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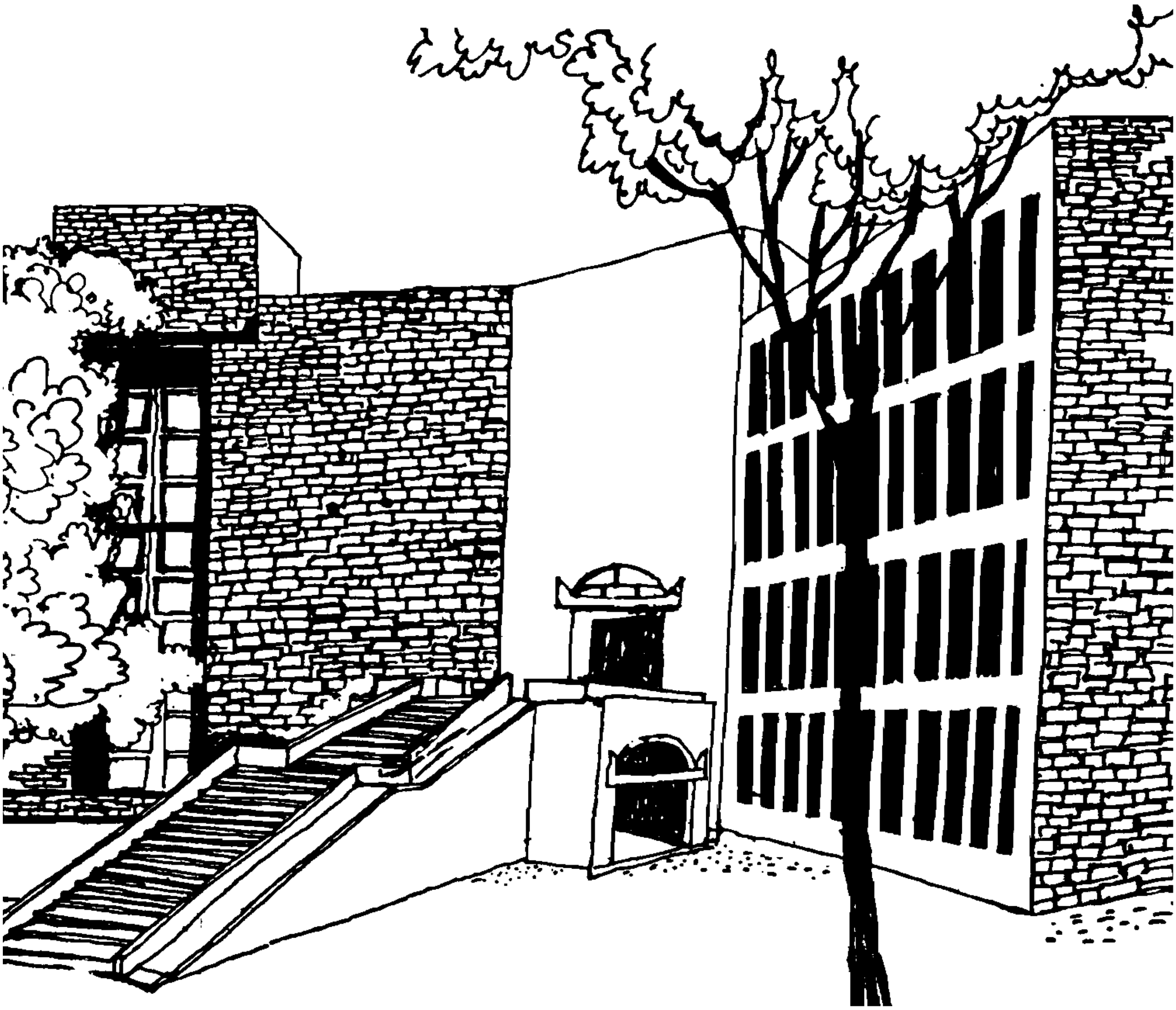

A MULTI-PERIOD OPTIMIZATION BASED DECISION SUPPORT SYSTEM FOR STRATEGIC AND OPERATIONS PLANNING

By

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A

Multi-Period

Optimization Based Decision Support System

for Strategic and Operations Planning

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ABSTRACT

We discusss how a generic multi-period optimization based decision support system (DSS) can be used for strategic and operational planning in a company with five fundamental elements, namely, Materials, Facilities, Activities, Times and Storage-Areas. This DSS which optimizes the company's activities over multiple-time horizon, having a multi-material, multi-facility, multi-activity system, requires little or no managerial knowledge of optimization techniques. This is the first reported attempt of an optimization based DSS in an integrated steel company in the USA with real world data. The result demonstrates significant profit and revenue improvement potential.

1. Introduction and Motivation

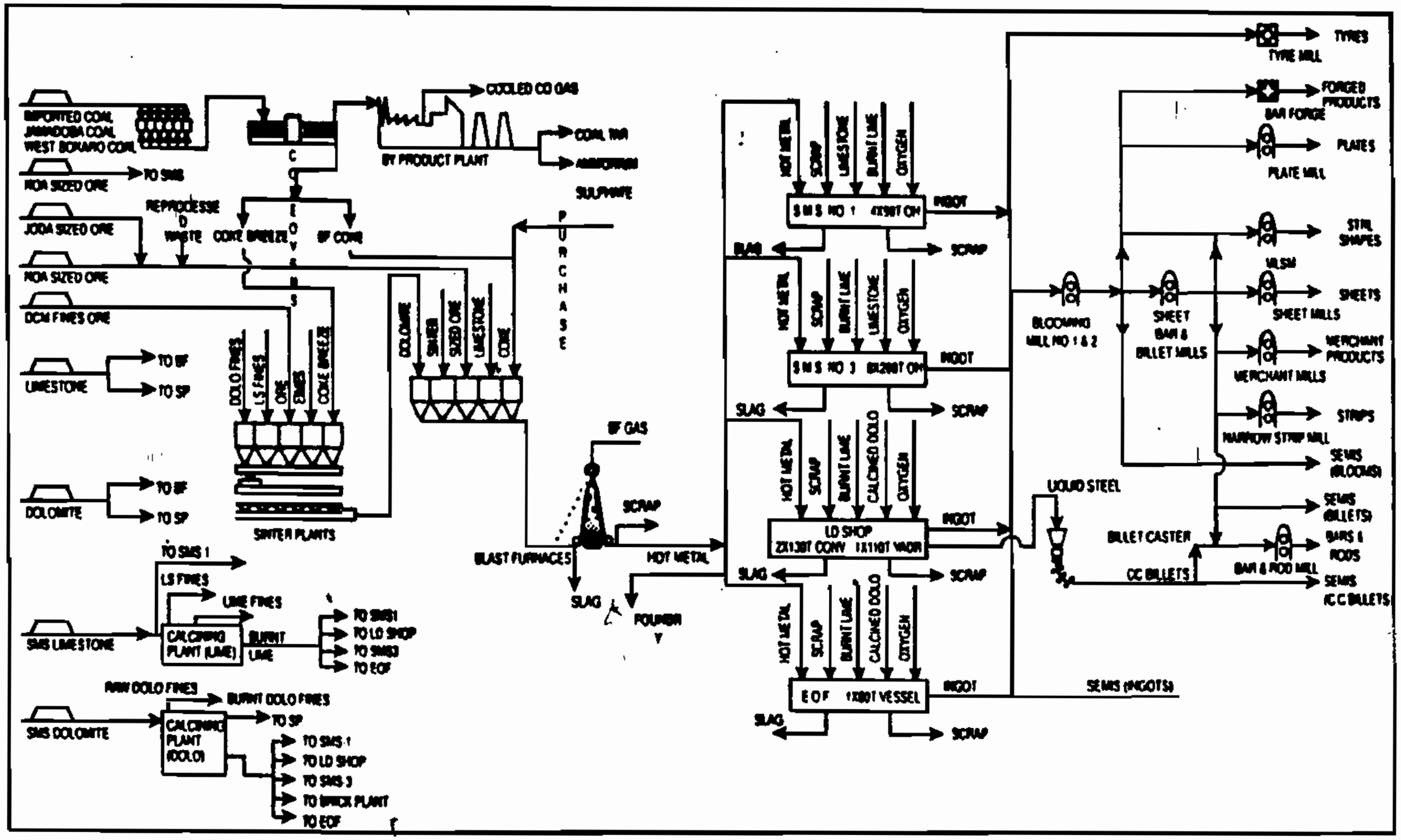
This research is motivated by our previous experience in modeling integrated steel plants in Asia and the North America. A series of publications (Dutta et. al.,1994; Sinha et. al., 1995; Dutta et. al , in review) reports the conceptualization, development and real-world implementation of one of the largest strategic planning models in India whose development took about 20 person years. This work resulted in a 58 percent increase in profitability (or a direct financial benefit of 73 million dollars) during the last six months of fiscal year 1986-87. The work has accrued similar proportional benefit in later years. During the same time an initiative at Northwestern University (Fourer, 1997) with the support of AISI (American Iron and Steel Institute) has resulted in the development of a generic model for strategic and operational planning in US steel plants. However in both cases, the models were single-period and did not discuss the effect of time varying parameters in the decision making.

