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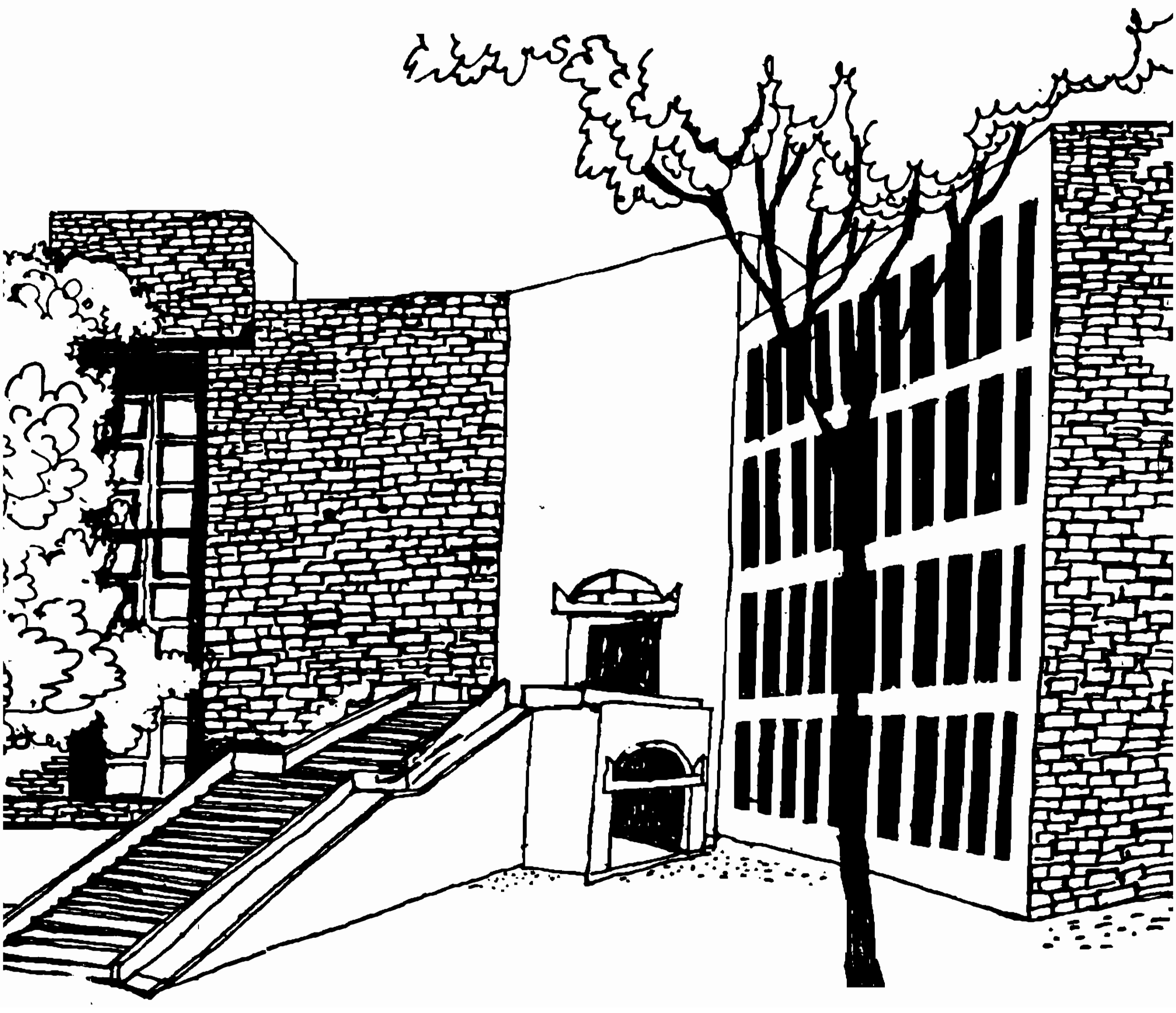

USING DEA TO EVALUATE 29 CANADIAN TEXTILE COMPANIES--CONSIDERING RETURNS-TO-SCALE

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1 Introduction

In recent years, the Canadian textile industry has experienced upheaval. Imports have captured more than half of the market for apparel in Canada, and they seriously threaten other large segments of industry. The question of what should be done for the textile industry in Canada has been hotly debated in recent years. One view holds that this sort of labor-intensive, "sunset" industry should be allowed to wither away as resources and capital are shifted to more promising "sunrise" industries. The opposing view recommends that trade barriers be strengthened to protect the industry while it automates some production operations. However, researchers (Dertouzos, Lester and Solow, 1989) have revealed serious flaws in these popular views of the causes of the decline. The research concluded that this aging, labor-intensive industry can be revived by adjusting structure, strategies, and technology.

Motivated by the observations described above, in this paper, we provide an analysis of the productivity (efficiency) performance of 29 Canadian textile companies in 1994 by using the Data Envelopment Analysis (DEA) approach developed by Charnes, Cooper and Rhodes (1978). The DEA results provide us with the efficiency scores, the efficiency frontier and the returns-to-scale. We then focus on the returns-to-scale and address the alternatives of reducing inefficient inputs in order to be DEA efficient. For the decision making units (DMUs) in increasing returns, we propose to increase the output further by increasing the inputs. For the DMUs in decreasing returns, we develop a vertical integration strategy to utilize the inefficient inputs. To find the optimal expansion or integration level, we develop models to solve the problems. Although the data we have is limited, the results derived from this research do provide some interesting insights.

