main results in axiomatic bargaining can be found in Thomson [1994] and an economic interpretation of the problems can be found in Lahiri [1996]. Earlier forays into rational choice theory on such domains are those of Peters and Wakker [1991], and Lahiri [1998].

1. Introduction: This paper grew out of a reading of an interesting exposition by Danilov and Sotskov [1997], where general one to one correspondences are established between binary relations and choice correspondences. Our purpose in this paper is to establish similar and more specific bijections in choice situations that arise in social choice theory. The kind of domain we consider here, has not been considered by Danilov and Sotskov [1997].

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2. Some Preliminary Results: Let X be a universal set of alternatives. A nonempty subset S of X is called an issue. Let Σ be any non-empty set of issues. A choice correspondence C on Σ associates to each issue $S \in \Sigma$ a non-empty subset C(S) of S;

i.e. $C : \Sigma \dashrightarrow X$ such that

$\phi \neq C(S) \subset S \; \forall \, S \in \Sigma.$

A choice correspondence C on Σ is said to satisfy the Arrow Axiom (AA) if

$\forall S, T \in \Sigma, [S \subset T \; and \; C(T) \cap S \neq \phi\,] \rightarrow C(S) = C(T) \cap S.$

Given a choice correspondence C on Σ let

$R_c = \bigcup_{S \in \Sigma} [C(S) \times S]$ and $R_c^* = \bigcup_{S \in \Sigma} [C(S) \times (S \backslash C(S))].$

A choice correspondence C on Σ is said to satisfy the Weak Axiom of Revealed Preference (WA) if

$x, y \in X, (x,y) \in R_c \rightarrow (y,x) \notin R_c^*.$

It is said to satisfy Richter - Sen Weak Congruence Axiom (WCA) if given

$S\in\Sigma, y\in C(S), [x\in S \text{ and } (x,y)\in R_c] \rightarrow x\in C(S)$.

It is easy to see that (WCA) $\leftrightarrow$ (WA) $\rightarrow$ AA.

Say that Σ is closed under intersection if $S, T\in\Sigma \rightarrow S\cap T\in\Sigma$. If Σ is closed under intersection and C is a choice correspondence on Σ, then C satisfies (AA) if and only if C satisfies (WA).

We say that a choice function C on Σ is rational if it satisfies (WA).

Given C a choice correspondence on Σ let

$G(S,R_c) = \{x\in S: (x,y)\in R_c \ \forall\, y\in S\} \ \forall\, S\in\Sigma$.

<u>Observation 1</u>:- Let C be a choice correspondence on Σ. Then $\forall S\in\Sigma, C(S)\subset G(S,R_c)$.

<u>Proof</u>: Obvious.

Observation 2:- Let C be a choice correspondence on Σ and R a binary relation on X such that $\forall S \in \Sigma, C(S) = G(S,R)$. Then $C(S) = G(S,R_c)\ \forall S \in \Sigma$.

Proof:- Let $x \in G(S,R_c)$. Thus $(x,y) \in R_c\ \forall\ y \in S$.

Let $y \in S$. Thus $\exists T \in \Sigma : x \in C(T) = G(T,R)$ and $y \in T$

$\therefore (x,y) \in R$.

$\therefore x \in G(S,R) = C(S)$. Thus $G(S,R_c) \subset C(S)$.

Coupled with observation 1, we get observation 2.

Q. E. D.

Definition 1:- A choice correspondence C on Σ is said to be normal if $C(S) = G(S,R_c)\ \forall\ S \in \Sigma$.

Claim 1:- Let C be a rational choice correspondence. Then C is normal.

Proof:- Let $S \in \Sigma$ and $x \in G(S, R_c)$

$\therefore (x, y) \in R_c \forall y \in S.$

Suppose $x \notin C(S)$. Let $z \in C(S)$.

$\therefore (z, x) \in R_c^*$ and $(x, z) \in R_c$ contradicting rationality of C.

$\therefore x \in C(S)$. Thus $G(S, R_c) \subseteq C(S)$.

Coupled with observation 1, this proves the claim.