The application of linear programming in integrated steel companies was first reported in Kaiser Steel Plant, USA (Fabian, 1958). Some other applications (Beale, Coen and Flowerdew, 1965; Bandopadhay, 1969; Bielfield, Klaus and Wartmann, 1986; Hanai, 1994, Sarma, 1995;Sharma and Sinha, 1990) have since been reported in UK, Japan, Germany, and other western countries. However, a comprehensive survey of mathematical programming models (Dutta and Fourer, 1996) indicates that very little work has been done in the area of multi-period linear programming models. *In North America, applications of linear programming are in short-term scheduling models in one facility.*

Against this background, we discuss in this paper, the development of a generic model that can be customized by any process industry in the world. We demonstrate that a simple deterministic model can have a significant impact on the bottom line of the company. In section 2, we discuss the basic approach of modeling a process industry. The elements of the database required to define the mathematical model are discussed in section 3. In section 4, we discuss the application of the model in a steel company in North America. The paper concludes with the scope of further work in section 5. The model formulation is discussed in Appendix.

2. Basic Approach to Modeling a Process Industry

The material flow diagram of an integrated steel industry is shown in Figure 1. From this material flow diagram, we see the basic structure of a steel or process industry. Normally raw materials (such as coal, ore and limestone) can only be bought, but finished products (pipes, axles, wheels) can only be sold. Intermediates can often neither be bought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SHEET BAR & BILLET MILLS
LIQUID STEEL
BILLET CASTER
CC BILLETS
TYRE MILL
TYRES
BAR FORGE
FORGED PRODUCTS
PLATE MILL
PLATES
STRL SHAPES
SHEET MILLS
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MERCHANT MILLS
MERCHANT PRODUCTS
NARROW STRIP MILL
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BAR & ROD MILL
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SEMIS (C C BILLETS)

Figure No. 1

nor sold. Practically all material can be inventoried. However, for practical considerations, the storage and inventory of hot metal, liquid steel is bounded by the mixer capacity. At any time, we can set products bought, sold or inventoried to zero to indicate that no buying, selling or inventorying is possible

For each material the model also specifies a list of conversions to other materials. Each conversion has a yield and cost at any time. Conversion takes care of recycling and will typically be used for recycling of gas, coke-oven gas and the scrap.

The production of any product is much more difficult than a simple conversion. We define a collection of facilities at which transformation occurs. At any time, each facility houses one or more activities, which use and produce material in certain proportions. We assume the production system to be continuously linear and hence we use linear models. The following information is provided for each unit activity at each facility at each time:

1. The amount of each input required for an activity
2. The amount of each output resulting from activity
3. The cost per unit of activity
4. Upper and lower limits on the number of units of each activity
5. The number of units of activity that can be accommodated by one unit of the facility's own capability. We call this the *Facility Activity Ratio.*

In defining an activity we have two different cases. First we take the rolling mill as an example where we produce various outputs like billets and bars. The rolling of each product is modeled as a separate activity, since each activity produces a separate output. The units of each activity are tons, but capacity of each facility is in hours. Thus the model specifies the Facility Activity Ratio as tons per hour. For example, the capacity of a rolling mill (for ingot production) may be 150 tons/hour. In one time period in one facility more than one activity is present. The other case is the blast furnace where in one facility only one activity is present. The production of liquid hot metal is an example of this type.

Another important factor in this modeling is the definition of time. We take the time unit to be flexible from one day to one year. For long term capital budgeting and business planning, we would use a year, month or quarter as the unit of time, whereas for the short term operational model, we use one week or one day as the unit of time. In long term planning or capital budgeting we need to calculate the discounted cash flow and the interest rate is an important factor.

3. Optimization Model Structure

We optimize a generalized network-flow linear program based on five fundamental elements: Materials, Facilities, Activities, Storage-Areas and Times. The details of the computer implementation is beyond the scope of this paper and would be discussed in a forthcoming paper by the same authors. The appendix discusses the formulation of the model.

3.1 Definitions

Times: These are the periods of the planning horizon, represented by discrete numbers (1, 2, 3 . . .N).

Materials: Any product in the steel company in any stage of production (input, intermediate, output) is considered to be a Material.

Facilities: A facility is a collection of machines which produces some materials from others. For example, a Hot Mill that produces sheets from slabs is a facility.

Activities: At any time, each facility houses one or more activities, which use and produce material in certain proportions. In each activity at each time, we have one or more input materials being transformed to various output materials. Production of hot metal at blast furnace, production of billets at rolling mill, pickling, and galvanizing at sheet mill are examples of activities.

Storage-Areas: These are the warehouses where raw materials, intermediate products, and finished products are stored.

The model is a generalized network-flow model that maximizes the contribution to profit (nominal or discounted) of a company subject to the following constraints for all time periods:

1. Material Balance
2. Facility Capacity (with soft capacity options as artificial variable)
3. Facility Input
4. Facility Output
5. Storage-Area Capacity

3.2 Assumptions

The model is based on the following assumptions.

1. There are several facilities, which are in series, in parallel, or in combination of series or parallel.

2. In each facility, there are either one or more than one activities.

3. There can be purchase, sale and storage of materials at the raw materials stage, at the finishing stage or at the intermediate processing stages.

4. The purchase price of raw materials, the selling price of finished goods, and the inventory carrying costs vary over time.