DEA has gained increasing popularity as a tool for evaluating efficiency of a DMU relative to its peers. Applications have been numerous and most applications can be found in public sectors such as health care, education, transportation and bank institutions. Unlike parametric models, DEA does not require specification of any functional relationship between inputs and outputs, or *apriori* specification of weights of inputs and outputs. It provides an efficiency score for each DMU based on observed input-output data for a set

While the literature that addresses the returns-to-scale is sparse, research on using DEA to evaluate productivity performance in manufacturing sectors is also limited. Charnes, Cooper and Li (1989) used DEA as a tool to evaluate the economic performance of 28 Chinese cities from 1983 to 1984. Their results show that the cities that play a critical role in the economic development of China did make significant economic progress after the reform. Banker, Datar and Kemerer (1991) extended the stochastic DEA approach to estimate a production frontier and to study the effects of several productivity factors. They also performed sensitivity analyses to alternative forms of the production function to examine the quality of the production function proposed in the research. Ahn et al. (1991) provided an initial study on the effects of the economic reforms in the textile industry in China. Cooper et al. (1995) reports the results of a study on the impact of the 1978 Chinese economic reforms for the period of 1986-1988 in Chinese textile, chemical and metallurgical industries.

In this research, we examine the usefulness of the returns-to-scale obtained from the CCR model. Considering the difficulties in reducing inefficient inputs in reality, we develop alternatives for the DMUs in increasing or decreasing returns for them to be DEA efficient. Although the purpose of this research is not to provide direct guidance to the DMUs, the results of the research are quite useful for planning purposes.

3 CCR Input Model and Its Computational Results

In this section, we provide a brief introduction of the CCR input model and discuss the application of the CCR model. In the CCR input model, we assume that there are n decision making units (DMU) to be evaluated. Each DMU consumes varying amounts of m different inputs to produce s different outputs. Specifically, DMU_j consumes amounts $X_j = \{x_{ij}\}$ of inputs ($i = 1, ..., m$) and produces amounts $Y_j = \{y_{rj}\}$ of outputs ($r = 1, ..., s$). The CCR model evaluates the efficiency score of each DMU called DMU_o ($o = 1, 2, ..., n$) relative to other DMUs. The model can be described as below:

$$\min\ \theta - \sum_{i=1}^{m} s_i^- - \sum_{r=1}^{s} s_r^+ \quad (1)$$

subject to:

$$\sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- = \theta x_{io} \qquad i = 1, .2, ..., m \quad (2)$$

$$\sum_{r=1}^{s} y_{rj}\lambda_j - s_r^+ = y_{ro} \qquad r = 1, .2, ..., s \quad (3)$$

$$\theta,\ s_i^-,\ s_r^+,\ \lambda_j\ \geq\ 0 \qquad \forall\ i, r,\ j \quad (4)$$

where θ is an efficiency ratio, x_{ij} represents the amount of input i, $(i = 1, ..., m)$ used by DMU_j, $(j = 1, ..., n)$, and y_{rj} represents the amount of output r, $(r = 1, ..., s)$ used by DMU_j. x_{io} and y_{ro} are the amounts of input i and output r obtained from DMU_o respectively.

The objective function is to minimize the efficiency score, θ, and to maximize input and output slacks (also called input surplus and output slack) of DMU_o. Constraint (2) specifies that the optimal input of i for DMU_o should be equal to the linear combination of the inputs of a set of efficient DMUs plus an input surplus of i consumed by DMU_o. Constraint (3) states that the optimal output r of DMU_o should be equal to the linear combination of the outputs of the same set of efficient DMUs minus an output slack of DMU_o

3.1 Using CCR to Evaluate 29 Canadian Textile Companies

In this section, we discuss the application of the CCR input model to evaluate the productivity (efficiency) performance of 29 Canadian textile companies We collected various data from the Canadian Textile Companies with 6 in the spinning process, 10 in the weaving process and 13 in the dyeing process for the year 1994. The initial input data collected for each DMU is: the number of employees, the hourly wage rates (in dollars), the percentage of time of machine breakdown, the average annual investment (in dollars) in the last 10 years, the plant size per worker, the percentage of absentees, the product diversities, the number of process stages, the job classification, and the raw material inventory (in dollars). The initial output data for each DMU is: the defect rates, the average finished goods inventory, the annual sales and the work-in-process. Obviously, we could not use all the collected data since we have only a small number of DMUs for each process. Thus, we conducted statistical tests to examine the correlations among the input and the output data and chosed the following as the output and inputsfor our research:

Output:

- Annual sales values (in 00,000 dollars) – a measure of annual sales volumes in production in 1994

Inputs:

- Number of employees - number of staff and workers employed in each DMU in 1994
- Average annual investment over last 10 years (in 00,000 dollars) - annual amount of additional money used for acquisition of facilities or technologies

Since the 29 textile companies are in three different processes, we divide the 29 DMUs into three sets and evaluate their efficiencies separately. Following the CCR model, a DEA efficient DMU, j, can be defined to satisfy the following two conditions:

(i) $\theta = 1.0$; and

(ii) all s_i^- and $s_r^+ = 0$ $\qquad \forall i, r$

If θ is less than 1.0, it means that to achieve DEA efficiency, the inputs of DMUj should be reduced to at least θ% of the amount originally used. Note that in DEA, efficiency is only a relative measure, which means that efficient DMUs perform relatively better than the other DMUs. If a DMU is inefficient, the CCR model identifies the input surplus and the output slack as below:

Input surplus: $s_i^- = \theta x_{io} - \sum_{j=1}^{n} x_{ij}\lambda_j \qquad \forall i$

Output slack: $s_r^+ = \sum_{r=1}^{s} y_{rj}\lambda_j - y_{ro} \qquad \forall r$

The optimal inputs and outputs (also called virtual inputs and virtual output) of DMU$_o$ are represented by $\sum_{j=1}^{n} x_{ij}\lambda_j$ and $\sum_{r=1}^{s} y_{rj}\lambda_j$, which are the linear combinations of the inputs and outputs of a set of efficient DMUs identified by the CCR model.

For the selected inputs and output, we then use the CCR model to evaluate the efficiency of the 29 Canadian textile companies. For security reasons, we can not disclose the real names of the 29 textile companies. Instead, we represent 6 DMUs in the spinning process by S01 to S06, 10 DMUs in the weaving process by W01 to W10 and 13 DMUs in the dyeing process by D01 to D13. In Tables 1a, 1b and 1c, we present the DEA results from the CCR model for the 29 textile companies in the spinning, weaving and dyeing processes respectively.