Q. E. D.

Definition 2:- A binary relation R is called pseudo-transitive if

$x, y \in S \in \Sigma, (y, z) \in R \forall z \in S$ and $(x, y) \in R \rightarrow (x, z) \in R \forall z \in S.$

Claim 2:- Let R be pseudo-transitive and let $G(S, R) \neq \phi$

whenever $S\epsilon\Sigma$. Let $C(S) = G(S,R)\ \forall\ S\epsilon\Sigma$. Then C is rational.

__Proof__:- Suppose $(x,y)\epsilon R_c$ and towards a contradiction assume

$(y,x)\epsilon R_c^*$

$\therefore \exists T\epsilon\Sigma$: $y\epsilon G(T,R)$ *and* $x\epsilon T\backslash G(T,R)$

$\therefore (y,z)\epsilon R \forall z\epsilon T$

But $(x,y)\epsilon R_c \rightarrow \exists S\epsilon\Sigma$: $x\epsilon G(S,R)$ *and* $y\epsilon S$.

$\therefore (x,y)\epsilon R$

Since R is pseudo transitive, we get $(x,z)\epsilon T \forall z\epsilon T$ $\therefore x\epsilon G(T,R)$ which is a contradiction.

$\therefore C$ is rational.

__Q.E.D.__

__Definition 4__:- A binary relation R is said to be star-shaped if $\phi \neq C(S) = G(S,R) \forall S \in \Sigma$ *implies* $R = R_c$

In general R_c as defined here is said to be the star-shaped part of R and $R_c \subset R$:

$(x,y) \in R_c \leftrightarrow \exists S \in \Sigma : x \in C(S)$ *and* $y \in S$

$\leftrightarrow \exists S \in \Sigma : x \in G(S,R)$ *and* $y \in S$

$\rightarrow (x,y) \in R.$

<u>Claim 3</u>:- Let C be a rational choice correspondence on Σ.

Then R_c is pseudo-transitive.

<u>Proof</u>:- Let $x, y \in S \in \Sigma$, $(x,y) \in R_c$ *and* $(y,z) \in R_c \forall z \in S$.

Suppose $y \notin C(S)$; *let* $\bar{z} \in C(S)$.

Thus $(\overline{z}, y) \in R_c^{*}$ and $(y, \overline{z}) \in R_c$ contradicting rationality of C.

$\therefore y \in C(S)$.

By (WCA), $x \in C(S)$

$\therefore (x, z) \in R_c \; \forall \; z \in S$.

$\therefore R_c$ is pseudo-transitive.

Q.E.D.

Theorem 1:- Let C be a choice correspondence on Σ. C is rational $\leftrightarrow \exists$ a binary relation R:

(a) $C(S) = G(S, R) \; \forall \; S \in \Sigma$

(b) R is pseudo transitive

(c) $R = R_c$.

Proof:- Suppose C is rational (and hence normal by Claim 1).

Thus $C(S) = G(S, R_c) \quad \forall S \in \Sigma$.

Set $R = R_c$

Clearly R_c is pseudo transitive by Claim 3.

On the other hand, suppose $\exists$ a binary relation R : (a), (b) and (c) hold. Towards a contradiction assume, $(x,y) \in R_c \wedge (y,x) \in R_c^*$ $\therefore \exists S \in \Sigma : x \in G(S,R) \wedge y \in S$. Further $\exists T \in \Sigma : y \in G(T,R) \wedge x \in T \setminus G(T,R)$.

$\therefore (y,z) \in R = R_c \ \forall \ z \in T \ and \ (x,y) \in R = R_c$. But $x \in T$. By pseudo transitivity of R, $(x,z) \in R \forall z \in T$. Thus $x \in G(T,R)$, which is a contradiction. Thus C is rational.

Q. E. D.

Theorem 1 is our understanding of a corresponding theorem in Danilov and Sotskov [1997]. However, since this theorem will be used in the sequel, we thought it was desirable to present it here. Before we close this section let us introduce one more axiom.

Given a binary relation R on X, let $R^{(1)} = R$ and for $\tau \in N$ $\tau \geq 2$, let $R^{(\tau)} = R^{(\tau-1)} R = \{(x,y) \in X x X : \exists\, z \in X$ with $(x,z) \in R^{(\tau-1)} \wedge (z,y) \in R\}$.

Let $T(R) = \bigcup_{\tau \in N} R^{(\tau)}$

Given a choice correspondence C on Σ say that it satisfies Houthakker's Axiom of Revealed Preference (HA) if $\forall x,y \in X,\ (x,y) \in T(R_c) \rightarrow (y,x) \notin R_c^*$.

