Capacity management of Intensive Care Units in a multi-specialty hospital in India

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Abstract

In this paper, we describe the capacity management of Intensive Care Units (ICUs) in a 300-bed multi-speciality hospital where the alternative ICU is utilized when the appropriate ICU is full for a set of two types of ICUs. Inter-arrival time and service time distributions in these ICUs have been tested and found to be exponentially distributed.

While most capacity management models are deterministic in nature, we have developed a queuing model to provide a basis for decision-making in the design and management of these ICUs. The model results in around 19800 linear steady state equations, which are solved using the CPLEX linear optimization solver. Based on real data available from a hospital in India, the results demonstrate that the utilization of the ICU beds will improve up to 28 percent when admissions to the alternative ICU are permitted.

Keywords: OR in health services; Queuing; Productivity and competitiveness,

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1. Introduction

An Intensive Care Unit (ICU) is a specific part of a hospital that provides one-to-one nursing care to patients requiring special attention. The concept of ICU originated during the poliomyelitis epidemics in the 1950s, when polio patients were managed in a specific part of the hospital and received one-to-one nursing care. From then on, there was a gradual development in this concept, until the ICU was a recognizable component of most general hospitals. Now modern ICUs are special nursing units designed, equipped and staffed with specially skilled personnel for treating critically ill patients or those requiring specialized care and equipment.

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With the advancement in medical care and the availability of facilities for highly specialized treatments, separate sophisticated ICUs are available for admitting patients according to their age, surgical procedures, diagnosis etc. In particular, Surgical Intensive Care Unit (SICU) provides care to post-surgical patients and those patients who develop complications after surgery and require close nursing observation and care. Medical Intensive Care Unit (MICU) provides care for emergency patients suffering from coma, shock, hemorrhage, convulsions, respiratory and other medical problems. In Pediatric Intensive Care Unit (PICU) neonates and children are admitted, while the Coronary Intensive Care Unit (CICU) cares for patients with acute cardiac conditions utilizing electronic monitoring and therapy equipment. In Neuro Intensive Care Unit (NICU) patients with epilepsy, head injury, paralysis, etc. are admitted

Capacity management in an ICU involves optimal utilization of available resources (doctors, nursing staff, equipment, operating theatres and beds). The bed of an ICU is a scarce and expensive resource because a fixed number of beds are allotted to each ICU.

We undertook this study in a multi-speciality hospital where a patient may be admitted to an alternative ICU when the appropriate ICU is full. We have developed a queuing model where every ICU is considered as a service station and each ICU bed as a server. Here we consider a system where patients qualify for any of these two service stations can be interchanged to a certain extent. We also show that there is an increase in capacity utilization of up to 28% when admissions to alternative ICUs are permitted.

The remainder of this paper is organized as follows. The next section deals with a brief review of available literature on ICUs; a detailed discussion on the problems faced by the hospital administration is given in section three. Methodology and statistical models are presented in sections four and five respectively. Sections six and seven provide the demonstration of computer implementation and the results. The last section gives the concluding remarks and extension for further research.

2. Literature Survey

A wide range of literature is available on capacity management and bed allocation models in a hospital. Here, we focus only on the literature related to a hospital's ICU. Some of the literature relies on queuing theory, while some discuss simulation and prediction models.

The estimation and forecasting of the number of beds required in an ICU is an important issue for new planners and for those who want to expand existing unit beds.

Cooper and Corcoran (1974) estimated bed needs by means of queuing theory in a hospital that was planning to increase acute Coronary Care Unit (CCU) beds and also establish a new intermediate CCU. Sissouras and Moores (1976) report that the number of beds a CCU requires depends on medical and operational criteria. They use simulation procedures to provide a guide to planners of prospective CCU's to determine the number of beds most appropriate for various projected admission rates.

While forecasting is an important issue, another equally important consideration is the efficient utilization of available ICU beds. This has an impact on a patient's welfare in terms of quality of care provided, and on the hospital's cost effectiveness. Ridge et al. (1998) use a simulation model, which shows that there is a non-linear relationship between numbers of beds, mean occupancy level, and the number of patients who have to be transferred due to lack of bed space. Kim et al. (1999) analyze the admission-and-discharge data of a specific ICU of a public hospital within a steady state queuing framework and through a computer simulation model of that ICU. Kim et al. (2000) suggest that the way to minimize the number of cancelled surgeries is to reserve some of the unit's beds for the exclusive use of the elective-surgery patients. Based on a hospital ICU's historical data, they evaluate various bed-reservation schemes via a simulation model.

We know that for good patient care in an ICU, optimum staffing level of nursing staff is required. Hence, management of staff is also a challenging issue. Hashimoto et al. (1987) designed a computer program to simulate a 12-bed Medical/Critical ICU workload and staffing system. Nursing staffing policy costs, availabilities, and a table of past patient acuity points per shift were input; total overstaffing, understaffing, and cost per year for full-time nursing equivalents for direct patient care were output for different staffing levels. Issues of financial cost, quality of care, and staff working preferences were used to evaluate optimal staffing levels.

Another important point is the prediction of the risk associated with the transfer of a patient from ICU to a ward because the ICU is full. When a CCU becomes full, an existing patient is transferred into a ward in order to make room for the next arrival. The patient transferred may have suffered a heart attack and still be at risk whilst the next patient admitted may subsequently be diagnosed as having nothing more serious than indigestion. McNeer et al. (1975) present their experience with 522 consecutive patients with acute myocardial infarction admitted directly to the CCU. They suggest that it would be feasible and ethically justified to make a trial of early discharge in patients who meet the given criteria. Wharton (1996) used the queuing theory to develop a model which predicts the proportion of patients from each diagnostic or risk category that would be prematurely transferred as a function of size of the unit, number of risk categories, mean arrival rates and length of stay.

Advances in intensive care have made it possible to prolong the lives of patients with little expectation that they will survive. Futile treatment is not only costly but also prolongs suffering for patients and families. Predictive models to identify patients in ICU who will die depend on physiological data obtained at the time of admission or in the first 24 hours. Atkinson et al. (1994) describe a dynamic scoring system based upon daily organ failure scores – APACHE II scores corrected for the duration and number of organs in failure- with an algorithm designed to make daily predictions of individual outcomes in 3600 patients.

If the length of ICU stay can be predicted, patients can be scheduled so as to improve the use of existing ICU beds. Tu et al. (1994) develop a predictive index for length of stay in the ICU following cardiac surgery. Univariate and multivariate logistic regression analysis of a cohort of 1404 patients divided into a derivation set of 713 patients and a validation set of 691 patients was conducted. A predictive index was created by assigning risk scores based on the odds ratios of the significant variables in the logistic regression analysis. The predictive index was found to predict lengths of ICU stay greater than 2, 4, 7, 10 days, and patient death in the validation set.

In the literature survey we found that Ridge et al. (1998) and Kim et al. (1999, 2000) have reported work on a single ICU. Generally, they have considered it to be the only ICU (General ICU) or one of the many ICU’s of the hospital. To the best of our knowledge, no work is available for multiple ICUs where admissions to alternative ICUs are permitted when the appropriate ICU is full. Moreover, in the Indian context, practically no research work has been done in this area.