In Table 1a, the number of companies in the spinning process is only 6, and 3 out of 6 DMUs are DEA efficient and the efficiency scores of the three inefficient DMUs are well below 0.5. This indicates that the performances of the 6 DMUs in the spinning process are split into two extremes. Further investigation might be required to find the causes. In Tables 1b and 1c, the results show that 3 DMUs (W01, W08 and W09) in the weaving process and 2 DMUs (D05 and D06) in the dyeing process are DEA efficient. For the weaving process, except 1 DMU with a score as low as 0.19 and 1 DMU with a score of 0.973 (See Table 1b), the rest of the inefficient DMUs have efficiency scores around 0.5. This implies that the performances of the majority of DMUs in the weaving process are only 50% as efficient as the best performers. In Table 1c, we find that the efficiency scores for the inefficient DMUs in the dyeing process range from .392 to .876 with only two inefficient DMUs whose scores are below 0.5. Probably we can conclude that the DMUs in the dyeing process perform relatively more uniformly than those in the spinning and weaving processes. Variations in the efficiency performances among the DMUs in the three different processes may be due to various reasons such as the differences in process technologies, employee skills and others that might be worth studying.

Note that the above results are obtained through the CCR input model which also provides information on returns-to-scale. In the next section, we focus on the returns-to-scale and discuss its implication.

4 Returns-to-Scale

As we mentioned earlier, a production unit characterized by the constant returns-to-scale reaches its total efficiency. A production unit characterized by increasing returns-to-scale may improve its efficiency by increasing its output since its output will increase more proportionally with the increase of its inputs. A production unit characterized by decreasing returns-to-scale may improve its efficiency by reducing its output because its output will increase less proportionally with the increase of its inputs. Banker and Thrall (1992) provided the following theorem to identify the returns-to-scale through the CCR model:

Theorem 1 *The conditions for returns-to-scale are as follows:*

If $\sum_{j=1}^{n} \lambda_j^* = 1$ in any alternate optima, the constant returns-to-scale prevail.

If $\sum_{j=1}^{n} \lambda_j^* > 1$ for all alternate optima, the decreasing returns-to-scale prevail.

If $\sum_{j=1}^{n} \lambda_j^* < 1$ for all alternate optima, the increasing returns-to-scale prevail.

Fare, Grosskopf and Lovell (1985) proposed another approach to examine the returns-to-scale. Banker, Chang and Cooper (1995) show that these two models are equivalent. In this paper, we employ the model in Banker and Thrall (1992) to identify the returns-to-scale of all DMUs in the three processes and Tables 2*a*, 2*b* and 2*c* present the results. The efficient DMUs, as is well known, are characterized by constant returns-to-scale. From Tables 2*a*, 2*b* and 2*c*, we notice that 3 in the spinning, 3 in the weaving and 2 in the dyeing processes are identified as the constant returns-to-scale. In addition, 3 in the spinning, 3 in the weaving and 8 in the dyeing are in increasing returns. We also notice that no DMUs in the spinning, 5 in the weaving and 3 in the dyeing are in decreasing returns. In the next section, we will present a model used to consider the trade-off between the increase and the decrease in inputs for the DMUs in increasing returns.

4.1 DMUs with Increasing Returns

The CCR models (both input and output models) suggest that all inefficient DMUs should reduce their inputs or increase their outputs. In reality, we know that most inputs might be difficult to reduce. A typical example is the number of the employees in each DMU. Even with downsizing, the company may find it extremely difficult to lay off its workers. In this section, we examine the possibility of improving inefficiency by taking the returns-to-scale into account. Since increase in the outputs is always desirable for any DMU in increasing returns, we suggest that the DMU should increase its output by increasing the inputs or reducing the inputs less. For example, purchasing more advanced process technology or re-training existing workforce can greatly increase the productivity of DMUs. Thus, a company may want to weigh the trade-off between removing the input surplus to maintain the current output level or adding more inputs to achieve an even higher output level.

To find an optimal output level for the DMU in increasing returns, we locate the *best* projection of the DMU in increasing returns onto the efficiency frontier. The *best* projection is defined as follows: it (i) maximizes the increase in outputs, and (ii) minimizes the increase in inputs (or maximizes the reduction in inputs). Tables $2a, 2b$ and $2c$ provide information about the facets that form the efficiency frontiers for the three processes. For example, the efficiency frontier of the weaving process is formed by two facets. One is the combination of W01 and W08 and the other is the combination of W01 and W09. The efficiency frontier of the dyeing process consists of only one facet, a combination of D05 and D06.

We now present a model (called P1) to find the best projection of DMU_o in increasing returns onto the efficiency frontier. The objective of P1 is different from that of the DEA models including the CCR model. The objective of the DEA models is to find the best efficiency score for each DMU. P1 assumes that the efficiency frontier has been identified by the CCR model and its objective is to find a projection of DMU_o in increasing returns onto the efficiency frontier such that the output can be further increased at a minimal increase of the inputs. In other words, in P1 we consider the trade-off between the increase and decrease in inputs by measuring the amount of the output that can be increased. Figure 1 provides an example which shows the differences in the projections of a DMU in increasing returns onto the efficiency frontier by the CCR model and model P1. In Figure 1 (see also Page 38, Charnes, Cooper, Lewin and Seiford 1994), D1 to D6 represent 6 DMUs and the two values in the brackets represent the input and the output of the 6 DMUs respectively. Following the CCR model, D1 (represented by D1) is projected to the efficiency frontier formed by D2 and is required to reduce its input. Our model P1 would project D1 to the point of D2 and require D1 to increase its input to reach constant returns-to-scale. We now introduce some notation before we present model P1:

K : the number of facets in an efficiency frontier

J_k : is the set of the efficient DMUs that form kth facet of the efficiency frontier

$$I_k = \begin{cases} 1 & \text{if kth facet is chosen} \\ 0 & \text{otherwise} \end{cases}$$

so_{rk} : is the increase in output r if kth facet is chosen for projection

si^+_{ik} : is the increase in input i if kth facet is chosen for projection

si^-_{ik} : is the decrease in input i if kth facet is chosen for projection

Model P1 can be described as below:

[P1]

$$\max \sum_{k=1}^{K} I_k \left(\sum_{r=1}^{s} so_{rk} + \sum_{i=1}^{m} \left(si^-_{ik} - si^+_{ik} \right) \right) \tag{5}$$

subject to:

$$\sum_{k=1}^{K} I_k \sum_{j \in J_k} x_{ij} \lambda_{jk} = x_{io} + si^+_{ik} - si^-_{ik} \qquad for\ i = 1, 2, ..., m \tag{6}$$

$$\sum_{k=1}^{K} I_k \sum_{j \in J_k} y_{rj} \lambda_{jk} = y_{ro} + so_{rk} \qquad for\ r = 1, 2, ..., s \tag{7}$$

$$\sum_{k=1}^{K} I_k \sum_{j \in J_k} \lambda_{jk} = 1 \tag{8}$$

$$\sum_{k=1}^{K} I_k = 1 \tag{9}$$

$$I_k \in \{0, 1\} \qquad for\ k = 1, 2, ..., K \tag{10}$$

$$so_{rk}, si^+_{ik}, si^-_{ik}, \lambda_{jk} \geq 0 \qquad \forall r, i, j, k \tag{11}$$

The objective of (5) is to select a facet that maximizes the increase in output and minimizes the increase in inputs (or maximizes the reduction in inputs). Constraints (6) and (7) provide the optimal virtual inputs and outputs for DMU_o by projecting it onto a facet obtained from the CCR model. Constraint (8) states that the chosen facet is the convex combination of the DEA efficient DMUs in that facet. Constraint (9) guarantees that only one facet is chosen. Note P1 is used separately for each of the three processes. To solve P1, we need to examine all the facets in each efficiency frontier and find the best one. P1 is a nonlinear integer program and is difficult to solve. Since the number of facets in each efficiency frontier is quite few, we propose to solve P1 K times with K as the number of facets in an efficiency frontier. Then, it becomes straightforward to solve P1 as a linear programming model.

In solving P1, we eliminate 6 DMUs in the spinning process due to too few DMUs. Hence, Tables 3*a* and 3*b* provide the results of P1 for the weaving and the dyeing processes respectively. The first column provides the DMUs identified as increasing returns by the CCR model. The second column provides the best "facet" on the efficiency frontier of the projection obtained from P1. Columns 3 to 7 provide the increment of the output and the increment (or reduction) of the inputs required by P1. The last three columns indicate λ values in P1, representing a convex combination of the efficient DMUs in that particular facet. It is interesting to notice that in Table 3*a*, although the DMUs in increasing returns in the weaving process are all projected to the same facet formed by W01 and W08 in the CCR and P1 models, they are projected to different points. For example, in the CCR model, W10 has an efficiency ratio of 0.513 and following the CCR model, W10 should reduce its inputs to 51.3% of the amounts originally used in order to achieve an output of 2.8. The results of P1 show that W10 should be projected to the point of W08 on the same facet with an increment of 167.2 in output and increments of 12.0 and 65.8 in inputs. Clearly, P1 provides a much better alternative for W10. In other words, following the example of W08, W10 will achieve much higher labor productivity with a total labor input of 15.0 and an output of 170. Probably, the result of P1 will motivate W10 to invest in more advanced technologies to improve its labor productivity.

For the dyeing process, all DMUs in increasing returns are also projected to the same facet of D06. Similarly, P1 specifies that the DMUs are projected to different points from those obtained by CCR model. For example, in the CCR model, D01 is projected to a point on the facet of D06 and indicates that at efficiency, D01 should reduce its labor from 12.0 to 6.432 and its capital from 5.0 to 2.68 to obtain an output of 10.0. The results from P1 indicate that D01 is projected to the same facet of D06 but to the point where D06 is located. Instead of reducing its inputs, D01 should increase its output from 10.0 to 140.0 and increase its input of labor from 12.0 to 90.0 and input of capital, from 5.0 to 30.0. As we can see, the output increases 14 times but inputs only increase by 6 to 8 times. Clearly, this is a much better alternative for D01 to improve its efficiency. Of course, how to accomplish the improvement will become a very challenging task for the managers in those DMUs. As mentioned in Section 1, one major issue facing the Canadian textile industry is how to restructure the infrastructure and to develop new manufacturing strategies for its technology and workforce. Instead of laying off its workforce, retraining and purchasing highly advanced new process technologies probably could revamp the performance of the Canadian textile industry.

In summary, in this section, we propose a model to find the best point on the efficiency frontier for each DMU in increasing returns. The results show that instead of the projected point specified by the CCR model, other points on the efficiency frontier may provide more attractive alternatives in terms of output increase. In the next section, we discuss the benefits and costs when the output and the inputs are increased or reduced.

4.1.1 Projection Considering Cost and Revenue Parameters

In this section, we discuss the impact of the incorporation of cost and benefit parameters in P1. As we know, the inputs and the outputs may not have equal importance to all DMUs in reality. An increase in one output may provide more benefit to a DMU than the other inputs A typical example would be sales revenues in contrast to defect ratios. Similarly, reduction in one input may also save more money than other inputs such as investment versus the number of employees. Such costs and benefits can be found in increasing or decreasing labor, sales volumes and investments. For example, the benefit of reducing labor input may be savings in annual salary and the cost can be firing costs and/or compensations. To increase labor input through hiring, the related cost can be salary plus hiring cost (administrative and training cost etc.). On the other hand, increasing capital input may result in opportunity costs in terms of interest rates. Similarly, increasing various outputs could lead to different benefits. In our study, we treat unit revenue as the benefit of the output. As a result of this, we propose to modify P1 to find a new projection of the DMU in increasing returns by considering the costs and benefits. We first introduce the following notation:

p_j : the revenue per unit of the output in DMU j

f_j : the cost of firing an employee in DMU j

h_j : the cost of hiring an employee in DMU j

w_j : the salary per employee in DMU j

b_j : the opportunity cost per dollar in the investment in DMU j

so_k : the increase in output if the DMU$_o$ is projected to *kth* facet

si^+_{Lk} : the increase in labor input if the DMU$_o$ is projected to *kth* facet

si^-_{Lk} : the reduction in labor input if the DMU$_o$ is projected to *kth* facet

si^+_{Ck} : the increase in capital input if the DMU$_o$ is projected to *kth* facet

si^-_{Ck} : the reduction in capital input if the DMU$_o$ is projected to *kth* facet