5. At any time, one or more materials are used as input and output in a facility. Generally more than one material is used to produce one product. The relative proportion of various inputs and outputs (generally called technological coefficients) in an activity remains the same in a period. Technological coefficients vary with time.

6. The capacity of each facility and each storage-area is finite.

7. Since the facilities will have different patterns of preventive maintenance schedules, the capacity of the machines will vary over period of time.

8. Essential features of the production planning problem can be captured in a deterministic, linear optimization model.

3.3 Implementation

The model is implemented in the powerful 4D relational database management system (core of this system). STEEL-TIME is the name of the structure of the database, which is shown in Figure 2. There are six steps in optimization: data collection and storage, constraint generation, variable generation, matrix generation, solution in XMP solver (Marsten, 1981), optimal result reading and display. This is described in Figure 3.

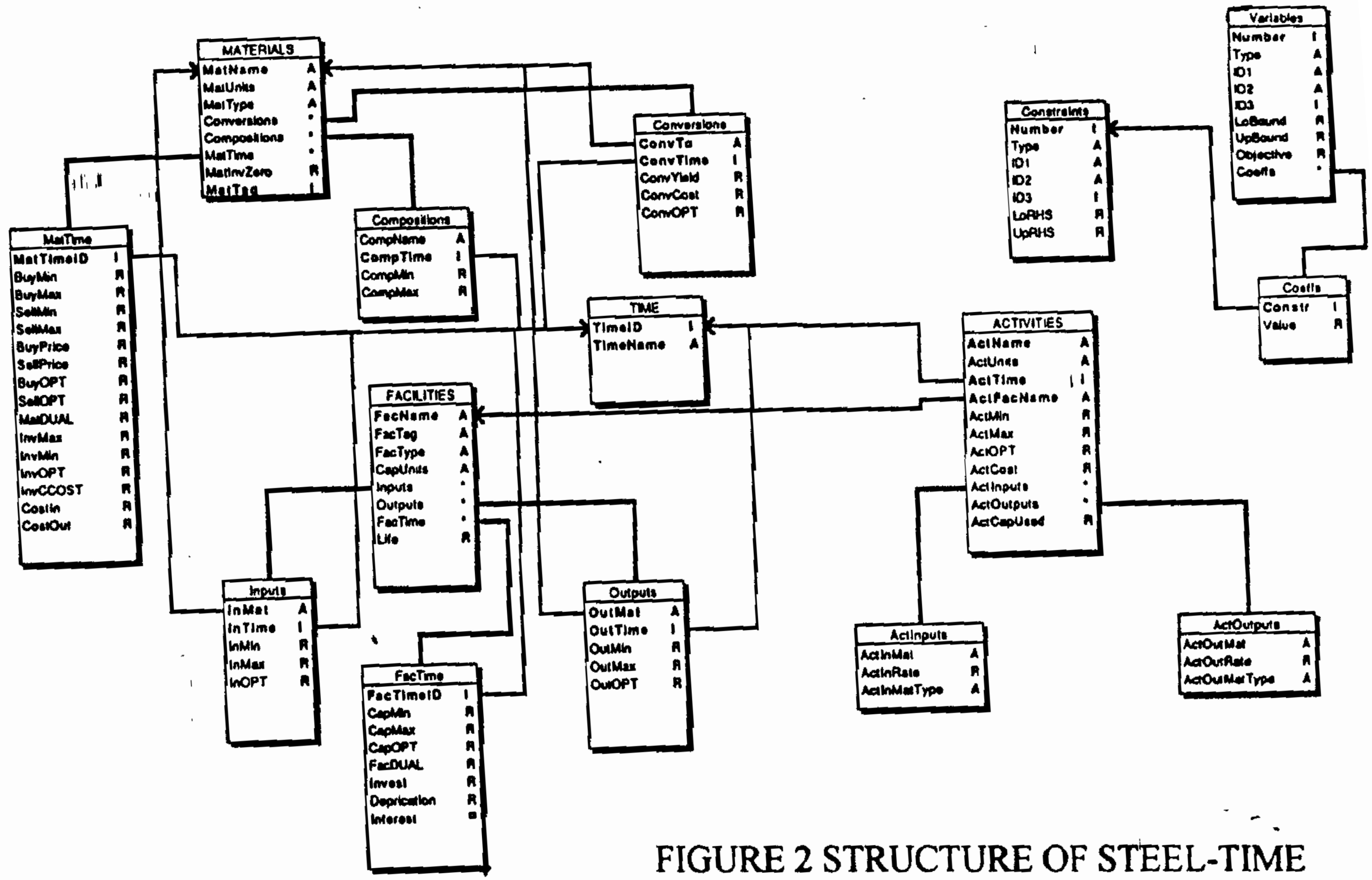

FIGURE 2 STRUCTURE OF STEEL-TIME

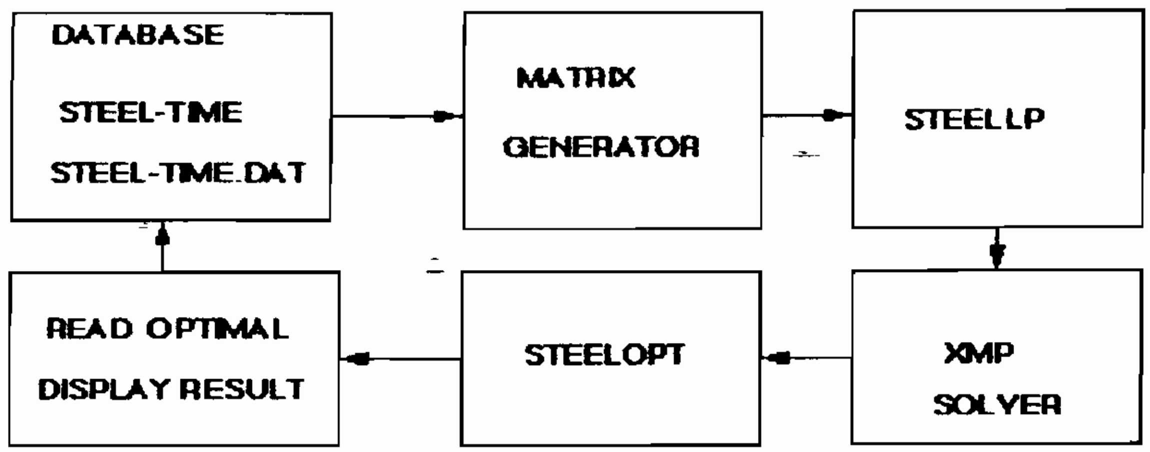

FIGURE 3 OPTIMIZATION STEPS

4. Application to an American Steel Company

We have customized our DSS for strategic and operational planning of an American steel plant. This is a 1.4 billion dollar (sales revenue per year) company producing 860,000 tons of marketable steel. This company has two manufacturing facilities, one located in the eastern part of the USA and another in the midwest. In this chapter we will discuss how we modeled the real world data of the company. We also discuss the difficulties involved and the impact the DSS can make in improving the bottom line of the company.

The specific questions we are interested in studying are:

1. What is the opportunity of increasing the profit of the company?
2. What are the facilities which have high dual price?
3. What are the products that need the attention of the management?

4.1 Converting the Company's Data to Model Data

The following agreement was reached with the company representatives regarding confidentiality of the data.