3. Problem Description

We undertook this study in a hospital situated in a prime location of the city of Ahmedabad in western India. This hospital was built in 2001 with a bed capacity of 250 in the beginning. Subsequently the bed strength was increased to 300. It is a multi-speciality hospital offering medical services in general medicine, general surgery, cardiology, neurology, pulmonary medicine, orthopedics, gynecology and obstetrics, pediatrics, skin, ophthalmology etc. It also has a recognized medical college under Gujarat University, India.

A brief study of different activities in this hospital enabled us to have detailed discussions on the different problems being faced by the physicians and hospital administrators and seek suitable solutions. The hospital faces a heavy patient load over ICU beds as compared to general beds, particularly over MICU. The hospital administration was sure that utilization of alternative ICUs would increase the capacity-utilization of the hospital, but they were not sure about the extent of such an improvement.

In this hospital five ICU's are available for admitting the patients separately, i.e., SICU, MICU, NICU, CICU and PICU. Distribution of beds in these ICUs for June 2004 is given in **Table 1**.

Table 1 Distribution of beds among different ICUs

Type of ICU	No. of beds
Surgical ICU	8
Medical ICU	14
Coronary ICU	9
Neuro ICU	10
Pediatric ICU	8

This hospital guarantees elective admission dates and does not turn away any emergency admission. This creates a situation where, when there is no empty bed in the appropriate ICU, a sufficiently recovered patient is transferred to the ward in order to create a vacancy for the new referral. If an expedited transfer is not possible, the new referral is admitted to an alternative ICU. This process is feasible only for MICU, CICU and NICU. As **Figure 1** indicates, a new referral that qualified for any of these three ICUs (MICU, CICU, and NICU) could not be admitted to SICU or PICU, since the chances of infection were high in SICU and PICU.

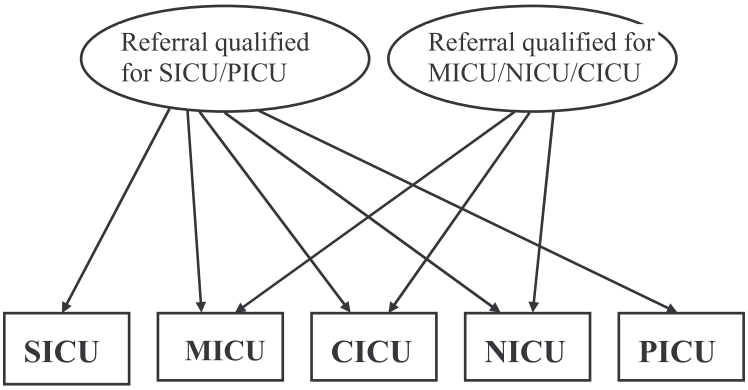


Figure 1. Feasible movements of new referral in ICUs.

MICU, CICU and NICU work as a network - in case of an emergency the same treatment and care is provided in an alternative unit **(Figure 2)**. In the beginning, the original MICU had 23 beds where patients qualified for MICU and also for CICU were admitted together. In June 2004, the original MICU was divided into a new 14-bed MICU and a 9 bed CICU. One more CICU is being planned.

We decided to restrict this study to the network of two ICUs (23-bed original MICU and 10-bed NICU) over the continuous four-month period before June 2004 i.e. from February – May

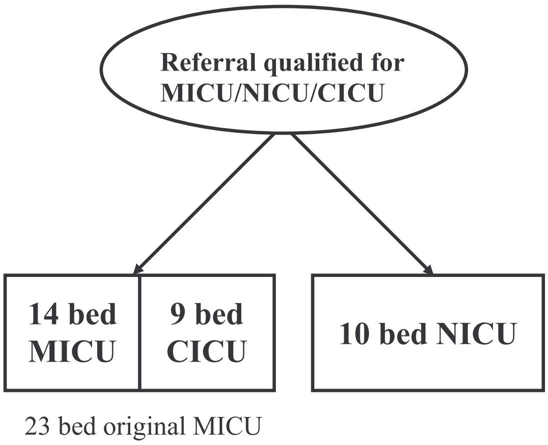


Figure 2. Network of three ICUs

2004. A new referral who qualifies for the MICU is the right type for MICU and if admitted to the NICU, he/she is the wrong type for NICU. Similarly a new referral who qualifies for NICU is the right type for NICU and if admitted to the MICU, he/she is the wrong type for MICU. Now we consider two possibilities. When,

i. wrong type of patients are not allowed, i.e., alternative ICU admissions are not permitted, so two ICUs work as independent service stations (*case I*),

ii. wrong type of patients are allowed, i.e., alternative ICU admissions are permitted so two ICUs work as dependent service stations (*case II*).

Initially we have set up the following goals:

1. To obtain steady-state queuing models for these two cases
2. To obtain expected number of patients in the system for both the cases.

To achieve the above goals, real data is required to study the following:

i. The average arrival rate per day for these ICUs.
ii. The arrival pattern of the patients in these ICUs.
iii. The average length of stay (service time) in each ICU.
iv. The service time distribution of the patients in these ICUs.

4. Methodology

Our data collection exercise was very challenging. In every ICU, details of each arriving patient were written manually in different registers by the nursing staff. They maintained different registers- such as Admission Book (for details of patients admitted to the ICU), Discharge Book (for details of patients discharged directly), Transfer Book (for details of patients transferred), Death Book (for details of patients who die), and Remaining Book (the number of patients in the ICU at midnight, after accounting for the day's arrivals, transfers, discharges). The data were, therefore, scattered in various registers which were dumped in a record-room at the hospital. The following are the different data details that we have collected from the different registers:

1. Total number of arrivals per day (one day is counted from midnight to midnight) in each ICU.
2. Age, sex, and diagnosis of each patient.
3. Arrival and discharge/transfer date and time of each patient.
4. Survival outcome of the patient.

The ICU that is the focus of this study receives virtually all of its patients from four different sources: (1) Ward; (2) Casualty; (3) Operating Theatre; and (4) Admission office. Patients are referred to the ICU by their physicians. Very often, a physician will call at the admission office of the hospital when a patient is to be admitted, giving the name of the patient, his accommodation preferences, and a brief diagnosis, so that the admitting clerk will know in advance where to assign the patient expected. When the patient comes directly to the hospital without any physician's reference, he/she will go through the assessment process in the 3-bed casualty, where the appointed physician or the hospital's on-call physician will decide his accommodation preferences. The number of patients admitted is not necessarily an indication of the correct assessment of the cases when first seen. Patients are admitted throughout the day and night but usually discharged in the morning.

In ICU all the patients can be classified into two groups according to the medical speciality of the required treatment: (1) Medicine, and (2) Surgery. For each ICU these specialties can be divided into sub-specialties **(Table 2)**.

Table 2 Sub-speciality groups in each ICU

Type of ICU	**Medicine**	**Surgery**
MICU	Internal	General
	Infectious Disease	Orthopedic
	Oncology	Onco surgery
	Gestroentrology	Gastric surgery
	Nephrology	Urology
	Respiratory	Plastic surgery
		ENT
CICU	Cardiology	Cardiothoracic surgery
NICU	Neurology	Neuro surgery

The problem could be modeled in two approaches (analytical/queuing and simulation). In the analytical approach, the actual arrival and service time distributions are approximated using one of the statistical distributions known, i.e. Poisson, exponential, etc., to describe the expected value of various operational characteristics of the queuing process. We first looked at the arrival and service-time data and attempted to determine their distributions.