We then calculate the costs and benefits in increasing or decreasing outputs and inputs as below:

The revenue generated from increasing *so* units of output: $p_o so_k$

The cost associated with hiring si^+_L units of labor: $(h_o + w_o)\, si^+_{Lk}$

The savings from reducing the workforce by si^-_L units: $(w_o - f_o)\, si^-_{Lk}$

The benefit from reducing the investment by si^-_C units: $b_o si^-_{Ck}$

The cost of increasing the investment level by si^+_C units: $b_o si^+_{Ck}$

The objective function of P1 can be revised as below:

[P1']

$$\max \sum_{k=1}^{K} I_k \left[p_o so_k + (w_o - f_o)\, si_{Lk}^- - (h_o + w_o)\, si_{Lk}^+ + b_o \left(si_{Ck}^- - si_{Ck}^+\right)\right] \tag{12}$$

Subject to

(6), (7), (8), (9), (10) and (11)

Similar to the solution procedure for model P1, in P1', we run the model for each DMU in increasing returns as many times as the number of facets in an efficiency frontier. Tables 4*a* and 4*b* show the results of P1' for the DMUs in the weaving and dyeing processes with the following given parameters: $p_j = 1$, $w_j = 27$, $f_j = 25, h_j = 5$ and $b_j = 5$. It is interesting to note that with the inclusion of the costs and benefits in P1', being DEA efficient may not be the best choice. As shown in Table 4*b*, for the given cost and benefit parameters, D01, D02, D12 and D13 will incur loss by being DEA efficient. The reason can be explained as follows. Since all four DMUs are in small scale (see Table 1*c*), the DMUs have to either reduce or increase significant amounts of inputs in order to be DEA efficient. However, the costs incurred due to increasing or reducing the inputs can not be made up by the benefit resulting from the increment of the output. Note that D01, D02, D012 and D013 are now projected to the point where D05 is located instead of D06 as shown in Table 3*b*. This is because between D05 and D06, D05 has a smaller scale than D06 so that D01, D02, D12 and D13 probably will incur less cost by becoming efficient. For the given cost and benefit data, our results, in general, show that most DMUs (see Tables 4*a* and 4*c*) benefit from becoming efficient. It is interesting to notice that the best projection also changed when costs and benefits are considered.

Clearly, P1' adds a planning feature to DEA models. The CCR model and the DEA model identify DEA efficient DMUs and the efficiency frontier formed by those efficient DMUs. P1 provides an alternative to the DMUs in increasing returns to be DEA efficient. P1' considers the benefit and cost in increasing or decreasing inputs and outputs and computes the most profitable way for the DMU in increasing returns to become DEA efficient. By incorporating the costs and benefits, P' also provides the flexibility of considering various alternatives. For example, if management feels it difficult to lay off employees, we can impose a very high value to the firing cost. This does not necessarily mean we would like to exclude the possibility of laying off employees. It means that management has to lay off employees at a high cost in order to become an efficient DMU. For example, Table 4*b* shows that for the given cost and benefit parameters, D07 has to reduce its employees by 42 in order to be DEA efficient.

4.2 DMUs with Decreasing Returns

In this section, we focus on the strategic utilization of input surplus for the DMUs in decreasing returns. For those DMUs, increasing their outputs by increasing inputs obviously is not an attractive strategy since the outputs increase less proportionately with the increase of the inputs. Following the CCR model, DMUs in decreasing returns should reduce their inputs. However, as stated before, in reality it may not always be possible to bring down the levels of inputs due to some practical reasons (say, a labor union). Thus, we propose that the DMUs in decreasing returns could expand vertically to fully utilize their input surplus. In fact, the weaving and dyeing processes in the textile industry are closely related (the output of the weaving process can be the input of the dyeing process) and the infrastructure and skills required are very similar too. The DMUs in decreasing returns could shift their inefficient inputs to a new upstream or downstream process. In other words, by utilizing its inefficient inputs, the DMUs in the dyeing process may extend to the weaving process and *vice-versa*. With the two related processes combined together, we refer to it as vertical integration. We believe that diversifying vertically into the other process is a good strategy because not only the inefficient inputs can be utilized but also more inputs will be added if necessary. Then the key issue is the appropriate level to vertically expand. We now discuss how to use the DEA cone ratio model (see Charnes, Cooper, Wei and Huang (1989)) to address this issue. One of the major features of the DEA cone ratio model is to enforce additional restrictions bounds on ratios of multipliers and require multipliers to belong to given closed cones. To illustrate the general approach, suppose we wish to incorporate additional inequality constraints of the following form:

$$\mu a_k^o + \nu a_k^i \le 0, \qquad k = 1, ..., K$$

where a_k^o is the s-vector of coefficients for the output multipliers, μ, and a_k^i is the m-vector of coefficients for the input multipliers, ν.

Such constraints, of course, may be included in any of the DEA models including the CCR model. In our study, we assume that the ratios between the two inputs of all the DMUs in one process must fall in a specified range. When a new DMU is created for that process, the ratios between the two inputs of that new DMU should also be satisfied.