1. The financial figures of the company like the buy price, and sell price of the materials and the cost of an activity were multiplied by a factor. This factor was not disclosed to us.

2. The exact values of input and output materials of each facility, yields for each activity and the capacities of different facilities were supplied to us.

3. The actual names of the products and facilities were not disclosed, but the products and facilities were given code-names that were supplied to us.

4. The data indicating the yield, capacity and facility activity ratios were the annual average values for the previous year.

Considering all of the above points we would like to refer to data as the disguised real world data. In this chapter all the results are based on the disguised data.

Since the route of each product is different, the products at different stages were distinctly identified as different materials or different records in the [Materials] file. Let us suppose that a product P1 is processed at F4 and then at F5. Therefore, product P1 after facility F4 was identified as "P1 after F4" and the product after facility F5 was identified as "P1 after F5". Although the chemical composition of the material is the same in both cases, "P1 after F4" is a different material in the [Materials] file from "P1 after F5" (as they are physically different and their dimensions are different).

The facilities included electric steel making units, coiling machines, secondary steel making, continuous casting machines, hot mills, pickling machines, coating machines and finish trimmers. Typical activities were production of steel, production of slabs, hot rolling of slabs, cold rolling of slabs, trimming of sheets, and pickling of sheets.

We decided to keep units of all materials in tons. The capacity of each facility was in hours and capacity of each activity was also kept in tons. The facility activity ratio was thus expressed in tons per operating hour. For example, if 100 tons of cold rolled steel could be galvanized in 2 hours then the facility activity ratio was set to 50 tons per operating hour. The capacity of the facility, entered [Facilities]FacTime'CapMax was taken to be the expected operating hours available. The company did not supply a corresponding minimum number of operating hours, so [Facilities]FacTime'CapMin was set to zero.

It is expected that in a company where there is a wide variety of selling prices, yields of products, rolling rates and capacities, a linear programming based DSS will be more useful than where these figures are similar. The yield values of different activities varied between 81% and 100%. The ratio of maximum selling price to minimum selling price was 5.7. The similar ratio for raw materials was 126.87.

In this context, we would like to draw attention of previous study in a steel plant in India. In this Indian steel company, the yield values of different activities varied between 45 to 99 per cent. The ratio of maximum selling price to minimum selling price was 12.5. The similar ratio for purchase price of raw materials was 170. In this case, there was difficulty of getting the right data from multiple sources. (Dutta et. al.,1994). Moreover the reliability of each piece of data was in question. We had to use our own judgment to find whether the data were right or not right. So we had to do some amount of data filtering to find the exact price, yield, variable cost and other values.

However, with this American Company, we did not have to go through the process of data filtering. The company official who compiled the data did the data filtering and handed the data to us.

4.2 Issues in Modeling

In this section, we will discuss the project we did for this company. We will also discuss some concepts of modeling in this company.

We were supplied with the data for a single period model of the production and financial parameters. Although the DSS was multi-period, this steel company decided to test the DSS for one year and supplied the data for one period. Therefore issues regarding inventories and discounting could not be tested.

The [Materials] file had 443 records, 104 were marketable products. There were 58 facilities and 560 activities in a single time period describing the complete production details of 3 product-types and 23 product groups. These data translate to 1909 constraints and 2838 variables.

4.3 Activity Definition and the Scrap Rate

Let us consider a hot mill operating with 100 tons of SLAB as input and producing 96 tons of finished PRIME plates and 4 tons of SCRAP. After discussion with company officials we decided to treat 50% of the SCRAP as recoverable and 50% as not recoverable. Therefore the activity input rate (or [Activities]ActInputs'ActInRate) is 1.00 for SLAB, the activity output rate (or [Activities]ActOutputs'ActOutRate) is 0.96 for PLATE and 0.02 for SCRAP. In this case, the unit of activity is one ton of the input of SLAB getting rolled. In the context of material balance equations, this is the same as 1/0.96 tons of input producing 1 ton of PLATE and 0.02/0.96 tons of SCRAP. However, in this case the unit of activity is defined as production of one ton of plate or one ton of PRIME output.

In a generic model the definition of activity is flexible. We discussed the matter with the representatives of the company and decided that we would follow the practice of the company that one ton of PRIME output would be considered as the unit of activity.

4.4 Multiple Processing in the Same Facility

The second issue encountered in this case study is that of multiple processing in the same facility. This is explained in Figure 4 which shows the facility CM1 is used twice (once after P1 and another time after P4). In this case the same Product-Type CS7 is processed twice on the same machine CM1. This can be incorporated in the model, by defining two different activities at two stages: Stage 1 and Stage 2. In the first stage, the input material is CS-7 after P1 and output material is CS7 after CM1/CM2/CM5-ST1. In stage-2, the input material is CS-7 after P4 and output material is CS-7 after CM1/CM2/CM5-ST2. So the two stages of the same product are competing for the same facility capacity.