5. Model Development

For the period February to May 2004, approximately 4600 inpatients were admitted in the hospital. Of these, 1125 patients were admitted to the MICU and NICU. Out of the 1125 patients, 43 came twice and 3 came thrice to the ICU during the length of their stay in the hospital. The classification of patients according to their ICU preference and medical sub-specialties is given in **Table 3**.

Table 3 Admission and survival rates according to medical sub-specialties in each ICU.

Type of ICU	Sub-speciality Groups	Admitted patients	Admission rate (%)	Died patients	Survival outcome (%)	Patient(s) came		Total arrivals
						Twice	Thrice	
MICU	Inter. Medicine	61	5.42	7	88.52	3	-	64
	Infec. Disease	7	0.62	1	85.71	-	-	7
	Respiratory	41	3.64	7	82.93	2	-	43
	Gestroentrology	9	0.80	-	100.00	1	-	10
	Nephrology	9	0.80	2	77.78	-		9
	Oncology	4	0.36	-	100.00	-	-	4
	Cardiology	694	61.69	27	96.11	22	2	720
	General surgery	20	1.78	4	80.00	1	-	21
	Orthopedic	13	1.16	2	84.62	1	-	14
	Onco surgery	-	-	-	-	-	-	-
	Gastric surgery	2	0.18	-	100.00	-	-	2
	Urology	5	0.44	-	100.00	-	-	5
	Plastic surgery	1	0.09	-	100.00	-	-	1
	ENT	4	0.36	-	100.00	1	-	5
	Cardio Th. Surg	82	7.29	1	98.78	7	-	89
NICU	Neurology	105	9.33	11	89.52	3	-	108
	Neuro surgery	68	6.04	7	89.70	2	1	72
Total		**1125**		**69**		**43**	**3**	**1174**

5.1 Distribution of Arrival Time

We tested the data to check if the arrival process could be following a Poisson distribution. If the probability distribution of a number of random arrivals in a fixed time interval follows a Poisson distribution (or if the arrival process follows the Poisson distribution), then the distribution of the intervals between successive arrivals (defined as inter-arrival time) follows the (negative) exponential distribution and vice-versa.

An exponential distribution has the property that its variance is equal to the square of the mean. We therefore compute mean inter-arrival times and variances for each ICU. We also carry out Chi-square tests of the hypothesis that inter-arrival times are exponentially distributed and find that the hypothesis cannot be rejected at the 1% level of significance for MICU and 5% level of significance for NICU.

5.2 Distribution of Service Time

The service time in the ICU means the length of time a patient stays in ICU before being discharged/transferred. We compute the means and variances of the service time for each of the ICUs. We again conduct Chi-square tests to test the hypothesis that service-time distributions are exponential. The results show that the hypothesis for exponential distribution cannot be rejected at the 1% level of significance for MICU and 5% level of significance for NICU. **Table 4** shows mean arrival and mean service rates in both the ICUs.

Table 4 Mean arrival rates and mean service rates for each ICU

Type of ICU	Mean arrival rate λ = 1/ (mean inter-arrival time) patients per day	Mean service rate μ =1/ (mean service time) patients per day
MICU	λ_1=7.97	μ_1=0.46
NICU	λ_2=1.44	μ_2=0.33

5.3 Obtain the System of Steady State Equations

Consider MICU as a system of M servers (beds) in which patients arrive, from outside the system, to each server i, i =1,…,M, in accordance with independent Poisson processes at rate λ_1; then they join the queue until their turn at service comes; the service rate per server is μ_1. Similarly NICU as a system of N servers (beds) in which patient arrive, from outside the system, to each server k, k =1,…,N, in accordance with independent Poisson processes at rate λ_2; then they join the queue until their turn at service comes; the service rate per server is μ_2.

The derivation of this model is based on certain assumptions about the queuing system:

(i) Patients arrive according to Poisson processes at average rates of λ_1 and λ_2 patients per unit of time in MICU and NICU respectively.

(ii) Patients are admitted on a First Come First Served (FCFS) basis at any of the servers in MICU and NICU.

(iii) These servers are identical, each serving according to exponential distribution with average rates μ_1 and μ_2 patients per unit of time in MICU and NICU respectively.

(iv) No queue is allowed (i.e. finite capacity).

Case I: Those patients, who are qualified for MICU (or NICU), admitted to MICU (or NICU) only, notwithstanding the empty beds in NICU (or MICU). If all the servers in MICU (or NICU) are busy, a sufficiently recovered patient is transferred to the ward. If an expedited transfer is infeasible, the new referral joins a queue. This movement can be easily understood by the flow chart given in **Figure 3**.

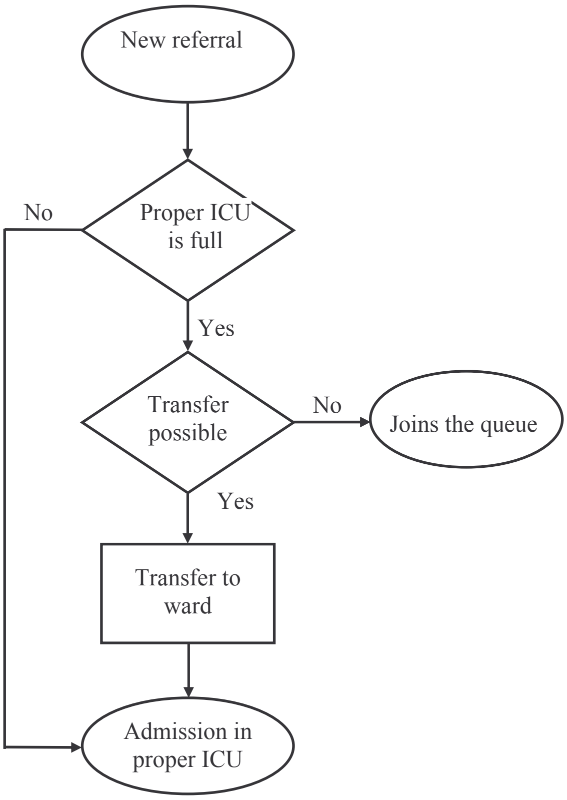


Figure 3. Flow chart of the admission process for new referral in ICUs

Define

M = Number of servers in MICU

N = Number of servers in NICU

λ_1 = Arrival rate of the patients in MICU

λ_2 = Arrival rate of patients in NICU

μ_1 = Service rate per bed in MICU

μ_2 = Service rate per bed in NICU

P_i = Steady state probability of i patients in MICU

P_k = Steady state probability of k patients in NICU

$P_{(i, k)}$ = Steady state joint probability of (i + k) patients when i patients in MICU and k patients in NICU

Figures 4a and **4b** show the movement of patients among several steady states. Under steady state conditions for i, k>0, the expected rates of flow into and out of state (i, k) must be equal. The steady state balance equations are:

$$[\lambda_1 + \lambda_2 + i\mu_1 + k\mu_2]\, P_{(i, k)} = \lambda_1 P_{(i-1, k)} + \lambda_2\, P_{(i, k-1)} + (i+1)\mu_1\, P_{(i+1, k)} + (k+1)\mu_2\, P_{(i, k+1)} \quad (1)$$

Where i = 1,…,M-1,   k = 1,…,N-1.