Given a choice correspondence C on Σ say that it satisfies Richter-Sen's Strong Congruence Axiom (SCA) if $\forall x,y \in S \in \Sigma, [y \in C(S) \wedge (x,y) \in T(R_c)] \rightarrow x \in C(S)$.

It is an established result that $(HA) \leftrightarrow (SCA)$. Further $(HA) \rightarrow (WA)$.

Given a binary relation R on X, we say that

(i) R is complete if $x,y \in X, x \neq y \rightarrow [(x,y) \in R \vee (y,x) \in R]$.

(ii) R is reflexive if $x \in X \rightarrow (x,x) \in X$

(iii) R is transitive if $[(x,y) \in R \wedge (y,z) \in R] \rightarrow (x,z) \in R$

A binary relation satisfying the above properties is called an ordering.

Richter [1967, 1971] contains the following landmark result:

Theorem 2:- Given a choice correspondence C on Σ, C satisfies $(HA) \leftrightarrow \exists$ ordering R on X : $C(S) = G(S,R) \; \forall S \in \Sigma$.

A choice correspondence which satisfies (HA) is said to be strongly rational.

Let C be a choice correspondence on Σ. Then

$C(S) \subset G(S, T(R_c))\ \forall\ S \in \Sigma$. Given $S \in \Sigma$, let $C^{\cdot}(S) = G(S, T(R_c))$.

<u>Proposition 1</u>:- Let C be a choice correspondence on Σ. Then C satisfies (HA) only if $R_c = R_c^*$ Further if C is rational and $R_c = R_c^*$ then C satisfies (HA).

<u>Proof</u>:- $(x, y) \in R_c \rightarrow \exists\ S \in \Sigma\ ;\ x \in C(S) \wedge y \in S$

$\rightarrow \exists\ S \in \Sigma\ ;\ (x, z) \in R_c \forall z \in S \wedge y \in S$

$\rightarrow \exists\ S \in \Sigma\ ;\ (x, z) \in T(R_c) \forall z \in S \wedge y \in S$

$\rightarrow \exists\ S \in \Sigma\ :\ x \in C^{\cdot}(S) \wedge y \in S$

$\rightarrow (x, y) \in R_c^*$

$\therefore R_c \subset R_c\cdot$ (irrespective of whether C satisfies (HA)).

Now suppose C satisfies (HA).

$(x,y) \in R_c^* \leftrightarrow \exists\, S \in \Sigma : x \in C^*(S) \wedge y \in S$

$\rightarrow \exists\, S \in \Sigma : (x,z) \in T(R_c)\, \forall z \in S \wedge y \in S$

Suppose $x \notin C(S)$. Let $\bar{z} \in C(S)$. Thus $(\bar{z},x) \in R_c^* \wedge (x,z) \in T(R_c)$. This contradicts (HA). Thus $x \in C(S)$.

$\therefore (x,y) \in R_c$.

$\therefore R_c^* \subset R_c$.

Hence $R_c^* = R_c$.

Now suppose $R_c^* = R_c$. Towards a contradiction assume $\exists\, x,y \in X: (x,y) \in T(R_c) \wedge (y,x) \in R_c^*$. Thus there exists

$S\in\Sigma$: $y\in C(S)\wedge x\in S\backslash C(S)$. $\therefore (y,z)\in T(R_c)\forall z\in S$. Thus $y\in C^*(S)$.

But $(x,y)\in T(R_c)\wedge y\in C^*(S)\rightarrow x\in C^*(S)$.

Thus $(x,y)\in R_c{*}=R_c$. But $(x,y)\in R_c\wedge(y,x)\in R_c^*$ contradicts rationality of C. Thus C satisfies (HA).

<u>Theorem 3</u>:- Let C be a choice correspondence C on Σ and let C^* be defined as in Proposition 1. Then C is strongly rational $\leftrightarrow\exists$ a binary relation

R : (a) $C(S) = G(S,R)\forall S\in\Sigma$.

(b) R is pseudotransitive

(c) $R = R_c = R_{c^*}$

<u>Proof</u>:- Suppose C is strongly rational. Then by Theorem 1 and Proposition 1, (a), (b), (c) is satisfied.