From Tables 1b and 1c, we notice that the weaving and dyeing processes use a different input mix. It seems that the dyeing process is more labor intensive than the weaving process. Tables 5*a* and 5*b* provide the ratios between the two inputs in the weaving and the dyeing processes respectively. In Tables 5*a* and 5*b*, labor to capital ratios range from 0.22 to 4.71 in weaving, and from 1.00 to 8.00 in dyeing. Hence, for any DMU in the dyeing to extend to the weaving process, the ratio of the inputs should fall within the proposed ranges of 0.22 to 4.7, as indicated in Table 5*a*. Tables 6*a* and 6*b* provide the initial results from the CCR model. Columns 2 to 4 provide the original input data and DEA efficiency ratios. Columns 5 and 6 provide the amount of inefficient inputs that should be reduced and Column 7 provides the ratios between the two inefficient inputs. Columns 8 and 9 calculate the amounts of inputs to be added to meet the ratio ranges specified in Tables 5*a* and 5*b*. As we can see, all the ratios in columns 8 and 9 in Tables 6*a* and 6*b* fall within the ratio ranges indicated in Tables 5*a* and 5*b*.

We now propose to use the DEA cone ratio output model to find an optimal vertical integration level. The procedure is briefly summarized as below:

Step 0: Obtain the DEA results from the CCR model and identify the amounts of inputs to be removed, s_i^-, ($\forall$ i) for each DMU in decreasing returns for one process (either weaving or dyeing).

Step 1: Create a new DMU with the inputs of s_i^- and assign a value (Note: Any arbitrary value could be used.) as the output to the new DMU. Include the data of the new DMU to the data set in the other process.

Step 2: Use the cone ratio model to find the best input and output levels for the new DMU. (Note that the DEA cone ratio model obtains the DEA results by comparing the data of the new DMU with the performances of DEA efficient DMUs in the other process.)

Tables 7*a* and 7*b* provide the results from the cone ratio model. Columns 2 and 3 provide the amount of inputs to be reduced based on the results from the CCR model. Columns 4, 5 and 6 provide the optimal amounts of the inputs and output specified by the cone ratio model in order to be an efficient DMU in the other process. For example, D09 is identified as in decreasing returns (see Table 2*c*) by the CCR model. The amounts of labor and capital for D09 to be reduced are 11.87 and 2.32 respectively (see Table 6*b*). Considering 11.87 and 2.32 as the inputs of a new DMU in the weaving process, we assign an arbitrary value (say, 1.0) as the output to the new DMU. Then we add the new DMU to the original data set of the DMUs in the weaving process. The DEA cone ratio model provides the best output and input values for the new DMU to be DEA efficient (by referencing the performance of the efficient DMUs in the weaving process). That is, to start a new weaving process, the desired inputs for D09 in the new weaving process are 10.92 and 2.32 and the desired output is 13.65. However, D09 has 0.95 units of labor out of 11.87 units that still have to be reduced even adding a new weaving process. This means that although vertical integration can not help utilize all the inefficient inputs, it does open a way to effectively utilize the inefficient inputs. In the case of D09, 10.92 out of 11.87 inefficient inputs from the dyeing process can be further utilized by launching a vertical integration.

5 Conclusion

DEA has been widely used as a tool to evaluate relative performance of DMUs. While DEA models are extremely helpful in identifying efficiency performance, the results of the DEA models suggest removal of inefficient inputs without considering their quality and potential usages. In this paper, we recognize that returns-to-scale is the key factor that helps companies to better utilize their inputs (resources). Thus, we focus on returns-to-scale to explore the alternatives to reducing inefficient inputs. For the DMUs in increasing returns, we consider the trade-off between an increase or non-increase in inputs by evaluating the amount of the output that can be increased. We develop a mathematical model to find the best expansion plan in terms of increments in outputs and inputs. We then modify the model to incorporate the costs and benefits in expansion. For the DMUs in decreasing returns, we suggest that they should explore vertical integration to utilize inefficient inputs that might still be valuable to them. The data of the 29 Canadian textile companies in 1994 show that most Canadian textile companies did not perform well, with a few being DEA efficient and the rest very poor performers. Due to the big gap in efficiency scores, we suggest that the inefficient DMUs need to revamp their performance by initiating significant changes in their structure, strategy and capacity plans.

As an extension of this research, we are in the process of studying the performance of the textile industry in Pakistan. We have collected extensive data from about 60 Pakistan textile companies. Unlike Canada, the textile industry in Pakistan is ranked as the first important industry and has been developing very fast in recent years. Thus, we are very interested in studying the performance of Pakistan industry and comparing their performance with their counterparts in Canada.

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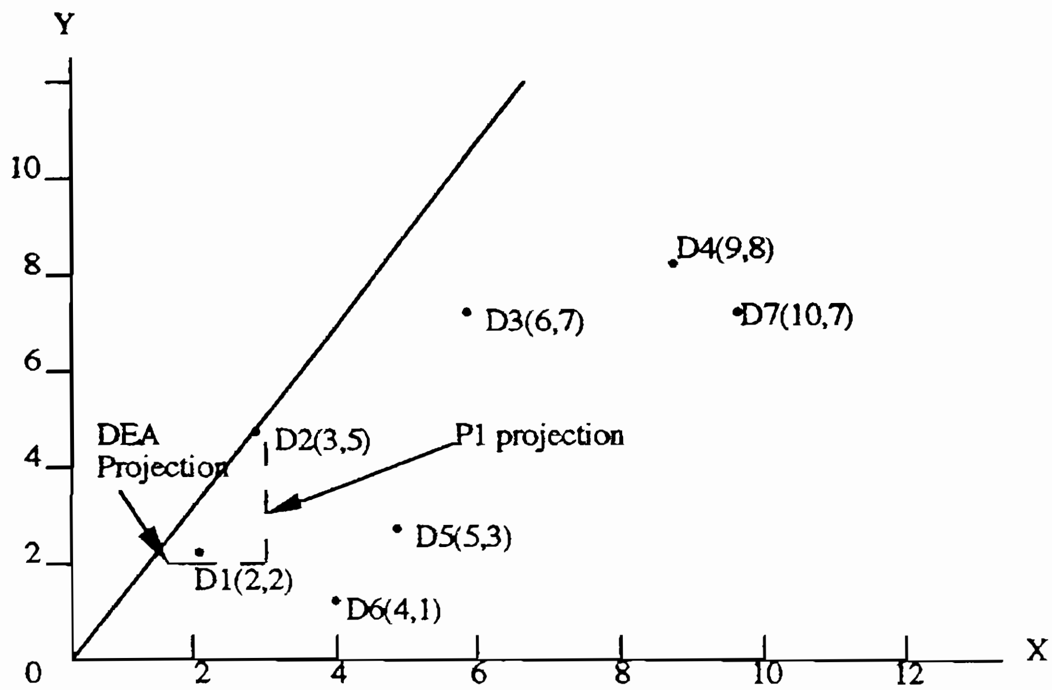