The balance equations associated with boundary states are:

For i = 0,

$$[\lambda_1 + \lambda_2 + k\mu_2]\, P_{(0, k)} = \lambda_2\, P_{(0, k-1)} + \mu_1\, P_{(1, k)} + (k+1)\mu_2\, P_{(0, k+1)} \quad (2)$$

For k = 0,

$$[\lambda_1 + \lambda_2 + i\mu_1]\, P_{(i, 0)} = \lambda_1 P_{(i-1, 0)} + (i+1)\mu_1\, P_{(i+1, 0)} + \mu_2\, P_{(i, 1)} \quad (3)$$

For i = 0, k = 0,

$$[\lambda_1 + \lambda_2]\, P_{(0, 0)} = \mu_1\, P_{(1, 0)} + \mu_2\, P_{(0, 1)} \quad (4)$$

For i = M,

$$[\lambda_2 + M\mu_1 + k\mu_2]\, P_{(M, k)} = \lambda_1 P_{(M-1, k)} + \lambda_2\, P_{(M, k-1)} + (k+1)\mu_2\, P_{(M, k+1)} \quad (5)$$

For k = N,

$$[\lambda_1 + i\mu_1 + N\mu_2]\, P_{(i, N)} = \lambda_1 P_{(i-1, N)} + \lambda_2\, P_{(i, N-1)} + (i+1)\mu_1\, P_{(i+1, N)} \quad (6)$$

For i = M, k = N,

$$[M\mu_1 + N\mu_2]\, P_{(M, N)} = \lambda_1 P_{(M-1, N)} + \lambda_2\, P_{(M, N-1)} \quad (7)$$

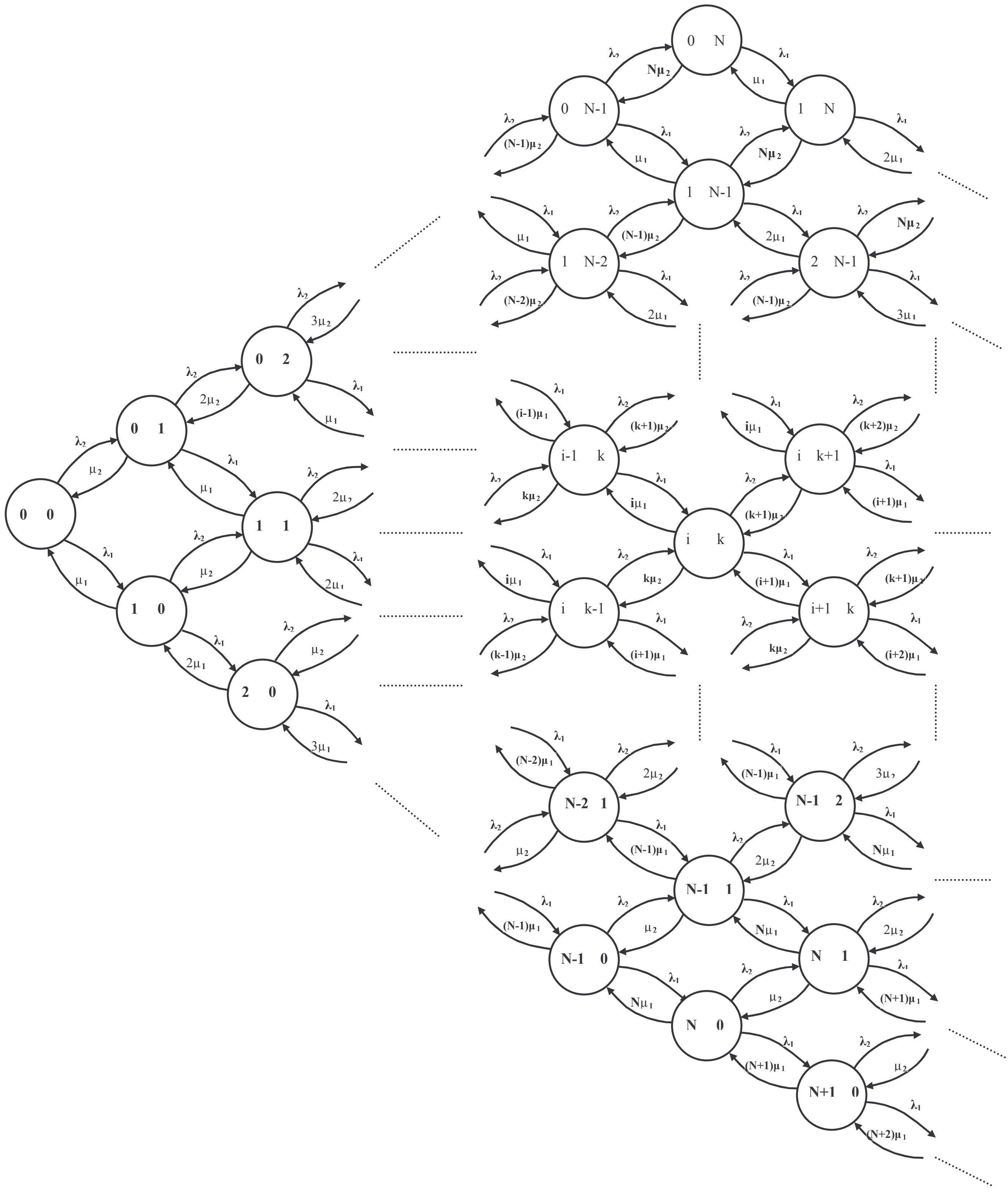
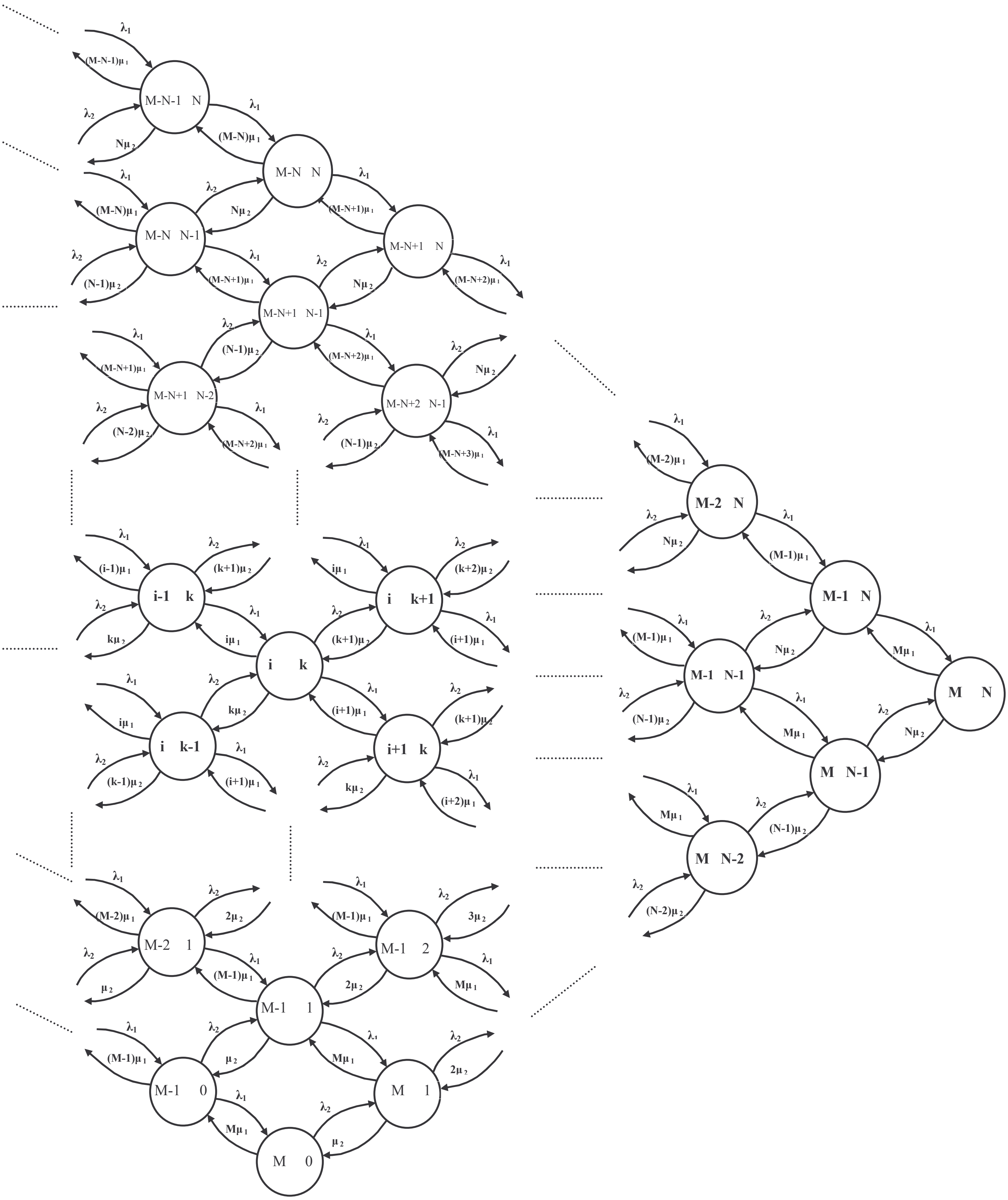