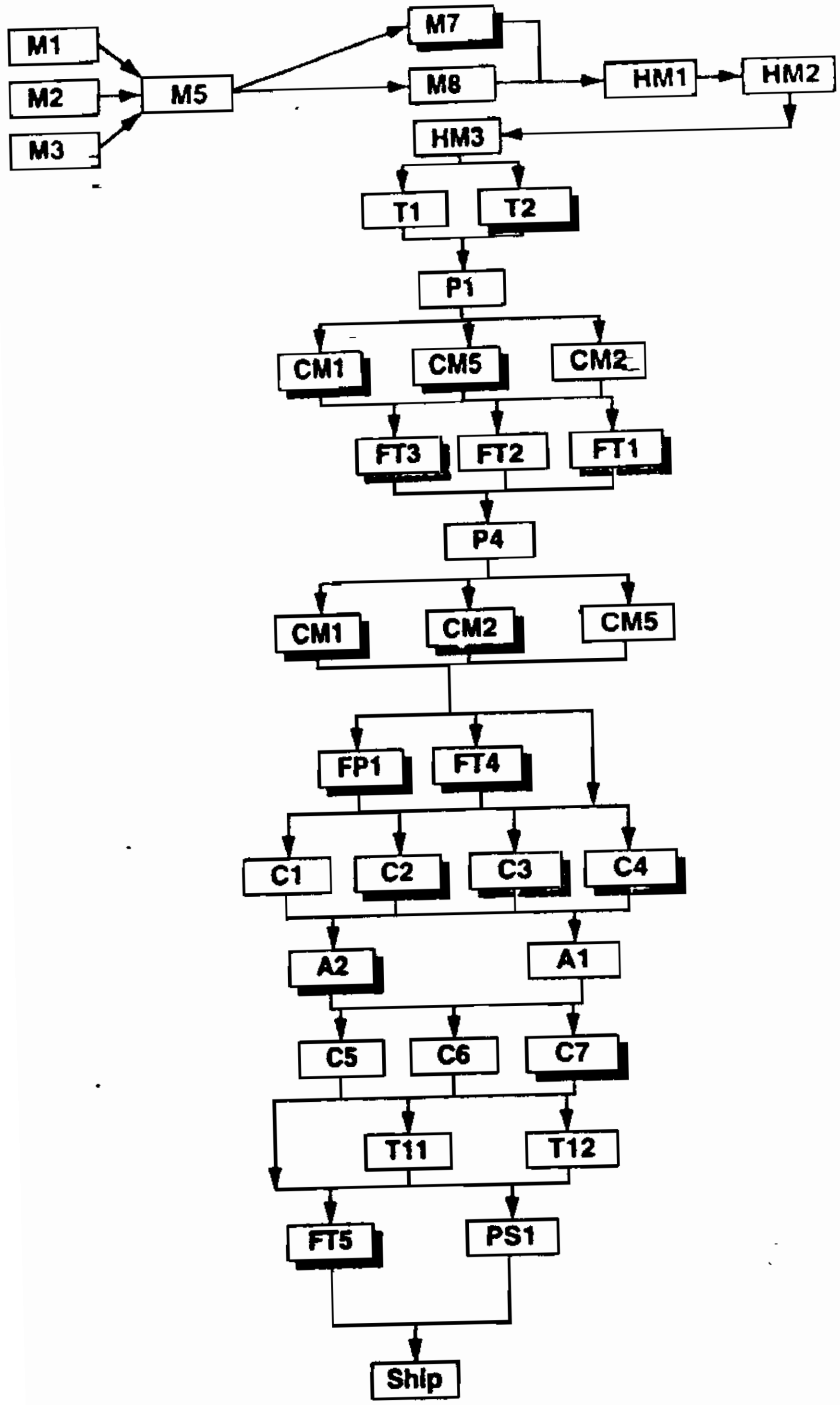

FIGURE 4 ROUTING OF ONE PRODUCT

4.5 Modeling Re-treatment

Let us consider a facility where an activity takes 100 tons of input producing 100 tons per hour of PRIME product. Since all products are good there is no re-treatment and the capacity of the activity is not affected. If the capacity of the machine is (100+X) tons per hour, (100+X) tons of PRIME products will be produced. This is shown in the Figure 5.

Now let us consider that we have re-treating. In this case, in one hour, (100+X) tons of product as input produces 100 tons of output of PRIME product and X tons of product as non-PRIME (or DEFECTIVE) which must be re-processed. This is illustrated in the Figure 6. After re-processing, 100 tons of PRIME product yields (100-X) tons of PRIME that was never re-treated, and X tons of non-PRIME which was retreated and will be converted to PRIME. In this case (Figure 7), although the capacity of the facility is (100+X) tons per hour, it is working at 100 tons of PRIME output per hour. This is equivalent to reducing the capacity of the machine from (100+X) tons per hour to 100 tons per hour. So the re-treatment can be effectively handled by multiplying the capacity by a factor of 100/(100+X). For practical purposes, when the value of X is small, we can approximate a non-linear system by a linear system. We have 443 materials, 58 facilities and 631 activities. The data in the database translate to 1909 constraints and 2838 variables. It takes about 25 minutes to go through all the six steps in PowerMac 9200 (90MHz). Solution time will further reduce after compilation of the database and with the latest versions of Macintosh. We discussed several issues with the company officials to match our work to suit their needs. Example points follow.

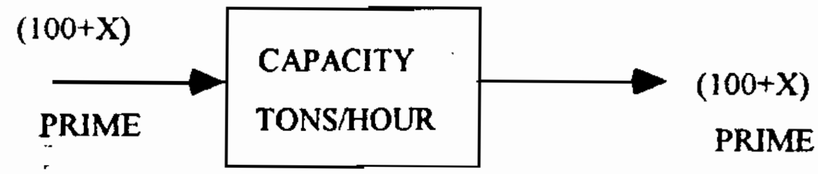

FIGURE 5

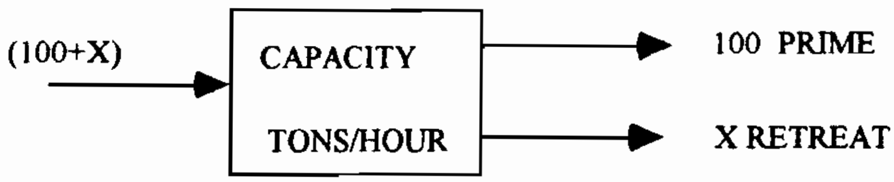

FIGURE 6

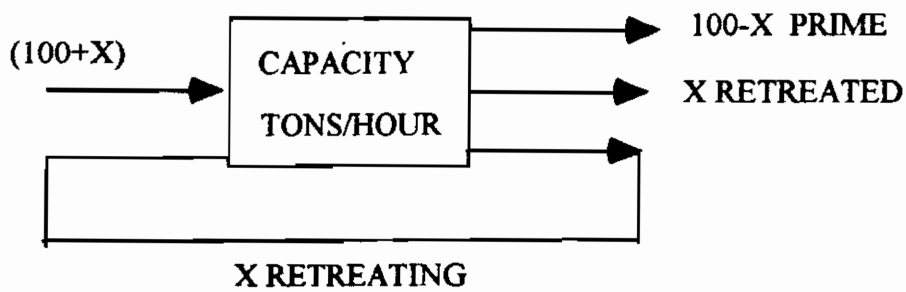

FIGURE 7

4.6 Contact Points

To check that the model represents reality, contact points have been identified. These points are functions of the variables in the model and at the same time measurable quantities in real life. We consider the following figures and their respective units:

1. Total production figures of the steel making units (tons)
2. Production figures of the hot mills (tons)
3. Total production of *Product Group ES* (tons)
4. Total production of *Product Group SS* (tons)

5. Total production of *Product Group CS* (tons)

6. Total revenue (dollars)

7. Total cost of purchases (dollars)

8. Total cost of activities (dollars)

9. Total profit (dollars).

4.7 Experiments on the Model and Results

We took the actual production limits for the company for financial year 1994 and found the optimal result. Then we removed the production limits and found the optimal result. We considered optimal results for four different cases, which are as follows:

Case 1: With the company's upper and lower bounds

Case 2: With the company's upper bounds and the lower bounds removed

Case 3: With the company's lower bounds and the upper bounds removed

Case 4: With the company's lower and upper bounds both removed

The results of the four different cases were tabulated and listed in Table 1. These results clearly demonstrate that:

1. When all the bounds of the company are removed, the net revenue of the company is increased by 31 percent. In this time the cost of purchases increases by 52 percent and the cost of activity increases by 11 percent giving a profit improvement of 30 percent. During this period, the production of the company also increases from 869685 to 973154 tons.