Figure 1: An Example of DEA Projection and P1 Projection

Table 1. Results of DEA runs

1a. Spinning

DMU NO	INPUTS		OUTPUT	INPUT SURPLUS		OUTPUT SLACKS	EFFICIENCY
	LABOR	CAPITAL	(SALES)	LABOR	CAPITAL		
S01	35.00	40.00	40.00	0.000	0.000	0.000	0.279
S02	103.00	500.00	1000.00	0.000	0.000	0.000	1.000
S03	75.00	20.00	80.00	0.000	0.000	0.000	0.438
S04	200.00	6.00	150.00	0.000	0.000	0.000	1.000
S05	61.00	20.00	175.00	0.000	0.000	0.000	1.000
S06	150.00	71.00	92.35	0.000	0.000	0.000	0.199

1b. Weaving

DMU NO	INPUTS		OUTPUT	INPUT SURPLUS		OUTPUT SLACKS	EFFICIENCY
	LABOR*	CAPITAL**	(SALES)+	LABOR	CAPITAL		
W01	31.00	40.00	140.00	0.000	0.000	0.000	1.000
W02	72.00	60.00	135.00	0.000	0.000	0.000	0.599
W03	56.00	120.00	180.00	0.000	0.000	0.000	0.507
W04	110.00	150.00	500.00	0.000	0.000	0.000	0.973
W05	165.00	135.00	300.00	0.000	0.000	0.000	0.589
W06	56.00	200.00	100 00	0.000	0.000	0.000	0.190
W07	48 00	80.00	130.00	0.000	0.000	0.000	0.509
W08	15.00	67.00	170.00	0.000	0.000	0.000	1.000
W09	4.00	0.85	5.00	0 000	0.000	0.000	1.000
W10	3.00	1.20	2.80	0.000	0.000	0.000	0.513

1c. Dyeing

DMU NO	INPUTS		OUTPUT	INPUT SURPLUS		OUTPUT SLACKS	EFFICIENCY
	LABOR*	CAPITAL**	(SALES)+	LABOR	CAPITAL		
D01	12.00	5.00	10.00	0.000	0.536	0.000	0.536
D02	8.00	1.00	6.00	1.200	0.000	0.000	0.800
D03	129.00	45.00	120.00	0.000	1.196	0.000	0.598
D04	99.00	60.00	87.50	0.000	15.341	0.000	0.568
D05	52.00	8.00	60.00	0.000	0.000	0.000	1.000
D06	90.00	30.00	140.00	0.000	0.000	0.000	1.000
D07	132.00	80.00	100.00	0.000	17.432	0.000	0.487
D08	191.00	45.00	187.50	0.000	0.000	0.000	0.734
D09	92.00	18.00	100.00	0.000	0.000	0.000	0.871
D10	150.00	35.00	175.00	0.000	0.000	0.000	0.876
D11	41.00	30.00	25.00	0.000	6.402	0.000	0.392
D12	15.00	15.00	20.00	0.000	8.571	0.000	0.857
D13	29.00	30.00	30.00	0.000	13 522	0.000	0.665

* Number of employee

** Average annual investment (in 000,000 $)

\+ Annual sales (in 000,000 $)

Table 2: Returns to Scale

2a. Spinning

DMU NO		Returns-to--Scale	DMUs in Facet
S01	0.148	Increasing	S02, S05
S02	1.000	Constant	S02
S03	0.462	Increasing	S04, S05
S04	1.000	Constant	S04
S05	1.000	Constant	S05
S06	0.483	Increasing	S02, S05

2b. Weaving

DMU NO	$\Sigma\lambda_j$	Returns-to--Scale	DMUs in Facet
W01	1.000	Constant	W01
W02	5.371	Decreasing	W01, W09
W03	1.177	Decreasing	W01, W08
W04	3.537	Decreasing	W01, W08
W05	12.411	Decreasing	W01, W09
W06	0.606	Increasing	W01, W08
W07	0.887	Increasing	W01, W08
W08	1.000	Constant	W08
W09	1.000	Constant	W09
W10	0.327	Increasing	W01, W08

2c. Dyeing

DMU NO	$\Sigma\lambda_j$	Returns-to--Scale	DMUs in Facet
D01	0.071	Increasing	D06
D02	0.100	Increasing	D05
D03	0.857	Increasing	D06
D04	0.625	Increasing	D06
D05	1.000	Constant	D05
D06	1.000	Constant	D06
D07	0.714	Increasing	D06
D08	2.179	Decreasing	D05, D06
D09	1.390	Decreasing	D05, D06
D10	2.055	Decreasing	D05, D06
D11	0.179	Increasing	D06
D12	0.143	Increasing	D06
D13	0.214	Increasing	D06

Table 3: Projections for inefficient DMUs with Increasing Returns

3a. Weaving

DMU NO	DMUs in Facet	Increase in Output	Increase in Inputs		Decrease in Inputs		Obj Ftn Value	λ		
			Labor	Capital	Labor	Capital		W01	W08	W09
W06	W01, W08	70.00	0.00	0.00	41.00	133.00	244.00	0.0	1.0	0.0
W07	W01, W08	40.00	0.00	0.00	33.00	13.00	86.00	0.0	1.0	0.0
W10	W01, W08	167.20	12.00	65.80	0.00	0.00	89.40	0.0	1.0	0.0