Figure 4a Steady state diagram for independent service-stations queuing system

Figure 4b Steady state diagram for independent service-stations queuing system



The normalization is provided by

$$\sum_{0\le i\le M}\ \sum_{0\le k\le N} P_{(i,\,k)} = 1 \qquad (8)$$

$$\text{where } P_i = \sum_{k=0}^{N} P_{(i,\,k)} \qquad \text{for } i = 0,1,\ldots..,M \qquad (9)$$

$$\text{and } P_k = \sum_{i=0}^{M} P_{(i,\,k)} \qquad \text{for } k = 0,1,\ldots..,N \qquad (10)$$

When M = 23 and N = 10, then the total number of possible steady states is 264. Hence a total of 264 balance equations exist corresponding to 264 steady states.

Case II: Those patients, who are qualified for MICU (or NICU), admitted to MICU (or NICU) only. If all the servers are busy in MICU (or NICU) a sufficiently recovered patient is transferred to the ward. If an expedited transfer is infeasible, the new referral is admitted to NICU (or MICU) and the same treatment and care is provided in NICU (or

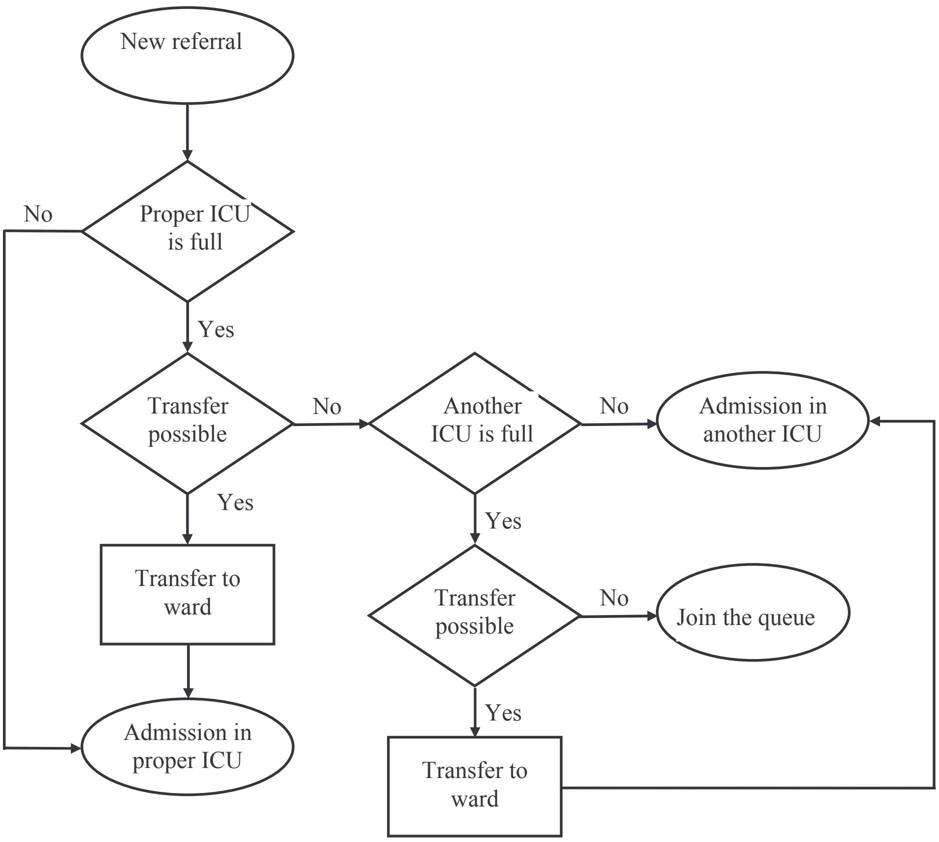


Figure 5. Flow chart of the admission process for new referral in ICUs

MICU). If all the servers in both MICU and NICU are busy, a sufficiently recovered patient is transferred to the ward and if an expedited transfer is infeasible, the new referral joins a queue. The flow of the patients is given in **Figure 5**.