2. Since all the figures of production, profit and revenue have gone up from case1 to case 4, it would not be right to compare the total figures. The optimal results were converted to the per unit results. Lower portion of Table 1 displays the impact of optimization on the basis of per ton of liquid steel. *It shows a potential of 16 percent profit improvement per ton of steel and 17 percent revenue improvement per ton of steel.*

Table 1
Impact of Optimization-I

VARIABLES	Unit	CASE 1	CASE2	CASE 3	CASE 4	Change
1. Revenue	Million Dollar	977.86	1037.07	1176.56	1289.20	31.84
2. Cost of Purchase	Million Dollar	270.30	306.50	322.46	412.84	52.74
3. Cost of Activities	Million Dollar	245.80	250.40	290.40	274.20	11.55
4. Net Profit	Million Dollar	461.76	480.17	563.70	602.16	30.41
5. Total Steel	Tons	869685	906229	972203	973154	11.90
6. Total Hot Mill Prodn.	Tons	638109	636652	697548	573793	-10.08
7. Total ES Production	Tons	422601	451451	351127	304504	-27.95
8. Total CS Production	Tons	235516	205905	284150	203738	-13.49
9. Total SS Production	Tons	211568	463111	336744	463111	118.89
1. Unit Revenue	(Dollar/ton)	1124.38	1144.38	1210.20	1324.77	17.82
2. Unit Cost of Purchase	(Dollar/ton)	310.80	338.22	331.68	424.23	36.50
3. Unit Cost of Activities	(Dollar/ton)	282.63	276.31	298.71	281.76	-0.31
4. Unit Net Profit	(Dollar/ton)	530.96	529.86	579.82	618.78	16.54

Case 1 — With the Company's Upper and Lower Bounds
Case 2 — With the Comapany's Upper Bound but Lower Bounds Removed
Case 3 — With the Companys Lower Bound ,but Upper Bounds Removed
Case 4 — With the Company's both Upper and Lower Bounds Removed
Change — Indicates the percentage improvement of Case 4 Over Case 1

Bounds are limits on the production of the company.

Case 4 is the desired goal of the company

Case 4 have 11% higher production than Case 1, profit and revenure are up by 30% and 31 %

With Optimization, profit per ton increases by 16% and Revenue per ton by 17%

4.8 Relaxing the Bounds

We also found the final products for which optimal values were at their upper limits ([Materials]MatTime'SellOPT= [Materials]MatTime'SellMax). We increased the upper bound by 5 % for each final product. We re-ran the optimization model and noted the optimal results of the model. The same procedure was repeated twice more with an additional 5% increase each time. The results of this study are shown in Table 2.

These results show that with the same level of liquid steel production, the profit and revenue can be improved by changing the [Materials] MatTime'Sellmax limit of the company. In other words, even if the same amount of steel is made in the steel melting shop, optimization at the finishing mills can have a substantial impact on the organization. *By increasing the company's upper bounds by 15.7 % the profit increases by 1.1% and revenue increases by 0.5% without any change in cost of purchases. This improvement comes entirely from using different production routes through the plant.*

4.9 Critical Facilities

We found the bottleneck areas of different facilities. We found that the facilities HM3 and T3 were the bottlenecks having the highest dual values. They would be the best candidates for further investment.

4.10 Comparison with Indian Steel Plant Case Study

This case study demonstrates that a *simple deterministic linear programming model can have significant impact in an integrated steel company in North America.* In this case, we draw a comparison with our previous experience in India. Whereas the primary focus of the work done in the USA is in the area of product -mix optimization, the primary focus of the work in India was in the area of optimal power distribution. In the Indian case of optimal power distribution, profit actually obtained (and verified by annual reports) with an optimization model was 2.25 times that of profit without an optimization model. The production of steel improved by 42 percent. This resulted in a per ton profit improvement of 58 percent. In the case of the American steel company the profit and revenue improvement potentials are only 16 and 17 percent respectively. **However for a company of 1.4 billion dollars in annual revenue, this is a very significant benefit.**

Table 2

VARIABLES	Unit	CASE 1	CASE2	CASE 3	CASE 4	Change
1. Revenue	Million Dollar	977.86	979.61	981.49	983.36	0.56
2. Cost of Purchase	Million Dollar	270.30	270.30	270.30	270.30	0.00
3. Cost of Activities	Million Dollar	245.80	245.93	246.07	246.21	0.17
4. Net Profit	Million Dollar	461.76	463.38	465.07	466.84	1.10
5. Total Steel	Tons	869685	869685	869685	869685	
6. Total Hot Mill Prodn.	Tons	638109	638109	638109	638109	
7. Total ES Production	Tons	422601	422601	422601	422601	
8. Total CS Production	Tons	235516	235516	235516	235516	
9. Total SS Production	Tons	211568	211568	211568	211568	

Case 1	With the Company's Upper and Lower Bounds (market limit normal)
Case 2	With the Company's Upper and Lower Bounds (market limit 5% increased over case1) *
Case 3	With the Company's Upper and Lower Bounds (market limit 5% increased over case 2) *
Case 4	With the Company's Upper and Lower Bounds (market limit 5% increased over case 3) *
Change	Indicates the percentage improvement of Case 4 Over Case 1

* We increase the limits of those products which have positive dual price

With the same liquid steel and the same hot mil production, the profit increases with changing market bounds

The mix of raw-materials the total purchase cost of raw materails is same in all four cases

PRODUCTS THAT NEED ATTENTION OF MANAGEMENT

1. ES5 after SP2
2. ES5 after T8/T10
3. ES5 after P8
4. ES5 after T456

5. CS4 after CM6
5. CS4 after FT5/PS7
7. CS4 after CM1/2/7

(ALL COST, REVENUE DATA have been multiplied by a factor to maintain confidentiality)

5. Extension for Future Research

In this paper, we have studied a generic model for process industries, which is multi-material, multi-facility, multi-activity and which optimizes the net profit (nominal/discounted) of the company subject to the constraints of the industry. This problem can be visualized as single period multi-scenario or multi-period single scenario. We would like to extend it to a multi-period, multi-scenario model. This requires that we define the variables as stochastic with probability distributions. This will be harder problem as the constraint generation and variable generation time will increase proportionally with the number of scenarios. Moreover, we need to study the issues related to the interface of database and optimization in the stochastic case as well as in the case of the following extensions.

A second extension of the model will be non-linearity of the model. Most of the industrial cost curves are non-linear or at best can be represented as having a piece-wise linear behavior. It will be interesting to study how to represent this non-linearity while retaining the model's user-friendliness.

A third extension of the model will be to have multiple objective linear programs and represent them in the database. This can be done by changing the model management system. For example, the current model can be changed to cost minimization, revenue maximization, maximization of marketable products (revenue or production), maximization of the utilization of the facilities etc. It is possible to have a menu driven program in this DSS which optimizes over different objectives.

A fourth and interesting extension will be to study the paradigm neutrality (Geoffrion, 1989) of this data structure for the multiple period model. Although the model is designed for the mathematical programming paradigm, we can extend it for inventory control and also for scheduling, vehicle routing and queuing applications. We have parameters for all materials at all times. We can determine the ordering and holding cost for all material and hence try to find optimal order quantities. However, the batch size will be decided by practical consideration like the heat size of the steel making shop, the capacity of the vehicle carrying the products and the capacity of the loading and unloading facility. Given that we have the batch size and lead-time of all materials produced the present model can be extended to a scheduling model of each product in each time.

Appendix-1

Optimization Model

As explained earlier, the model has the five fundamental elements: Materials, Facilities, Activities, Storage-Areas, and Times. The model is a generalized network flow model with the objective of maximizing the net profit the company. The user has a choice of changing the objective function from maximizing the net profit of the company to maximizing the net discounted profit of the company. The optimization is performed with the following constraints:

1. Material Balance
2. Facility Inputs
3. Facility Outputs
4. Facility Capacity
5. Storage Capacity
6. Storage Total

In addition to the above constraints, the each variable of the model (like the amount you can buy in particular period) is bounded by a upper bound and a lower bound.

A.1 Times Data

T is the set of times planning periods indexed by t

N= Planning Horizon

T = {1, 2, 3, ...N}

int = The interest rate between two time periods.