Conversely suppose (a), (b), (c) is satisfied. Then by Theorem 1, C is rational. This coupled with $R_c = R_{c^*}$ and Proposition 1 implies, C is strongly rational.

<u>Q. E. D.</u>

3. <u>Rational Choice in Social Choice Domains</u>:- Let $n \in \mathbf{N}$, and $X = \mathbf{R}^n_+$. A social choice problem S is any nonempty subset of $\mathbf{R}^n_+$ satisfying the following properties:

(i) S is compact and convex,

(ii) S is comprehensive, i.e. $0 \leq x \leq y \in S \rightarrow x \in S$.

Let Σ now denote the set of all social choice problems.

Given $S \in \Sigma$, let $P(S) \equiv \{x \in S / y > x \rightarrow y \notin S\}$. P(S) is called the Pareto set of S.

We endow Σ with the Hausdorff topology: Let $\{S_\nu\}_{\nu \in \mathbf{N}}$ be a sequence in Σ and $S \in \Sigma$. We say that $\lim_{\nu \to \infty} S_\nu = S$ if and only if $S = \{x \in \mathbf{R}^n_+ / \forall v \in \mathbf{N}, \exists\, x_v \in S_v \wedge \lim_{v \to \infty} x_v = x\}$

We say that a choice correspondence C on Σ is closed if $\bigcup_{S \in \Sigma} [C(S) \times \{S\}]$ is closed in the product topology on $X x \Sigma$.

<u>Claim 4</u>:- Let C be a rational choice correspondence on Σ.

If C is closed then R_c is closed.

<u>Proof</u>:- Let $\{(x_v, y_v)\}_{v \in N}$ be a sequence in R_c with

$\lim_{v \to \infty} (x_v, y_v) = (x, y) \in X x X$

Let $S_v = cch\, \{x_v, y_v\} \in \Sigma\, \forall v \in \mathbf{N}$.

Clearly $x_v \in C(S_v)\ \forall v \in \mathbf{N}$ by rationality of C.

Let $S = cch\{x, y\} \in \Sigma$.

$\lim_{\nu \to \infty} S_\nu = S$.

Since C is closed, $x \in C(S)$.

$\therefore (x, y) \in R_c$.

<u>Q. E. D.</u>

<u>Claim 5</u>:- Let C be a rational choice correspondence on Σ.

If R_c is closed then C is closed.

<u>Proof</u>:- Let $\{S_\nu\}_{\nu \in N}$ be a sequence in Σ such that

$\lim_{\nu \to \infty} S_\nu = S \in \Sigma$ and let $x_\nu \in C(S_\nu)\ \forall\ \nu \in \mathbf{N}$. *Suppose*

$\lim_{\nu \to \infty} x_\nu = x$.

Let $y \in S$. Thus $\forall\, v \in \mathbb{N}, \exists\, y_v \in S_v$ such that $\lim_{v \to \infty} y_v = y$.

$\therefore (x_v, y_v) \in R_c \;\forall\, v \in \mathbb{N}$.

Since R_c is closed $(x,y) \in R_c$. This is true $\forall y \in S$. Since C is rational, $x \in C(S)$.

<u>Lemma 1</u>:- Let C be a choice correspondence on Σ such that $C(S) \subset P(S) \,\forall S \in \Sigma$. Further suppose C is rational and closed. If $(x,y) \in R_c$ then for any $z \in co\,\{x,y\}$ it follows that $(z,y) \in R_c$. (Here as else-where co. stands for convex hull).

<u>Proof</u>:- Suppose not. Thus there exists $z_0 \in co.\,\{x,y\}$ such that $z_0 \notin C\,(cch\{z_0,y\})$. (Here as else-where cch stands for comprehensive convex hull).

Now $x \in C\,(cch\{x,y\})$. Further $C(cch\{z_0,y\}) \subset co.\{z_0,y\} \subset co.\{x,y\}$

since $C(S) \subset P(S) \forall S \in \Sigma$. Thus since C is closed, there exists a neighborhood ϑ of z_0 such that $C(cch\{z,y\}) \cap \vartheta = \phi$ whenever $z \in co.\{x,y\}$. Thus there exists two nonempty disjoints subsets A and B of co. {x, y} such that $\bigcup_{z \in A} C(cch\{z,y\})$ and $\bigcup_{z \in B} C(cch\{z,y\})$ are separated by ϑ (: it once again follows from the closedness of C that for a given $z \in co.\{x,y\}, C(cch\{z,y\})$ cannot intersect both sides of ϑ simultaneously). Clearly there exists $\bar{z} \in A \cup B$ such that every neighbourhood of $\bar{z}$ intersects both A and B. Without loss of generality assume $\bar{z} \in A$. Then choose a sequence in B converging to A. We will then be contradicting the fact that C is closed.