Define

M = Number of servers in MICU for right type of patients

m = Number of servers in MICU for wrong type of patients

N = Number of servers in NICU for right type of patients

n = Number of servers in NICU for wrong type of patients

λ_1 = Arrival rate of right type of patients in MICU and wrong type of patients in NICU

λ_2 = Arrival rate of right type of patients in NICU and wrong type of patients in MICU

μ_1 = Service rate per bed in MICU for right type of patients and in NICU for wrong type of patients

μ_2 = Service rate per bed in NICU for right type of patients and in MICU for wrong type of patients

P_i = Steady state probability of i right type of patients in MICU

P_j = Steady state probability of j wrong type of patients in MICU

P_k = Steady state probability of k right type of patients in NICU

P_l = Steady state probability of *l* wrong type of patients in NICU

$P_{(i, j, k, l)}$ = Steady state joint probability of (i + j + k + *l*) patients when i right type and j wrong type patients in MICU, k right type and *l* wrong type patients in NICU

The steady state diagrams in **Figures 6a, 6b, 6c, 6d** show the movement of patients among several states. Under steady state conditions for i, j, k, $l > 0$, the expected rates of flow into and out of state (i, j, k, *l*) must be equal. The steady state balance equations are:

$$[\lambda_1 + \lambda_2 + i\mu_1 + j\mu_2 + k\mu_2 + l\mu_1]\, P_{(i, j, k, l)} = \lambda_1 P_{(i-1, j, k, l)} + \lambda_2\, P_{(i, j, k-1, l)} + (i + 1)\mu_1\, P_{(i+1, j, k, l)} + (k + 1)\mu_2\, P_{(i,, j, k+1, l)} + (j + 1)\mu_2\, P_{(i, j+1, k, l)} + (l + 1)\mu_1\, P_{(i,, j, k, l+1)} \quad (11)$$

Where i = 1,…,M-1, j = 1,…,m-1, k = 1,…,N-1, *l* = 1,…,n-1.

$$[\lambda_1 + \lambda_2 + i\mu_1 + j\mu_2 + k\mu_2 + l\mu_1]\, P_{(i, j, k, l)} = \lambda_1 P_{(i-1, j, k, l)} + \lambda_2\, P_{(i, j, k-1, l)} + \lambda_2\, P_{(i, j-1, k, l)} + (i + 1)\mu_1\, P_{(i+1, j, k, l)} + (j + 1)\mu_2\, P_{(i, j+1, k, l)} \quad (12)$$

Where i = 1,…,M-1, j = 1,…,m-1, k = 1,…,N, *l* = 1,…,n, and k + *l* = N.

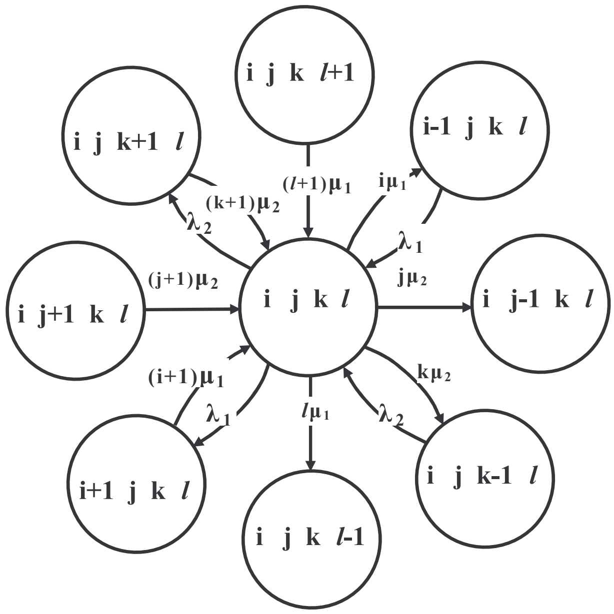


Figure 6a Steady state diagram for dependent service-stations queuing system (equation 11)

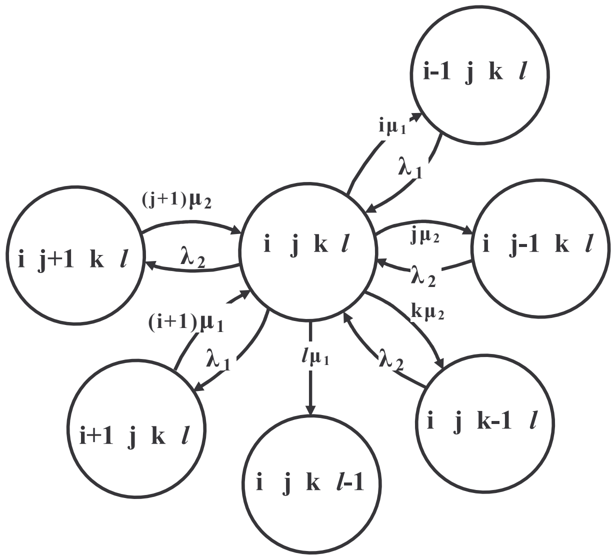


Figure 6b Steady state diagram for dependent service-stations queuing system when $k + l = N$ (equation 12)

$$[\lambda_1 + \lambda_2 + i\mu_1 + j\mu_2 + k\mu_2 + l\mu_1]\, P_{(i, j, k, l)} = \lambda_1 P_{(i-1, j, k, l)} + \lambda_2\, P_{(i, j, k-1, l)} + \lambda_1\, P_{(i, j, k, l-1)} + (k + 1)\, \mu_2\, P_{(i, ,j, k+1, l)} + (l + 1)\mu_1\, P_{(i, j, k, l+1)} \quad (13)$$

Where $i = 1,\ldots,M$, $j = 1,\ldots,m$, $k = 1,\ldots,N\text{-}1$, $l = 1,\ldots,n\text{-}1$, and $i + j = M$.

$$[i\mu_1 + j\mu_2 + k\mu_2 + l\mu_1]\, P_{(i, j, k, l)} = \lambda_1 P_{(i-1, j, k, l)} + \lambda_2\, P_{(i, j, k-1, l)} + \lambda_2\, P_{(i, j-1, k, l)} + \lambda_1\, P_{(i, j, k, l-1)} \quad (14)$$

Where i = 1,…,M,   j = 1,…,m,   k = 1,…,N, l = 1,…,n, and i + j = M,   k + l = N.

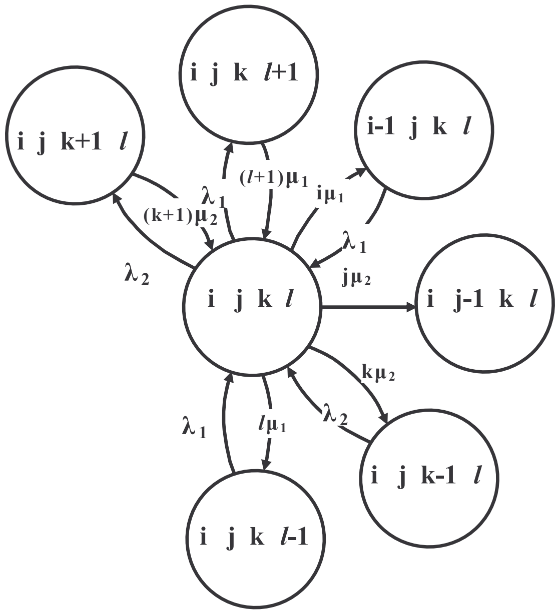


Figure 6c Steady state diagram for dependent service-stations queuing system when i + j = M (equation 13)