A.2 Materials Data

M is the set of materials indexed by j

A is the set of chemical constituents (like C, Si, Fe, FeSi etc.) indexed by α

l_{jt}^{buy} = the lower limit of purchases of material j for each j $\in M$ and t $\in T$

u_{jt}^{buy} = the upper limit on purchases of material j for each j $\in M$ and t $\in T$

c_{jt}^{buy} = the cost per unit of material j purchased for each j $\in M$ and t $\in T$

l_{jt}^{sell} = the lower limit on sales of material j for each j $\in M$ and t $\in T$

u_{jt}^{sell} = the upper limit on sales of material j for each j $\in M$ and t $\in T$

c_{jt}^{sell} = the revenue per unit of material j for each j $\in M$ and t $\in T$

l_{jt}^{inv} = lower limit of inventory of material j for each j $\in M$ and t $\in T$

u_{jt}^{inv} = upper limit on inventory of material j for each j $\in M$ and t $\in T$

h_{jt} = holding cost of the material j at time t for each j $\in M$ and t $\in T$

I_{j0}^{inv} = initial inventory of the material j for each j $\in M$

$M^{conv} \subseteq M \times M$ is the set of conversions

(j,j') $\in M^{conv}$ means that material j can be converted to material j', and j # j'

$a_{jj't}^{conv}$ = number of units of material j' that result from converting one unit of material j for each (j,j') $\in M^{conv}$ and t $\in T$

$c_{jj't}^{conv}$ = cost per unit of material j of conversion from j to j' for each (j, j') $\in M^{conv}$ and t $\in T$

$M^{comp} \subseteq M \times A$ is the set of compositions

(j, α) $\in M \times A$ means Material j has constituent α for each j $\in M$ and $\alpha \in A$

$Comp_{\alpha jt}^{(\min)}$ = Minimum composition of the constituent α for each (j, α) $\in M^{comp}$ and t $\in T$

$Comp^{(\max)}_{\alpha jt}$ = Maximum composition of the constituent α for each $(j, \alpha) \in M^{comp}$ and t $\in T$

A.3 Facilities Data

F is the set of facilities indexed by i

l^{cap}_{it} = the minimum amount of the capacity of facility i that must be used for each i $\in F$ and t $\in T$

u^{cap}_{it} = the maximum amount of the capacity of facility i that must be used for each i $\in F$ and t $\in T$

$F^{in} \subseteq F \times M$ is the set of facility inputs:

(i,j) $\subseteq F^{in}$ means that material j is used as an input at facility i

l^{in}_{ijt} = the minimum amount of material j that must be used as input to facility i, for each (i,j) $\in F^{in}$ and t $\in T$

u^{in}_{ijt} = the maximum amount of material j that may be used as input to facility i, for each (i,j) $\in F^{in}$ and t $\in T$

$F^{out} \subseteq F \times M$ is the set of facility outputs:

(i,j) $\subseteq F^{out}$ means that material j is produced as an output at facility i

l^{out}_{ijt} = the minimum amount of material j that must be produced as output from facility i for each (i,j) $\in F^{out}$ and t $\in T$

u^{out}_{ijt} = the maximum amount of material j that may be produced as output from facility i for each (i,j) $\in F^{out}$ and t $\in T$

C^{vend}_{it} = the cost of vendoring (outsourcing) a unit of capacity of facility i at time t.

A.4 Activities Data

F^{act} is the set of activities indexed by k

$(i,k) \in F^{act}$ means that k is an activity available at facility i

l^{act}_{ikt} = the minimum number of units of activity k that must be run at facility i for each $(i,k) \in F^{act}$ and $t \in T$

u^{act}_{ikt} = the maximum number of units of activity k that may be run at facility i for each $(i,k) \in F^{act}$ and $t \in T$

c^{act}_{ikt} = the cost per unit of activity k at a facility i, for each $(i,k) \in F^{act}$ and $t \in T$

r^{act}_{ikt} = the number of units of activity k that can be accommodated by one unit of capacity at facility i for each $(i,k) \in F^{act}$ and $t \in T$

$A^{in} \subseteq \{(i,j,k,t): (i,j) \in F^{in}, (i,k) \in F^{act} \text{ and } t \in T\}$ is a set of activity inputs

$(i,j,k,t) \in A^{in}$ means that input material j is used by activity k at facility i at time t

a^{in}_{ijkt} = the number of units of input material j used by one unit activity k at facility i for each $(i,j,k,t) \in A^{in}$

$A^{out} \subseteq \{(i,j,k,t): (i,j) \in F^{out}, (i,k) \in F^{act} \text{ and } t \in T\}$ is a set of activity outputs

$(i,j,k,t) \in A^{out}$ means that output material j can be produced by activity k at facility i at time t

a^{out}_{ijkt} = the number of units of output material j that can be produced by one unit of activity i at time t for each $(i,j,k,t) \in A^{out}$.

A.5 Storage-Areas Data

S is the set of Storage-Areas indexed by s.

l^{inv}_{st} = lower limit of the material stored in Storage-Area s at time t for each $s \in S$ and $t \in T$

u^{inv}_{st} = upper limit of the material stored in Storage-Area s at time t for each $s \in S$ and $t \in T$

A.6 Variables

x_{jt}^{buy} = units of material j bought at time t for each j $\in M$ and t $\in T$

x_{jt}^{sell} = units of material j sold at time t for each j $\in M$ and t $\in T$

x_{jt}^{inv} = units of material j inventoried at time t for each j $\in M$ and t $\in T$

x_{j0}^{inv} = inventory of the material j at time 0 for each j $\in M$

x_{jst}^{inv} = units of material j inventoried at time t in storage s for each j $\in M$, t $\in T$ and s $\in S$

$x_{jj't}^{conv}$ = units of material j converted to material j' for each (j,j') $\in M^{conv}$ and t $\in T$

x_{ijt}^{in} = units of material j used as an input by facility i for each (i,j) $\in F^{in}$ and t $\in T$

x_{ijt}^{out} = units of material j produced as an output by facility i for each (i,j) $\in F^{out'}$ and t $\in T$

x_{ikt}^{act} = units of activity k operated at facility i for each (i,k) $\in F^{act}$ and t $\in T$

x_{it}^{vend} = units of capacity vendored / outsourced at facility i at time t for each i $\in F$ and t $\in T$

A.7 Objective Function

The objective of this model is to maximize revenue from sales, less the cost of purchasing, converting, running activities, vendoring and holding inventories over all periods of time.

$$Z(t) = \sum_{J \in M} c_{jt}^{sell} x_{jt}^{sell} - \sum_{J \in M} c_{jt}^{buy} x_{jt}^{buy} - \sum_{(j,j')M^{conv}} c_{jj't}^{conv} x_{jj't}^{conv} - \sum_{(i,k) \in F^{act}} c_{ikt}^{act} x_{ikt}^{act} - \sum_{j \in M} h_{jt} x_{jt}^{inv} - \sum_{i \in F} c_{it}^{vend} x_{it}^{vend} \quad (1)$$

$$Z = \sum_{t \in T} Z(t) \quad (2)$$

The first term in equation (1) is the revenue from sales. The second, third and fourth terms are the cost of purchasing, conversion and activities respectively. The fifth term is the inventory holding cost. The last term is the outsourcing cost. Equation (2) is the sum of Equation (1) over all periods of Time. Constraints The various constraints for this model are described next.