Q. E. D.

Lemma 2:- Let C be a rational correspondence on Σ

such that $C(S) \subset P(S)\ \forall\ S \epsilon \Sigma$, and suppose C is closed. If $(x, t) \epsilon R_c \wedge t \epsilon co.\{y, z\}$ then, either $(x, y) \epsilon R_c \veebar (x, z) \epsilon R_c$.

<u>Proof</u>:- Let $a \epsilon C(cch\{x, y, z\})$. Since $C(S) \subset P(S)\ \forall S \epsilon \Sigma$, $a \epsilon co.\{x, y, z\}$. Now $t \epsilon co.\{y, z\} \rightarrow$ either $a \epsilon co.\{x, y, t\} \veebar a \epsilon co.\{x, z, t\}$.

Let $a \epsilon co.\{x, y, t\}$. Let $b \epsilon C(cch\{x, z, t\})$. Once again, $b \epsilon co.\{x, z, t\}$. Now co. {a, b} intersects co. {x, t} at 'c' say. $(a, b) \epsilon R_c \rightarrow (c, b) \epsilon R_c$ by Lemma 1. By rationality of C, $c \epsilon C(cch\{x, z, t\})$. Further $(x, t) \epsilon R_c \rightarrow x \epsilon C(cch\{x, t\})$ by rationality of C. Thus $(x, c) \epsilon R_c$. By rationality of C, $x \epsilon C(cch\{x, z, t\})$. Thus $(x, z) \epsilon R_c$.

Q. E. D.

Lemma 3:- Let C be a rational choice correspondence Σ such that $C(S) \subset P(S)\ \forall\ S \in \Sigma$. Then $(x,y) \in R_c$, $(y,t) \in R_c \forall t \in co.\{x,z\}$ implies $(x,z) \in R_c$.

Proof:- Let S = cch {x, y, z}

Let us show that $y \in C(S)$.

Now $C(S) \subset P(S) \subset co.\{x,y,z\}$.

Let us show that $(y,t) \in R_c \forall t \in co.\{x,y,z\}$.

Let $t \in co.\{x,y,z\}$. Then $\exists \bar{t} \in co.\{x,z\}$ such that $t \in co.\{y,\bar{t}\}$.

Now $(y,\bar{t}) \in R_c \rightarrow y \in C(cch.\{y,\bar{t}\})$ (by rationality of C.) Thus $(y,t) \in R_c$.

Hence $y \in C(S)$ by rationality of C.

But $(x,y) \in R_c$. Hence $x \in C(S)$ by rationality of C. Thus

$(x,z) \in R_c$.

<u>Q. E. D.</u>

Let R be a binary relation X. Say that R satisfies

(a) <u>Starness</u> if $(x,y) \in R, t \in co.\{x,y\} \rightarrow (x,t) \in R$

(b) <u>Reflexivity</u> if $(x,x) \in R \forall x \in X$

(c) <u>Almost Transitivity</u> if

$(x,y) \in R \wedge (y,t) \in R \ \forall \ t \in co.\{x,z\} \rightarrow (x,z) \in R$

(d) <u>Concavity</u> if

$(x,t) \in R \wedge t \in co.\{y,z\} \rightarrow$ [*either* $(x,y) \in R \ \underline{\vee} \ (x,z) \in R$].

(e) <u>Closedness</u> if R is closed.

We know that if C on Σ is a rational choice correspondence then R_c satisfies Starness and we have shown that if also $C(S) \subset P(S)\ \forall\ S \epsilon \Sigma$, then R_c satisfies almost transitivity. If in addition C is closed, then R_c satisfies concavity. Reflexivity of R_c is true if $C(S) \subset P(S)\ \forall\ S \epsilon \Sigma$, (simply take S = cch {x}). Further, $C(S) \subset P(S)\ \forall\ S \epsilon \Sigma$, implies the following:

$$x, y \epsilon X, x > y \rightarrow (x, y) \epsilon R_c.$$

<u>Lemma 4</u>:- Let C be a rational choice correspondence on Σ such that $C(S) \subset P(S)\ \forall\ S \epsilon \Sigma$, and suppose C is closed. Then $\forall S \epsilon \Sigma, C(S)$ is a convex set.