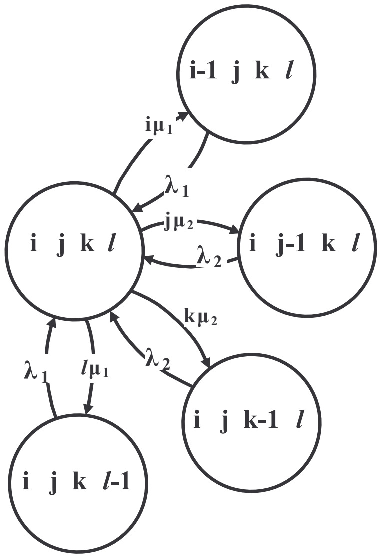


Figure 6d Steady state diagram for dependent service-stations queuing system when i + j = M, k + l = N (equation 14)

The balance equations associated with zero boundary states are:

For i = 0, substitute $P_{(i-1, j, k, l)} = 0$ in above equations. (15)

For j = 0, substitute $P_{(i, j-1, k, l)} = 0$ in above equations. (16)

For k = 0, substitute $P_{(i, j, k-1, l)} = 0$ in above equations. (17)

For l = 0, substitute $P_{(i, j, k, l-1)} = 0$ in above equations. (18)

The normalization is provided by

$$\sum_{\substack{i, j \geq 0 \\ i+j \leq M}} \sum_{\substack{k, l \geq 0 \\ k+l \leq N}} P_{(i, j, k, l)} = 1 \tag{19}$$

Where $$P_i = \sum_{j=0}^{M-i} \sum_{k=0}^{N} \sum_{l=0}^{N-k} P_{(i, j, k, l)}, \quad \text{for } i = 0 \ldots M, \tag{20}$$

$$P_j = \sum_{i=0}^{M-j} \sum_{k=0}^{N} \sum_{l=0}^{N-k} P_{(i, j, k, l)}, \quad \text{for } j = 0 \ldots m, \tag{21}$$

$$P_k = \sum_{i=0}^{M} \sum_{j=0}^{M-i} \sum_{l=0}^{N-k} P_{(i, j, k, l)}, \quad \text{for } k = 0 \ldots N, \tag{22}$$

$$P_l = \sum_{i=0}^{M} \sum_{j=0}^{M-i} \sum_{k=0}^{N-l} P_{(i, j, k, l)}, \quad \text{for } l = 0 \ldots n, \tag{23}$$

When M = 23, m = 23, N = 10 and n = 10, then the total number of possible steady states is 19800. Hence a total of 19800 balance equations exist corresponding to 19800 steady states.

5.4 Objective function and constraints

We assume the objective function of these queuing models to be the maximization of the expected number of patients served in the system.

L_M = Expected number of right type of patients in MICU
L_m = Expected number of wrong type of patients in MICU
L_N = Expected number of right type of patients in NICU
L_n = Expected number of wrong type of patients in NICU
By definition,

$$L_M = \sum_{i=0}^{M} i\, P_i$$

$$L_m = \sum_{j=0}^{m} j\, P_j$$

$$L_N = \sum_{k=0}^{N} k\, P_k$$

$$L_n = \sum_{l=0}^{n} l \, P_l$$

L_{Mq} = Expected number of patients qualified for MICU

L_{Nq} = Expected number of patients qualified for NICU

L_{Ms} = Expected number of patients served in MICU

L_{Ns} = Expected number of patients served in NICU

Then $L_{Mq} = L_M + L_n$

$L_{Nq} = L_N + L_m$

$L_{Ms} = L_M + L_m$

$L_{Ns} = L_N + L_n$

Maximize

$Z = L_{Ms} + L_{Ns}$

Subject to

Case I: Equations 1 to 10

Case II: Equations 11 to 23

6. Computer Implementation

For both the cases, obtained linear steady-state equations are modeled in AMPL (Fourer et al., 1993) and solved using the CPLEX solver. While this problem can also be solved with the help of any other scientific package, we have chosen AMPL/CPLEX (version 8.0) for the following advantages it has:

1) The model is generic. If we need to extend the study to another hospital, we can use the same model. We would only need to test the Markovian property of the arrival time and service time.
2) The model is independent of the data and the solver. If another hospital faces a similar problem, we can use the same model.

The AMPL model codes are given in **Appendix A** for *case I* and in **Appendix B** for *case II*. The expected number of patients qualified for MICU and NICU and served in MICU and NICU are given in **Table 5** with the variations in the value of M and N for both the cases. One of the important points addressed in this study is the computational time required to solve this model with AMPL/CPLEX (version 8.0). We found that for the largest model with

M,m=23	N,n=5	Case I	16.65738	0.00000	3.35473	0.00000	16.65738	+ 0.36%	3.35473	+ 25.50%	16.6
		Case II	16.2981	1.09223	3.11806	0.41906	16.71716		4.21029		17.3
M,m=23	N,n=6	Case I	16.65738	0.00000	3.73551	0.00000	16.65738	+ 1.35%	3.73551	+ 13.82%	16.6
		Case II	16.416	0.73463	3.51725	0.46637	16.88237		4.25188		17.1
M,m=23	N,n=7	Case I	16.65738	0.00000	4.00432	0.00000	16.65738	+ 2.12%	4.00432	+ 6.99%	16.6
		Case II	16.50268	0.46606	3.81824	0.50842	17.01110		4.28430		16.9
M,m=23	N,n=8	Case I	16.65738	0.00000	4.17607	0.00000	16.65738	+ 2.71%	4.17607	+ 3.18%	16.6
		Case II	16.56260	0.27943	4.02939	0.54587	17.10847		4.30882		16.8
M,m=23	N,n=9	Case I	16.65738	0.00000	4.27455	0.00000	16.65738	+ 3.14%	4.27455	+ 1.22%	16.6
		Case II	16.60154	0.15936	4.16743	0.57830	17.17984		4.32679		16.7
M,m=23	N,n=10	Case I	16.65738	0.00000	4.32511	0.00000	16.65738	+ 3.44%	4.32511	+ 0.33%	16.6
		Case II	16.62549	0.08731	4.25225	0.60499	17.23048		4.33956		16.7
M,m=24	N,n=10	Case I	16.85642	0.00000	4.27455	0.00000	16.85642	+ 2.22%	4.27455	+ 1.52%	16.8
		Case II	16.81701	0.13887	4.20069	0.41348	17.23049		4.33956		16.9
M,m=25	N,n=10	Case I	17.00659	0.00000	4.17607	0.00000	17.00659	+ 1.32%	4.17607	+ 3.91%	17.0
		Case II	16.95701	0.23384	4.10572	0.27348	17.23049		4.33956		17.1
M,m=22	N,n=10	Case I	16.40275	0.00000	4.32511	0.00000	16.40275	+ 4.74%	4.32511	+ 0.04%	16.4
		Case II	16.35686	0.10656	4.22024	0.82297	17.17983		4.32680		16.4
M,m=21	N,n=10	Case I	16.08768	0.00000	4.32511	0.00000	16.08768	+ 6.35%	4.32511	- 0.38%	16.0
		Case II	16.02342	0.13042	4.17841	1.08505	17.10847		4.30883		16.1
M,m=20	N,n=10	Case I	15.70953	0.00000	4.32511	0.00000	15.70953	+ 8.29%	4.32511	- 0.94%	15.7
		Case II	15.62216	0.15883	4.12547	1.38894	17.01110		4.28430		15.7
M,m=22	N,n=9	Case I	16.40275	0.00000	4.27455	0.00000	16.40275	+ 4.30%	4.27455	+ 0.80%	16.4
		Case II	16.32562	0.18501	4.12382	0.78286	17.10848		4.30883		16.5
M,m=21	N,n=8	Case I	16.08768	0.00000	4.17607	0.00000	16.08768	+ 4.94%	4.17607	+ 1.82%	16.0
		Case II	15.92453	0.34671	3.90516	0.95782	16.88235		4.25187		16.2

over 19800 variables, it took about 124 minutes. The variations in solution time and percentage increment in capacity utilization in *case II* with different values of M and N are shown in **Table 6.**