3b. Dyeing

DMU NO	DMUs in Facet	Increase in Output	Increase in Inputs		Decrease in Inputs		Obj Ftn Value	λ	
			Labor	Capital	Labor	Capital		D05	D06
D01	D05, DO6	130.00	78.00	25.00	0.00	0.00	27.00	0.0	1.0
D02	D05, DO6	134.00	82.00	29.00	0.00	0.00	23.00	0.0	1.0
D03	D05, DO6	20.00	0.00	0.00	39.00	15.00	74.00	0.0	1.0
D04	D05, DO6	52.50	0.00	0.00	9.00	13.00	91.50	0.0	1.0
D07	D05, DO6	40.00	0.00	0.00	42.00	50.00	132.00	0.0	1.0
D11	D05, DO6	115.00	49.00	0.00	0.00	0.00	66.00	0.0	1.0
D12	D05, DO6	120.00	75.00	15.00	0.00	0.00	30.00	0.0	1.0
D13	D05, DO6	110.00	0.00	0.00	0.00	0.00	49.00	0.0	1.0

Table 4. Projections for Inefficient DMUs with Increasing Returns in presence of cost and revenue parameters.

4a. Weaving

DMU NO	DMUs in Facet	Increase in Output	Increase in Inputs		Decrease in Inputs		Obj Ftn Value	λ		
			Labor	Capital	Labor	Capital		W01	W08	W09
W06	W01, W08	70.00	0.00	0.00	41.00	133.00	1727.00	0.0	1.0	0.0
W07	W01, W08	40.00	0.00	0.00	33.00	13.00	691.00	0.0	1.0	0.0
W10	W01, W08	167.20	12.00	65.80	0.00	0.00	1627.00	0.0	1.0	0.0

4b. Dyeing

DMU NO	DMUs in Facet	Increase in Output	Increase in Inputs		Decrease in Inputs		Obj Ftn Value		
			Labor	Capital	Labor	Capital		D05	D06
D01	D05, DO6	50.00	40.00	3.00	0.00	0.00	-595.00	1.0	0.0
D02	D05, DO6	54.00	44.00	7.00	0.00	0.00	-687.00	1.0	0.0
D03	D05, DO6	20.00	0.00	0.00	39.00	15.00	433.00	0.0	1.0
D04	D05, DO6	52.50	0.00	0.00	9 00	13 00	497.00	0.0	1.0
D07	D05, DO6	40.00	0.00	0.00	42.00	50.00	894 00	0.0	1.0
D11	D05, DO6	35.00	11.00	0.00	0.00	22.00	248.00	1.0	0.0
D12	D05, DO6	40.00	37.00	0.00	0.00	7.00	-589.00	1.0	0.0
D13	D05, DO6	30.00	23.00	0.00	0.00	22.00	-206.00	1.0	0.0

Table 5. Ratio between Inputs (Labor/Capital)

5a. Weaving

DMU	Labor	Capital	Ratio
W01	31.00	40.00	0.78
W02	72.00	60.00	1.20
W03	56.00	120.00	0.47
W04	110.00	150.00	0.73
W05	165.00	135.00	1.22
W06	56.00	200.00	0.28
W07	48.00	80.00	0.60
W08	15.00	67.00	0.22
W09	4.00	0.85	4.71
W10	3.00	1.20	2.50

b. Dyeing

DMU	Labor	Capital	Ratio
D01	12.00	5.00	2.40
D02	8.00	1.00	8.00
D03	129.00	45.00	2.87
D04	99.00	60.00	1.65
D05	52.00	8.00	6.50
D06	90.00	30.00	3.00
D07	132.00	80.00	1.65
D08	191.00	45.00	4.24
D09	92.00	18.00	5.11
D10	150.00	35.00	4.29
D11	41.00	30.00	1.37
D12	15.00	15.00	1.00
D13	29 00	30.00	0.97

Table 6. Amounts of Inputs to be invested in the other process for inefficient DMUs with Decreasing Returns to Scale

6a. Weaving

DMU NO	INPUTS IN USE		EFFI-CIENCY	TO BE SAVED		RATIO	MORE NEEDED	
	LABOR	CAPITAL		LABOR	CAPITAL		LABOR	CAPITAL
W02	72.00	60.00	0.599	28 87	24 06	1.20	0.00	0.00
W03	56.00	120.00	0.507	27.61	59.16	0 47	0.00	0.00
W04	110.00	150.00	0.973	2.97	4.05	0.73	0.00	0.00
W05	165.00	135.00	0.589	67.82	55.49	1.22	0.00	0.00

6b. Dyeing

DMU NO	INPUTS IN USE		EFFI-CIENCY	TO BE SAVED		RATIO	MORE NEEDED	
	LABOR	CAPITAL		LABOR	CAPITAL		LABOR	CAPITAL
D08	191.00	45.00	0.734	50.81	11.97	4.24	0 00	0.00
D09	92 00	18.00	0.871	11.87	2.32	5 11	0.00	0 00
D10	150 00	35 00	0.876	18 60	4.34	4.29	0.00	0.00

Table 7. New Process for inefficient DMUs with decreasing returns

7a. Weaving

DMU NO	INPUTS AVLBL		INPUTS REQD		EFFCNT OUTPUT	EXTRA INPUTS	
	LABOR	CAPITAL	LABOR	CAPITAL		LABOR	CAPITAL
W02	28.87	24.06	28.87	9.62	44.91	0.00	14.44
W03	27.61	59.16	27.61	9.20	42.95	0.00	49.96
W04	2.97	4.05	2.97	0.99	4.62	0.00	3.06
W05	67.82	55.49	67 82	22.61	105.50	0.00	32.88

7b. Dyeing

DMU NO	INPUTS AVLBL		INPUTS REQD		EFFCNT OUTPUT	EXTRA INPUTS	
	LABOR	CAPITAL	LABOR	CAPITAL		LABOR	CAPITAL
D08	50.81	11.97	50.81	11.97	67.07	0.00	0.00
D09	11.87	2.32	10.92	2.32	13.65	0.95	0.00
D10	18.60	4.34	18.60	4.34	24.42	0.00	0.00

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