<u>Proof</u>:- Let $y, z \epsilon C(S)$ and suppose $x \epsilon co.\{y, z\}$.

By reflexivity $(x, x) \epsilon R_c$. Thus by concavity of R_c, either $(x,y)\epsilon R_c \veebar (x,z)\epsilon R_c$.

By rationality of C and the fact that $y,z\epsilon C(S)$, we get $x\epsilon C(S)$.

Q.E.D.

Theorem 4:- Let C be a choice correspondence on Σ such that $C(S)\subset P(S)\ \forall\ S\epsilon\Sigma$, Then C is convex valued, closed and rational if and only if $\exists$ a binary relation R on X satisfying (a), (b), (c), (d) and (e) such that $C(S) = G(S,r)\ \forall\ S\epsilon\Sigma$.

Proof:- Let C be convex valued, closed and rational. Put $R=R_c$. We have shown that R_c satisfies (a), (b), (c) and (d) above and by Claim 1, $C(S) = G(S,R_c)\forall S\epsilon\Sigma$.

Now suppose $\exists$ a binary relation R on X such that

$C(S) = G(S,R) \forall S \in \Sigma$ and R satisfies (a), (b), (c) and (d).

First let us show C is rational. Let $x, y \in X$ with $(x,y) \in R_c$.

Thus $\exists S \in \Sigma$ such that $x \in C(S)$ *and* $y \in S$. Towards a contradiction

assume $(y,x) \in R_c^*$. Thus $\exists T \in \Sigma$ such that $y \in C(T)$ and

$x \in T \setminus C(T)$.

$\therefore (y,z) \in R \forall z \in T$

But $(x,y) \in R$

Further $(y,t) \in R \forall t \in co.\{y,z\}$ since T is convex.

Thus $(x,z) \in R \forall z$.

Thus $x \in G(T,R) = C(T)$ which is a contradiction. Thus C is rational.

Now R is closed. Let $\{(x_\nu, y_\nu)\}_{\nu \in N} \subset R_c$ and suppose

$\lim_{v \to \infty} (x_v, y_v) = (x, y)$. Clearly $(x, y) \in R$ since R is closed and $R_c \subset R$.

By rationality of C, for $S_v = cch\{x_v, y_v\}$, $x_v \in C(S_v)$.

Further $\lim_{v \to \infty} S_v = S = cch\{x, y\}$.

Let $\bar{z} \in S$. Then $\exists \{z_v\}_{v \in N}$, $z_v \in S_v \forall v$ such that $\lim_{v \to \infty} z_v = \bar{z}$.

Now $(x_v, z_v) \in R \forall v$ and $\lim_{v \to \infty} (x_v, z_v) = (x, \bar{z}) \to (x, \bar{z}) \in R$, by closedness of R.

Thus, $x \in C(S)$.

$\therefore (x, y) \in R_c$.

Hence R_c is closed.

By Claim 5, C is closed. By Lemma 4, C is convex valued.

Q.E.D.

Theorem 5:- Let C be a choice correspondence on Σ such that $C(S) \subset P(S)\ \forall\ S \in \Sigma$. Then C is convex valued, closed and strongly rational if and only there exists a binary relation R on X satisfying (a), (b), (c), (d), (e) and $R_c = R_c{*}$ such that $C(S) = G(S,R)\ \forall S \in \Sigma$.

Proof:- Follows easily from Theorem 3 and 4.

Q.E.D.

4. Conclusion:- In this paper we have established necessary and sufficient conditions for rational choice correspondences for social choice problems. The social choice problems we consider are standard in the current literature on axiomatic bargaining, except that unlike the predominant situation, it is not necessary for such a problem to contain a strictly positive vector. This becomes necessary since we invoke reflexivity and closedness for our brand of rational choice. For basic results and notations used in this paper, a good source is Suzumura [1983]. It should be mentioned that Theorem 3 is definitely a new result in abstract rational choice theory. So is Proposition 1.

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