A.8 Constraints:

We now describe the various constraints

Material Balance

For all j $\in M$ and t $\in T$ the amount of material j made available by purchases, production and conversions and inventory must equal the amount used for sales , production, conversions and inventory:

$$x_{jt}^{buy} + \sum_{(i,j) \in F^{out}} x_{ijt}^{out} + \sum_{(j',j) \in M^{conv}} a_{j'jt}^{conv} x_{j'jt}^{conv} + x_{j,t-1}^{inv} =$$

$$\sum_{(i,j) \in F^{in}} x_{ijt}^{in} + x_{jt}^{sell} + \sum_{(j,j') \in M^{conv}} x_{jj't}^{conv} + x_{jt}^{inv} \quad (3)$$

Facility Inputs

For each (i,j) $\in F^{in}$ and t $\in T$, is the amount of input j used at facility i must equal the total consumption of all activities at facility i:

$$x_{ijt}^{in} = \sum_{(i,j,k,t) \in A^{in}} a_{ijkt}^{in} x_{ikt}^{act} \quad (4)$$

Facility Outputs

For each (i,j) $\in F^{out}$ and t $\in T$, the amount of output j produced at facility i must equal the total production of all activities at facility i:

$$x_{ijt}^{out} = \sum_{(i,j,k,t) \in A^{out}} a_{ijkt}^{out} x_{ikt}^{act} \qquad (5)$$

Facility Capacity

For each i $\in F$ and time t $\in T$, the capacity used by all activities at facility i must be within the specified limits:

$$l_{it}^{cap} \leq \sum_{(i,k) \in F^{act}} x_{ikt}^{act} / r_{ikt}^{act} \leq u_{it}^{cap} + x_{it}^{vend} \qquad (6)$$

Storage Capacity

For each s $\in$ S and time t $\in T$, the sum of all materials stored in storage-areas must be within the specified limits.

$$l_{st}^{inv} \leq \sum_{j \in M} x_{jst}^{inv} \leq u_{st}^{inv} \qquad (7)$$

Storage Total

For each j $\in$ M and time t $\in T$, the sum of material j inventoried in all Storage-Areas must be equal to the total amount of that material inventoried.

$$\sum_{s \in S} x_{jst}^{inv} = x_{jt}^{inv} \qquad (8)$$

Bounds

All variables must lie within the relevant limits defined by the data:

$$l_{jt}^{buy} \le x_{jt}^{buy} \le u_{jt}^{buy} \quad \text{for each } j \in M \text{ and } t \in T \tag{9}$$

$$l_{jt}^{sell} \le x_{jt}^{sell} \le u_{jt}^{sell} \quad \text{for each } j \in M \text{ and } t \in T \tag{10}$$

$$l_{jt}^{inv} \le x_{jt}^{inv} \le u_{jt}^{inv} \quad \text{for each } j \in M \text{ and } t \in T \tag{11}$$

$$0 \le x_{jj't}^{conv} \text{ for each } (j,j') \in M^{conv} \text{ and } t \in T \tag{12}$$

$$0 \le x_{it}^{vend} \text{ for each } i \in F \text{ and time } t \in T \tag{13}$$

$$0 \le x_{jst}^{inv} \text{ each } j \in M,\ s \in S \text{ and time } t \in T \tag{14}$$

$$l_{ijt}^{in} \le x_{ijt}^{in} \le u_{ijt}^{in} \text{ for each } (i,j) \in F^{in} \text{ and } t \in T \tag{15}$$

$$l_{ijt}^{out} \le x_{ijt}^{out} \le u_{ijt}^{out} \text{ for each } (i,j) \in F^{out} \text{ and } t \in T \tag{16}$$

$$l_{ikt}^{act} \le x_{ikt}^{act} \le u_{ikt}^{act} \text{ for each } (i,k) \in F^{act} \text{ and } t \in T \tag{17}$$

Initial Conditions

$$x_{j0}^{inv} = I_{j0} \quad \text{for each } j \in M \tag{18}$$

A.9 Discounted Objective Function

The objective function can be changed to a discounted net profit maximization, by changing all cost profit parameters to discounted cost. If Zd is the discounted objective of nominal objective function Z (defined in 4.2), then the nominal and the discounted objective functions are related as follows:

$$Zd = \sum_{t \in T} Z(t)(1 + \text{int})^{-t} \tag{19}$$

References

Beale E.M. L.; Coen P.J.; and Flowerdew A. D. J. 1965, "*Separable Programming Applied to an Ore Purchasing Problem,*" Applied Statistics (U.K.) Vol. 14, pp. 89-101.

Bandopadhay R., 1969, " *Open Hearth and Basic Oxygen Furnaces: An Allocation Model for Production Planning,*" IEEE Transaction on System Science and Cybernetics, SSC-5, Vol. 12, April, pp. 115-124.

Bielfield F. W.; Klaus D. W.; and Wartmann R., 1986, " *A Computer Based Strategic Planning System for Steel Production,*" Interfaces, Vol. 16, No. 4, July-August, pp. 41-46.

Dutta G.;and Fourer R., 1996, "*A Survey of Mathematical Programming Applications In an Integrated Steel Plant* " Abstract Booklet, IFORS-96, pp.84.

Dutta G.; Sinha G. P; Roy P.N ; and Mitter N., 1994," *A Linear Programming Model for Distribution of Electrical Energy in a Steel Plant,*" International Transactions in Operational Research, Vol. 1, No 1. pp. 17-29.

Dutta G., Sinha G. P., and Roy P.N., 1997, "*A Product-Mix Optimizer for an Integrated Steel Plant,*" in preparation.

Fabian T. 1958, "*A Linear Programming Model of Integrated Iron and Steel Production,*" Management Science, Vol. 4, No. 4, pp. 415-449.

Fourer R ., 1997; *Database Structure for Mathematical Programming Models ,*" to appear in *Decision Support Systems,*

Geoffrion A. M., 1989, " *Computer-based Modeling Environments,*" European Journal in Operations Research, Vol. 41, pp. 33-43.

Hanai H., 1994, "*Linear Programming Applied to the Planning of Raw Material Purchases,*" Unpublished Internal Report, Kawasaki Steel Corporation.

Marsten R.E., 1981. *The Design Of XMP Linear Programming Library,*" ACM Transactions on Mathematical Software. Vol. 7, pp. 481-497.

Sharma A. and Sinha S. K., 1991, "*Product Mix Optimization : A Case Study of Integrated Steel Plants of SAIL,*" OPSEARCH, Vol. 28, No. 3, pp. 188- 201.

Sarma G.V, 1995, "*Application of Lexicographical Goal Programming to Solve a Product-Mix Decision in a Large Steel Manufacturing Unit -A Case Study* " OPSEARCH, Vol. 32 No 1, pp. 55-78.

Sinha G. P.; Chandrashekaran B.S.; Mitter N.; Dutta G.; Singh S.B.; Roy P.N.; and Roy Choudhary A., 1995, "*Strategic and Operations Management with Optimization at Tata Steel*" Interfaces, Vol. 25, No. 1, pp. 6-19.