Table 6 Percentage increment in the capacity utilization of both the ICUs

Values of M and N	No. of Equations		Elapsed time in solution (in seconds)		**Z**		
	Case I	Case II	Case I	Case II	Case I	Case II	% increment
M,m = 23, N,n = 5	144	6300	1	134	20.01211	20.92745	4.57%
M,m = 23, N,n = 6	168	8400	1	307	20.39289	21.13425	3.64%
M,m = 23, N,n = 7	192	10800	1	733	20.66170	21.29540	3.07%
M,m = 23, N,n = 8	216	13500	1	1514	20.83345	21.41729	2.80%
M,m = 23, N,n = 9	240	16500	1	3740	20.93193	21.50663	2.75%
M,m = 23, N,n = 10	264	19800	1	8700	20.98249	21.57004	2.80%
M,m = 24, N,n = 9	250	17875	1	4142	21.13097	21.29544	0.78%
M,m = 25, N,n = 8	234	15795	1	2980	21.18266	21.57005	1.83%
M,m = 22, N,n = 10	253	18216	1	3660	20.72786	21.50663	3.76%
M,m = 21, N,n = 10	242	16698	1	2074	20.41279	21.4173	4.92%
M,m = 20, N,n = 10	231	15246	1	1320	20.03464	21.29540	6.29%
M,m = 22, N,n = 9	230	15180	1	1751	20.67730	21.41731	3.58%
M,m = 21, N,n = 8	198	11385	1	582	20.26375	21.13422	4.30%

7. Conclusion and Extension

We found that there is an increase in the number of beds utilized, if admissions are permitted to alternative ICUs when the appropriate ICU is full. In both the ICUs in *case II*, between 1-6% more beds are utilized as compared to *case I* (**Table 6**). For only NICU the highest increment in bed utilization may be as much as 28%, at the loss of 1% patients qualified for NICU (**Table 5**). We, therefore, recommend that when it is practicable to admit patients into alternative ICUs, the hospital must do so.

The work can be extended in many ways. The first one is to extend it to another hospital and examine the results obtained. The other extension is to model different types of possible movements of patients within a hospital.

Those patients, who are qualified for MICU (or NICU), admitted in MICU (or NICU) only. If all the servers are busy in MICU (or NICU), a sufficiently recovered patient is transferred to NICU (or MICU), and if an expedited transfer is infeasible, the new referral is admitted to NICU (or MICU) temporarily and the same treatment and care is provided in NICU (or MICU) until a vacancy is created in the MICU (or NICU). If all the servers in both MICU and NICU are busy, a sufficiently recovered patient is transferred to the ward and if an expedited transfer is infeasible, the new referral joins a queue.

The corresponding stochastic process is not Markovian since the future behavior of the process depends not only on the current state but also on the time spent in the previous ICU. As this process is not Markovian, it may not lend itself to modeling by standard queuing models. Either a new analytical model may be developed or we could use simulation techniques.

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Appendix A: AMPL model file codes for *case I*

```
param  M;

param  N;

param  m_arate  {0..M} >= 0;

param  n_arate  {0..N} >= 0;

param  m_srate  {0..M} >= 0;

param  n_srate  {0..N} >= 0;

var Prob  {0..M, 0..N} >=0, <= 1;

maximize exp_m: sum {m in 0..M, n in 0..N} m* Prob[m,n];

subject to balance  {m in 0..M, n in 0..N}:
Prob[m,n] * (m_arate[m] + n_arate[n] + m_srate[m] + n_srate[n]) =
(if m = 0 then 0 else m_arate[m-1] * Prob[m-1,n]) +
(if n = 0 then 0 else n_arate[n-1] * Prob[m,n-1]) +
(if m = 23 then 0 else m_srate[m+1] * Prob[m+1,n]) +
(if n = 10 then 0 else n_srate[n+1] * Prob[m,n+1]);

subject to total: sum {m in 0..M, n in 0..N} Prob[m,n] = 1;
```

Appendix B: AMPL model file codes for *case II*

```
param M1;
param M2;
param N1;
param N2;
param m1_arate {0..M1} >= 0;
param m2_arate {0..M2} >= 0;
param n1_arate {0..N1} >= 0;
param n2_arate {0..N2} >= 0;
param m1_srate {0..M1} >= 0;
param m2_srate {0..M2} >= 0;
param n1_srate {0..N1} >= 0;
param n2_srate {0..N2} >= 0;

var Prob {m1 in 0..M1, m2 in 0..M2-m1, n1 in 0..N1, n2 in 0..N2-n1}
>= 0, <= 1;

maximize exp_m1:sum{m1 in 0..M1, m2 in 0..M2-m1, n1 in 0..N1, n2 in
0..N2-n1} m1 * Prob[m1,m2,n1,n2];

subject to balance{m1 in 0..M1, m2 in 0..M2-m1, n1 in 0..N1, n2 in
0..N2-n1}:
Prob[m1,m2,n1,n2]*(m1_srate[m1]+ m2_srate[m2]+ n1_srate[n1]+
n2_srate[n2]+ if m1+m2 = 20 and n1+n2 = 10 then 0 else
(m1_arate[m1]+n1_arate[n1]))=
(if m1 = 0 then 0 else m1_arate[m1-1] * Prob[m1-1,m2,n1,n2])+
(if n1 = 0 then 0 else n1_arate[n1-1] * Prob[m1,m2,n1-1,n2])+
(if m2 = 0 or n1+n2 <> 10 then 0 else m2_arate[m2-1] * Prob[m1,m2-
1,n1,n2])+
(if n2 = 0 or m1+m2 <> 23 then 0 else n2_arate[n2-1] * Prob[m1,m2,
n1,n2-1])+
(if m1 = 23 or m1+m2 = 23 then 0 else m1_srate[m1+1] * Prob[m1+1,m2,
n1,n2])+
(if m2 = 23 or m1+m2 = 23 then 0 else m2_srate[m2+1] * Prob[m1,m2+1,
n1,n2])+
(if n1 = 10 or n1+n2 = 10 then 0 else n1_srate[n1+1] * Prob[m1,m2,
n1+1,n2])+
(if n2 = 10 or n1+n2 = 10 then 0 else n2_srate[n2+1] * Prob[m1,m2,
n1,n2+1]);

subject to total: sum{m1 in 0..M1, m2 in 0..M2-m1, n1 in 0..N1, n2
in 0..N2-n1} Prob[m1,m2,n1,n2] = 1;
```

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